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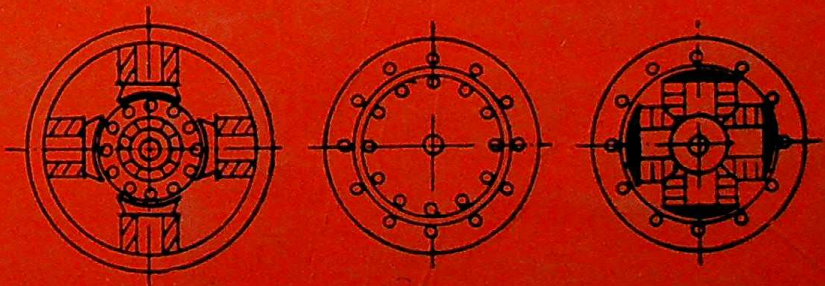

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FUNDAMENTALS OF ELECTRICAL ENGINEERING AND ELECTRONICS

IN

INTERNATIONAL SYSTEM (SI) OF UNITS

(Incorporating rationalized M.K.S.A. system)

[For the Examinations of Sec. A of A.M.I.E. (I) ; A.M.Ae.S.I. ; City & Guilds (London) & Diploma (Elect. & Mech.), L.E.E., and L.M.E.]

B. L. THERAJA

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to

My Wife

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PREFACE TO THE TWENTY-FIRST EDITION

Time, he explained, is the prime factor
Governing human advancement :
The heart of measurement is time ;
Measurement is the blood of Science

Cal Clothier in *Knowledge is Power*

In response to many suggestions received from numerous readers including A.M.I.E. (I) students and others preparing for their Diploma and Degree in Engineering, following major additions/ changes have been made in the present edition :

(*i*) the combined chapter on **'Alternators and Synchronous Motors'** has been split into two independent chapters thereby enabling the author to treat each more extensively and deeply appropriate to the level required by engineering students for whom this book has been written.

(*ii*) chapter on **'Single-phase Motors'** has been updated by the addition of 'Motor Trouble Chart' and fresh material on split-phase motors used in home air-conditioners and refrigerators.

(*iii*) the number of **Questions and Answers on Electric Machinery** has been almost doubled from 38 to 68.

(*iv*) in the chapter entitled **'Semiconductors and Diodes,'** detailed presentation of Semiconductor Theory and P-N Junction Theory (including the phenomenon of junction breakdown) has been given since it is of vital importance in understanding the working of solid-state devices.

(*v*) on persistent demand of the readers, a new chapter dealing exclusively with **Junction Transistors** has been added. Transistor theory including transistor biasing rules, formulas for different configurations, leakage currents, different transistor models and equivalent circuits and their use in amplifier and oscillator circuits has been presented in a very interesting and easy-to-understand manner. The sheer simplicity and novelty of this presentation should earn the author high approbation from his readers.

(*vi*) a totally new but much-needed chapter on **'Illumination'** has been added incorporating latest internationally-approved terminology employed in **Illumination Engineering.**

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(*vii*) past A.M.I.E. (I) examination papers upto **Winter 1986** have been included in the book for the benefit of A.M.I.E. students.

(*viii*) on the persistent demand of my esteemed readers, particularly from the states of Maharashtra, Karnataka, Kerala and Tamil Nadu, a separate chapter on **Thermoelectricity** has been added in this edition.

It is earnestly hoped that with these extensive additions and revisions, the present edition will prove even more useful to my esteemed readers than the earlier ones.

In the end, it now remains for me to thank all those who helped me, in one way or the other, in the preparation and ultimate publication of the book. Amongst them, I must gratefully mention my publishers especially Padam Shree L. Shyam Lal Gupta and Shree Ravinder Kumar Gupta—fast upcoming clear-headed young man who runs and controls the entire business of printing and binding with the ease and equanimity of a person twice his age. I am deeply indebted to their General Manager Sh. P. S. Bhatti who brings almost every three months like a Santa Claus, bagful of book orders on his return from extensive sale tours of West Asia (including African continent) and S-E Asia and to their cool-headed Printing Manager Sh. H. C. Saxena who neither misses a heart beat nor allows his B. P. to exceed 120/80 even under the most trying conditions and goes about his job with the rythmic regularity of a mechanical engine. In the end, I must offer my sincere thanks to quick-witted Kerala-born minicomputer Ms Philomina Perepatt for her perseverance and patience in typing and retyping various versions of the text and to Mr. Cal Clothier, Lecturer Leeds Polytechnic U.K. for borrowing a few beautiful lines from his famous poem 'Knowledge is Power .

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SYLLABUS FOR THE PAPER

"Elements of Electrical Engg. & Electronics"

For Associate Membership Examination of Institution of Engineers (India)

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Electrostatics : Electric charge, potential, field and capacitance. Potential gradient in the dielectric field. Electric force, flux density and permittivity. Parallel plate and concentric cylindrical capacitor, capacitors in series and in parallel. Energy stored in an electric field.

Magnetism : Magnetic field and flux, permanent magnets and flux density.

Current Electricity : Electromotive force and potential difference, electric current and resistance. Ohm's law ; resistivity and calculation of resistance. Temperature coefficient of resistance. Thermo-e.m.f. Heating effect of current. Energy and power and their units in electrical, thermal and mechanical systems. Efficiency calculations. Kirchhoff's laws. Wheatstone bridge. Simple potentiometers. Insulation and insulation resistance. Faraday's laws of electrolysis. Lead-acid and nickel-iron alkaline cells. Charging and discharging of cells. Charging and discharging currents for capacitors with resistance in series.

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Alternating Current : Production of alternating current. Instantaneous, average and r.m.s. values of alternating current and voltage. Vector representation of sinusoidal current and voltage. Simple circuit calculations involving combinations of resistance, inductance and capacitance. Power and power factor ; use of capacitance for power factor improvement.

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Electrical Machines : Principles and action of single-phase transformers and simple d.c. and a.c. generators and motors. Polyphase circuits and systems.

Measuring Instruments : Principles of moving-coil, moving-iron, dynamometer and thermal ammeters and voltmeters. Electrostatic voltmeters. Wattmeters and energy meters. Megger. The d.c. potentiometer and its uses.

Electronics. Thermionics. Secondary, photo electric and field emissions. Vacuum diode, triode, tetrode, pentode and gas-filled tubes such as voltage regulator tubes and thyratrons—their construction, operation and characteristics. Intrinsic and extrinsic semiconductors. Current carriers in semiconductors. P-N junction diode. Transistor. Zener diode and thyristor—their properties and characteristics. Simple amplifier and oscillator circuits.

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SYMBOLS USED

A, ampere ; area
B, magnetic flux density (induction density)
C, capacitance
D, electric flux density
d, diameter
E, electromotive force, electric field strength
e, base of natural logarithms, induced e.m.f. ; instantaneous value of alternating e.m.f.
F, force
f, frequency
G, conductance
g, gravitational acceleration
H, magnetic field strength or magnetising force
I, current, moment of inertia
i, instantaneous value of alternating current
J, intensity of magnetisation
j, vector operator ; $\sqrt{(-1)}$
K_f, form factor
ϵ_0, permittivity of free space
ϵ_r, relative permittivity of a medium
L, self-inductance
l, length
M, mutual inductance, molecular weight
m, mass
μ_0, permeability of free space
μ_r, relative permeability of a medium
N, number of turns, speed in rev/min, number
P, power, number of poles in a machine
p, instantaneous power in an a.c. circuit
Q, electric charge
R, resistance
r, radius
S, reluctance
s, slip
T, time period, torque, absolute temperature
t, time
V, potential difference
v, velocity, voltage
W, work, energy
X, reactance
Y, admittance
Z, impedance, number of armature conductors, electrochemical equivalent
α, temperature coefficient of resistance, angle
β, angle
γ, angle
η, efficiency
θ, angle ; temperature
λ, wavelength
μ, amplification factor
Ω, ohm
ρ, specific resistance or resistivity
Σ, sum of
σ, charge density, conductivity
Φ, magnetic flux
ϕ, phase angle
Ψ, electric flux
ω, angular velocity

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ABBREVIATIONS USED

A	ampere	m	metre
A.C.	alternating current	min	minute
Ah	ampere-hour	μF	microfarad
AT	ampere-turns	M.K.S.	metre-kilogram second
C	coulomb		
°C	degree centigrade	MΩ	megohm
Cal	calorie	m.m.f.	magnetomotive force
D.C.	direct current	MW	megawatt
E.C.E.	electrochemical equivalent	N	newton(s)
		P.D.	potential difference
E.M.F.	electromotive force	p.f.	power factor
eV	electron volt	rad.	radian
F	farad	rev.	revolution
g	gram	R.M.S.	root mean square
H	henry	r.p.m.	revolutions per minute
H.P.	horse-power	r.p.s.	revolutions per second
°K	degree Kelvin	V	volt(s)
kg	kilogram	VA	volt-ampere
kcal	kilocalorie	VAR	volt-ampere reactive
kW	kilowatt (s)	W	watt (s)
kWh.	kilowatt-hour (s)	Wb	weber (s)
kVA	kilovolt-ampere	Wh	watt-hour (s)

GREEK ALPHABET

α	alpha	ρ	rho
β	beta	σ	sigma
γ	gamma	Σ	capital sigma
δ	delta	ϕ	phi
η	eta	Φ	capital phi
θ	theta	ψ	psi
λ	lambda	Ψ	capital psi
μ	mu	ω	omega
π	pi	ϵ	epsilon

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INTRODUCTION

International System of Units

1. Brief History

An international organisation of which most advanced and developing countries including India, are members, called the General Conference of Weights and Measures (*CGPM*) has been entrusted with the task of prescribing definitions of fundamental units of weights and measures which are the very basis of science and technology today.

These definitions are adopted all over the world for purposes of science, technology, industry and commerce. The 11th General Conference of Weights and Measures which met in October 1960 recommended a unified, rational, coherent and comprehensive system of fundamental, supplementary and derived units for international use. This system, called International System of Units (Systéme International d' Unite's) and designated by the abbreviation, *SI*, is now legally compulsory in about twenty-five countries.

The *SI* system is, in fact, simply the rationalized *MKSA* system expanded by the addition of degree Kelvin and Candela as basic units, to include the units used in heat and photometry.

2. The SI System of Units

As said above, the system is an expanded and improved version of the rationalized *MKSA* (metre-kilogram-second-ampere) system which has four basic or fundamental units. The *SI* system has six fundamental units, two supplementary units and a large number of derived units.

(*a*) The six basic units of *SI* system along with their names and symbols are given in Table 1 below.

Table 1. BASIC UNITS

Unit	*Name*	*Symbal*
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	°K
luminous intensity	candela	cd

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The definitions of the six basic units as recognised by the General Conference of Weights and Measures are given below :

(*i*) **Length** : The metre is the length equal to 1,650, 763·73 wavelengths in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom.

(*ii*) **Mass** : One kilogram is equal to the mass of the international prototype of the kilogram.

(*iii*) **Time** : The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

(*iv*) **Electric current** : One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

(*v*) **Thermodynamic temperature** : The kelvin is the fraction 1/273·16 of the thermodynamic temperature of the triple point of water.

(*vi*) **Luminous intensity** : The candela is the luminous intensity, in the perpendicular direction, of a surface 1/600,000 square metre of a black body at the temperature of freezing platinum under a pressure of 101,325 newton per square metre.

(*b*) The two supplementary units along with their names and units are given in Table 2 below :

Table 2. SUPPLEMENTARY UNITS

Unit	*Name*	*Symbol*
Plane angle	radian	rad
Solid angle	steradian	sr

(*c*) Some of the derived units are tabulated below :

Table 3. DERIVED UNITS

Unit	*Name*	*Symbol*
Area	square metre	m^2
Volume	cubic metre	m^3
Frequency	hertz	Hz
Density	kilogram per cubic metre	kg/m^3

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Unit	*Name*	*Symbol*
Velocity	metre per second	m/s
Angular velocity	radian per second	rad/s
Acceleration	metre per second squared	m/s^2
Angular acceleration	radian per second squared	rad/s^2
Force	newton	N
Pressure	newton per square metre	N/m^2
Dynamic viscosity	newton-second per square metre	$N-s/m^2$
Kinematic viscosity	square metre per second	m^2/s
Work, energy, quantity of heat	joule	J (N-m)
Power	watt	W
Electric charge	coulomb	C
Potential, potential difference	volt	V
Electric field strength	volt per metre	V/m
Resistance	ohm	Ω
Capacitance	farad	F
Magnetic flux	weber	Wb
Inductance	henry	H
Magnetic flux density	tesla	T (Wb/m^2)
Magnetic field strength	ampere per metre	A/m
Magnetomotive force	ampere	A(AT)
Luminous flux	lumen	lm
Illumination	lux	lx(lm/m^2)
Entropy	joule per degree kelvin	J/°K
Specific heat	joule per kilogram kelvin	J/kg-°K
Thermal conductivity	watt per metre kelvin	W/m-°K

3. Multiples and Sub-multiples

Recommended prefixes for the formation of multiples and sub-multiples of units are as shown in Table 4.

Table 4. RECOMMENDED PREFIXES AND SYMBOLS

Fraction	*Prefix*	*Symbol*
10^{-1}	deci	d*
10^{-2}	centi	c*
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
13^{-15}	femto	f
10^{-18}	atto	a

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Multiple	*Prefix*	*Symbol*
10	deca	da*
10^2	hecto	h*
10^3	kilo	k
10^6	mega	M
10^9	giga	G
10^{12}	tera	T

*To be restricted as much as possible.

4. Advantages of SI Units

(*i*) The decimal relationship between units of the same quantity makes it possible to express any small or large quantity as a power of 10.

(*ii*) Units for many different quantities are related through a series of simple and basic relationships.

(*iii*) Being an absolute system, it avoids the factor '*g*' *i.e.* acceleration due to gravity in several equations in Science and Engineering which has been a nuisance in all numericals in Physics and Engineering.

(*iv*) Being a rationalized system, it ensures all the advantages of rationalized *MKSA* system in the fields of electricity, magnetism, electrical engineering and electronics.

(*v*) Joule has been taken as the unit of energy of *all forms*. Not only does it save lot of labour in calculations, it additionally provides a useful link between the mechanical *MKS* units and the electrical rationalized *MKSA* units.

(*vi*) The watt as the sole unit of power, excludes a host of other power units used in various branches of Science and Engineering.

(*vii*) In electricity, all the practical units (like volt, ohm, ampere, henry, farad, coulomb, joule and watt) accepted in industry and laboratories all over the world for well over a century have become absolute in their own right in the *SI* system without the need for any more practical units.

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ELECTRIC CURRENT AND OHM'S LAW

1. Nature of Electricity 2. Modern Electron Theory 3. Idea of Electric Potential 4. Resistance 5. Unit of Resistance 6. Laws of Resistance 7. Unit of Resistivity 8. Conductance 9. Effect of Temperature on Resistance 10. Temperature Coefficient of Resistance 11. Value of α at Different Temperature 12. Variation of Resistivity with Temperature 13. Resistors 14. Linear and Non-linear Resistors 15. Uses of Non-linear Resistors 16. Stranded Wires and Cables 17. Ohm's Law 18. Relations Derived From Ohm's Law 19. Resistances in Series 20. Resistances in Parallel 21. Highlights.

1-1 Nature of Electricity

It is comparatively easy to describe what electricity can do than to give a simple and direct answer to the question : What exactly is meant by electricity ? Electricity has become such a universal medium for transmission and utilization of energy that almost every one is familiar with its innumerable uses right from the earliest childhood. Electric energy is variously utilized as for lighting, transportation, communication, for operating electric furnaces, elevators and for driving various kinds of machine tools etc. It can be easily stored and concentrated to produce extremely high temperatures as in welding and electric furnaces, arc lights and spark plugs etc.

Turning back to the question regarding the nature of electricity, it may be noted that ancient Greeks were the first to observe that when amber[1] is rubbed against a piece of silk cloth, it attracts light objects like small pieces of paper etc. The agency that endowed this attracting property to amber was given the name of electricity by Gilbert[2] in 1600 A.D. The name electricity is derived from the word 'electron' which is the Greek name of amber.

To explain the few observed phenomena about electricity, Benjamin Franklin advanced in 1749, what was called **One Fluid**

1. It is a yellow brown gum that hardens into hard stone-like material.

2. William Gilbert (1544—1603), noted British physician and experimenter, was the first to investigate electric and magnetic phenomena. He was the first to recognize the magnetic field surrounding a magnet and the first person to demonstrate that earth is in reality a large permanent magnet.

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Theory of electricity. According to this theory, electricity was assumed to be a sort of an invisible and intangible fluid which was associated with matter in different degrees. The normal state of matter (in which it exhibited no electrical properties) was associated with a certain definite amount of this fluid. Any disturbance of this assumed normal distribution of fluid was supposed to result in the body being either positively or negatively charged corresponding to an excess or deficit of this normal amount of fluid. This theory was successful in explaining the production of positive and negative electricity when two bodies are rubbed together. Because, according to this theory, rubbing of two bodies together led to an unequal redistribution of their fluids whose total sum was supposed to remain constant. One body which got more of this fluid became positively charged and the other which got less became negatively charged.

It will be noted that this theory does not assume two types of fluids, but simply two states of electrification resulting from an excess or deficit of the same fluid. This theory, however, did not hold ground for long since it could not satisfactorily explain many electrical phenomena including induction.

In the year 1735, Dufay advanced the **Two Fluid Theory** which was later modified and restated in a more satisfactory manner by Symmer in 1759. According to this theory, two different types of fluids resided simultaneously in matter corresponding to two states of electrification *i.e.* positive and negative. The type and degree of electrification was measured by the excess of one type of fluid over the other. The normal non-electrical state of matter was assumed to be due to the presence of equal quantities of these two fluids which neutralized each other.

The theory was successful for some time, but was later on, discarded because (*i*) it was based on the untenable assumption of two invisible fluids present in the matter and (*ii*) it was found inadequate to fully explain many of the facts about electricity discovered afterwards.

1-2 Modern Electron Theory

Modern research has established that all matter whether solid, liquid or gaseous, consists of minute particles called **molecules** which are themselves made up of still minute particles known as **atoms**. Those substances whose molecules consist of similar atoms are known as **elements** and those whose molecules consist of dissimilar atoms are called **compounds**. The number of elements so far discovered is 106 whereas the number of compounds is unlimited.

An atom is taken to consist of the following :

(*i*) It has a hard central core known as **nucleus**. It contains two types of particles[1] ; one is known as **proton** and carries positive charge, the other is **neutron** (discovered by

1. In addition to protons and neutrons, there have been discovered other particles like mesons and neutrino etc. and electrons inside the nucleus. All these particles within the nucleus are known as **nucleons**.

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Chadwick in 1932) which is electrically neutral *i.e.* it carries no charge though it is as heavy as proton. The protons and neutrons are very closely held together with tremendous nuclear forces.

(*ii*) Revolving round the relatively massive nucleus, in more or less elliptical orbits (or shells), are infinitesimally small particles known as **electrons**. These electrons carry the smallest negative charge and have a negligible mass. The mass of an electron is approximately 1/1840 that of a proton.

Such a view of an atom, known as Bohr-Rutherford model, is shown in Fig. 1-1. It has been found that an atom is like a miniature solar system, a heavy positively charged nucleus taking the place of the Sun at the centre with orbital electrons acting like planets. The planetary electrons revolve at distances which are much greater than the size of nucleus or electrons themselves, hence most of the space occupied by an atom is empty ! In fact, a solid may be thought of as a sponge-like structure in which nucleus and electrons occupy but little of the space taken up by the solid.

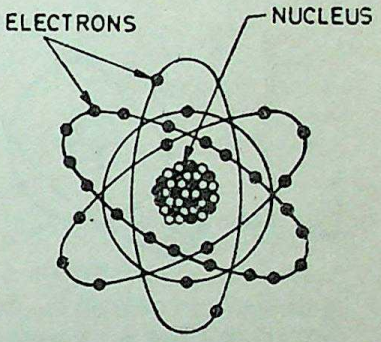

Fig. 1-1

It has also been found that the effective diameters of the atoms are of the order of 10^{-10} metre and the diameters of their nuclei of the order of 10^{-15} metre. Hence the diameter of an atom is roughly 100,000 times greater than the diameter of the nucleus. Some rough idea of the vast emptiness existing within an atom can be got by imagining that model of hydrogen atom may consist of a cricket ball with a small soap bubble revolving round it at a distance of 3 km or so !

The particles discussed above form the fundamental bricks of which all matter is made. Atoms of all substances consist of identical protons, neutrons, and electrons etc., the only difference being in their number and relative configuration.

It has been found that the positive charge on a proton is numerically equal to the negative charge of an electron. Normally an atom is electrically neutral, because it consists of as many protons as electrons. The number of protons in the nucleus of an atom gives the atomic number (Z) of the substance whose atom it is. The total weight of a nucleus (*i.e.* protons plus neutrons) is called the atomic mass number (A). If the number of protons in a nucleus is changed, then transmutation of one element into another can be achieved.

The simplest atom is that of hydrogen. It consists of one positive proton and one orbital electron as shown in Fig. 1-2 (*a*).

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Next is the helium atom which has two planetary electrons and whose nucleus contains two protons and two uncharged neutrons as shown in Fig. 1-2 (*b*). Similarly, in Fig. 1-2 (*c*) and (*d*) are shown atoms of lithium and beryllium. Under each figure is given the symbol used in Nuclear Physics. The subscript is the atomic

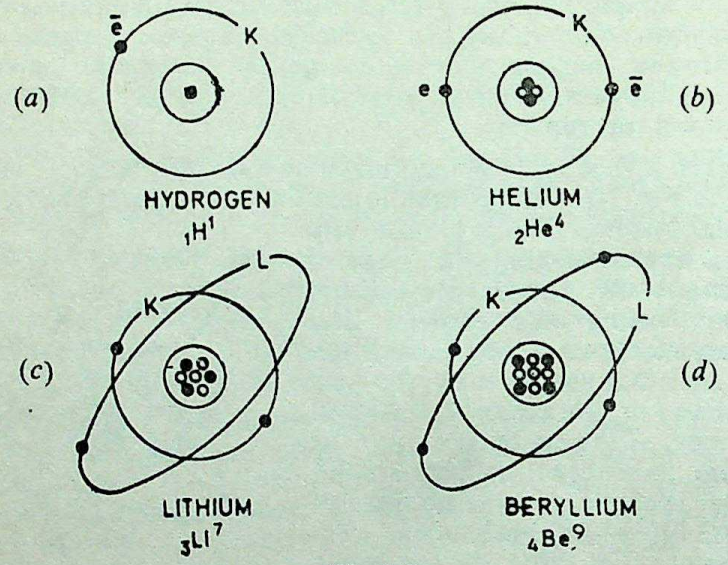

Fig. 1-2

number and the upper number represents the atomic weight. Oxygen atom ($_8O^{16}$) has 8 orbital electrons and its nucleus consists of 8 protons and 8 neutrons. Heaviest atom is that of a newly-discovered element (still un-named) with $Z = 106$.

The following important points about atomic structure should be understood clearly :—

(*i*) The mass of a proton is $1{\cdot}66 \times 10^{-27}$ kg and that of an electron $9{\cdot}1 \times 10^{-31}$ kg. Though the charge carried by an electron is the natural unit of electricity, but it is so extremely small that to adopt it as a unit of electricity would be like adopting the grain as a unit for measuring sand. The practical unit of charge or quantity of electricity adopted by general agreement is one coulomb[1] which is equal to the charge of $6{\cdot}242 \times 10^{18}$ electrons. Hence, the charge of a single electron is $1/6{\cdot}242 \times 10^{18}$ *i.e.* $1{\cdot}602 \times 10^{-19}$ coulomb.

(*ii*) The orbits are more or less elliptical in shape and lie in all planes and not in one, though for convenience they are so shown in the figures to follow. These orbits are marked *K*, *L*, *M*, *N* etc. or are designated by their principal quantum number *n* where $n = 1, 2, 3, 4$ etc.

1. After Charles A. Coulomb (1736—1806), French philosopher distinguished for his investigations in electricity and mathematics. He derived both mathematically and experimentally, the laws governing forces between magnetic poles and electric charges.

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(*iii*) The maximum number of electrons possible in any extra-nuclear orbit or shell is fixed and is given by $2n^2$ subject to the condition that maximum number of electrons in any orbit will not exceed 32. Counting these orbits from nucleus outwards, the first orbit can have a maximum number of 2 electrons ; second orbit 8 electrons ; third orbit 18 electrons and so on. For example, a copper atom has 29 electrons which will be distributed as follows (Fig. 1-3).

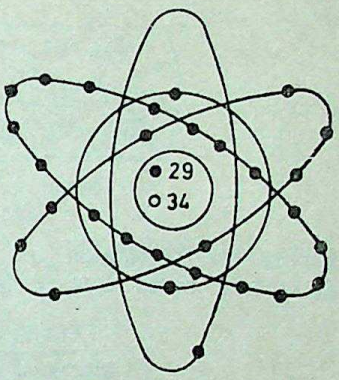

Fig. 1-3

1st orbit : 2 electrons
2nd orbit : 8 electrons
3rd orbit : 18 electrons
4th orbit : 1 electron

Similarly, silver atom has 47 electrons which will be grouped as 2, 8, 18, 18, 1.

(*iv*) The centripetal force necessary to keep electrons rotating in elliptical orbits round the nucleus is supplied by the force of attraction between their charges as given by Coulomb's laws. It is obvious that nearer the electron is to the nucleus, greater is the force with which it is bound to it.

The electrons in the outermost orbit experience a very weak force of attraction for two reasons : (*a*) force varies inversely as the square of the distance between two charges (*b*) the presence of a large number of electrons in the intermediate orbits acts as a partial screen between the nucleus and the outermost electrons. This screening or shielding action results in reduced attraction between the two. It is found that in metals the outermost electrons are very loosely attached to the atom. In fact, they can be hardly said to be attached to one parent atom, they very freely move from one atom to another and behave very much like the molecules of a gas in a draught-free room. They wander about with random motion between atoms, continuously colliding with one another but not moving in any particular direction. These free-moving and unattached electrons form what is known as *electron gas*. The condition of these electrons in a copper wire is shown diagrammatically in Fig. 1-4. The atoms are arranged in a particular pattern called crystal lattice. The + signs indicate that they are charged positively (because of having lost some electrons) *i.e.* they are now ions. Although these ions can oscillate about their mean position, yet for the present, they are shown stationary.

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The electrons shown as black dots with arrows, wander about in all sorts of manner and directions. When this wire is joined a

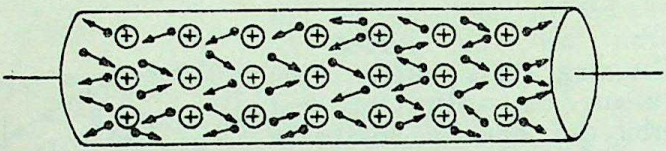

Fig. 1-4

across the terminals of a battery, the electrons experience an attractive force due to the anode and a repulsive force due to

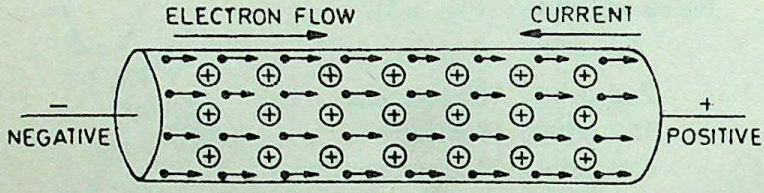

Fig. 1-5

cathode, with the result that they start drifting from cathode to anode as shown in Fig. 1-5. When some external force (*i.e.* potential difference) is applied to these atoms, the outermost one or more electrons get easily detached from the parent atom and start drifting along and so give rise to flow of electrons.

This continuous flow of electrons constitutes an electric current. It is found that those substances whose atoms have their outermost orbits incomplete act as good conductors of electricity *i.e.* they permit an easy detachment of their outermost electrons and offer very little hindrance to their flow through their atoms. Such substances are known as *good conductors*. But substances whose electrons are rigidly held to their atoms are termed as *bad conductors*. In their case, a very large force (*i.e.* potential difference) is required to detach their electrons and even then the number of electrons detached and set drifting is comparatively small.

1-3 The Idea of Electric Potential

In Fig. 1-6 is shown a simple voltaic cell. It consists of a copper plate (known as anode) and a zinc rod (*i.e.* cathode) immersed in dilute sulphuric acid (H_2SO_4) contained in a suitable vessel. The chemical action taking place within the cell causes the electrons to be removed from Cu plate and to be deposited on the zinc rod (for details see Art. 2-4). This transfer of electrons is accomplished through the agency of the diluted H_2SO_4 which is known as an electrolyte. The result is that zinc rod becomes negative due to the deposition of electrons on it and the Cu plate becomes positive due to the departure of electrons from it. The large number of electrons collected on the zinc rod is being attracted by anode,

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but is prevented from returning to it by the force set up by the chemical action within the cell. But if the two electrodes

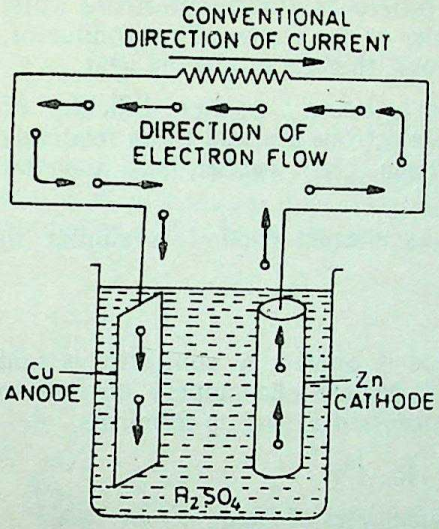

Fig. 1-6

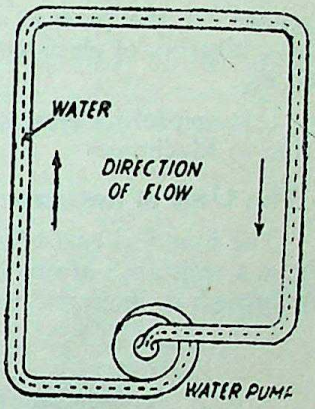

Fig. 1-7

are joined by a wire *externally* then electrons rush to the anode, thereby equalizing the charges of the two electrodes. However, due to the continuity of chemical action, a continuous difference in the number of electrons on the two electrodes is maintained which keeps up a continuous flow of current through the external circuit. The action of an electric cell is similar to that of a water pump which, while working, maintains a continuous flow of water *i.e.* water current through the pipe (Fig. 1-7).

It should be particularly noted that the direction of *electronic* current is from zinc to copper in the external circuit. However, the direction of *conventional* current (which is given by the direction of flow of *positive* charge) is from Cu to zinc. In the present case, there is no flow of positive charge as such from one electrode to another. But we can look upon the arrival of electrons on copper plate (with subsequent decrease in its positive charge) as equivalent to an actual departure of positive charge from it.

When zinc is negatively charged, it is said to be at negative potential with respect to the electrolyte, whereas anode is said to be at positive potential relative to. the electrolyte. Between themselves, Cu plate is assumed to be at a higher potential than the zinc rod. This difference in potential is continuously maintained by the chemical action going on in the cell which supplies energy to establish this potential difference.

1-4 Resistance

It may be defined as the property of a substance due to which it opposes the flow of electricity (*i.e.* electrons) through it.

Metals (as a class), acids and salt solutions are good conductors of electricity. This, as discussed earlier, is due to the presence

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of a large number of free or loosely-attached electrons in their atoms. These vagrant electrons assume a directed motion on the application of an electric potential difference. These electrons while flowing pass *through* the molecules or the atoms of the conductor, collide with other atoms and electrons, thereby producing heat.

Those substances which offer relatively greater difficulty or hindrance to the passage of these electrons are said to be relatively poor conductors of electricity like mica, glass, rubber, oils and dry wood etc.

It is helpful to remember that electric friction is similar to friction in Mechanics.

1-5. The Unit of Resistance

The practical unit of resistance is ohm[1]. A conductor is said to have a resistance of one ohm if it permits one ampere current to flow through it when one volt is impressed across its terminals.

TABLE No. 1-1

Multiples and Submultiples of Ohm

Prefix	*Its meaning*	*Symbol*	*Equal to*
Mega—	One million	MΩ	10^6 Ω
Kilo—	One thousand	KΩ	10^3 Ω
Centi—	One hundredth	—	—
Milli—	One thousandth	mΩ	10^{-3} Ω
Micro—	One millionth	μΩ	10^{-6} Ω

For insulators whose resistances are very high, a much bigger unit is used *i.e.* megohm $= 10^6$ ohm (the prefix 'mega' or 'mego' meaning a million) or kilohm $= 10^3$ ohm (kilo means thousand). In the case of very small resistances, smaller units like milliohm $= 10^{-3}$ ohm or microhm $= 10^{-6}$ ohm are used. The symbol for ohm is Ω.

1-6 Laws of Resistance *(A.M.I.E. May 1973)*

The resistance R offered by a conductor depends on the following factors :

(*i*) It varies directly as its length.

If, for example, one metre of a copper wire has a resistance of 0·04 Ω, then two metres will have a resistance of 0·08 Ω and 50 metres will have a resistance of 2·0 Ω.

(*ii*) It varies inversely as the cross-section of the conductor.

1. After George Simon Ohm (1787—1854), a German mathematician who in about 1827 formulated the law known after his name as Ohm's Law. He made notable contribution to the development of other basic laws of electricity.

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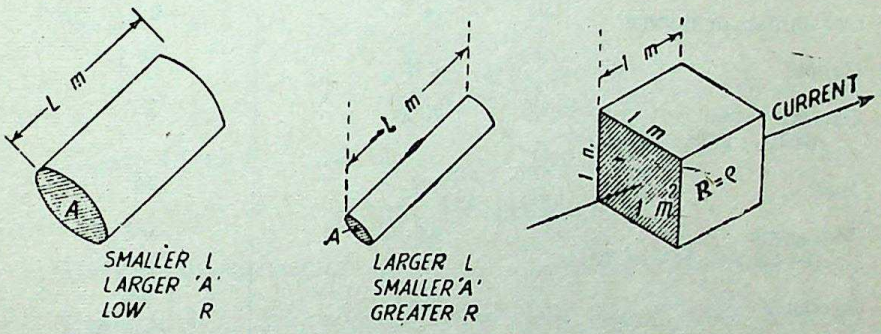
(*iii*) It depends on the nature of the material.

(*iv*) It also depends on the temperature of the conductor.

Neglecting the last factor for the time being, we can say that

$$R \propto \frac{l}{A} \qquad \text{or} \qquad R = \rho \frac{l}{A} \qquad \ldots(i)$$

where l is the length, A the area of cross-section of the conductor and ρ is a constant depending on the nature of the material of the conductor and is known as the *specific resistance* or *resistivity*.



(*a*) (*b*) (*c*)

Fig. 1-8

If in Eq. (*i*) above, we put

$$l = 1 \text{ m and } A = 1 \text{ m}^2 \text{ (}i.e.\text{ 1 m}^3 \text{ of a material)}$$

then $R = \rho$ [Fig. 1-8(*c*)]

Hence, specific resistance of a material may be defined as :

the resistance between the opposite faces of a metre cube of that material.

1-7 Unit of Resistivity

From Eq. (*i*), we have $\rho = \dfrac{AR}{l}$

In the rationalized M.K.S.A. system of units,

$$\rho = \frac{A \text{ metre}^2 \times R \text{ ohm}}{l \text{ metre}} = \frac{AR}{l} \text{ ohm-metre}$$

The above unit of ohm-metre (Ω—m) is sometime wrongly but frequently written as ohm/metre^3 or Ω/m^3.

Values of resistivity and temperature coefficients for various materials are given in Table No. 1-2. The resistivities of commercial materials may differ by several per cent due to impurities etc.

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TABLE No. 1-2

Resistivities and Temperature Coefficients

Material	*Resistivity in ohm-metre at 20°C* $\times 10^{-8}$	*Temperature coefficient at 20° C* $\times 10^{-4}$
Aluminium ...	2·69	40·3
Brass ...	6–8	20
Carbon ...	7000	−5
Constantan or Eureka ...	49	+0·1 to −0·4
Copper ...	1·72	39·3
German Silver (84 Cu ; 2 Ni ; 1 Zn) ...	20·2	2·7
Iron ...	9·8	65
Manganin (84 Cu ; 12 Mn ; 4 Ni) ...	42–44	0·25
Mercury ...	95·8	8·9
Nichrome (80 Cu ; 20 Cr) ...	108·5	1·4
Platinum ...	9—15·5	36·7
Silver ...	1·64	38
Tungsten ...	5·5	47
Amber ...	5×10^{14}	
Bakelite ...	10^{10}	
Glass ...	10^{10}—10^{12}	
Mica ...	10^{15}	
Rubber ...	10^{16}	
Shellac ...	10^{14}	
Sulphur ...	10^{15}	

Example 1-1. *The resistance of a Cu wire 200 metres long is 21 Ω. If its thickness is 0·44 mm, calculate its specific resistance in ohm-metre (Ω-m).*

Solution. Formula used : $R = \rho \frac{l}{A}$

Here $l = 200$ m, $R = 21$ Ω, diameter $d = 0{\cdot}44 \times 10^{-3}$ m

$\therefore A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (0{\cdot}44 \times 10^{-3})^2$ m^2

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Now, $\rho = AR/l$

$$= \frac{\pi(0{\cdot}44 \times 10^{-3})^2 \times 21}{4 \times 200} = \mathbf{1{\cdot}597 \times 10^{-8}\ ohm\text{-}m}$$

Example 1-2. *A current of 0·2 A is passed through a coil of iron wire which has a cross-sectional area of 0·01 cm². If the resistivity of iron is 14×10^{-8} Ω-m and the p.d. across the ends of the coil is 21 volts, what is the length of the wire? What is the conductivity of the wire and rate of generation of heat in the coil?*

Solution. Resistance of coil

$$R = 21/0{\cdot}2 = 105\ \Omega$$

$$A = 0{\cdot}01\ cm^2 = 0{\cdot}01 \times 10^{-4}\ m^2$$

$$\rho = 14 \times 10^{-8}\ \Omega\text{-m}$$

Now $R = \rho \dfrac{l}{A}$

$$\therefore \quad l = \frac{AR}{\rho} = \frac{105 \times 0{\cdot}01 \times 10^{-4}}{14 \times 10^{-8}} = \mathbf{750\ m}$$

Conductivity $\sigma = \dfrac{1}{\rho} = \dfrac{1}{14 \times 10^{-8}} = \dfrac{10^8}{14}$

$$= \mathbf{7{\cdot}14 \times 10^6\ mho/metre}$$

Heat produced/second $= \dfrac{VI}{4{\cdot}2} = \dfrac{21 \times 0{\cdot}2}{4{\cdot}2} = \mathbf{1\ cal/s.}$

Example 1-3. *Two wires, having equal lengths and made of the same material, have resistances of 25 Ω and 49 Ω respectively. Find their relative diameters.*

Solution. $R = \rho \dfrac{l}{A} = \rho \dfrac{l}{\pi d^2/4}$

Since the lengths of the wires and their material are the same, ρ and l are constant,

$\therefore \quad R \propto \dfrac{1}{d^2}$ —where d is the diameter of the wire.

$\therefore \quad R_1 \propto \dfrac{1}{d_1^2}$ and $R_2 \propto \dfrac{1}{d_2^2}$ $\therefore \quad \dfrac{R_2}{R_1} = \dfrac{d_1^2}{d_2^2}$

$$\therefore \quad \frac{d_1}{d_2} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{49}{25}} = \frac{7}{5}\ ; \quad \mathbf{d_1 = 1{\cdot}4\ d_2}$$

1-8 Conductance (G) and Conductivity (σ)

(A.M.I.E. Nov. 1971)

Conductance (G) is reciprocal of resistance.[1] Whereas resistance of a conductor measures the *opposition* which it offers to the flow of current, the conductance measures the *inducement* which it offers to its flow.

From Eq. (i) of Art. 1-6, we have

$$R = \rho \frac{l}{A}$$

$$G = \frac{1}{\rho} \cdot \frac{A}{l} = \frac{\sigma A}{l} \qquad \ldots(i)$$

1. In a.c. circuits, it has a slightly different meaning.

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where σ is called the *conductivity or specific conductance* of a conductor. Its unit is mho/metre. The unit of conductance is mho (*i.e.* ohm—read backwards).

1-9 Effect of Temperature on Resistance (*A.M.I.E. Winter 1985*)

The effect of rise in temperature is :

(*i*) to *increase* the resistance of the pure metals. The increase is large and fairly regular for normal ranges of temperature. The temperature/resistance graph is a straight line (Fig. 1-9). As would be presently clarified, metals have a *positive* temperature-coefficient of resistance.

(*ii*) to *increase* the resistance of alloys, though, in their case, the increase is relatively small and irregular. For some high resistance alloys like Eureka (60% Cu and 40% Ni) the increase in resistance is (or can be made) negligible over a considerable range of temperature.

(*iii*) to *decrease* the resistance of electrolytes, insulators (such as paper, rubber, glass, mica etc.) and partial conductors such as carbon. Hence, insulators are said to possess a *negative* temperature-coefficient of resistance.

1-10 Temperature-coefficient of Resistance

(*A.M.I.E. Winter 1979*)

Let a metallic conductor having a resistance of R_0 at 0°C be heated to t°C and let its resistance at this temperature be R_t. Then considering normal ranges of temperature, it is found that the increase in resistance $\triangle R = R_t - R_0$ depends

(1) directly on its initial resistance

(2) directly on the rise in temperature

(3) on the nature of the material of conductor.

or $$R_t - R_0 \propto R_0 \times t$$

or $$R_t - R_0 = \alpha R_0 t \qquad \ldots(i)$$

where α (alpha) is a constant and is known as the *temperature-coefficient of resistance* of that conductor.

Rearranging Eq. (*i*) above, we get

$$\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\triangle R}{R_0 \times t}$$

If $R_0 = 1\ \Omega$, $t = 1$°C, then $\alpha = \triangle R$

Hence, temperature-coefficient at 0°C may be defined as

the change in resistance per ohm per degree change in temperature from 0°C.

Now, $\triangle R/R_0$ represents the *fractional* change in resistance *i.e.* the change in resistance expressed as a fraction of the original resistance. Hence, temperature-coefficient of resistance may also be defined as :

the fractional change in resistance per degree change in temperature from 0°C.

From Eq. (*i*) above, we find that $R_t = R_0 (1 + \alpha t)$

Since this formula is true both for rise and fall in temperature, in general, it may be written as

$$R_t = R_0 (1 \pm \alpha t)$$

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It should be remembered that the above equation holds good for both rise and fall in temperature. As temperature of a conductor is decreased, its resistance is also decreased. In Fig. 1-9 is shown the temperature/resistance graph for copper which is practically a straight line. If this line were extended backwards, it would cut the temperature axis at a point where temperature is −234·5°C (a number quite easy to remember). It means that theoretically, the resistance of a copper conductor will become zero at this point though as shown by solid line, in practice, the curve departs from a straight line at very low temperatures. From the two similar triangles of Fig. 1-9, it is seen that

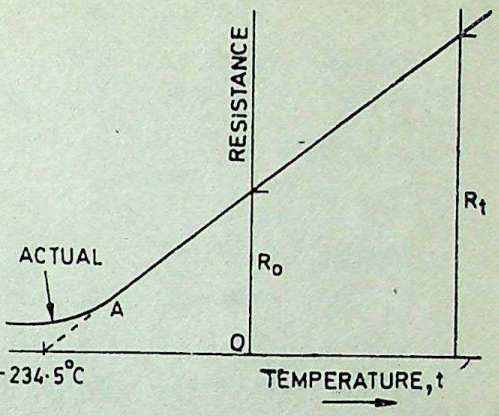

Fig. 1-9

$$\frac{R_t}{R_0}=\frac{t+234{\cdot}5}{234{\cdot}5}=\left(1+\frac{t}{234{\cdot}5}\right)$$

$$R_t=R_0\left(1+\frac{t}{234{\cdot}5}\right) \text{ or } R_t=R_0\,(1+\alpha t)$$

where $\alpha=1/234{\cdot}5$ for copper.

1-11 Value of α at Different Temperatures

(A.M.I.E. Winter 1979)

So far we did not make any distinction between values of α at different temperatures. But it is found that value of α itself is not constant but depends on the initial temperature on which the increment in resistance is based. When the increment is based on the resistance measured at 0°C, then α has the value of α_0. At any other initial temperature t°C, value of α is α_t and so on. It should be remembered, that for any conductor, α_0 has the maximum value.

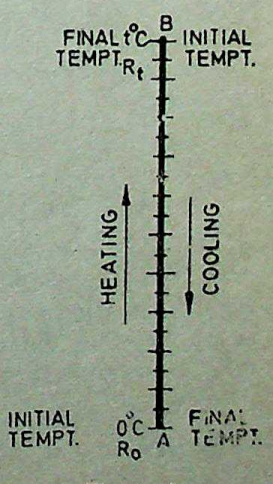

Fig. 1-10

With reference to Fig. 1-10 the value of R_0 in terms of R_t during cooling is given by

$$R_0=R_t[1+\alpha_t(-t)]$$

$$=R_t(1-\alpha_t t) \qquad \ldots(i)$$

where α_t is the temperature coefficient at t°C. It should be noted that here the initial starting point is B and temperature is decreased from t°C to 0°C at point A.

Now, taking point A as the initial point, we get

$$R_t=R_0\,(1+\alpha_0 t) \qquad \ldots(ii)$$

where α_0 is temperature-coefficient as referred to 0°C.

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From Eq. (*i*) above, we have

$$\alpha_t = \frac{R_t - R_0}{R_t \times t}$$

Substituting the value of R_t from Eq. (*ii*), above, we get

$$\alpha_t = \frac{R_0(1+\alpha_0 t) - R_0}{R_0(1+\alpha_0 t) \times t} = \frac{\alpha_0}{1+\alpha_0 t}$$

$$\therefore \quad \alpha_t = \frac{\alpha_0}{1+\alpha_0 t} \qquad \ldots(iii)$$

In general, let

α_1 = tempt-coefficient at t_1°C

α_2 = tempt-coefficient at t_2°C

Then, from Eq. (*iii*) above, we get

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \quad \text{or} \quad \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_1}{\alpha_0}$$

Similarly $\frac{1}{\alpha_2} = \frac{1+\alpha_0 t_2}{\alpha_0}$

Subtracting one from the other, we get

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = (t_2 - t_1) \quad \text{or} \quad \frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1)$$

$$\therefore \quad \alpha_2 = \frac{1}{1/\alpha_1 + (t_2 - t_1)}$$

Values of α for copper at different temperatures are given in Table No. 1-3.

TABLE No. 1-3

Different Values of α for Copper

Tempt. in °C	0	5	10	20	30	40	50
α	0·00427	0·00418	0·00409	0·00393	0·00378	0·00364	0·00352

In view of the dependence of α on the initial temperature, we may define the temperature coefficient of resistance at a given temperature as the change in resistance per ohm per degree centigrade change in temperature from the given temperature.

In case R_0 is not given, then relation between the known resistance R_1 at t_1°C and the unknown resistance R_2 at t_2°C can be found as follows :

$$R_2 = R_0(1+\alpha_0 t_2) \quad \text{and} \quad R_1 = R_0(1+\alpha_0 t_1)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1+\alpha_0 t_2}{1+\alpha_0 t_1} \qquad \ldots(iv)$$

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The above expression can be simplified by a little approximation as follows :

$$\frac{R_2}{R_1} = (1+\alpha_0 t_2)(1+\alpha_0 t_1)^{-1}$$

$$= (1+\alpha_0 t_2)(1-\alpha_0 t_1) \quad \text{[Using Binomial Theorem for expansion and neglecting squares and higher powers of } (\alpha_0 t_1)\text{]}.$$

$$= 1+\alpha_0(t_2-t_1) \quad \text{[Neglecting product } \alpha_0{}^2 t_1 t_2\text{]}$$

$$\therefore \quad R_2 = R_1[1+\alpha_0(t_2-t_1)] \qquad \ldots(v)$$

For more accurate calculations, however, Eq. (*iv*) should be used.

Note. We could also use the equation $R_2 = R_1[1+\alpha_1(t_2-t_1)]$

1-12 Variation of Resistivity with Temperature

(*A.M.I.E. May 1973*)

Not only resistance but specific resistance or resistivity of metallic conductors also increases with rise in temperature and *vice versa*. The relation between ρ_0 and ρ_t within normal ranges of temperature is given by

$$\rho_t = \rho_0(1+\alpha_0 t)$$

where ρ_0 = resistivity at 0°C

ρ_t = resistivity at t°C.

Example 1-4. *The base of an incandescent lamp with a tungsten filament is marked 120-V, 60-W. Measurement on a Wheatstone bridge of the resistance of the lamp at 20°C indicates 20 Ω. What is the normal temperature of incandescence if the resistance-temperature coefficient of tungsten is 5×10^{-3} per °C at 20°C ?*

(*A.M.I.E. Nov. 1970*)

Solution. Working current of the lamp is $= 60/120 = 0{\cdot}5$ A

Hot resistance of the lamp $= 120/0{\cdot}5 = 240\ \Omega$

Let t°C be the normal working temperature of the lamp.

Then $R_t = 240\ \Omega$; $R_{20} = 20\ \Omega$; $\alpha_{20} = 5\times10^{-3}$/°C

Now $R_t = R_{20}[1+\alpha_{20}(t-20)]]$

$\therefore \quad 240 = 20[1+5\times10^{-3}(t-20)]$; $t = \mathbf{2220°C}$

Example 1-5. *A conductor has a cross-section of 10 cm² and specific resistance of 7·5 μ Ω-cm at 0°C. What will be its resistance in ohm per km when the temperature is 40°C ? Take the tempt-coeff. of the material = 0·005 per °C.* (*A.M.I.E. Summer 1976*)

Solution. It will be assumed that tempt. coeff. of 0·005 per °C represents α_0.

$$\rho_{40} = \rho_0(1+40\,\alpha_0) = 7{\cdot}5\,(1+40\times0{\cdot}005)$$

$$= 9\ \mu\Omega\text{-cm} = 9\times10^{-8}\ \Omega\text{-m}$$

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$$R_{40}=\rho_{40}\frac{l}{A}$$

Here, $l=1$ km $=10^3$ m ; $A=10$ cm$^2=10\times10^{-4}=10^{-3}$ m^2

$$R_{40}=9\times10^{-8}\times10^3/10^{-3}=\mathbf{0{\cdot}09\ \Omega}$$

Example 1-6. *A semicircular ring of copper has an inner radius of 8 cm, radial thickness 4 cm and axial thickness 6 cm. Calculate the resistance of the ring at 50°C between its two end faces. Specific resistance of copper at 20°C $=1{\cdot}724\times10^{-6}$ Ω-cm. Resistance-temperature coefficient of copper at 0°C $=0{\cdot}0043$/°C.* (*A.M.I.E. Dec. 1974*)

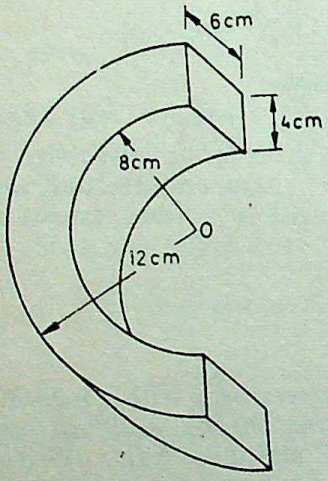

Fig. 1-11

Solution. The semicircular ring is shown in Fig. 1-11.

Mean radius $=(12+8)/2=10$ cm

Length of the semicircular ring

$=\pi R=10\pi$ cm $=31{\cdot}4$ cm

Area of cross-section

$=6\times4=24$ cm^2

Now, we have to find ρ_{50} from given value of ρ_{20}. For this purpose, we need α_{20}.

$$\alpha_{20}=\frac{\alpha_0}{1+20\alpha_0}=\frac{0{\cdot}0043}{1+20\times0{\cdot}0043}$$

$$=0{\cdot}00396/°C$$

$$\rho_{50}=\rho_{20}[1+\alpha_{20}(50-20)]$$

$$=1{\cdot}724\times10^{-6}(1+30\times0{\cdot}00396)$$

$$=1{\cdot}91\times10^{-6}\ \Omega\text{-cm}$$

$$R_{50}=\frac{\rho_{50}\times l}{A}=\frac{1{\cdot}91\times10^{-6}\times31{\cdot}4}{24}=\mathbf{2{\cdot}5\times10^{-6}\ \Omega}$$

Example 1-7. *It is found that the resistance of a coil of wire increases from 50 Ω at 15°C to 60 Ω at 55°C. Calculate the resistance-temperature coefficient at 0°C of the conductor material.*

Solution. Formula used : $R_t=R_0(1+\alpha_0 t)$

$$\therefore\quad R_{15}=50=R_0(1+15\alpha_0)$$

$$R_{55}=60=R_0(1+55\,\alpha_0)$$

$$\therefore\quad \frac{60}{50}=\frac{1+55\alpha_0}{1+15\alpha_0}\quad\text{or}\quad \alpha_0=\mathbf{\frac{1}{185}\ \text{per °C}}$$

Example 1-8. *Two coils connected in series have resistances of 600 Ω and 300 Ω and tempt. coeff. of 0·1% and 0·4% respectively at 20°C. Find the resistance of the combination at a tempt. of 50°C. What is the effective temperature coeff. of combination ?*

Solution. Resistance of 600 Ω resistor at 50°C is

$$=600[1+0{\cdot}001\,(50-20)]=618\ \Omega$$

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Similarly, the resistance of 300 Ω resistor at 50°C is

$$=300[1+0{\cdot}004(50-20)]=336\ \Omega$$

Hence, total resistance of combination at 50°C is

$$=618+336=\mathbf{954}\ \Omega$$

Let β = resistance-temperature coefficient at 20°C

Now, combination resistance at 50°C = 900 Ω

Combination resistance at 20°C = 954 Ω

$\therefore$ $954=900[1+\beta(50-20)]$ $\therefore$ $\beta=\mathbf{0{\cdot}002\ per\ °C}$

Example 1-9. *A resistor of 80 Ω resistance having a temperature coefficient of 0·0021 per degree C is to be constructed. Wires of two materials of suitable cross-sectional area are available. For material A, the resistance is 80 ohms per 100 metres and the temperature coefficient is 0·003 per degree C. For material B, the corresponding figures are 60 ohms per 100 metres and 0·0015 per degree C. Calculate suitable lengths of wires of materials A and B to be connected in series to construct the required resistor. All data are referred to the same temperature.*

Solution. Let R_a and R_b be the resistances of suitable lengths of materials A and B respectively which when joined in series will have a combined temperature coeff. of 0·0021. Hence, combination resistance at any temperature is (R_a+R_b). Suppose we heat these materials through t°C.

When heated, resistance of wire A increases from R_a to $R_a(1+0{\cdot}003\ t)$. Similarly, resistance of wire B increases from R_b to $R_b(1+0{\cdot}0015\ t)$.

$\therefore$ Combination resistance after being heated through t°C

$$=R_a(1+0{\cdot}003\ t)+R_b(1+0{\cdot}0015\ t)$$

The combination α being given, value of combination resistance can be found directly as

$$=(R_a+R_b)(1+0{\cdot}0021\ t)$$

$$\therefore\quad (R_a+R_b)(1+0{\cdot}0021\ t)=R_a(1+0{\cdot}003\ t)+R_b(1+0{\cdot}0015\ t)$$

Simplifying the above, we get

$$\therefore\quad \frac{R_b}{R_a}=\frac{3}{2} \qquad \ldots(i)$$

Now $$R_a+R_b=80\ \Omega \qquad \ldots(ii)$$

Substituting the value of R_b from (i) into (ii), we get

$$R_a+(3/2)\ R_a=80$$

$$\therefore\quad R_a=32\ \Omega \quad \text{and} \quad R_b=48\ \Omega$$

If L_a and L_b are the required lengths in metres, then

$$L_a=\frac{100}{80}\times 32=\mathbf{40\ m} \quad \text{and} \quad L_b=\frac{100}{60}\times 48=\mathbf{80\ m}$$

Example 1-10. *Two materials have resistance-temperature coefficient of 0·004 and 0 0002 respectively at given temperature. In what ratio should the wires made of these materials be connected in*

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series so as to have an overall tempt-coefficient of 0·002 ?
(A.M.I.E. Summer 1980)

Solution. Let R_1 and R_2 be the resistances of wires of the two given materials which are to be joined in series. Their ratio may be found by the simple technique illustrated in Fig. 1-12.

$$R_1/R_2 = 0{\cdot}0018/0{\cdot}002 = 9/10$$

Hence, R_1 is 9/10 of R_2. If resistance per unit length of the two wires were given, we could find the ratio of their respective lengths also.

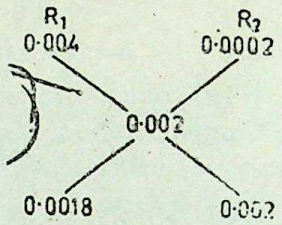

Fig. 1-12

Tutorial Problems—1

1. Calculate the resistance of a 100 m length of wire having a uniform cross-section of 0·1 mm² if the wire is made of manganin having a resistivity of 50×10^{-8} Ω−m.

If the wire is drawn out to three times its original length, by how many times would you expect its resistance to be increased ? [500 Ω ; 9 times]

2. A cube of material of side one cm has a resistance of 0·001 Ω between its opposite faces. If the same volume of the material has a length of 8 cm and a uniform cross-section, what will be the resistance of this length ? [0·064 Ω]

3. A lead wire and an aluminium wire are connected together in parallel. The current flowing in the respective wires is in the ratio of 39 : 40. The lead wire is 65 per cent longer than the aluminium wire and the ratio of their specific resistances is 98 : 13. Find the ratio of their cross-sectional areas. [12·22 : 1]

4. A coil of Cu wire has a resistance of 25 Ω at a temperature of 15°C, a coil of carbon wire has a resistance of 25·5 Ω at a temperature of 15°C. At what temperature will the resistance be the same in both coils ? α for Cu $= 4{\cdot}28 \times 10^{-3}$ per °C, α for carbon $= -0{\cdot}52 \times 10^{-3}$ per °C. [19·4 °C]

5. An electric radiator is required to dissipate 1 kW when connected to a 230 V supply. If the coils of the radiator are of wire 0·5 mm in diameter having resistivity of 60×10^{-8} Ω−m, calculate the necessary length of wire, [17·32 m]

6. Ten cm³ of copper are (*a*) drawn into a wire 100 m long (*b*) rolled into a square sheet of 10 cm side. Find the resistance of the wire and the resistance between opposite faces of the plate if the specific resistance of the copper is $1{\cdot}7 \times 10^{-8}$ Ω−m. [(*a*) 17 Ω (*b*) $1{\cdot}7 \times 10^{-9}$ Ω]

7. Two conductors, one of copper and the other of iron, are connected in parallel and at 20°C carry equal currents. What proportion of current will pass through each if the temperature is raised to 100°C ? For copper $\alpha = 0{\cdot}0043$/°C and for iron $\alpha = 0{\cdot}0063$/°C. [52·8% ; 47·2%]

8. An aluminium wire 5 metre long and 2 mm diameter is connected in parallel with a copper wire 3 metre long. The total current is 4 A and that in the aluminium wire is 2·5 A. Find the diameter of the copper wire. The respective resistivities of copper and aluminium are 1·7 and 2·6 μΩ-cm. [0·97 mm]

9. A potential difference of 200 V is applied to a copper field coil a temperature of 15°C and the current is 10A. What will be the mean temperature of the coil when the current has fallen to 5A, the applied voltage being the same as before ? Given $\alpha = 1/234{\cdot}5$ per °C at 0°C. [265·5°C

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10. The current through an electrical conductor is 1A when the temperature of the conductor is 0°C and 0·7A when the temperature is 100°C. What would be the current when the temperature of the conductor is 1200°C and what is the temperature coefficient of resistance of the conductor ?
[$\alpha = 0{\cdot}0043/°C$; 0·16 A]

11. The shunt winding of a certain d.c. generator has resistance of 135 Ω at a temperature of 15°C. Given that α for copper $= 4{\cdot}28 \times 10^{-3}$ per °C, calculate the resistance of the winding when the temperature has reached 52°C.
[155 Ω]

12. A 230-V, 60-W lamp has a filament whose normal working temperature is 2000°C. Find the current the cold lamp will take when first switched on. Assume that the temperature of the lamp when cold is 20°C and α for the filament is 0·0045 per °C. [2·58 A]

1-13. Resistors

Different resistors, whether fixed or of variable type are commonly made of metals and alloys. The resistance of an incandescent lamp is due to its coil which is made of tungsten wire. The heating elements of toasters, irons etc. are made of nickel-chromium alloy in wire or ribbon form. The small-sized colour-banded resistors employed in radio and TV receivers use a carbon composition as the resistance material.

A rheostat is a variable wire resistance having two connections, one fixed and the other movable.

Those variable resistors which have three terminals, two at the ends and one movable contact are called potentiometers.

1-14. Linear and Non-linear Resistors

Linear resistors are those in which current produced is *directly* proportional to the applied voltage. Their current *Vs.* voltage graph is straight and linear. In other words, their resistance remains constant.

Non-linear resistors are those whose current does not change linearly with changes in applied voltage. It is so because current flow always results in the production of heat which either increases (as in metals) or decreases their resistance (as in insulators). Because of this change in resistance, the current through such a resistor is not directly proportional to the impressed voltage. Circuits of this type are called *non-linear circuits* and the current-carrying elements are known as *non-linear resistors*. In Fig. 1-13, *A* represents the case of a linear resistor, whereas *B* and *C* represent non-linear resistors. In case of *B*, current increases more than proportionately with applied voltage which means that for this material resistance decreases with rise in temperature. It is examplified by Thyrite which is a non-metallic non-linear resistor.

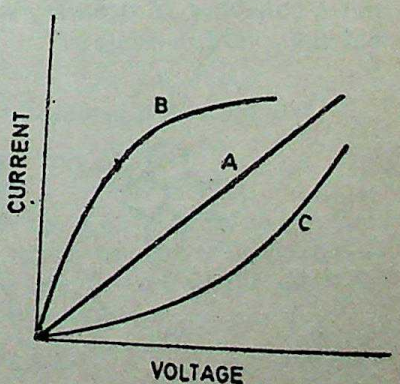

Fig. 1-13

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Curve *C* also represents a non-linear resistor but one whose resistance increases more than proportionately with applied voltage. Example of such a resistor is a semiconductor material chemically similar to ceramic oxide.

1-15. Uses of Non-linear Resistors

Mostly, they are used as protective devices in various electrical circuits as explained below.

(*i*) **Thyrite.** It is non-metallic and is made by combining silicon carbide with a binder under high pressure and temperature. Electric contacts are made to the material by spraying a metal coating on its both surfaces. At normal temperatures, the resistance of Thyrite is extremely high. However, when impressed voltage is doubled, its current increases 10 to 100 times. Because of this property, a Thyrite resistor is connected directly across the winding terminals of a motor or a generator in order to limit the induced discharge e.m.f. and thereby protect the winding insulation against failure when the impressed voltage is suddenly withdrawn.

(*ii*) **Thermistor.** Its resistance is comparatively low at normal temperatures and remains nearly constant up to a certain critical point beyond which it rises sharply. Usually, a thermistor (about the size of an aspirin tablet) is kept in direct contact with a motor winding and is connected into a control circuit. For normal operating temperature, its resistance is low enough to permit motor operation. But as soon as temperature exceeds permissible limit, its resistance increases sharply which results in opening the control circuit thereby stopping the motor.

1-16. Stranded Wires and Cables

Stranding of wires and cables is done for achieving greater flexibility. Following strandings are employed :

(*i*) **Bunch Stranding.** A bunch-stranded wire is nothing else but a collection of ordinary wires without any particular geometrical pattern.

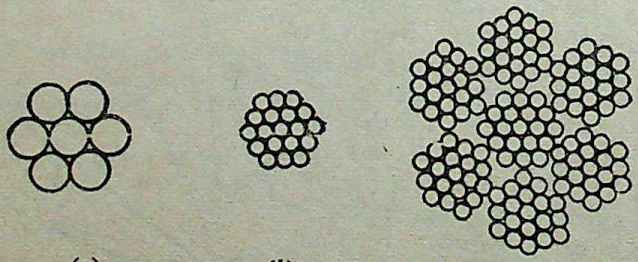

Fig. 1-14

(*ii*) **Concentric Stranding.** In this case, a centre wire is stranded by one or several definite layers of wires. It is so arranged that each layer contains six more wires than the layer immediately below it [Fig. 1-14 (*a*)]. When several layers are employed, each layer has a twist opposite to that of the layer beneath it [Fig. 1-14 (*b*)].

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(*iii*) **Rope-stranded Cable.** It is a concentric-stranded assembly made up of several concentric cables twisted together [Fig. 1-14 (*c*)].

1-17. Ohm's Law *(A.M.I.E. May 1970)*

Whenever electric current flows through a conductor, the following three factors are present :

(*i*) The pressure or potential difference V across the conductor (measured in volts) causing the current to flow.

(*ii*) The opposition or resistance R of the conductor (measured in ohms) which must be overcome.

(*iii*) The current strength I (measured in amperes) which is maintained in the conductor as a result of pressure overcoming the resistance.

There exists a definite relationship between the three quantities involved and is known as **Ohm's Law.** Itmay be stated thus :

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided the temperature of the conductor does not change.

In other words,

$$\frac{V}{I}=\text{constant} \quad \text{or} \quad \frac{V}{I}=R$$

where R is the resistance of the conductor between the two points considered.

Put in another way, it simply means that provided R is kept constant, current is directly proportional to the potential difference across the ends of a conductor.

If V is measured in volts and I in amperes, then R is given in ohms. For calculations, the following three forms of the Ohm's law should be remembered.

(*i*) $I=\dfrac{V}{R}$ *i.e.* current $=\dfrac{\text{potential difference}}{\text{resistance}}$

(*ii*) $V=IR$ *i.e.* p.d. $=\text{current}\times\text{resistance}$

(*iii*) $R=\dfrac{V}{I}$ *i.e.* resistance $=\dfrac{\text{p.d.}}{\text{current}}$

This law is applicable not only to d.c. circuits but to a.c. circuits as well provided account is taken of the induced e.m.f. resulting from the self-inductance of the circuit and of the distribution of current in the cross-section of the circuit.

1-18. Relations Derived from Ohm's Law

Following additional relationships connected directly or indirectly with Ohm's law are worthnoting.

(*i*) **Power.** It is given by the product of voltage (V) and current (I).

$$W=VI$$

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Its unit is watt.

Other forms of the above formula are

$$W = V^2/R \quad \text{—eliminating } I$$
$$= I^2R \quad \text{—eliminating } V$$

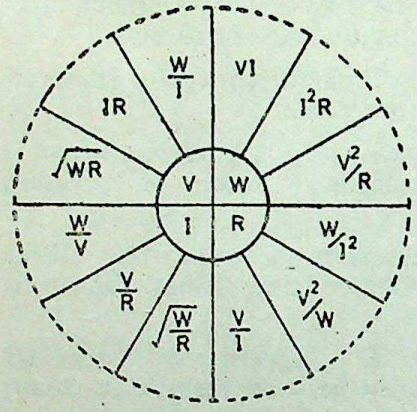


Fig. 1-15

(*ii*) **Resistance**

$$R = V/I$$
$$= V^2/W$$
$$= W/I^2$$

(*iii*) **Current**

$$I = V/R$$
$$= W/V$$
$$= \sqrt{W/R}$$

(*iv*) **Voltage**

$$V = IR$$
$$= W/I$$
$$= \sqrt{WR}$$

All the above relationships have been summarized in Fig. 1-15.

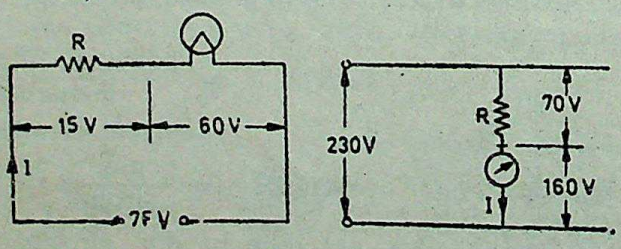
Example 1-11. *An incandescent projector lamp has the rated voltage of 60 volts and hot resistance of 20 Ω. Find the series resistance required to operate the lamp from a 75 volt supply.*

Solution. Rated value of lamp current $= 60/20 = 3$ A

Let R be the required series resistance [Fig. 1-16(*a*)].

Then the excess voltage of $(75 - 60) = 15$ V is to be dropped on A.

$\therefore \quad R = 15/3 = 5\ \Omega$



(*a*) (*b*)

Fig. 1-16

Example 1-12. *A voltmeter has a resistance of 20,000 Ω. When connected in series with an external resistanae across a 230-V supply, the instrument reads 160 V. What is the value of external resistance ?*

Solution. The circuit is shown in Fig. 1-16(*b*). The voltage drop across external resistance $R = 230 - 160 = 70$ V

Circuit current $I = 160/20{,}000 = 1/125$ A

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Now $IR = 70$

$$\therefore \quad \frac{1}{125} \times R = 70 \qquad \text{or} \qquad R = 8{,}750\ \Omega$$

1-12. Resistances in Series

When some conductors having resistances of R_1, R_2 and R_3 etc. are joined end on end as in Fig. 1-17, they are said to be connected

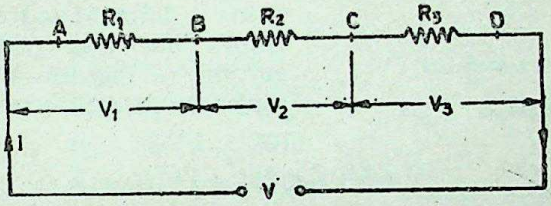

Fig. 1-17

in series. It can be proved that the equivalent resistance or total resistance between points A and D is equal to the sum of the three individual resistances. Being a series circuit, it should be remembered that (*i*) current is the same through all the three conductors (*ii*) but voltage drop across each is different due to its resistance

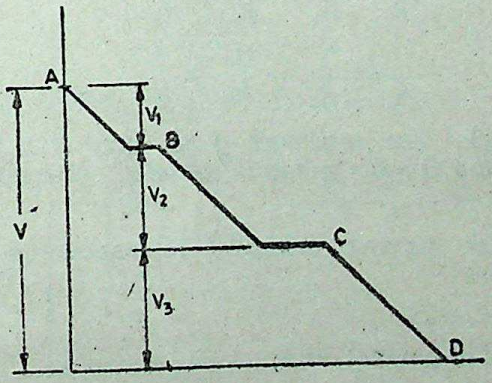

Fig. 1-18

being different and is given by Ohm's law and (*iii*) sum of the three voltage drops is equal to the voltage applied across the three conductors (Fig. 1-18). There is a progressive fall in potential as we go from point A to D as shown in Fig. 1-18.

$$\therefore \quad V = V_1 + V_2 + V_3$$

$$= IR_1 + IR_2 + IR_3 \qquad \text{...Ohm's Law}$$

But $V = IR$

where R is the equivalent resistance of the series combination.

$$\therefore \quad IR = IR_1 + IR_2 + IR_3$$

or $$R = R_1 + R_2 + R_3$$

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Note : In parallel combination, the equivalent resistance is less than the *least* among the resistors.

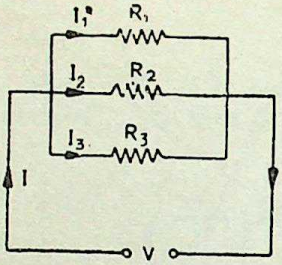

Fig. 1-19

1-20. Resistances in Parallel

Three resistances, as joined in Fig. 1-19, are said to be connected in parallel. In this case (*i*) p.d. across all resistances is the same (*ii*) current in each resistor is different and is given by Ohm's law and (*iii*) the total current is the sum of the three separate branch currents.

$$\therefore \quad I = I_1 + I_2 + I_3$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now $$I = \frac{V}{R}$$

where R = equivalent resistance of the parallel combination

$$\therefore \quad \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Example 1-13. *The resistance of two wires is 25 Ω when connected in series and 6 Ω when joined in parallel. Calculate the resistance of each wire.*

Solution. Let the two unknown resistances be R_1 and R_2. Then when in series

$$R_1 + R_2 = 25 \qquad \ldots(i)$$

When joined in parallel

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} \quad \text{or} \quad 6 = \frac{R_1 R_2}{R_1 + R_2} \qquad \ldots(ii)$$

Putting the value of R_2 from Eq. (*i*) in Eq. (*ii*), we have

$$6 = \frac{R_1(25 - R_1)}{25}$$

or $R_1^2 - 25R_1 + 150 = 0$ or $(R_1 - 15)(R_1 - 10) = 0$

$\therefore$ $R_1 = 10\ \Omega$, so $R_2 = 15\ \Omega$ or $R_1 = 15\ \Omega$, so $R_2 = 10\ \Omega$

Hence, the two wires have resistances of **10 Ω** and **15 Ω**.

Example 1-14. *The equivalent resistance of four resistors joined in parallel is 20 Ω. The currents flowing through them are 0·6, 0·3, 0·2*

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and 0·1 A. Find the value of each resistor.

(A.M.I.E. Winter 1976)

Solution. Total current in the circuit is the sum of the four branch currents. Its value is $=0·6+0·3+0·2+0·1=1·2$ A.

The common voltage across parallel resistors $= 20 \times 1·2 = 24$ V (Fig. 1-20)

$\therefore$

$$R_1 = 24/0·6 = 40\ \Omega$$
$$R_2 = 24/0·3 = 80\ \Omega$$
$$R_3 = 24/0·2 = 120\ \Omega$$
$$R_4 = 24/0·1 = 240\ \Omega$$

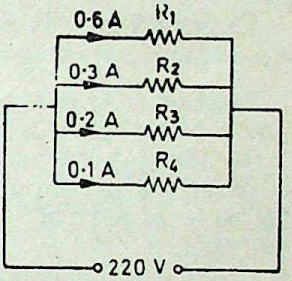

Fig. 1-20

Example 1-15. *Calculate the effective resistance of the following combination of resistances and the voltage drop across each resistance when a p.d. of 60 V is applied between points A and B. Figures represent resistances in ohms.*

Solution. Resistance between A and C (Fig. 1-21)

$$= \frac{6 \times 3}{6+3} = 2\ \Omega$$

Resistance of branch $ACD = 18+2 = 20\ \Omega$

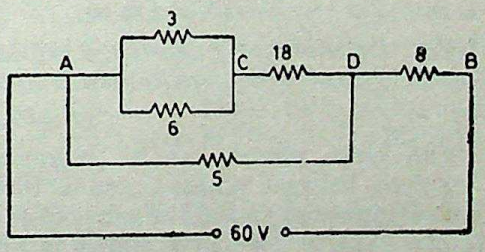

Fig. 1-21

Now, there are two parallel paths between points A and D of resistance 20 Ω and 5 Ω. Hence, resistance between A and D

$$= \frac{20 \times 5}{20+5} = 4\ \Omega$$

$\therefore$ Resistance between A and $B = 4+8 = 12\ \Omega$

Total circuit current $= 60/12 = 5$ A

Current through 5 Ω resistor $= 5 \times \frac{20}{25} = 4$ A

Current in branch $ACD = 5 \times \frac{5}{25} = 1$ A

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P.D. across 3 Ω and 6 Ω resistors $= 1 \times 2 = 2$ V

P.D. across 18 Ω resistor $= 18 \times 1 = 18$ V

P.D. across 5 Ω resistor $= 5 \times 4 = 20$ V

P.D. across 8 Ω resistor $= 8 \times 5 = 40$ V

Example 1-16. *In the circuit shown in Fig. 1-22, find thevoltage across and the current in each element.* (*A.M.I.E. Summer 1975*)

Solution. As seen, there are two parallel paths between points *A* and *B*; one path is *ACB* and has a total resistance of $(9+3) = 12$ Ω. The other is *ADB* and has a total resistance of $(2+6) = 8$ Ω. Both paths have a p.d. of 24 V applied across them.

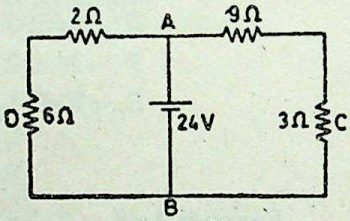

Fig. 1-22

Current through path *ACB* is $= 24/12 = 2$ A

Drop across 9 Ω $= 2 \times 9 = 18$ V

Drop across 3 Ω $= 2 \times 3 = 6$ V

Current through path *ADB* is $= 24/8 = 3$ A

Drop across 2 Ω $= 3 \times 2 = 6$ V

Drop across 6 Ω $= 3 \times 6 = 18$ V

Note. Drop across different elements can also be found without the help of current. For example, there is a drop of 24 V over 12 Ω resistance in path *ACB*. Hence, voltage drop per ohm $= 24/12 = 2$ V/Ω. Accordingly, by proportion, drop over 9 Ω resistance $= 9 \times 2 = 18$ V and so on.

Example 1-17. *Determine the current I in the circuit shown in Fig. 1-23 (a) below. Figures denote resistances in ohm.*

(*A.M.I.E. Summer 1975*)

Solution. The given circuit can be redrawn as shown in Fig. 1-23 (*b*). As seen, between points *C* and *D* there are two parallel paths; one of resistance 12 Ω and the other of $(4+2) = 6$ Ω. Their combined resistance $= 12 \times 6/18 = 4$ Ω as shown in Fig. 1-24 (*a*). Now

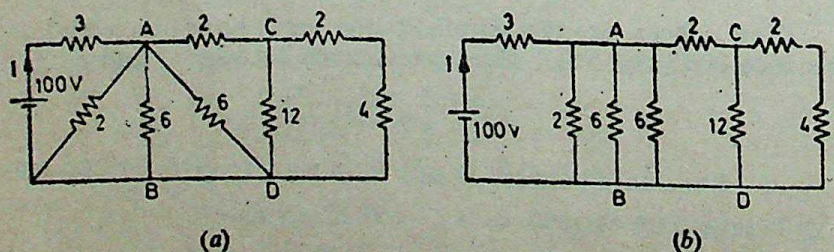

Fig. 1-23

as seen from Fig. 1-24 (*b*), the 2-Ω resistance and 4-Ω resistance have been combined into a single 6-Ω resistance which is connected in parallel with other three resistances between points *A* and *B*, their

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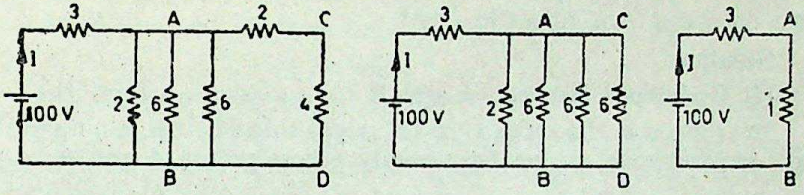
combined resistance equals 1 Ω. As seen from Fig. 1-24(*c*), $I = 100/4 = 25$ A.

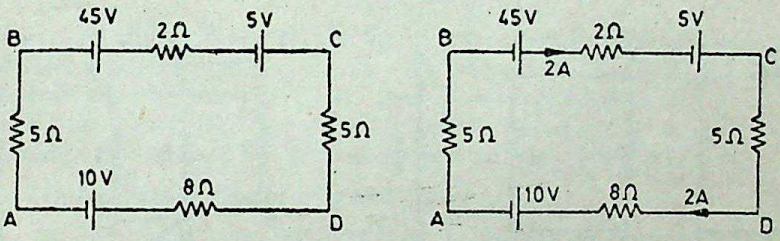


(*a*) (*b*) (*c*)

Fig. 1-24

Example 1-18. *In the circuit shown in Fig. 1-25 (a), determine the voltage rise from A to C and the power absorbed by the portion AD.* (*A.M.I.E. Dec. 1974*)

Solution. It should be noted that 45-V and 5-V batteries are connected in additive series whereas 10-V battery is connected in opposition to them *i.e.* in subtractive series.



(*a*) (*b*)

Fig. 1-25

Hence, net driving voltage around the circuit $= (45 + 5) - 10 = 40$ V. Total resistance $= (2 + 5 + 8 + 5) = 20$ Ω. Circuit current $= 40/20 = 2$ A. As shown, it flows clockwise round the circuit. Since current flows from *C* to *D* to *A*, it is obvious that *C* is at a higher electrical potential than *A*.

As we go from *A* to *C via* point *D*, we meet the following voltages :

(*i*) Since we go from —ve to +ve terminal of the 10-V battery, there is an *increase* of 10 V.

(*ii*) There is a voltage drop of $8 \times 2 = 16$ V over the 8-Ω resistance. This represents an *increase* or rise in voltage because we are going upstream *i.e.* opposite to the direction of flow of current.

(*iii*) Similarly, there is a *rise* in voltage of $2 \times 5 = 10$ V as we go from *D* to *C*.

Total increase or rise in voltage is

$$= 10 + 16 + 10 = 36 \text{ V}$$

Note. If we go from *A* to *C* via point *B*, the change in voltage is

$$= -(5 \times 2) + 45 - (2 \times 2) + 5 = +36 \text{ V}$$

The positive sign indicates that it is a rise in voltage.

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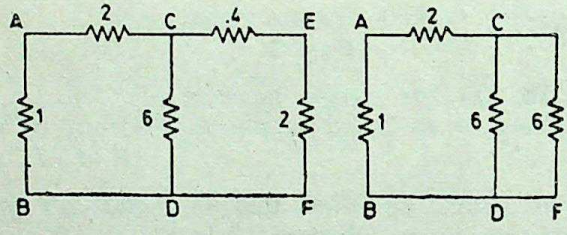
Example 1-19. *Find the equivalent resistance of the circuit given in Fig. 1-26 (a) between the following points (i) A and B (ii) C and D (iii) E and F (iv) A and F and (v) A and C. Figures against resistances represent their values in ohms.*

Solution.

(*i*) Resistance Between A and B

In this case, the circuit to the right side of AB is in parallel with 1 Ω resistance connected directly across points A and B.



(*a*) (*b*)

Fig. 1-26

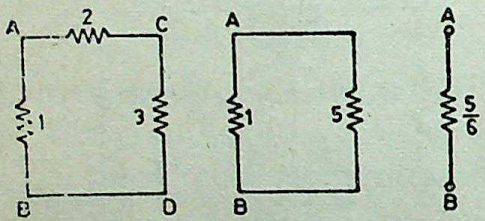
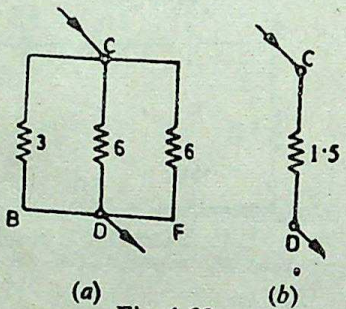


Fig. 1-27



(*a*) (*b*)

Fig. 1-28

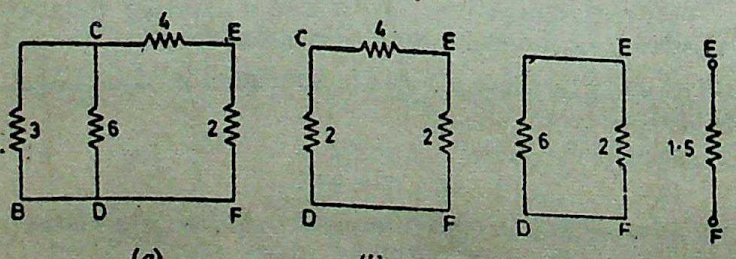
The different steps to simplify the circuit are shown in Fig. 1-26 (*b*) and Fig. 1-27. The equivalent resistance R_{AB}=**5/6 ohm.**

(*ii*) Resistance Between C and D

In this case, there are three parallel paths between points C and D. One is CD itself, the second is CFD and the third is CBD [Fig. 1-28 (*a*)].

It is so because a current entering at point C will have three paths to go to D as shown. The equivalent resistance R_{CD} as shown in Fig. 1-28 (*b*) equals 1·5 Ω.

(*iii*) Resistance Between E and F



(*a*) (*b*) (*c*) (*d*)

Fig. 1-29

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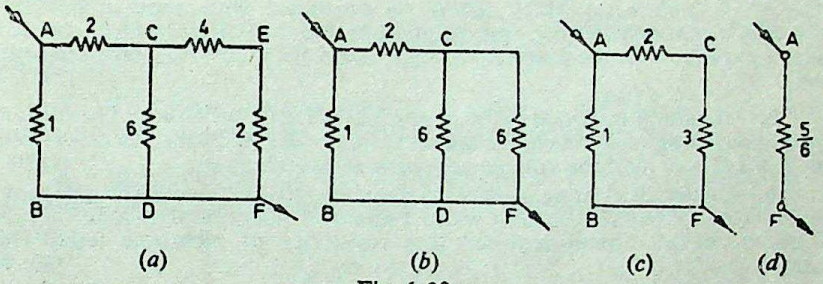
As seen from Fig. 1-29(*a*) in this case, the whole circuit to the left of *EF* is in parallel with the 2 Ω resistance connected directly across *EF*.

After various simplifications as shown in Fig. 1-29,

$$R_{EF} = 1{\cdot}5\ \Omega$$

(*iv*) Resistance Between A and F

As shown in Fig. 1-30(*a*), if a current enters the circuit at point *A* it has two choices : partly it goes along *AC* and partly along



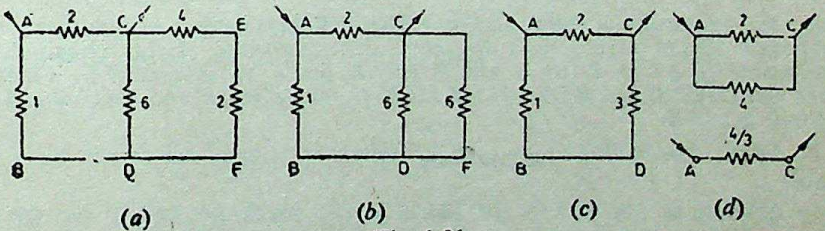
(*a*) (*b*) (*c*) (*d*)

Fig. 1-30

ABDF. At point *C*, there are again two parallel paths : one along *CEF* and the other along *CDF*. As shown in Fig. 1-30 (*b*), equivalent resistance between points *C* and *F* is $= 6/2 = 3\ \Omega$. Fig. 1-30 (*c*) shows that there are two parallel paths between *A* and *F*. Their equivalent resistance $= 5/6\ \Omega$ as shown in Fig. 1-30 (*d*).

(*v*) Resistance Between A and C

In this case, as shown in Fig. 1-31(*a*), if a current enters the circuit at point *A* (and leaves at *C*) it has two choices : one is directly



(*a*) (*b*) (*c*) (*d*)

Fig. 1-31

from *A* to *C* and the other is along *AD*. At point *D* there are again two parallel paths : one directly along *DC* and the other along *DFEC*. The different stages of simplification are shown in Fig. 1-31. It is seen that $R_{AC} = 4/3\ \Omega$.

Tutorial Problems—2

1. Two resistances of 10 Ω and 40 Ω respectively are connected in parallel. A third resistance of 5 Ω is connected in series with the combination and a d.c. supply of 240 V is applied to the ends of the complete circuit

(*i*) calculate the current in each resistance

(*ii*) what power would be spent in a fourth resistance of 20 Ω connected in parallel with the 5 Ω resistance. **[(*i*) 14·77 A ; 3·69 A ; 18·46 A (*ii*) 320 W]**

2. Two resistances of 4 Ω and 12 Ω respectively are connected in parallel with each other. Another resistance of 10 Ω is connected in series with the combination.

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Calculate the respective d.c. voltages which should be applied across the whole circuit

(*i*) to pass 6 A through the 10 Ω resistance

(*ii*) to pass 6 A through the 12 Ω resistance

Calculate also the total power from the supply in each case.

[(*i*) 78 V ; 468 W (*ii*) 312 V ; 7,488 W]

3. An armature has a resistance of 0·2 Ω at 15°C and the armature Cu loss is to be limited to 600 watts with a temperature rise of 55°C. If α_0 for Cu is 0·0043/°C, what is the maximum current that can be passed through the armature ? [50·8 A]

4. A bulb rated 110-V, 60-W is connected with another bulb rated 110-V, 100-W across a 220-V mains. Calculate the resistance which should be joined in parallel with the first bulb so that both the bulbs may take their rated power. [302·5 Ω]

5. A saloon is lighted by 30 lamps marked 110-V, 100-W. The distance between the saloon and the engine room switch-board is 55 m. If cable resistance is 1·2 Ω/km, find the voltage necessary at switch-board. [113·6 V]

6. An electric heating element to dissipate 450 watts on 250-V mains is to be made from nichrome ribbon of width 1 mm and thickness 0·05 mm. Calculate the length of the ribbon required (the resistivity of nichrome is 110×10^{-8} Ω – m). [631 cm]

7. For the circuit shown in Fig. 1-32, find the equivalent resistance between points *A* and *B*. [5 Ω]

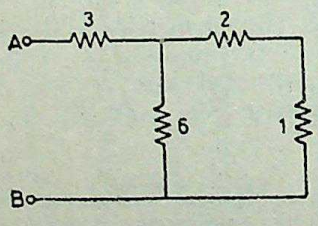

Fig. 1-32

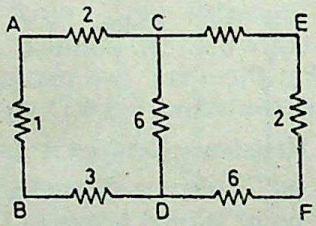

Fig. 1-33

8. In the circuit shown in Fig. 1-33, the figures against resistances represent their values in ohms. Find the equivalent resistance of this circuit between points (*i*) *A* and *B* (*ii*) *C* and *D* (*iii*) *E* and *F* (*iv*) *A* and *D* (*v*) *A* and *F* (*vi*) *A* and *C* (*vii*) *C* and *B* (*viii*) *E* and *D* (*ix*) *C* and *F* (*x*) *C* and *E* (*xi*) *B* and *D* and (*xii*) *B* and *E*.

Hint. For finding R_{AF} and R_{BE}, delta/star transformation will have to be used.

[(*i*) 9/10 Ω (*ii*) 12/5 Ω (*iii*) [26/15 Ω (*iv*) 2·4 Ω (*v*) 22/5 Ω (*vi*) 8/5 Ω (*vii*) 21/10 Ω (*viii*) 56/15 Ω (*ix*) 18/5 Ω (*x*) 44/15 Ω (*xi*) 20/10 Ω (*xii*) 381/90 Ω]

9. In Fig. 1-34, find the resistance between points (*i*) *C* and *D* (*ii*) *C* and *F* (*iii*) *A* and *B* (*iv*) *A* and *C* (*v*) *A* and *E* and (*vi*) *E* and *F*.

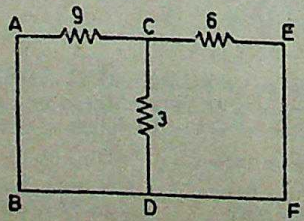

Fig. 1-34

[(*i*) 18/11 Ω (*ii*) 18/11 Ω (*iii*) zero (*iv*) 18/11 Ω (*v*) zero (*vi*) zero]

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HIGHLIGHTS

1. Laws of resistance are summed up as

$$R=\rho \frac{l}{A} \text{ ohm}$$

2. Specific resistance or resistivity is defined as the resistance between the opposite faces of a metre cube of that material.

3. Conductance is reciprocal of resistance (R). Its unit is mho (℧).

4. Conductivity (σ) is reciprocal of resistivity (ρ). Its unit is mho/metre.

5. Temperature coefficient is generally defined as the change in resistance per ohm per degree change in temperature.

6. In the case of good conductors, especially metals, the variations of resistance with temperature (within normal range) are iven by

$$R_t=R_0(1\pm\alpha_0 t)$$

7. It is found that α is not constant even for a given material. Its value depends on the initial temperature on which the increment in resistance is based. The values of α for various temperatures can be found from the relations

$$\alpha_t=\frac{\alpha_0}{1+\alpha_0.t} \quad \text{and} \quad \alpha_2=\frac{1}{1/\alpha_1+(t_2-t_1)}$$

8. In the same way, resistivity (ρ) of a given material is also not the same at all temperatures. It varies as

$$\rho_t=\rho_0(1+\alpha_0 t)$$

9. Ohm's law can be written as $I=V/R$. This law is applicable both to d.c. as well as a.c. circuits.

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DIVISION OF CURRENT 2

1. Primary Cell 2. Cell and Battery 3. E.M.F. and Terminal Potential Difference 4. The Simple Voltaic Cell 5. Polarisation 6. Local Action 7. Leclanche Cell 8. Standard Cell 9. Grouping of Cells 10. Series Grouping 11. Parallel Grouping 12. Mixed Grouping 13. Efficiency of a Cell 14. Maximum Power 15. Division of Current in Parallel Circuits 16. Theory of Shunt 17. Ammeter Shunt 18. Simple Potentiometer 19. Highlights.

2-1. Primary Cell

It essentially consists of two dissimilar conducting electrodes (one anode and the other cathode) immersed in a liquid called electrolyte which acts chemically on one of the two electrodes more readily than on the other. By using the energy released by chemical action, electrons are shifted from one electrode to another thereby creating a potential difference between the two electrodes. The value of *total* potential difference created between the electrodes, when *the cell is not connected* to an external circuit, is known as its *electromotive force* (*E.M.F.*).

Now, every cell has some internal resistance which depends upon the construction and condition of the cell. The internal resistance depends on the area of electrodes, the distance between the electrodes, the temperature, strength and density of the electrolyte. When the cell supplies current to an external circuit, there is always some internal voltage drop due to this internal resistance. Hence, the voltage available for external circuit is decreased by this amount. The net voltage available at the terminals for external circuit is known as the *terminal potential difference* (*T.P.D.*).

2-2. Cell and Battery

The word 'cell' means one unit or a combination of materials for converting chemical energy into electrical energy A 'battery' means a combination of these units or cells.

2-3. E.M.F. and Terminal Potential Difference

The e.m.f. of the cell, as said earlier, is the total potential difference established within the cell between the two electrodes

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when the cell is not supplying any current (so that there is no internal voltage drop). The e.m.f. can be measured by connecting a suitable voltmeter across the electrodes. But the terminal potential difference is equal to the e.m.f. *minus* the internal voltage drop. In other words, the *T.P.D.* is the potential difference available for external circuit. If '*i*' is the current supplied by the cell and *r* its internal resistance, then

$$\text{Terminal potential difference, } V = \text{e.m.f.} - ir$$

or
$$V = E - ir$$

where V = terminal potential difference

E = e.m.f. generated within the cell.

It should be noted that whereas e.m.f. E is constant, the terminal potential difference V is not, as it depends on load current supplied by the cell.

Also the term e.m.f. has local significance *i.e.* it is spoken of with reference to the generator itself, but potential difference is distributive. For example, we cannot say that an e.m.f. of 220 volts exists across an electric bulb. It is the potential difference of 220 V. In other words, p.d. can be distributed away from its source of generation.

2-4. The Simple Voltaic Cell

As shown in Fig. 2-1 such a cell consists of a copper (Cu) plate and a zinc (Zn) rod placed in dilute sulphuric acid (H_2SO_4). When terminals of the cell are joined together by wire, hydrogen is given off at the Cu plate and a current is found to flow from Cu to Zn in the external circuit and from Zn to Cu within the cell. Cu plate forms the positive electrode *i.e.* anode and zinc the negative electrode *i.e.* cathode. While the current is flowing, some of the zinc is found to dissolve in the acid.

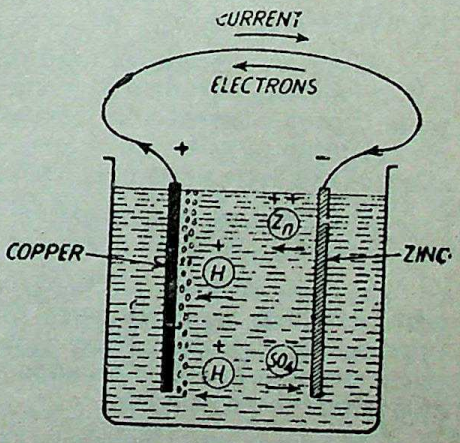

Fig. 2-1

The production of the e.m.f. can be explained with the help of contact difference of potential. Whenever a metal plate is brought in contact with an electrolyte, there always develops a tendency either for some of the metal to go into the solution in the form of positive ions, thus leaving the plate negatively charged or for some of the positive ions in the solution to be deposited on the plate which consequently becomes positively charged. In the case of a simple voltaic cell, when Zn is immersed in dilute H_2SO_4, some of the Zn enters the electrolyte in the form of positive ions which further combine with the neegative sulphions in the electrolyte to form zinc sulphat

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($\overset{++}{Zn}+\overset{--}{SO_4}=ZnSO_4$). However, this action stops when the contact potential between Zn and electrolytic solution reaches the value of 0·62 V, zinc being at the lower potential *i.e.* negative with respect to the solution film adjacent to it (Fig. 2-2).

Similarly, when Cu plate is placed in contact with the electrolyte, then the positive hydrogen ions in the solution have a tendency to get deposited on it until its potential rises nearly to 0·46 V above that of the solution. Hence, a total potential difference of 0·62—(−0·46)=1·08 V is developed between the two electrodes (Fig. 2-2).

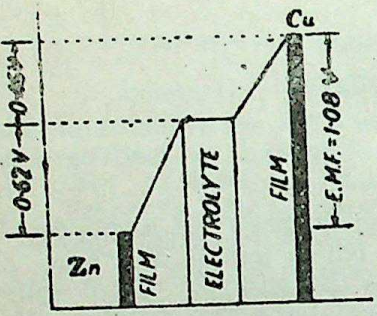

Fig. 2-2

As more and more of positive zinc ions pass into solution, an increasing number of electrons becomes available at the zinc electrode. These electrons run towards the copper anode *via* the external wire. There these electrons combine with the positive hydrogen ions, thus converting them into electrically neutral atoms. These atoms then combine in pairs to form molecules of hydrogen which bubble off from the copper plate. The chemical action taking place may be represented by the following equation

$$Zn+H_2SO_4=ZnSO_4+H_2$$

2-5. Polarisation

If an ammeter is included in the external circuit of the simple cell, it indicates a gradual decrease in the current flowing. After some time the current may cease altogether. The decrease is due to the collection of hydrogen bubbles on the surface of Cu plate. The effect of this layer of hydrogen is two-fold :

(*i*) It acts as an insulator, thus reducing the effective area of the Cu plate and thereby increasing the internal resistance of the cell.

(*ii*) The sticking layer of positive hydrogen ions on the Cu plate exerts a repulsive force on other hydrogen ions which are approaching the copper plate. Hence, the current is reduced. This phenomenon is called polarisation and the cell which is in this condition is said to be *polarised.*

Moreover, the hydrogen and zinc set up between them an e.m.f. which is *opposite* to that set up between zinc and copper. This e.m.f. is known as *back e.m.f.* The result of all these factors is that the current delivered by the cell diminishes to a very low value after a short time and hence the cell becomes useless for all practical purposes. The essential difference between various primary cells lies chiefly in the different methods employed to overcome this defect. The most widely used method is to surround the anode by a solid or liquid depolariser which oxidizes the hydrogen as soon as it is liberated.

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2-6. Local Action

It is found that even when the voltaic cell is not supplying any load current, zinc goes on continuously dissolving in the electrolyte. This is due to the fact that some traces of impurities like iron and lead in the commercial zinc form tiny local cells which are short-circuited by the main body of zinc. The action of these parasitic cells cannot be controlled so that there is some wastage of zinc. This phenomenon is known as *local action* and can be prevented by amalgamating the zinc plate *i.e.* by rubbing mercury over the zinc plate. Mercury is supposed to cover the impurities and maintain a film of zinc dissolved in mercury.

2-7. Leclanche Cell

It is the only primary cell of importance and is now-a-days mainly used in commercial practice. It is used, in large numbers, for domestic bell and telephone circuits and also in the 'dry' form for flash lamps and wireless batteries etc. A section of the cell is shown in Fig. 2-3. The cathode consists of a zinc rod placed inside a solution of ammonium chloride (sal ammoniac), NH_4Cl contained in a glass jar. The anode is a carbon rod which is surrounded by solid manganese dioxide (MnO_2) and crushed carbon in a porous pot. Manganese dioxide is rich in oxygen which easily combines with the hydrogen approaching the carbon anode. Crushed carbon is mixed with MnO_2 for improving its conductivity and hence the internal resistance of the cell.

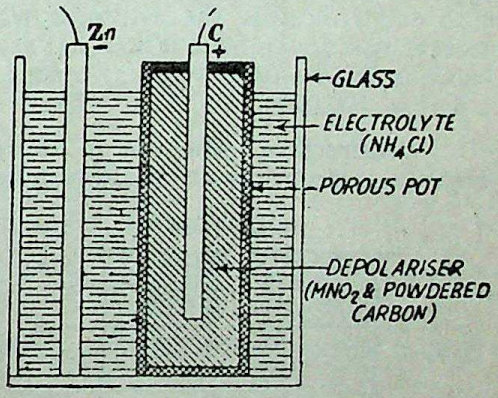

Fig. 2-3

Chemical Action. When the anode and cathode are joined by a wire, zinc dissolves in the electrolyte thereby producing zinc chloride ($ZnCl_2$), ammonia (NH_3) and hydrogen. The hydrogen released is in the form of H^+ ions :

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

Conversion of zinc into zinc chloride is the source of energy produced by the cell. The $ZnCl_2$ and NH_3 dissolve in the water present in the electrolyte. Then H^+ ions migrate towards the carbon rod through the porous pot and are converted into hydrogen atoms with the electrons reaching the carbon rod from the zinc rod *via* the external wire. Hydrogen attacks MnO_2 thus forming water

$$H_2 + 2MnO_2 = H_2O + Mn_2O_3$$

The main advantages of this cell are that the materials required for its construction are relatively cheap and its e.m.f. is 1·5 V. But

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its only serious disadvantage is that the solid depolariser (MnO_2) is slow in action *i.e.*, hydrogen is evolved too fast to be oxidised completely by MnO_2. The result is that polarisation does take place if current is drawn from the cell for too long a time. However, if the cell is disconnected from the circuit for some time, the hydrogen present is oxidised by MnO_2 and the cell recovers. Hence, this cell is useful in circuits where current is required intermittently for short periods as in bell circuits or in a torch.

2-8. Standard Cell

A standard cell is one whose e.m.f. varies very little with time and temperature. It is not intended as a source of energy but for use as a secondary standard of voltage in potentiometer circuits for laboratory calibration of voltmeters, ammeters etc. Such a cell maintains a very constant voltage over long periods of time provided no appreciable current is ever drawn from the cell. Most commonly used standard cell is the saturated Weston Cadmium Cell. If the current ever drawn from it does not exceed 0·1 mA, then its e.m.f. which is about 1·0183 V can be trusted to remain constant to about 1 part in 100,000.

Construction. The cell is housed in a *H*-shaped glass tube because its electrodes are liquid or semi-liquid (Fig. 2-4). Its anode is mercury and cathode is mercury cadmium amalgam (*i.e.*, 10 to 15% of cadmium dissolved in mercury). The electrolyte is a solution of cadmium sulphate ($CdSO_4$). The depolariser is a paste of mercurous sulphate (Hg_2SO_4) floating over the mercury anode and is an almost insoluble compound. The depolariser is effective only when extremely small currents are drawn from the cell. Hence, in ear y stages of balancing such a standard cell against a potentiometer wire, a protective resistance of the order of 100 kΩ is connected in series with the cell.

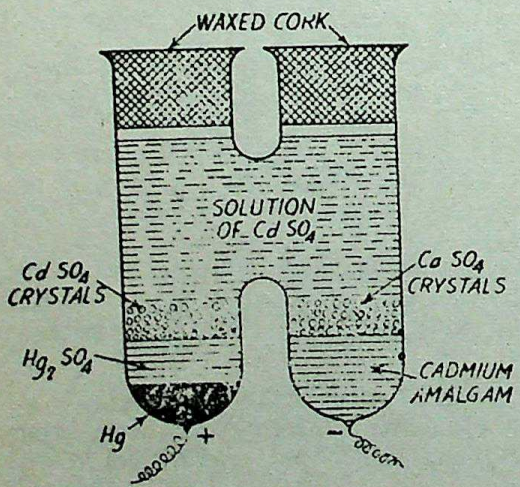

Fig. 2-4

2-9. Grouping of Cells

A given number of cells may be grouped or connected together either for increasing the e.m.f. or the current. Following are the three different ways of grouping cells.

2-10. Series Grouping

In series grouping, the positive electrode *i.e.* anode of one cell

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is connected to the cathode of the second cell and the anode of the second cell is connected to the electrode of the third cell and so on as shown in Fig. 2-5.

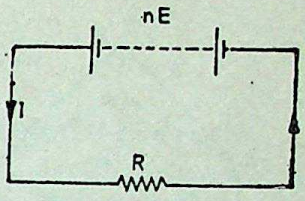

Fig. 2-5

Let

n = number of cells connected in series

r = internal resistance/cell

R = external load resistance

E = the e.m.f./cell

Then, total e.m.f. of the battery of n cells is $= nE$

Internal resistance of the battery $= nr$[1]

Total circuit resistance $= R + nr$

$$\text{circuit current } I = \frac{nE}{R+nr}$$

If R is negligible as compared to 'nr', then

$$I = nE/nr = E/r$$

$$= \text{current of one cell}$$

Hence, in this case, there is no increase of the current by joining n cells in series.

If, however, 'nr' is negligible relative to the external or load resistance R, then

$$I = nE/R = n \times E/R$$

$$= n \times \text{current due to one cell}$$

In this case, we find that current delivered by the battery has increased n times.

Hence, we conclude that series grouping results in maximum current when internal resistance of battery is negligible as compared to the external resistance.

2-11. Parallel Grouping

In this method of grouping, anodes of all the cells are joined together to give the anode of the combination and all cathodes are joined together to give the cathode of the combination (see Fig. 2-6).

Here, battery e.m.f. is the same as the e.m.f. of each cell. Moreover, the equivalent resistance of n resistances each of value r

---

1. The total e.m.f. of n cells in series is n times the e.m.f. per cell and total internal resistance is n times the internal resistance per cell.

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all connected in parallel is r/n. This equivalent resistance is in series with the external resistance R.

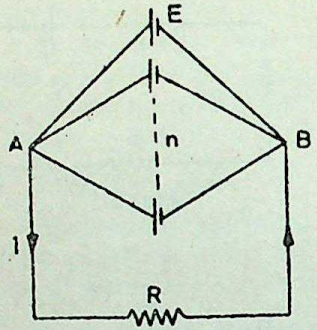

Fig. 2-6

Now, e.m.f. of the battery $= E$

Internal resistance of battery $= r/n$

Total circuit resistance $= R + \frac{r}{n}$

$\therefore$ Circuit current $I = \frac{E}{R + \frac{r}{n}}$

If 'r/n' is negligible relative to R, then $I = E/R$ *i.e.* equal to the current which we could have got even from one cell. Hence, under this condition, this type of grouping is not useful.

However, if R is negligible relative to r/n, then $I = nE/r$. This current is n times the current we could get from one cell.

Hence, this grouping is useful when external resistance is small as compared to the internal resistance of the battery.

2-12. Mixed Grouping

In this case, a few cells are connected in series and some such series groups are connected in parallel as shown in Fig. 2-7.

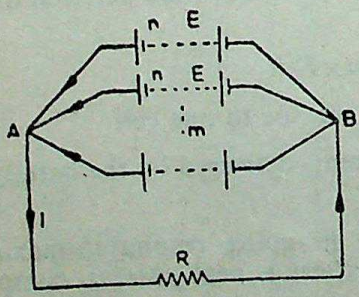

Fig. 2-7

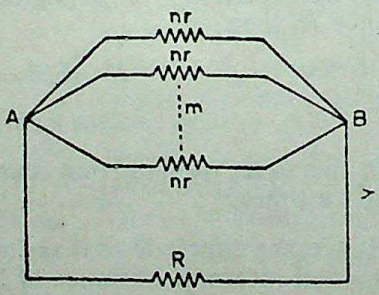

Fig. 2-8

Let the number of parallel rows be m and the number of cells, joined in series, in each row be n.

The internal resistance of this series-parallel group can be found thus (see Fig. 2-8).

Resistance of each row $= nr$

Equivalent resistance of such m rows connected in parallel

$= nr/m$

$\therefore$ total circuit resistance

$= R + \frac{nr}{m}$

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e.m.f. of the battery

$$= \text{e.m.f. of one row} = nE$$

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} = \frac{NE}{mR + nr}$$

where $N = m \times n =$ total number of given cells.

Now, numerator being constant, the circuit current will be maximum only when the denominator is minimum. The denominator will have minimum value[1] only when

$$mR = nr \quad \text{or} \quad R = \frac{nr}{m}$$

i.e. *external resistance = internal resistance of the battery.*

Hence, series-parallel combination will yield maximum current when external or load resistance equals internal resistance of the battery. It should, however, be noted that this arrangement has an electrical efficiency of 50% only, because internal and external resistances being equal, half the power developed is wasted within the battery itself.

Note. For finding a suitable arrangement for obtaining maximum current from a given number of cells, the following two equations should be used to find the number of parallel rows and the number of cells joined in series in each row.

(*i*) $mn = N$ and (*ii*) $mR = nr$

If m turns out to be unity or less, then it means that all the N cells should be joined in series.

If, on the other hand, n comes out to be unity, then all the cells should be joined in parallel.

2-13. Efficiency of a Cell

In general, the efficiency of any system may be defined by the relation,

$$\text{efficiency} = \frac{\text{output}}{\text{input}} \times 100 \text{ per cent}$$

Consider a cell having an e.m.f. of E, an internal resistance of r and delivering power to an external load resistance of R. If I is the circuit current, then useful power developed is I^2R watts and power lost within the cell is I^2r watts. Total power developed is $I^2R + I^2r = EI$ watts

$$\therefore \quad \eta = \frac{\text{useful power}}{\text{total power produced}}$$

$$= \frac{I^2R}{I^2R + I^2r} = \frac{R}{R + r}$$

1. It may be proved thus : Let it be equal to y.

Then $y = mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2 = (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mR}\sqrt{nr}$

Now, y will be minimum when right-hand side expression has a minimum value. The minimum value a squared quantity can have is zero.

$\therefore$ y is minimum when $(\sqrt{mR} - \sqrt{nr})^2 = 0$ or $mR = nr$.

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Theoretically, greater is the value of external resistance R, higher is the efficiency of the cell.

2-14. Maximum Power

Let us now consider how terminal potential difference V and the circuit current I vary with the external resistance R.

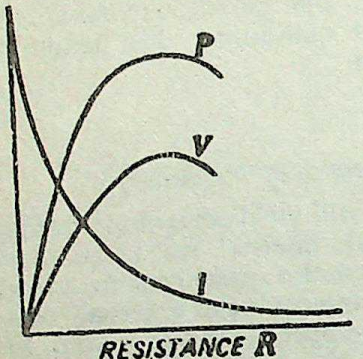

Fig. 2-9

Obviously, $V=IR$

Now $$I=\frac{E}{R+r}$$

$$\therefore \quad V=\frac{E}{R+r}R$$

$$=E\left(1-\frac{r}{R+r}\right)$$

As shown in Fig. 2-9, V increases as R increases. Similarly, $I=\frac{E}{R+r}$. As R increases, I decreases (Fig. 2-9). The useful power or power delivered to external resistance R is $P=VI$ watt.

By multiplying the ordinates for V and I, the third curve for power P can be drawn. It is seen from Fig. 2-9 that power P is maximum for a certain value of R. If external resistance is made more or less than this value, the power delivered is decreased.

Example 2-1. *A battery of 24 cells is required to be formed into a group in such a manner that it will send a maximum current through an external resistance of 5 Ω. Each cell has an e.m.f. of 2·1 V and an external resistance of 2 Ω. Find the best grouping and the value of the current through the external resistance.*

Solution. Let m=No. of rows in parallel

n=No. of cells in each row

then $$N=n\times m=24 \qquad ...(i)$$

Now, for maximum current

$$mR=nr$$

or $$m\times 5=n\times 2 \quad \text{or} \quad n=\frac{5m}{2} \qquad ...(ii)$$

Putting value of n from (ii) in (i), we get

$$\frac{5m^2}{2}=24 \quad \text{or} \quad m^2=\frac{48}{5}=9{\cdot}6$$

or $$m=3 \text{ (to the nearest whole number)}$$

$$\therefore \quad n=8$$

Hence, cells should be arranged in 3 rows each containing 8 cells.

Current through 5 Ω resistor $=\frac{2{\cdot}1\times 8}{5+16/3}=1{\cdot}63$ A (approx)

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Example 2-2. *Find the minimum number of cells connected in two rows in parallel required to pass a current of 6A through an external resistance of 0·7 Ω. Take the electromotive force of each cell as 2·1 volts and internal resistance of each cell as 0·5 Ω.*

Solution. Let 'n' be the number of cells in each of the two parallel rows.

E.M.F. of the battery $= 2{\cdot}1n$ volt

Internal resistance of the battery $= 0{\cdot}5n/2\ \Omega$

Total circuit resistance $= 0{\cdot}7 + (0{\cdot}5n/2)\ \Omega$

$$\text{Circuit current} = \frac{2{\cdot}1n}{0{\cdot}7 + (0{\cdot}5n/2)}$$

$$\therefore \quad 6 = \frac{2{\cdot}1n}{0{\cdot}7 + (0{\cdot}5n/2)} \qquad \therefore \quad n = 7$$

Hence, minimum number of cells is $= 2 \times 7 = \mathbf{14}$.

Example 2-3. (*a*) *Five batteries are connected in parallel to supply current to a load resistance for heating. Each battery consists of 12 cells connected in series. The e.m.f. per cell is 2 volts and the individual cell resistance is 0·15 Ω. What should be the resistance of the load so that power consumed in the load is 384 watts? For what load will the power consumption be maximum.*

(*b*) *Find the value of this maximum power.*

Solution. (*a*) Total e.m.f. of the group $= 12 \times 2 = 24$ V

Internal resistance of one row $= 12 \times 0{\cdot}15 = 1{\cdot}8\ \Omega$

Combined internal resistance of 5 rows $= \frac{12 \times 0{\cdot}15}{5} = 0{\cdot}36\ \Omega$

Let external resistance be $R\ \Omega$.

Then current $I = \frac{24}{R + 0{\cdot}36}$ A

Power consumption in $R = I^2R$ watts

$$= \left(\frac{24}{R + 0{\cdot}36}\right)^2 \times R \text{ watts}$$

$$\therefore \quad \left(\frac{24}{R + 0{\cdot}36}\right)^2 R = 384$$

or $$R^2 - 0{\cdot}78\,R + 0{\cdot}1296 = 0$$

or $$R = \frac{0{\cdot}78 \pm \sqrt{0{\cdot}78^2 - 4 \times 0{\cdot}1296}}{2}$$

$$= \frac{0{\cdot}78 \pm 0{\cdot}3}{2} = \mathbf{0{\cdot}54\ \Omega \text{ or } 0{\cdot}24\ \Omega}$$

Power consumption is maximum when

external resistance = internal resistance

i.e. $$R = \mathbf{0{\cdot}36\ \Omega}$$

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(*b*) In this case,

$$I=\frac{24}{0\cdot36+0\cdot36}=\frac{100}{3}\text{ A}$$

$\therefore$ maximum power consumed $=(100/3)^2\times0\cdot36=400$ W

Tutorial Problems—3

1. A battery of nine primary cells is connected

(*a*) all cells in series

(*b*) all cells in parallel

(*c*) three sets in parallel, each consisting of three cells in series.

Each cell has an e.m.f. of 1·4 V and an internal resistance of 0·45 Ω. The battery terminals are connected to a circuit of 7·2 Ω. Calculate in each case :

(*i*) the current in the 7·2 Ω resistance

(*ii*) the voltage drop across the resistance.

[(*a*) 1·12 A ; 8·07 V (*b*) 0·193 A ; 1·39 V (*c*) 0·549 A ; 3·95 V]

2. Thirty-six cells of 2 V/cell and of internal resistance 1·5 Ω/cell are given. Find how should they be joined as to send maximum current through an external resistance of 6 Ω. Find also the value of this current.

[3 rows of 12 cells each ; 2 A]

3. Eight cells are connected in series to an external resistance of 4 Ω but one of the cells is connected in opposition. If the e.m.f. and internal resistance/cell are 2 volts and 3 Ω respectively, find the current in the 4 ohm resistor. [3/7 A]

2-15. Division of Current in Parallel Circuits

In Fig. 2-10, two resistances are joined in parallel across a voltage V. The current in each branch, as given by Ohm's Law, is

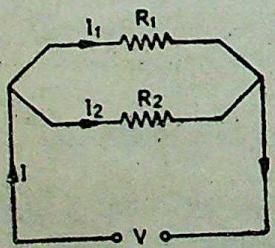

Fig. 2-10

$$I_1=V/R_1 \text{ and } I_2=V/R_2$$

$$\therefore \quad \frac{I_1}{I_2}=\frac{R_2}{R_1}$$

As $\quad \frac{1}{R_1}=G_1 \text{ and } \frac{1}{R_2}=G_2$

$$\therefore \quad \frac{I_1}{I_2}=\frac{G_1}{G_2}$$

Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current thus :

Now $\quad I_1+I_2=I \qquad \therefore \quad I_2=I-I_1$

Hence $\quad \frac{I_1}{I-I_1}=\frac{R_2}{R_1} \qquad \therefore \quad I_1R_1=R_2(I-I_1)$

or $\quad I_1=I\frac{R_2}{R_1+R_2}$

Similarly, $\quad I_2=I\frac{R_1}{R_1+R_2}$

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Take the case of three resistors connected in parallel across a voltage V (Fig. 2-11). The total current $I=I_1+I_2+I_3$. Let the equivalent resistance be R. Then

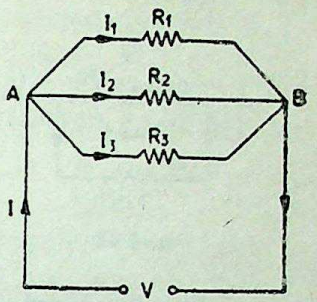

Fig. 2-11

$$V=IR$$

Also $$V=I_1R_1$$

$$\therefore \quad IR=I_1R_1$$

or $$\frac{I}{I_1}=\frac{R_1}{R}$$

or $$I_1=IR/R_1$$

Now $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

or $$R=\frac{R_1R_2R_3}{R_2R_3+R_3R_1+R_1R_2}$$

$\therefore$ From (*i*) above, $$I_1=I\left(\frac{R_2R_3}{R_1R_2+R_2R_3+R_3R_1}\right)$$

Similarly $$I_2=I\left(\frac{R_1R_3}{R_1R_2+R_2R_3+R_3R_1}\right)$$

and $$I_3=I\left(\frac{R_1R_2}{R_1R_2+R_2R_3+R_3R_1}\right)$$

Example 2-4. *An electric current of 5 A is divided into three branches, the lengths of the wires in the three branches being proportional to 1, 2 and 3. Find the current in each branch (the wires are of the same material and cross-section).*

Solution. Since resistance is directly proportional to the length of a wire, the resistances of three branches are in the ratio 1 : 2 : 3. Let

$$R_1=1\ \Omega, \text{ then } R_2=2\ \Omega \text{ and } R_3=3\ \Omega$$

As seen from Art. 2-15

$$I_1=I\left(\frac{R_2R_3}{R_1R_2+R_2R_3+R_3R_1}\right)$$

Now $$R_1R_2+R_2R_3+R_3R_1=1\times2+2\times3+3\times1=11$$

$$\therefore \quad I_1=5\times\frac{2\times3}{11}=\frac{30}{11}\text{A}$$

Similarly $$I_2=5\times\frac{3\times1}{11}=\frac{15}{11}\text{A}$$

$$I_3=5\times\frac{2}{11}=\frac{10}{11}\text{A}$$

2-16. Theory of Shunt

An instrument which detects electric current (by utilizing one of its effects) is known as a galvanometer. It is used for not only detecting the presence of current in any circuit but can also be used for measuring its relative strength and direction. The same galvanometer can be either used for measuring current strength *i.e.* as an ammeter or for measuring potential difference between two points *i.e.* as a voltmeter.

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Galvanometer itself can usually carry small currents. If the current is too large, then only a part of the total current should be passed through the galvanometer, the remainder passing through a small resistance S connected across the galvanometer as shown in Fig. 2-12. The low resistance S provides a 'bypath' for the current and is called a shunt and the galvanometer is then said to be 'shunted'. Shunted galvanometers can be made to record currents many times greater than their normal full-scale deflection currents. The ratio of the maximum current (with shunt) to the full-scale deflection current (without shunt) is known as the 'multiplying power' or 'multiplying factor' of the shunt.

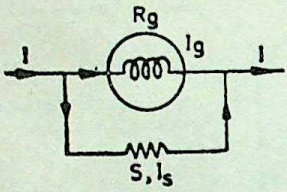

Fig. 2-12

Let

R_g = galvanometer resistance
S = shunt resistance
I = line current
I_g = full-scale deflection current of the galvanometer

As seen from Fig. 2-12, the voltage across the galvanometer and the shunt resistance is the same as both are joined in parallel.

$$\therefore \quad R_g \times I_g = S \times (I - I_g)$$

because current through shunt is $I_s = (I - I_g)$

$$\therefore \quad S = \frac{R_g I_g}{I - I_g}$$

Also $$\frac{I}{I_g} = \left(1 + \frac{R_g}{S}\right)$$

$$\therefore \text{ multiplying power of shunt} = \left(1 + \frac{R_g}{S}\right)$$

Obviously, lower the value of shunt, greater is its multiplying power.

2-17. Ammeter Shunt

The range of ammeters can also be extended by suitably adjusting the value of shunt resistance.

Example 2-5. *Calculate the resistance of the shunt which should be connected across a galvanometer of resistance 2100 Ω so that only 5% of the total current passes through it.*

Solution. Shunted galvanometer is shown in Fig. 2-13.

Let the line current be 100 A.

Then galvanometer current $= 5$ A

Current through shunt $= 100 - 5$

$= 95$ A

$$\therefore \quad 2100 \times 5 = S \times 95$$

$$S = 110.55\ \Omega$$

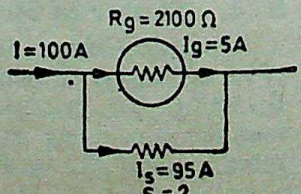

Fig. 2-13

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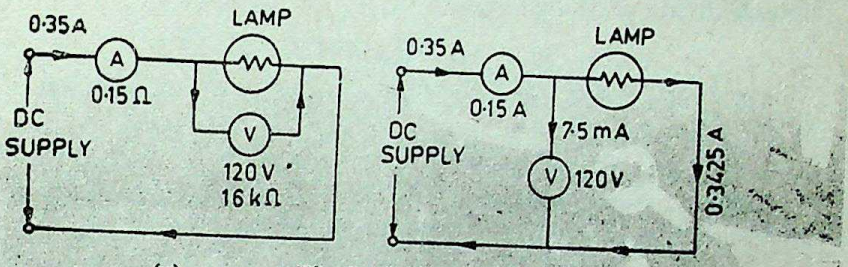
Example 2-6. *In the circuit of Fig. 2-14 (a), the resistance of ammeter A is 0·15 Ω and that of voltmeter V is 16 kΩ. The voltmeter reads 120 V and ammeter 0·35 A. What is (i) the apparent power read by the meters and (ii) actual power taken by the lamp.*

(A.M.I.E. Summer 1985)

Solution. The given circuit can be re-drawn as shown in Fig. 2-14 (*b*).



(*a*) Fig. 2-14 (*b*)

Current drawn by voltmeter = 120/16 K = 7·5 mA

(*i*) apparent power read by the instruments = 120 × 0·35 = **42 W**

(*ii*) current actually passing through the lamp

= 0·35 — 0·0075 = 0·3425 A

Actual power drawn by the 40-W lamp = 120 × 0·3425 = **41 W**

2-18. Simple Potentiometer

A simple potentiometer or as it is sometimes called, a potential divider is used for obtaining a variable voltage from a constant voltage supply. The whole of supply voltage is dropped across a resistance *AB* as shown in Fig. 2-15 and by changing the position of the sliding contact *C* over the potentiometer resistance *AB*, any voltage from zero upto that of the supply can be obtained for applying it across the load.

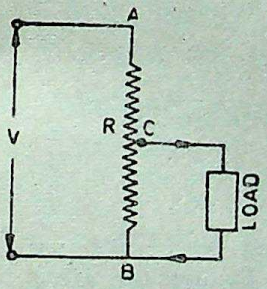


Fig. 2-15

Example 2-7. *A potentiometer of resistance 80 Ω is connected across a supply of 120 volts. A current of 2A is required in the 10 Ω resistor. Find the position of the tapping point C and calculate the resistance between points A and C.*

Solution. Let the resistance of potentiometer between points *C* and *B* = *R* Ω as shown in Fig. 2-16. The point *C* is such that for that p.d. across the coil, a current of 2 A passes through it. Now, p.d. across coil = 10 × 2 = 20 V. Same is the p.d. across resistance *R*. Hence, current through *CB* = 20/*R* amperes.

∴ Total current entering at point *A* = 2 + (20/*R*) amperes

This current while passing through portion *AC* produces a voltage drop of $(80 - R)\left(2 + \frac{20}{R}\right)$

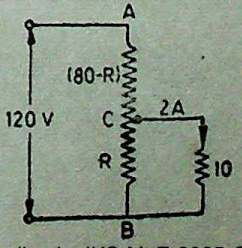


Fig. 2-16

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However, as voltage across $CB = 20\ V$, voltage drop across $AC = 120 - 20 = 100$ V.

$$\therefore \quad 100 = (80 - R)(2 + 20/R)$$

or $$R^2 - 20R - 800 = 0$$

or $$(R - 40)(R + 20) = 0 \quad \therefore \quad R = 40\ \Omega$$

Hence, the point C is situated at the middle point of the potentiometer resistance AB.

HIGHLIGHTS

1. The e.m.f. (E) of a cell is connected to its terminal p.d. V by the relation

$$E = V + ir.$$

2. A given number of cells may be joined (*i*) in series (*ii*) in parallel and (*iii*) in series-parallel combination.

3. For finding the best arrangement, the following two equations may be used :

(*i*) $N = m \times n$

(*ii*) $mR = nr$.

4. In parallel combination of various resistances, the circuit current divides inversely in the ratio of the branch resistances.

(*a*)
$$I_1 = I \frac{R_2}{R_1 + R_2}$$
$$I_2 = I \frac{R_1}{R_1 + R_2}$$
} for 2-branched circuit

(*b*)
$$I_1 = I \frac{R_2 R_3}{\Sigma R_1 R_2}$$
$$I_2 = I \frac{R_3 R_1}{\Sigma R_1 R_2}$$
$$I_3 = I \frac{R_1 R_2}{\Sigma R_1 R_2}$$
} for 3-branched circuit

5. Shunt is a low resistance connected in parallel with a galvanometer for increasing its current range. Smaller the value of shunt resistance, greater is the extension in its range.

6. A simple potentiometer or potential divider is a suitable resistance from which any percentage of the applied voltage can be obtained.

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NETWORK ANALYSIS 3

1. General 2. Kirchhoff's Laws 3. Determination of Sign 4. Assumed Directions of Currents 5. Maxwell's Loop Current Method or Theorem 6. Thevenin's Theorem 7. Delta/Star Transformation 8. Star/Delta Transformation—Highlights.

3-1. General

Many laws and theorems are available for solving networks of conductors both active and passive. The main advantage of these theorems is that they save time and considerably reduce the laborious mathematical work involved in solving networks thereby minimising chances of error. The different laws and theorems discussed in the text are :

(*i*) Kirchhoff's Laws (*ii*) Maxwell's Loop Current Theorem (*iii*) Thevenin's Theorem and (*iv*) Delta/Star Transformation.

3-2. Kirchhoff's Laws[1] (*A.M.I.E. Winter 1986*)

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter. The two laws are :

(1) Kirchhoff's First Law or Point Law or Current Law

It states that :

in any network of conductors, the algebraic sum of the currents meeting at a point (or junction) is zero.

Put in another way, it simply means that the total current *leaving* a junction is equal to the total current *entering* the junction. It is obviously true because there is no accumulation or depletion of current at any junction of the network.

1. After Gustav Robert Kirchhoff (1824—1887), a German physicist. In addition to the above laws, he solved many difficult problems connected with the laws of induced currents, magnetic distribution and electrical conduction in thin plates and in curved sheets etc. He originated the laws known after his name about the emissive and the absorptive powers of a radiating body. He made notable contributions in the field of spectrum analysis.

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Explanation

Consider the case of a few conductors meeting at a point A as in Fig. 3-1. Some conductors have currents leading to point A whereas others have currents leading away from point A. Assuming the incoming currents to be positive and the outgoing currents negative, we have

$$I_1+(-I_2)+(-I_3)+I_4+(-I_5)=0$$

or $$I_1+I_4-I_2-I_3-I_5=0$$

or $$I_1+I_4=I_2+I_3+I_5$$

or **Incoming currents = Outgoing currents**

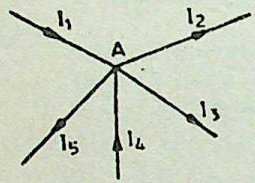

Fig. 3-1

We can express the above conclusion thus

$\Sigma I=0$ .....at a junction

(2) **Kirchhoff's Second Law or Mesh Law or Voltage Law**

It states that :

the algebraic sum of the products of current and resistance in each of the conductors in any closed mesh (or path) in a network plus the algebraic sum of the e.m.fs. in that path is zero.

In other words,

$\Sigma IR+\Sigma e.m.f.=0$ .....round a mesh

The basis of this law is this : If one starts from a particular junction and goes round the mesh till one comes back to the starting point, then one must be at the same potential with which one started. Hence, it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

3-2. Determination of Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs., otherwise results will come out to be wrong. Following sign convention is suggested :

A *rise* in voltage should be given a +ve sign and a *fall* in voltage a —ve sign. Keeping this in mind, it is clear that as we go from the —ve terminal of a battery to its +ve terminal, there is a *rise* in potential, hence this voltage should be given a +ve sign. If, on the other hand, we go from +ve terminal to —ve terminal, then there is a *fall* in potential, hence this voltage should be preceded by a —ve sign. *It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.*

Now, take the case of a resistor. If we go through a resistor in the *same* direction as the current, then there is a *fall* in potential because current flows from higher to lower potential. Hence, this voltage fall should be taken —ve. However, if we go in a direction *opposite* to that of the current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

It is clear that the sign of voltage drop across a resistor depends on the direction of current through that resistor.

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Consider the closed path *ABCDA* in Fig. 3-2. Different voltage drops will have the following signs :

I_1R_1 is —ve (fall in potential)
I_2R_2 is —ve ,, ,, ,,
I_3R_3 is —ve ,, ,, ,,
I_4R_4 is +ve (rise in potential)
E_2 is —ve (fall in potential)
E_1 is +ve (rise in potential)

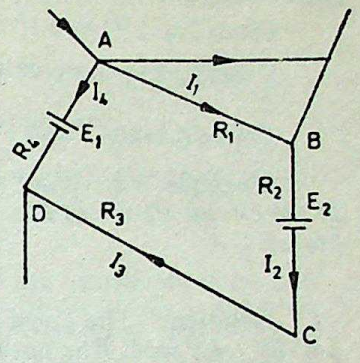

Fig. 3-2

Using Kirchhoff's Second Law, we get

$$-I_1R_1-I_2R_2-I_3R_3+I_4R_4-E_2+E_1=0$$
$$I_1R_1+I_2R_2+I_3R_3-I_4R_4=E_1-E_2$$

3-4. Assumed Direction of Current

In applying Kirchhoff's laws to electrical networks, the question of assuming proper direction of current usually arises. The direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, this current will be found to have a minus sign. If the answer is positive, then assumed direction is the same as actual direction (see examples 3-1 and 3-5). *However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.*

Note. It should be noted that Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account.

Example 3-1. *A Wheatstone bridge ABCD is arranged as follows :*

Resistances between A-B, B-C, C-D, D-A, and B-D are 10, 2, 8, 4 and 5 ohm respectively. A 100-V supply is connected between terminals A and C. Determine the currents in branches AB and AD of the circuit and the total current taken from the supply.
(A.M.I.E. Summer 1977)

Solution. The directions of the various currents are as shown in Fig. 3-3. Applying Kirchhoff's Second Law to various closed circuits, we have

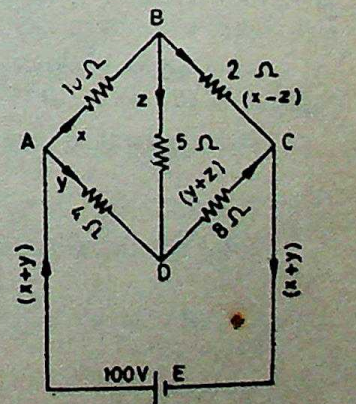

Fig. 3-3

Circuit ABDA

$$-10x-5z+4y=0$$
or $$10x-4y+5z=0 \quad \ldots(i)$$

Circuit BCDB

$$-2(x-z)+8(y+z)+5z=0$$
or $$2x-8y-15z=0 \quad \ldots(ii)$$

Circuit ADCEA

$$-4y-8(y+z)+100=0$$
or $$3y+2z=25 \quad \ldots(iii)$$

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Multiplying Eq. (*ii*) by 5 and subtracting it from Eq. (*i*), we get $5y+2z=0$ ...(*iv*)

From Eq. (*iii*) and (*iv*), we get, $y=250/21$ A, $z=-75/14$A

Substituting these values in Eq. (*i*), we have, $x=625/84$ A

Current taken from supply $= \frac{625}{84} + \frac{250}{21} = \frac{1625}{84}$ A

Example 3-2. *Find the value of R and the current through it in the circuit shown in Fig. 3-4 (a) below when the current is zero in branch OA.*

(A.M.I.E. Winter 1979)

Solution. The given circuit can be redrawn as shown in Fig. 3-4 (*b*). As seen, it is nothing else but a Wheatstone bridge circuit.

Since there is no current in branch *OA*, it means that the given bridge is balanced.

Under balanced conditions, the products of the resistances of the opposite arms of such a bridge are equal (Art. 15-47).

$$\therefore \qquad R \times 1 = 4 \times 1{\cdot}5$$

or $$R = 6\ \Omega$$

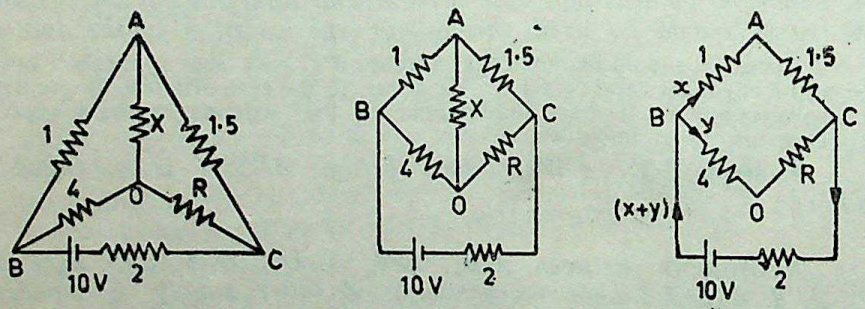

Fig. 3-4

It is given that there is no current in the branch *OA*. In that case, the resistance *X* of the branch *OA* can be removed as shown in Fig. 3-4(*c*) without in any way electrically affecting the circuit.

Now, there are two parallel paths between points *B* and *C* of resistances $(6+4) = 10\ \Omega$ and $(1+1{\cdot}5) = 2{\cdot}5\ \Omega$.

Total resistance between points *B* and *C* is

$$= \frac{10 \times 2{\cdot}5}{(10+2{\cdot}5)} = 2\ \Omega$$

Total circuit resistance

$$= 2+2 = 4\ \Omega$$

Circuit current $= 10/4 = 2{\cdot}5$ A

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This current divides into two parts at point B, one part going along BAC and the other along BOC.

Current going along path BOC is

$$y = 2{\cdot}5 \times \frac{2{\cdot}5}{12{\cdot}5} = 0{\cdot}5 \text{ A}$$

The same current, obviously, passes through the given resistance R.

Example 3-3. *Two batteries A and B are connected in parallel and a load of 10 Ω is connected across their terminals. A has an e.m. f. of 12 V and internal resistance of 2 Ω ; B has an e.m. f of 8V and an internal resistance of 1 Ω. Using Kirchhoff's laws, determine the values and directions of the currents flowing in each of the batteries and in the external resistance. Also determine the potential difference across the external resistance.*

(A.M.I.E. Winter 1980)

Solution. The two batteries are connected in parallel as shown in Fig. 3-5. It should be noted that unless stated otherwise, *the similar ends of the two batteries are assumed to be joined together.* Let the directions of the two branch currents be as shown.

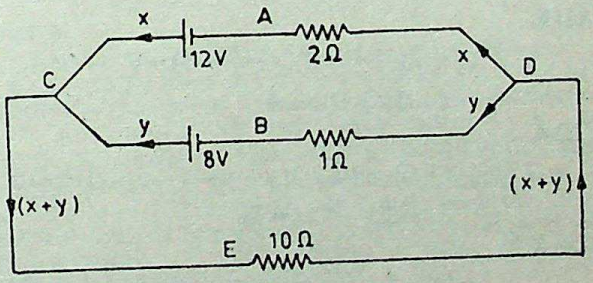

Fig. 3-5

Applying Kirchhoff's laws to the two loops, we get the following equations :—

Loop CEDAC. Starting from point C anticlockwise, we get

$-10(x+y)-2x+12=0$ or $6x+5y=6$ —(*i*)

Loop CEDBC. Starting from point C, we get

$-10(x+y)-1\times y+8=0$ or $10x+11y=8$ ...(*ii*)

Solving for x and y from (*i*) and (*ii*), we get

$x=13/8=1{\cdot}625$ A (**discharge**), $y=-0{\cdot}75$ A (**charge**)

Since y turns out to be *negative*, its actual direction of flow is opposite to that shown in Fig. 3-5. It is a *charging* current and not discharging one.

Current in 10 Ω resistor $=(1{\cdot}625-0{\cdot}75)=0$ 875 A

P.D. across external resistor $=10\times 0{\cdot}875=8{\cdot}75$ V.

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Example 3-4. *Determine the current x in the 4-ohm resistance in the circuit shown in Fig. 3-6 (a) below.*

(A.M.I.E. Winter 1975)

Solution. The assumed distribution of current is shown in Fig. 3-6 (*b*). Applying Kirchhoff's laws to different closed loops, we get

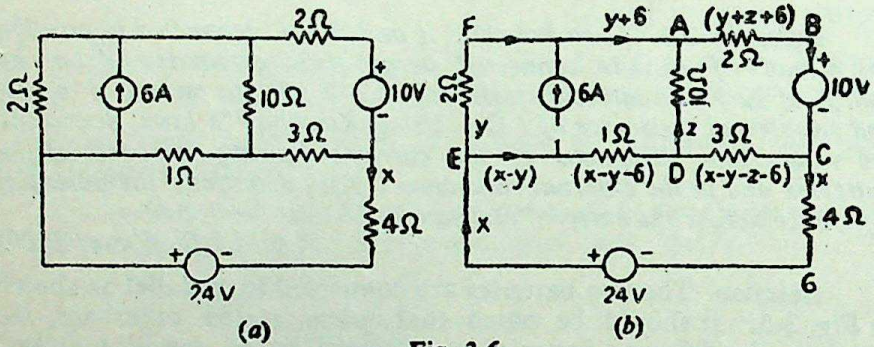

Fig. 3-6

Circuit E FADE

$$-2y+10z+(x-y-6)=0$$

or $$x-3y+10z=6 \quad \ldots(i)$$

Circuit ABCDA

$$-2(y+z+6)-10+3(x-y-z-6)-10z=0$$

or $$3x-5y-15z=40 \quad \ldots(ii)$$

Circuit EDCGE

$$-(x-y-6)-3(x-y-z-6)-4x+24=0$$

or $$8x-4y-3z=48 \quad \ldots(iii)$$

Multiplying Eq. (*i*) by 5 and Eq. (*ii*) by 3 and then subtracting Eq. (*ii*) from Eq (*i*), we get

$$-4x+95z=-90$$

$$4x-95z=90 \quad \ldots(iv)$$

Next, multiplying Eq. (*ii*) by 4 and Eq. (*iii*) by 5 and subtracting Eq. (*iii*) from Eq. (*ii*), we get

$$-28x-45z=-80$$

or $$28x+45z=80 \quad \ldots(v)$$

Multiplying Eq. (*iv*) by 45 and Eq. (*v*) by 95 and adding the two, we get

$$284x=1165 \quad \text{or} \quad x=1165/284=4{\cdot}1 \text{ A}$$

Example 3-5. *Formulate the Kirchhoff voltage law equations for the circuit of Fig. 3-7 and find the values of* I_1, I_2, I_3.

(A.M.I.E. Winter 1984)

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Solution. Circuit ABFGHA

$$-I_1 \times 1 - 3(I_1+I_2) - 6(I_1+I_2+I_3) - 0{\cdot}5\ I_1 + 12 = 0$$

$$\therefore \quad 7I_1 + 6I_2 + 4I_3 = 8 \qquad \ldots(i)$$

Circuit BCDFB. Starting from point B, we get

$$+0{\cdot}4I_2 - 10 - 1{\cdot}5I_3 + 3(I_1+I_2) = 0$$

$$\therefore \quad 30I_1 + 34I_2 - 15I_3 = 100 \qquad \ldots(ii)$$

Circuit FDEGF

$$+0{\cdot}3(I_2+I_3) - 6 + 6(I_1+I_2+I_3) + 1{\cdot}5I_3 = 0$$

$$\therefore \quad 20I_1 + 21I_2 + 26I_3 = 20 \qquad \ldots(iii)$$

Solving for the three currents, we get

$I_1 = -1{\cdot}25$ A ;

$I_2 = 3{\cdot}54$ A

$I_3 = -1{\cdot}13$ A

The negative signs mean that actual directions of flow of I_1 and I_3 are opposite to those shown in Fig. 3-7.

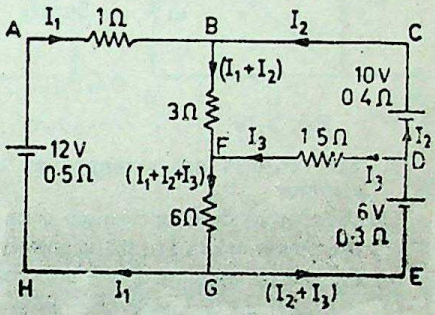

Fig. 3-7

Tutorial Problems-4

1. A Wheatstone bridge $ABCD$ is arranged as follows : resistances between $A-B$, $B-C$, $C-D$, $D-A$ and $B-D$ are 10, 20, 15, 5 and 40 Ω respectively, A 20−V battery of negligible internal resistance is connected between terminals A and C. Determine the current in each resistor,

[AB = 0·645 A ; BC = 0·678 A ; AD = 1·025 A ; DC = 0·992 A ; DB = 0·033 A]

2. Two batteries A and B are connected in parallel. The e.m.f. and internal resistance of A are 110 V and 6 Ω respectively and the corresponding values for B are 130 V and 4 Ω respectively. A resistance of 20 Ω is connected across the battery terminals. Calculate (*a*) the terminal voltage and (*b*) the value and direction of the current in each battery.

[(*a*) 108·9 V (*b*) 0·1785 A discharge ; 5·27 A discharge]

3. Two batteries A and B are connected in parallel and a load of 10 Ω is connected across their terminals. A has an e.m.f. of 12 V and an internal resistance of 2 Ω ; B has an e.m.f. of 8 V and an internal resistance of 1 Ω. Using Kirchhoff's laws, determine the values and directions of currents flowing in each battery and the external resistance. Also find the p.d. across the load resistance. **[I_A = 1·625 A ; I_B = −0·75 A ; I = 0·875 A ; 8·75 V]**

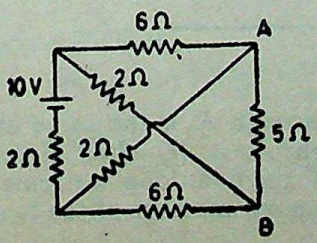

Fig. 3-8

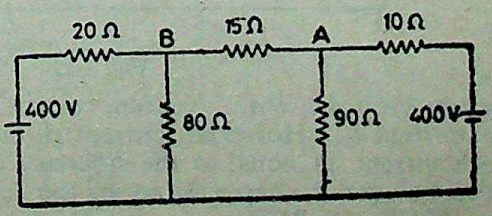

Fig. 3-9

4. Using Kirchhoff's laws, calculate the voltage across AB in Fig. 3-8 and indicate the polarity of this voltage. **[point B is 2 V above point A]**

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5. Using Kirchhoff's laws, find the magnitude and direction of current flowing through the 15-Ω resistance of Fig. 3-9.

[1 A from A to B]

6. A battery of 20 cells, all connected in series, is connected across the terminals *AB* of the circuit shown in Fig. 3-10. If each cell has an open-circuit e.m.f. of 2 V and an internal resistance of 0·08 Ω, calculate the current in each branch of the circuit and the potential difference between (*i*) *AB*, (*ii*) *CB*. **[10 A ; 6 A ; 4 A ; 3·6 A ; 2·4 A ; 24 V, 14·4 V]**

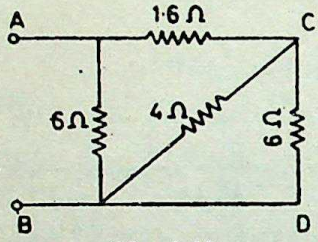

Fig. 3-10

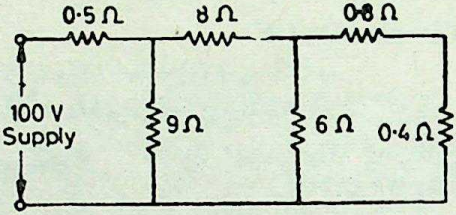

Fig. 3-11

7. Six resistors are arranged as shown in Fig. 3-11 across a 100-V supply. Calculate

(*a*) the current flowing through each resistor

(*b*) the power taken from the supply

(*c*) the energy consumed in ½ hour in joules

(*d*) the cost of running the system for 24 hours if energy costs 10 paise/kWh.

[(*a*) 20 A ; 10 A ; 10 A ; 1·67 A ; 8·33 A (*b*) 2 kW (*c*) 36×10^5 J (*d*) Rs. 4/80]

3-5. Maxwell's Loop Current Method or Theorem

In this method, instead of taking *branch* currents (as in Kirchhoff's law) *loop* currents are taken which are assumed to flow in the *clockwise* direction. Of course, where required branch currents can be found in terms of the loop currents. Sign conventions for the *IR* drops and battery e.m.fs. are the same as for Kirchhoff's law.

Consider the three-looped circuit of Fig. 3-12 where loop currents of I_1 and I_2 and I_3 have been assumed. Take the closed

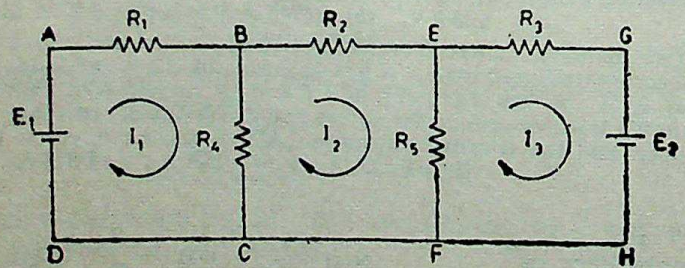

Fig. 3-12

circuit or loop *ABCDA*. As seen, current through R_1 is the same as loop current I_1. However, current through R_4 is (I_1-I_2). Here, branch current is equal to the *difference* of the two loop currents. It is so because R_4 happens to be the common member of the two loops *ABCD* and *BEFC*.

Consider the second loop *BEFCB*. Current through R_2 is I_2. But current through R_5 is (I_2-I_3) and that through R_4 is (I_2-I_1)

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and *not* (I_1-I_2) *as was the case when we went round* the first loop a little earlier.*

Similarly, current through R_3 is I_3 and that through R_5 is (I_3-I_2) and *not* (I_2-I_3) *as was the case for the second loop.*

The voltage equations for the three loops are as under :

Loop ABCDA

Starting from point A, we have

$$-I_1R_1-R_4(I_1-I_2)+E_1=0$$

Simplifying it further, we have

$$I_1(R_1+R_4)-I_2R_4=E_1 \qquad ...(i)$$

Loop BEFCB

Starting from point B, we have

$$-I_2R_2-R_5(I_2-I_3)-R_4(I_2-I_1)=0$$

or $$I_1R_4-I_2(R_2+R_4+R_5)+I_3R_5=0 \qquad ...(ii)$$

Loop EGHFE

Starting from point E, we get

$$-I_3R_3-E_3-R_5(I_3-I_2)=0$$

or $$I_2R_5-I_3(R_3+R_5)=E_2 \qquad ...(iii)$$

The three equations given above may be used for finding the values and directions of flow of loop currents I_1, I_2 and I_3.

Example 3-6. *Find I_1, I_2 and I_3 in the network shown in Fig. 3-13, using loop-current method. Figures againstresistances indicate their values in ohms.*

Solution. Different loops would be taken one after another.

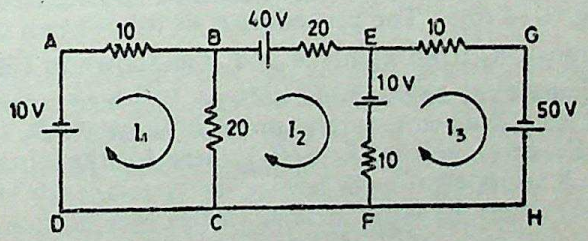

Fig. 3-13

Loop ABCDA

$$-10I_1-20(I_1-I_2)-10=0$$

or $$3I_1-2I_2=-1 \qquad ...(i)$$

Loop BEFCB

$$40-20I_2+10-10(I_2-I_3)-20(I_2-I_1)=0$$

or $$2I_1-5I_2+I_3=-5 \qquad ...(ii)$$

Loop EGHFE

$$-10I_3+50-10(I_2-I_3)-10=0$$

or $$I_2-2I_3=-4 \qquad ...(iii)$$

*The general rule for finding branch current through a common member is this : while going round a loop along with the loop current, always subtract the other opposing loop current from the loop current you are going along with.

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Multiplying Eq. (*ii*) by 2 and adding it to Eq. (*iii*), we get

$$4I_1 - 9I_2 = -14 \qquad \ldots(iv)$$

Solving for I_1 and I_2 from Eq. (*i*) and (*iv*), we get

$$I_1 = 1\text{A} \quad \text{and} \quad I_2 = 2\text{A}$$

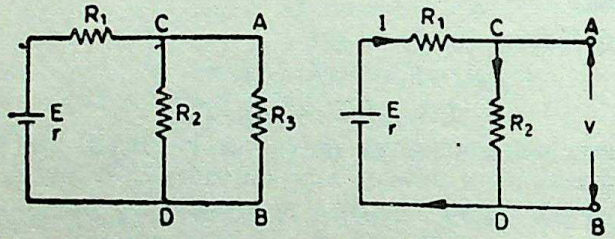
Substituting these values in Eq. (*iii*), we have, $I_3 = 3\text{A}$

3-6. Thevenin's Theorem

In this method, the entire network (excluding the branch resistance through which current is required) is reduced to a single source (known as Thevenin's source) whose

(*i*) e.m.f. is equal to the open-circuit voltage V between the two points from where the resistance under consideration is removed.

(*ii*) whose internal resistance is equal to the equivalent circuit resistance R as viewed from these two points.



(*a*) (*b*)

Fig. 3-14

Suppose it is required to find current flowing through R_3 as shown in Fig. 3-14 (*a*). The procedure is as follows :

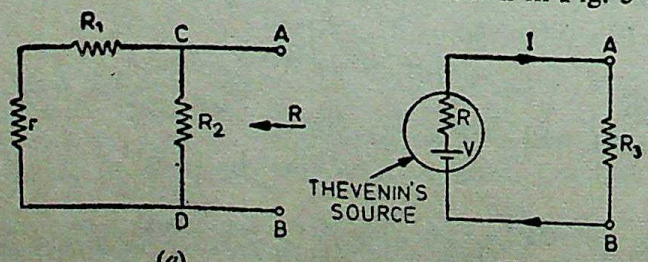
(*i*) Remove R_3 from points A and B as shown in Fig. 3-14 (*b*).

(*ii*) Calculate the open-circuit voltage V which appears across terminals A and B when they are opened *i.e.* when R_3 is removed. Obviously, V = drop across $R_2 = IR_2$ where I is the circuit current when A and B are open.

$$I = \frac{E}{R_1 + R_2 + r}$$

$$\therefore \quad V = IR_2 = \frac{ER_2}{r + R_1 + R_2}$$

(*iii*) Now, imagine the battery to be removed from the circuit leaving its internal resistance r behind as shown in Fig. 3-15 (*a*).



(*a*) (*b*)

Fig. 3-15

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When viewed inwards from points A and B, the circuit consists of two parallel paths : one containing resistance R_2 and the other containing two resistances R_1 and r in series. The equivalent resistance of the circuit as viewed from terminals A and B is given by

$$R = \frac{R_2(R_1+r)}{R_2+(R_1+r)}$$

Consequently, as seen from points A and B, the whole network can be reduced to a single source (called Thevenin's source) whose e.m.f. equals V and whose internal resistance equals R as shown in Fig. 3-15 (*b*).

(*iv*) R_3 is now connected back across terminals A and B from where it was removed earlier. Current flowing through R_3 is given by

$$I = \frac{V}{R+R_3}$$

As seen, use of Thevenin's theorem involves finding of the following two things :

(*i*) open-circuit voltage V and

(*ii*) internal or equivalent resistance R of the network as viewed from open-circuited terminals A and B.

After this replacement of the whole network by a single source of e.m.f. and a single resistance has been accomplished, it is easy to find current in the load resistance joined across terminals A and B.

Hence, Thevenin's theorem as applicable to d.c. circuits may be stated as under :—

The current flowing through a load resistance R_L connected across any two terminals A and B of a network is given by $V/(R+R_L)$ where V is the open-circuit voltage (*i.e.* voltage across the two terminals when R_L is removed) and R is the internal or equivalent resistance of the network as viewed from terminals A and B with all voltage sources replaced by their internal resistances if any.

Example 3-7. *With reference to the network of Fig. 3-16, by applying Thevenin's theorem, find the following :*

(*i*) *the equivalent e.m.f. of the network when viewed from terminals A and B*

(*ii*) *the equivalent resistance of the network when looked into from terminals A and B*

(*iii*) *current in the load resistance* R_L *of 15* Ω.

Solution. (*i*) Current in the network after load resistance has been removed [Fig. 3-16 (*b*)] $= 24/(12+3+1) = 1{\cdot}5$ A.

$\therefore$ voltage across terminals $AB = V = 12 \times 1{\cdot}5 =$ **18 V**

Hence, so far as terminals A and B are concerned, the network has an e.m.f. of 18 volt (and not 24 V).

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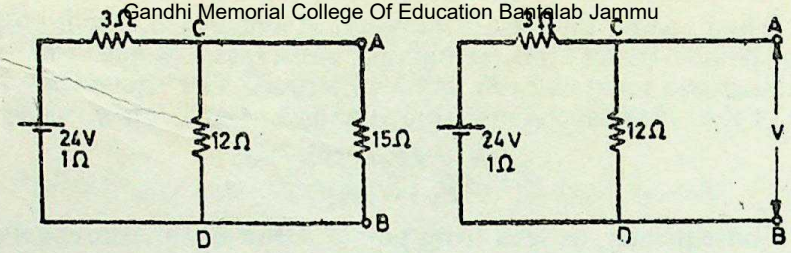
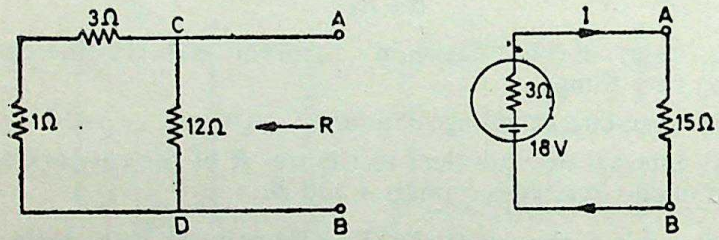


(*a*) Fig. 3-16 (*b*)

(*ii*) There are two parallel paths between points *A* and *B* [Fig. 3-17(*a*)]. Imagine that battery of 24 V is removed but not its internal resistance. Then equivalent resistance of the circuit as looked into from points *A* and *B* is

$$R=\frac{12\times 4}{12+4}=3\ \Omega$$



(*a*) Fig. 3-17 (*b*)

(*iii*) When load resistance of 15 Ω is connected across the terminals, then the network is reduced to the structure shown in Fig. 3-17 (*b*).

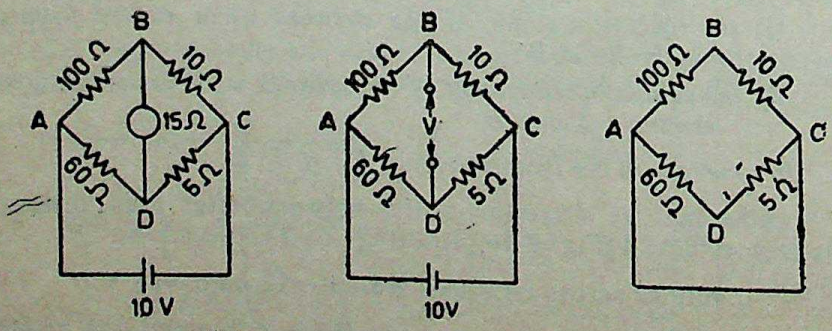
$$\therefore \qquad I=18/(15+3)=1\ \text{A}.$$

Example 3-8. *The four arms of a Wheatstone bridge have the following resistances in ohms :*

$$AB=100\ ;\ BC=10\ ;\ CD=5\ ;\ DA=60$$

A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC. (*A.M.I.E. May 1971*)

Solution. (*i*) Remove the galvanometer connected between points *B* and *D* in Fig. 3-18 (*a*) thus obtaining Fig. 3-18 (*b*).



(*a*) (*b*) (*c*)

Fig. 3-18

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(*ii*) Let us determine the p.d. across points *B* and *D*. Remembering that in Fig. 3-18 (*b*), *ABC* and *ADC* are potential dividers on each of which a voltage drop of 10 V takes place, we have

drop across $AB = 10 \times 100/110 = 9{\cdot}09$ volt

drop across $AD = 10 \times 60/65 = 9{\cdot}23$ volt

p.d. between *B* and *D* is

$$V = 9{\cdot}23 - 9{\cdot}09 = 0{\cdot}14 \text{ volt}$$

(*iii*) Remove the battery from the circuit leaving its internal resistance behind which, in this case, is zero. Hence, it amounts to placing a short-circuit between points *A* and *C* as, shown in Fig. 3-18 (*c*).

(*iv*) Next, let us find the internal resistance of the whole network when viewed from points *B* and *D*. It can be found easily by noting that *BA* is in parallel with *BC* and *AD* is in parallel with *CD* as shown in Fig. 3-19 (*a*).

As seen from Fig. 3-19 (*c*), $R = 13{\cdot}7\ \Omega$

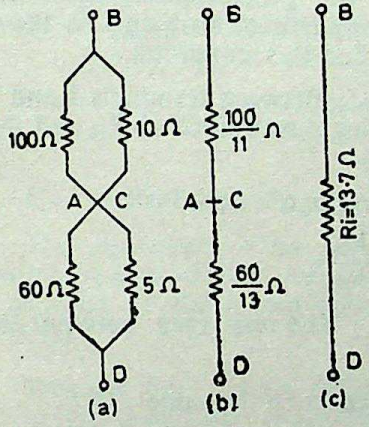

Fig. 3-19

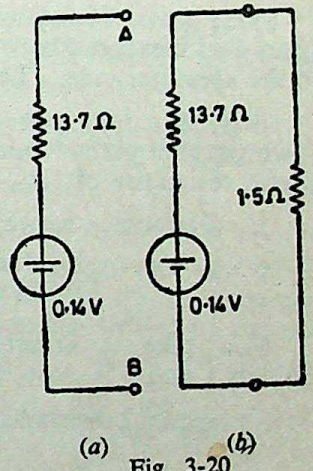

Fig. 3-20

(*v*) Now, so far as points *B* and *D* are concerned, the network has a source of e.m.f. 0·14 V and an equivalent resistance of 13·7 Ω. The Thevenin's source along with its internal resistance is shown in Fig. 3-20 (*a*).

(*vi*) Connect the 15-Ω galvanometer (initially removed) to this Thevenin's source and calculate the current flowing through it [Fig. 3-20 (*b*)].

$$I = \frac{0{\cdot}14}{15 + 13{\cdot}71} = 0{\cdot}00487 \text{ A} = 4{\cdot}87 \text{ mA}$$

3-7. Delta/Star Transformation

In solving networks (having considerable number of branches) by the application of Kirchhoff's laws or by Loop method, one sometimes experiences great difficulty due to a large number of simultaneous equations that have to be solved. However, such complicated networks can be simplified by successively replacing delta meshes by equivalent star systems and *vice versa*.

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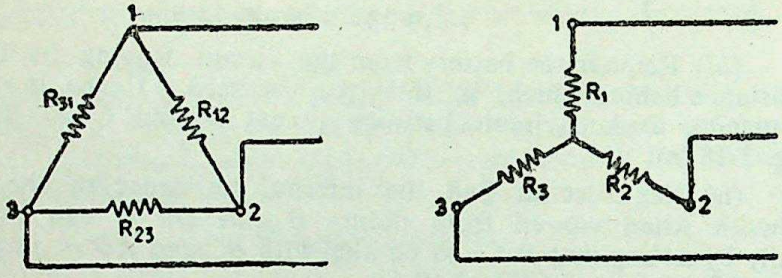
Suppose we are given three resistances R_{12}, R_{23} and R_{31} connected in delta fashion between terminals 1, 2 and 3 as shown in Fig. 3-21 (*a*). So far as the respective terminals are concerned, these three resistances can be replaced by the three resistances R_1, R_2 and R_3 connected in star as shown in Fig. 3-21 (*b*).



(*a*) Fig. 3-21 (*b*)

These two arrangements will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements. Let us find this condition.

First, take delta connection : Between terminals 1 and 2, there are two parallel paths : one having a resistance of R_{12} and the other having a resistance of $(R_{23}+R_{31})$.

$\therefore$ Resistance between terminals 1 and 2 is

$$\frac{R_{12}\times(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})}$$

Now, take star connection : The resistance between the same terminals 1 and 2 is (R_1+R_2).

As terminal resistances have to be the same

$$\therefore \quad R_1+R_2=\frac{R_{12}\times(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \qquad ...(i)$$

Similarly, for terminals 2 and 3 and terminals 3 and 1, we get

$$R_2+R_3=\frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} \qquad ...(ii)$$

and

$$R_3+R_1=\frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \qquad ...(iii)$$

Now, subtracting (*ii*) from (*i*) and adding the result to (*iii*), we get

$$R_1=\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_2=\frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$$

and

$$R_3=\frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

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How to remember ?

It is seen from above that each numerator is the product of the two sides of the delta which meet at the point in star terminals. Hence, it should be remembered that : *resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its ends divided by the sum of the three delta resistances.*

3-8. Star/Delta Transformation

This transformation can be easily done by using equations (*i*), (*ii*) and (*iii*) given above. Multiplying (*i*) and (*ii*), (*ii*) and (*iii*), (*iii*) and (*i*) and adding them together and then simplifying them, we get

$$R_{12}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_3}=R_1+R_2+\frac{R_1R_2}{R_3}$$

$$R_{23}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1}=R_2+R_3+\frac{R_2R_3}{R_1}$$

$$R_{31}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}=R_1+R_3+\frac{R_1R_3}{R_2}$$

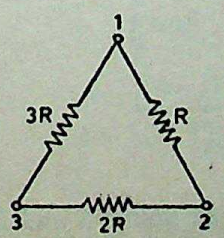
How to Remember ?

The equivalent delta resistance between any two terminals is given by the sum of star resistances between those terminals plus the product of these two star resistances divided by the third star resistance.

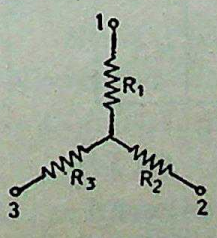
Example 3-9. *Three resistances R, 2R and 3R are connected in delta [Fig. 3-22 (a)]. Determine the resistances for an equivalent star connection.*

In Fig. 3-23, 160 volts are applied to the terminals AB. Determine the resistance (a) between the terminals A and B and (b) the current.

Solution. The three resistances are joined in delta in Fig. 3-22 (*a*).



(*a*)



(*b*)

Fig. 3-22

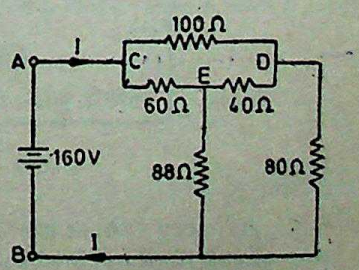


Fig. 3-23

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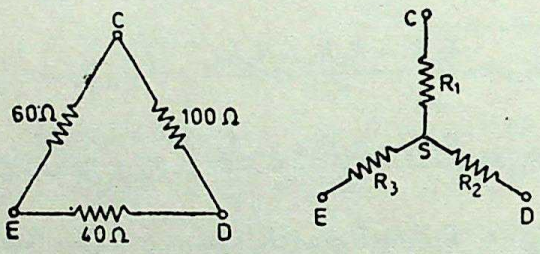
Remembering the rule given in Art. 3-7, we have in Fig. 3-22(*b*)

$$R_1 = \frac{R \times 3R}{R + 2R + 3R} = \frac{R}{2}$$

$$R_2 = R \times 2R/6R = \frac{R}{3}$$

$$R_3 = 2R \times 3R/6R = R$$

Take the network of Fig. 3-23. The three resistances of 100 Ω, 60 Ω and 40 Ω are delta connected between terminal points *C*, *D* and *E* as shown in Fig. 3-24 (*a*). They can be converted into equivalent star connection as shown in Fig. 3-24 (*b*).



( ) (*b*)

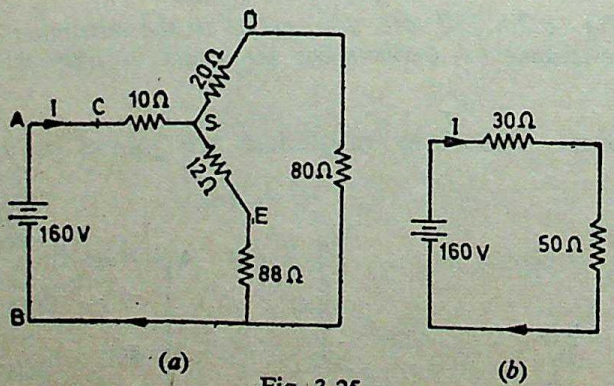
Fig. 3-24

$$R_1 = \frac{60 \times 100}{100 + 60 + 40} = 30\ \Omega$$

$$R_2 = 100 \times 40/200 = 20\ \Omega$$

$$R_3 \equiv 40 \times 60/200 = 12\ \Omega$$

Then the network of Fig. 3-23 is reduced to a simple structure of Fig. 3-25 (*a*)



(*a*) (*b*)

Fig. 3-25

As seen, there are two parallel paths between points *S* and *B*, one of resistance (20+80) = 100 Ω and the other also of (12+88) = 100 Ω. Hence, equivalent resistance between points *S* and *B*

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$= 100 \times 100/200 = 50\ \Omega$

The whole network is reduced to a simple circuit of Fig. 3-25 (*b*).

(*a*) $\therefore$ Resistance between points *A* and *B*

$= 30 + 50 = \mathbf{80\ \Omega}$

(*b*) Current $I = 160/80 = \mathbf{2\ A}$

Tutorial Problems—5

1 Using Thevenin's theorem, calculate the current flowing through 5 Ω resistance of Fig. 3-26. [2/3 A]

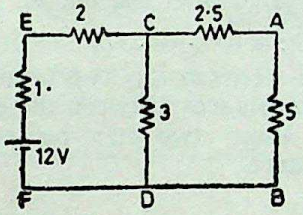

Fig. 3-26

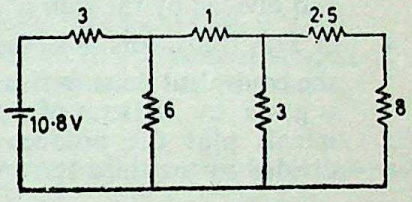

Fig. 3-27

2. With the help of Thevenin's theorem, find current through 8 Ω resistor of Fig. 3-27. [0·3 A]

3. Calculate the magnitude and direction of flow of current through the 12 ohm resistance of Fig. 3-28. Use Thevenin's theorem for the purpose. What would be the current if 100-V battery connections were reversed ? Also if 120-V battery is reversed ?

[2·25 A from A to B ; 9·75 A same direction ; 9·75 A from B to A]

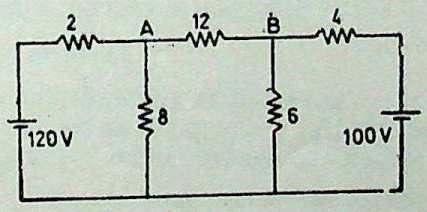

Fig. 3-28

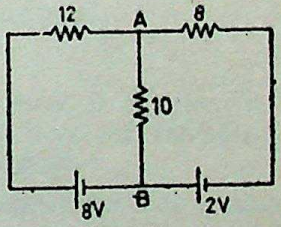

Fig. 3-29

4. Using Thevenin's theorem, find the current flowing through the 10-Ω resistor of Fig. 3-29. Also calculate the current when (*i*) 2-V battery connection is reversed and (*ii*) 8-V battery connection is reversed.

[0·135 A from A to B (*i*) 0·3 A from A to B (*ii*) 0·3 A from B to A]

HIGHLIGHTS

1. Two laws of Kirchhoff can be briefly stated as below :—

(*a*) Point Law or Current Law

$\Sigma I = 0$ —at a point or junction

(*b*) Mesh Law or Voltage Law

$\Sigma IR + \Sigma e.m.f. = 0$ —round a mesh.

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2. Thevenin's theorem may be stated as follows :

the current flowing through a load resistance R_L connected across any two terminals A and B of a network is given by $V/(R+R_L)$ where V is the open-circuit voltage and R is the internal resistance of the network as viewed back into the network from the terminals A and B with all voltage sources removed, leaving their internal resistance behind.

3. In $\triangle/Y$ transformation, the rule is :

resistance of each arm of the star is given by the product of the resistances of the two delta sides that meet at its end divided by the sum of the three delta resistances.

4. In $Y/\triangle$ transformation, the rule is :

the equivalent delta resistance between any two terminals is given by the sum of star resistances between these terminals plus the product of these two star resistances divided by the third star resistance.

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WORK, POWER AND ENERGY 4

1. Heating Effect of Electric Current 2. Unit of Heat 3. Joule's Law of Electric Heating 4. Thermal Efficiency 5. General Formula 6. Quantity of Electricity 7. Electric Power 8. Electric Energy 9. Some Units in M.K.S.A. System 10. Highlights.

4-1 Heating Effect of Electric Current

It is a matter of common experience that a conductor when carrying current, becomes hot after some time. As explained earlier, an electric current is just a directed flow or drift of electrons through a substance. The moving electrons as they pass *through* the molecules or atoms of that substance, collide with other electrons. This electronic collision results in the production of heat. This explains why passage of current is always accompanied by generation of heat. The heat so produced is measured in the following units.

4-2 Unit of Heat

The unit generally employed in scientific work is calorie. A calorie is defined as the quantity of heat that will raise the temperature of 1 gram of water through one degree centigrade.

For very accurate work, the particular degree has been specified as from 14·5°C to 15·5°C.

In M.K.S.A. system, as well as SI system, the unit of heat is kilocalorie (kcal) which is defined as the amount of heat required to heat one kg of water through 1°C or 1°K.

4-3 Joule's Law of Electric Heating

The amount of work required to maintain a current of I amperes through a resistance of R ohm (Fig. 4-1) for t seconds is

$$\text{W.D.} = I^2Rt \text{ joules}$$
$$= VIt \text{ joules} \qquad (\because \; R = V/I)$$
$$= Wt \text{ joules} \qquad (\because \; Watt = V \times I)$$
$$= \frac{V^2t}{R} \text{ joules} \qquad (\because \; I = V/R)$$

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This work is converted into heat and is dissipated away. Heat produced is

$$H = \frac{\text{work done}}{\text{mechanical equivalent of heat}}$$

$$= \frac{\text{W.D.}}{J}$$

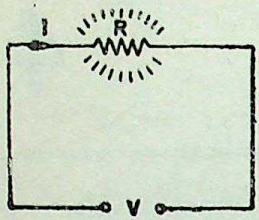

Fig. 4-1

where $J = 4{\cdot}186$ joules/cal
$= 4{\cdot}2$ joules/cal (approx)
$= 4{,}186$ joules/kcal
$= 4{,}200$ joules/kcal (approx)

$$\therefore \quad H = \frac{I^2Rt}{4{\cdot}2} \text{ cal} = \frac{I^2Rt}{4{,}200} \text{ kcal}$$

Other expressions for heat produced are

$$H = \frac{VIt}{4{,}200} \text{ kcal} = \frac{Wt}{4{,}200} \text{ kcal} = \frac{V^2t}{4{,}200R} \text{ kcal}$$

4-4 Thermal Efficiency

It is defined as the ratio of the heat actually utilised to the total heat produced electrically. Consider the case of the electric kettle used for boiling water. Out of the total heat produced (*i*) some goes to heat the apparatus itself *i.e.* kettle (*ii*) some is lost by radiation and convection etc. and (*iii*) the rest is utilised for heating the water. Out of these, the heat utilised for useful purpose is that in (*iii*). Hence, thermal efficiency of this electric apparatus is the ratio of the heat utilized to heat water to the total heat produced.

4-5 General Formula

Suppose m kg of water contained in a vessel of water equivalent w kg is to be heated electrically from t_1°C to t_2°C in t seconds. The heat required is

$$Q = (m+w)(t_2 - t_1) \text{ kcal}$$

Assuming an efficiency of η for the electric heater or other energy conversion apparatus used, we get

$$\frac{VIt}{4200} \times \eta = (m+w)(t_2 - t_1)$$

$$\frac{Wt}{4200} \times \eta = (m+w)(t_2 - t_1)$$

$$\frac{V^2t}{4200R} \times \eta = (m+w)(t_2 - t_1)$$

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Example 4-1. *An electric kettle was marked 500-W, 230-V and was found to take 15 minutes to bring 1 kg of water at 15°C to boiling point. Determine the heat efficiency of the kettle.*

Solution. $H=\frac{Wt}{4200}$ kcal

$\therefore$ $H=500\times 15\times 60/4200$ kcal

Heat actually absorbed by water

$=$ Mass $\times$ Sp. Heat $\times$ Rise in temperature

$=1\times 1\times(100-15)=85$ kcal

Heat efficiency $=\frac{\text{Heat absorbed}}{\text{Heat produced}}=\frac{85\times 4{,}200}{500\times 15\times 60}$

$=0{\cdot}79$ or 79%.

Example 4-2. *An immersion heater rated at 3 kW is used to heat a copper tank weighing 20 kg and holding 120 litres of water. How long will it take to raise the temperature of the water from 10°C to 60°C if 20 per cent of the energy supplied is wasted in heat losses ?*

Assume specific heat of copper $=0{\cdot}095$; $J=4{\cdot}2$ joules/calorie.

(A.M.I.E. May 1972)

Solution. Water equivalent of the copper tank

$=20\times 0{\cdot}095=1{\cdot}9$ kg of water

Effective amount of water $=120+1{\cdot}9=121{\cdot}9$ kg

Rise in temperature $=(t_2-t_1)=(60-10)=50°C$

Amount of heat reqd. $=Q=121{\cdot}9\times 50=6{,}095$ kcal

Since thermal efficiency is given as 80% or 0·8,

$\therefore$ $\frac{Wt}{4200}\times\eta=Q$ or $\frac{3000\,t}{4200}\times 0{\cdot}8=6095$

$\therefore$ $t=10{,}680$ second $=$ **2 h 58 min**

Example 4-3. *Determine the kW input necessary for an electric heater to raise the temperature of 4·536 kg of water from 15°C to 90°C in 10 minutes. Take the heat loss in radiation during this period as 65,000 joules and the water equivalent of the heater as 130 gram. Calculate approximately the efficiency of the operation. How long will it take to heat the water through the same range if the input kW were halved ?*

Solution. Mass of water $=4{\cdot}536$ kg

Water equivalent of heater $=0{\cdot}13$ kg

$\therefore$ Effective mass of water heated $=4{\cdot}666$ kg

Temperature rise $=90-15=75°C$

Heat absorbed in 10 minutes $=4{\cdot}666\times 75=349{\cdot}95$ kcal

$=349{\cdot}95\times 4180$

$=1{,}462{,}791$ J

(Assuming 1 kcal $=4180$ J)

Radiation loss in 10 minutes $=65{,}000$ J

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$\therefore$ Total energy consumed by water and calorimeter and loss
$= 1,462,791 + 65,000$
$= 1,527,791$ J

Rate of energy consumption $= \frac{1,527,791}{10 \times 60}$ J/s
$= 2546$ watt (approx.)

Rating of heater $= 2546/1000 = \mathbf{2{\cdot}546\ kW}$

Efficiency

Heat employed usefully = heat absorbed by water
$= 4{\cdot}536 \times 75$ kcal
$= 4{\cdot}536 \times 75 \times 4180$ J

$\therefore$ η of operation $= \frac{4{\cdot}536 \times 75 \times 4180}{1,527,791}$
$= 0{\cdot}931 = \mathbf{93{\cdot}1\%}$

When kW input is halved :

Input $= 2546/2 = 1273$ watt $= 1273$ J/s
Rate of radiation loss $= 65,000/600 = 108{\cdot}3$ J/s
$\therefore$ Net energy consumption/second
$= 1273 - 108{\cdot}3 = 1164{\cdot}7$ J/s
Energy absorbed by water and calorimeter
$= 4{\cdot}666 \times 75 \times 4180 = 1,462,791$ J
$\therefore$ Time needed $= 1,462,791/1164{\cdot}7 = 1256$ s (approx)
$= \mathbf{20\ min\ 56\ s}$

4-6. Quantity of Electricity

The difference between the rate of flow of electricity and the total quantity of electricity passing through a circuit in a given time, should be understood clearly. The rate of flow of electricity gives the current strength and is usually measured in amperes. The total quantity of electricity or charge is given by the product of current strength and time.

$\therefore$ $Q = I \times t$

If I is one ampere and it flows for one second, then

$Q = 1$ ampere $\times 1$ second $= 1$ A-s $= 1$ coulomb

Hence, one coulomb is that *quantity of electricity which flows per second past any point in a conductor when a current of one ampere flows through it.*

The bigger unit generally employed is ampere-hour (Ah)

One ampere hour is the charge transferred by a current of one ampere while flowing for one hour through a given circuit.

Ah $= 3600$ coulombs.

For example, a current of 5 ampere strength when flowing for 3 hours will carry over a charge of $5 \times 3 = 15$ Ah or $15 \times 3600 = 54000$ coulombs.

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4-7. Electric Power

Power is the rate of doing work and is independent of the total work to be done. The rate of working (or power) is found by dividing the work done by the time required to do it.

$$\therefore \text{ Electric power } = \frac{\text{Electric work done}}{\text{time taken}}$$

We have seen that work done electrically in time t seconds $= VIt$ joules

$$\therefore \qquad \text{Power} = \frac{VIt}{t} = VI$$

If V is in volts and I in amperes, then product VI is in watts.

$\therefore$ Power in watts = volts $\times$ amperes

One watt may be defined as *the rate of doing one joule per second.*

$\therefore$ 1 watt = 1 joule/second

Bigger units are :—

$$1 \text{ kW} = 1000 \text{ W} = 10^3 \text{ W}$$

$$1 \text{ MW} = 10^6 \text{ W}$$

Another unit but one which is being gradually discarded is

Horse-power (metric) = 75 m-kg/s

$= 735{\cdot}5$ joules/s or watt

4-8. Electric Energy

The unit of energy is Joule. Other units are :

1 watt-hour (Wh) = 1 watt $\times$ 1 hour

$= (1 \text{ J/s}) \times 3600 \text{ s} = 3600 \text{ J}$

1 kilowatt-hour (kWh)

$= 3600 \times 1000 = 36 \times 10^5 \text{ J}$

The bill for electric charges is based on the number of kWh consumed. This unit is also unofficially known as Kelvin.

4-9. Some Units in M.K.S.A. System

Each System of Units has two types of units known as (*i*) absolute units and (*ii*) gravitational units. The two units are interlinked by g whose value in the M.K.S.A. System of Units is 9·8 m/s^2. The absolute and gravitational units of force and work etc. are given as under :

(*a*) **Mass.** It is the quantity of matter contained in a body.

(*i*) Absolute unit is kg.

(*ii*) Gravitational unit is slug (though it is not used)

1 slug = 9·8 kg

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(*b*) **Force**

(*i*) Absolute unit of force is newton (N). Its definition may be obtained from Newton's Second Law of Motion *i.e.* $F=ma$

If $m=1$ kg and $a=1$ m/s², then $F=1$ newton.

Hence, one newton is that force which can give an acceleration of 1 m/s² to a mass of 1 kg.

(*ii*) Gravitational unit of force is kg-wt. It may be defined as follows :—

it is the force which can impart an acceleration of 9·8 m/s² to a mass of 1 kg

Or

it is the force which can impart an acceleration of 1 m/s² to a mass of 9·8 kg (*i.e.* one slug).

Obviously, 1 kg-wt = 9·8 N

(*c*) **Weight.** It is the force with which earth pulls a body downwards.

(*i*) Absolute unit of weight is newton.

(*ii*) Gravitational unit of weight is kg-wt.

Note. If a body has a mass of m kg then its weight

$$=mg=m\times 9{\cdot}8 \text{ N}$$

Now $\quad 1 \text{ kg-wt}=9{\cdot}8 \text{ N}$

$\therefore$ $\quad \text{weight}=m\times 9{\cdot}8/9{\cdot}8=m \text{ kg wt}$

Obviously, the mass of a body in absolute unit *i.e.* kg is numerically equal to its weight in gravitational unit *i.e.* kg-wt.

It should be clearly understood that a body weighing 500 kg *i.e.* 500 kg-wt has a mass of 500 kg.

(*d*) **Work.** If a force of F is applied in overcoming certain opposition or in moving a body through a distance S along the direction of force, then work done is

$$\text{W.D.}=F\times S$$

(*i*) Absolute unit of work is joule.

If $F=1$ N ; $\quad S=1$ m ; $\quad$ W.D. = 1 m-N or joule

Hence, one joule is the work done by a force of one newton when applied over a distance of one metre.

(*ii*) Gravitational unit of work is m-kg wt.

If $F=1$ kg-wt ; $\quad S=1$ m ; $\quad$ W.D. = 1 m-kg wt

Hence, one m-kg wt is the work done by a force of one kg wt. when applied over a distance of one metre.

Obviously, 1 m-kg wt = 9·8 joules.

Note : Usually the word 'wt' is omitted in which case unit of work becomes m-kg.

(*e*) **Torque.** It is defined as the turning moment of a force about an axis. It is given by the product of force and the radius at which it acts.

$\therefore$ $\quad T=F\times R$

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(*i*) *Absolute unit*

If $F=1$ newton and $R=1$ metre

then $T=1\ \text{N}\times 1\ \text{m}=1$ N-m

(*ii*) *Gravitational unit*

If $F=1$ kg wt and $R=1$ m

then $T=1$ kg wt $\times 1$ m $=1$ kg wt-m

Usually, the word 'wt' is omitted, hence the above unit is commonly written as kg-m.

Obviously, 1 kg-m $=9{\cdot}8$ N-m

(*f*) **Kilowatt-hour and Kilocalorie**

$1\ \text{kWh}=36\times 10^5$ joules

Also 1 kcal $=4{,}186$ joules

$\therefore$ $1\text{kWh}=36\times 10^5/4{,}186=860$ kcal

(*g*) **Power.** If a force of F newton imparts a velocity of v metre/second to a body, then power developed is

$=F\times v$ watt

If velocity v is in km/s, then

Power $=F\times v$ kilowatt.

Also, power $=T\omega$ watts

where T = torque in N-m

ω = angular velocity in radian/second

Example 4-4 *An electric kettle contains 1·5 kg. of water at 15°C. It takes 15 minutes to raise the temperature to 95°C. Assuming the heat losses due to radiation and heating the kettle to be 14 kcal, find the current taken. The supply voltage is 100 V.*

(*A.M.I.E. Winter 1976*)

Solution. The amount of heat required to raise the temperature of 1·5 kg of water from 15°C to 95°C is

$=1{\cdot}5\ (95-15)=120$ kcal

Total losses $=14$ kcal

Hence, total amount of energy taken from the supply is

$=120+14=134$ kcal

Now, 1 kWh $=860$ kcal or 1 kcal $=1/860$ kWh

$\therefore$ electric energy consumed

$=134\times 1/860=67/430\ \text{kWh}=67\times 1000/430$ Wh

Let I be the current drawn from the supply. Since time taken is 15 minutes *i.e.* 1/4 hour, energy consumed is

$VIt=100\ I\times 1/4=25I$ watt-hours

$\therefore$ $25\ I=6700/43$; $I=6{\cdot}23\text{A}$

Example 4-5. *Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75°C. Assume a heater efficiency of 90%. To what height could a weight of 5 tonnes be raised*

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with the same input energy as above. Assume a hoist efficiency of 70%.

Solution. Heat absorbed by water $=45\times75=3{,}375$ kcal

Heat produced $=3{,}375/0{\cdot}9=3{,}750$ kcal

$=3{,}750/860=$ **4·38 kWh**

Hoist efficiency $=0{\cdot}7$

Energy available for lifting the load

$=3{,}750\times4{,}200\times0{\cdot}7=11{\cdot}03\times10^6$ J

If h is the height through which the load of 5000 kg can be lifted with this energy, then

$$mgh=11{\cdot}03\times10^6$$

$$5000\times9{\cdot}8\times h=11{\cdot}03\times10^6$$

$$h=\mathbf{225\ m}$$

Example 4-6 *Two heaters A and B are in parallel across supply voltage V. Heater A produces 500 kcal in 20 minutes and B produces 1000 kcal in 10 minutes. The resistance of A is 10 Ω. What is the resistance of B? If the same heaters are connected in series across the same voltage, how much heat will be produced in 5 minutes?*

(A.M.I.E. Summer 1986)

Solution. Heat produced $H=V^2t/R$

For heater A : $500=V^2\times20/10$ ...(*i*)

For heater B : $1000=V^2\times10/R_2$ ...(*ii*)

Diving (*i*) by (*ii*), we get $R_2=2{\cdot}5\ \Omega$

Total resistance when connected in series $=12{\cdot}5\ \Omega$

Heat produced, $H=V^2\times5/12{\cdot}5$ ...(*iii*)

From (*i*) and (*iii*) we get, $H=$ **100 kcal.**

Example 4-7. *An electric heater is required to heat 13·61 kg of water from 15°C to the boiling point in 40 minutes. If the efficiency of the heater is 80 per cent, calculate (i) the energy consumed in kWh and (ii) the cost of energy if the charge is 10 paise per unit.*

(A.M.I.E. May 1973)

Solution. Heat required for raising the temperature of water

$=13{\cdot}61\times(100-15)=1157$ kcal

Heater efficiency $=80\%=0{\cdot}8$

Heat input $=1157/0{\cdot}8=1446$ kcal

Now, 1 kWh $=860$ kcal

(*i*) $\therefore$ energy consumed $=1446/860=$ **1·68 kWh**

(*ii*) Cost $=10\times1{\cdot}68=$ **16·8 paise**

Example 4-8. *An electric boiler has two heating elements each of 250 V, 5 kW rating and contains 8 litres of water at 30°C. Assum-*

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ing 10 per cent loss of heat from the boiler, find how long, after switching on the heater circuit, will the water boil at the atmospheric pressure (a) if the two elements are in parallel (b) if the two elements are in series. The supply voltage is 230 V ?

Mechanical equivalent of heat is 4,200 joules/kcal and 1 litre of water holds one kilogram of water.

Solution. Rating of each element

$$= 3{\cdot}5 \text{ kW} = 3500 \text{ W}$$

Combined rating of the two identical elements connected in parallel

$$= 3500 \times 2 = 7000 \text{ W}$$

Combined rating of two identical elements when connected in series

$$= 3500/2 = 1750 \text{ W} \qquad \text{(Why ?)}$$

Heat required $= 8(100 - 30) = 560$ kcal

(*i*) Elements in Parallel

$$\frac{Wt}{4200} \times \eta = Q \quad \text{or} \quad \frac{7000 \times t}{4200} \times 0{\cdot}9 = 560$$

$$t = \mathbf{373{\cdot}3 \text{ second}}$$

(*ii*) Elements in Series

$$\frac{1750\, t}{4200} \times 0{\cdot}9 = 560 \quad \text{or} \quad t = \mathbf{1493{\cdot}2}$$

Note. Since power rating is one-fourth, time taken would be four times that in the first case.

Example 4-9. *An immersion heater which is to operate on 230-V supply has to raise the temperature of 45·36 kg of water from 20°C to 95°C in 1 hour. Taking the efficiency of operation as 86%, determine (a) the rating of the heater and (b) the resistance of the heating elements in the heater. Calculate also the cost of the heating operation if energy costs 10 paise per kWh.*

Solution. Mass of water $= 45{\cdot}36$ kg

Tempt. rise $= 95 - 20 = 75°C$

Heat utilized $Q = 75 \times 45{\cdot}36$ kcal

Efficiency $= 0{\cdot}86$

Let W be the rating of the heater in watts.

Now, $$\frac{Wt}{4186} \times \eta = Q$$

$$\therefore \quad \frac{W \times 3600}{4186} \times 0{\cdot}86 = 75 \times 45{\cdot}36$$

(*a*) $$W = 4600 \text{ W} = \mathbf{4{\cdot}6 \text{ kW}}$$

(*b*) Now watts $= V^2/R$

$$\therefore \quad 4600 = 230^2/R \text{ or } R = 230^2/4600 = \mathbf{11{\cdot}5\ \Omega}$$

Energy input in one hour $= 4{\cdot}6 \times 1 = 4{\cdot}6$ kWh

Cost $= 4{\cdot}6 \times 10 = \mathbf{46 \text{ paise}}$

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Example 4-10. *A diesel-electric generating set supplies an output of 50 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35% (i) calculate the mass of oil required per hour and (ii) the electrical energy generated per tonne of the fuel.* (*A.M.I.E. Summer 1981*)

Solution. Output $=50$ kW

Overall efficiency $\eta=0{\cdot}35$

Input $=50/0{\cdot}35=142{\cdot}86$ kW

Input per hour $=142{\cdot}86$ kWh

$=142{\cdot}86\times860=122{,}860$ kcal

Since 1 kg of fuel oil produces 12,500 kcal

(*a*) mass of oil reqd. $=122{,}860/12{,}500=9{\cdot}83$ kg

(*b*) 1 tonne of fuel $=1000$ kg

Heat content $=1000\times12{,}500=12{\cdot}5\times10^6$ kcal

$=12{\cdot}5\times10^6/860=14{,}535$ kWh

Overall $\eta=0{\cdot}35$

Energy output $=14{,}535\times0{\cdot}35=5087$ kWh.

Example 4-11. *Calculate the current required by a 1,500 volt d.c. locomotive when drawing 100 tonne load at 45 km/hr with a tractive resistance of 5 kg/tonne along (a) level track (b) a gradient of 1 in 50. Assume a motor efficiency of 90 per cent.*

Solution. (*a*) As shown in Fig. 4-2 (*a*), in this case, only tractive resistance is to be overcome.

Tractive resistance $F=5\times100=500$ kg wt

$=500\times9{\cdot}81=4{,}905$ N

Velocity $=45$ km/hr $=45\times5/18=12{\cdot}5$ m/s

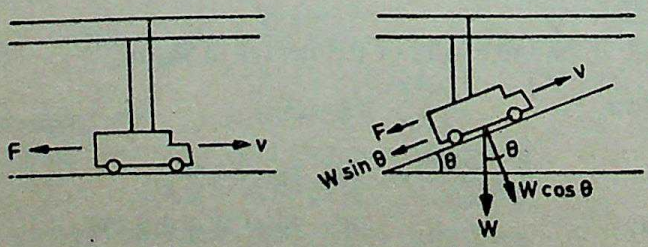
Power required $=4{,}905\times12{\cdot}5=61{,}310$ W

$\therefore$ locomotor power output $=61{,}310$ W

$\eta=0{\cdot}9$

Motor power input $=61{,}310/0{\cdot}9=68{,}125$ W

Current drawn $=68{,}125/1500=45{\cdot}4$ A



(*a*) (*b*)

Fig. 4-2

(*b*) In this case, downward component of load is also to be taken into consideration [Fig. 4-2 (*b*)]

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Load component acting downward along the gradient
$= W \sin\theta = 100 \times 1/50$ tonne
$= 2$ tonne $= 2{,}000$ kg wt
$= 2000 \times 9{\cdot}81 = 19{,}620$ N

Tractive resistance $= 4905$ N (as before)

Total force required $= 19{,}620 + 4905 = 24{,}525$ N

Power output required $=$ force $\times$ velocity $= 24{,}525 \times 12{\cdot}5$ W

Power input $= 24{,}525 \times 12{\cdot}5/0{\cdot}9$ W

Current drawn $= \dfrac{24{,}525 \times 12{\cdot}5}{1500 \times 0{\cdot}9} =$ **272 A**

Example 4-12. *In an electric furnace 100 kg of tin are to be melted in an hour from an initial temperature of 15°C. Find the power required and the cost of the melting with energy at 20 paise per unit.*

(Sp. heat of tin = 0·056 ; melting point of tin = 235°C ; latent heat of fusion of tin = 13·31 kilocalorie per kg ; thermal efficiency = 70 per cent ; $J = 0{\cdot}24 \times 10^{-3}$ calorie/joule).

Solution. Heat required to bring 100 kg of tin from 15°C to melting point of 235°C

$= 100 \times 0{\cdot}056 \times (235 - 15) = 1232$ kcal

Heat of fusion $= 100 \times 13{\cdot}31 = 1331$ kcal

Total heat needed $= 1232 + 1331 = 2{,}563$ kcal

Thermal efficieny $= 0{\cdot}7$

$\therefore$ Total heat produced electrically
$= 2{,}563/0{\cdot}7 = 3661{\cdot}4$ kcal

$J = 0{\cdot}24 \times 10^{-3}$ kcal/joule*

$\therefore$ Total energy produced/hour
$= 3661{\cdot}4/0{\cdot}24 \times 10^{-3} = 152{\cdot}56 \times 10^{5}$ J/hr

Rate of energy production
$= 152{\cdot}56 \times 10^{5}/3600$ J/s $=$ **4,238 watt**

Energy produced $= \dfrac{152{\cdot}56 \times 10^{5}}{36 \times 10^{5}} = 4{\cdot}238$ kWh

Cost $= 4{\cdot}238 \times 20 =$ **85 paise**

Example 4-13. *An electrically-driven pump lifts 15 tonne of water in a minute to a height of 10 m. Assuming an efficiency of 70% for the pump and 90% for the motor, calculate (i) motor output in kW (ii) input current to the motor if the supply voltage is 500 V.*

Also find the cost of running the pump for 3 hours a day for 30 days if the price of electrical energy is 10 paise/kWh.

---

*Normally, J is given as 4200 joules/kcal. Obviously J = 1/4200 kcal/joule or $J = 0{\cdot}24 \times 10^{-3}$ kcal/joule

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Solution. The motor-pump arrangement is shown in Fig. 4-3.

Wt. of water lifted $= 15,000$ kg wt

$= 15,000 \times 9{\cdot}81 = 147,150$ N

Height $= 10$ m

$\therefore$ Work done/minute

$= 147,150 \times 10$ joules/minute

$\therefore$ Output power $= 147,150 \times 10/60 = 24,525$ J/s or W

Pump efficiency $= 0{\cdot}7$

(*i*) Pump input or motor power output

$= 24,525/0{\cdot}7 = 35,036$ W $= \mathbf{35{\cdot}036}$ **kW**

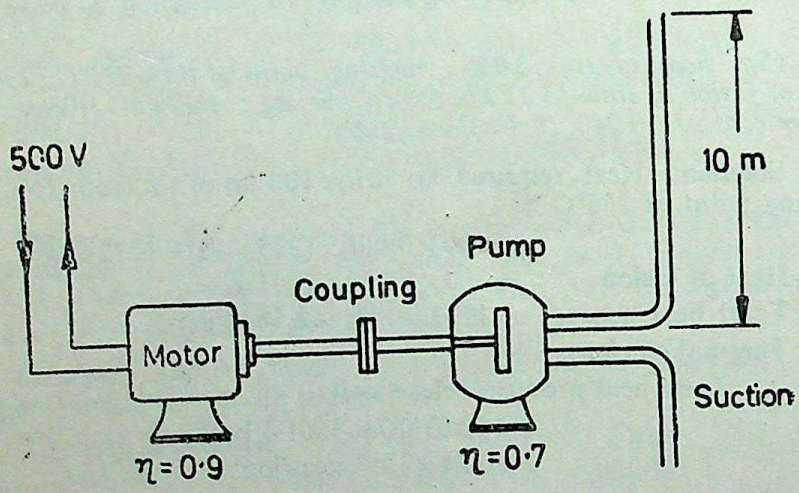

Fig. 4-3

(*ii*) Motor efficiency $= 0{\cdot}9$

Motor power input $= 35,036/0{\cdot}9 = 38,930$ W

Current drawn by motor $= 38,930/500 = \mathbf{77{\cdot}86}$ **A**

Cost

Running time of the pump

$= 30 \times 3 = 90$ hr

Energy consumed by the motor $= 38{\cdot}93 \times 90$ kWh

Cost $=$ Rs. $(38{\cdot}93 \times 90) \times 0{\cdot}1 =$ **Rs. 350 paise 37**

Example 4-14. *Calculate the number of kWh of electrica energy obtained per hour from a generating plant whose overall efficiency is 26 per cent, given :*

Amount of coal burnt/hour $= 4$ *tonne*

Calorific value of coal $= 6,000$ *kcal/kg*

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Solution. Total amount of heat produced

$$= 4{,}000 \times 6{,}000 = 24 \times 10^6 \text{ kcal/hr}$$

$$\eta = 0{\cdot}26$$

Amount of heat or thermal energy utilized

$$= 0{\cdot}26 \times 24 \times 10^6 = 624 \times 10^4 \text{ kcal}$$

Now $1 \text{ kWh} = 860 \text{ kcal}$

$\therefore$ Electric energy produced/hr is

$$= 624 \times 10^4 / 860 = \mathbf{6{,}615 \text{ kWh/hr}}$$

Example 4-15. *An electric lift with a cage (or body) weighing 1/2 tonne and a balance weight of 2 tonne takes a load of 3 tonne to a height of 20 metre in 30 seconds returning empty in the same time. The efficiency of the motor and gearing is 75% in either direction. If it makes 15 double journeys per hour, how many kWh will be consumed in an hour ?*

Solution. The lift is shown in Fig. 4-4

Wt. lifted during upward journey

$$= 3 + \frac{1}{2} - 2 = 1\frac{1}{2} \text{ tonne wt.}$$

$$= 1500 \text{ kg wt}$$

Wt. pulled down during downward journey

$$= 2 - \frac{1}{2} = 1\frac{1}{2} \text{ tonne wt}$$

$$= 1500 \text{ kg wt}$$

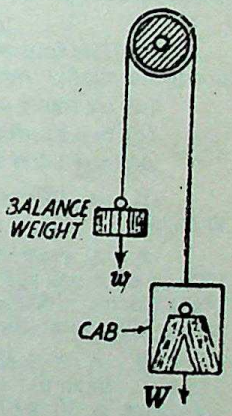

Fig. 4-4

Distance travelled either way

$$= 20 \text{ m}$$

Work done per double journey

$$= 2 \times (1500 \times 20) = 60{,}000 \text{ m-kg wt}$$

$$= 60{,}000 \times 9{\cdot}81 = 588{,}600 \text{ J}$$

Motor and gearing $\eta = 0{\cdot}75$

Energy consumed per double journey

$$= 588{,}600 / 0{\cdot}75 = 784{,}800 \text{ J}$$

No. of double journeys made per hour $= 15$

Energy consumed/hr $= 15 \times 784{,}800$ J

Now $1 \text{ kWh} = 36 \times 10^5$ J

$\therefore$ energy consumed/hr

$$= \mathbf{15 \times 784{,}800 / 36 \times 10^5 = 3{\cdot}27 \text{ kWh}}$$

Tutorial Problems—6

1. Calculate the length of wire of resistance 6 Ω/m required to construct a heating coil capable of producing 200 cal/min when connected to the 100-V supply mains. [119 m]

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2. It is required to make an immersion heater of 3 W which can be used on a 6-V supply. Determine the resistance of the coil to be used and also the number of calories of heat which would be generated per minute.
[12 Ω ; 42·9 cal]

3. A coil of resistance 100 Ω is immersed in a vessel containing 0·5 kg of water at 16°C and is connected to a 220-V electric supply. Calculate the time required to boil away all the water. Given J=4,200 joules/kcal ; latent heat of steam=536 kcal/kg.
[44 min 50 second]

4. An electric fire takes a power of 1 kW when connected directly across a 200-V main. If long leads having a total resistance of 1 Ω are used to connect the fire to a 200-V main, calculate (*a*) the power taken from the mains and (*b*) the power used in the fire itself. It may be assumed that the resistance of the fire remains constant.
[(*a*) 976 W (*b*) 952 W]

5. An aluminium kettle weighing 2 kg holds 2 litres of water and consumes electric power at the rate of 2 kW. If 40 % of the heat supplied is wasted, find the time taken to bring the kettle of water to boiling point from an initial temperature of 20°C. Specific heat of aluminium=0·2 kcal/kg/°C. J=4,200 joules/kcal.
[11·2 min]

6. An electric cooker is rated at a maximum of 3·45 kW at 230 V. Determine (*a*) the current when used at maximum rating (*b*) the resistance of the heating element (*c*) the cost of using the cooker for three hours, the first half hour being at maximum rating and the rest of the time at one-third maximum rating. Electric energy costs 10 paise/kWh.
[(*a*) 15 A (*b*) 15·33 Ω (*c*) 46 paise]

7. The following are the details of the load on a circuit connected through a supply meter :

(*a*) six lights of 40 W each working for 4 hours/day

(*b*) two fluorescent tubes of 125 W each working 2 hours per day

(*c*) one 1-kW heater working 3 hours/day

If one kWh costs 20 paise, what will be the total cost in a month of 30 days ?
[Rs. 26·76]

8. A centrifugal pump which is gear-driven by a d.c. motor delivers 1000 kg of water per minute to a tank 20 metre above the level of the sump. If the efficiency of the pump is 80%, that of the gearing 90% and that of the motor 85%, what does the motor take from 400-V supply ?
[13·4 A]

9. A 20-tonne tram-car driven by two motors ascends an incline of 3 in 100. Each motor delivers 20 kW when running on 600 V. The tractive resistance is 10 kg/tonne, gearing efficiency 90% and motor efficiency 80%. Find the speed of the car and the current taken by each motor.
[16·53 km/hr ; 41·7 A]

10. A current of 80 A flows for one hour in a resistance across which there is a voltage of 2 V. Determine the velocity in metres per second with which a weight of one tonne must move in order that its kinetic energy shall be equal to the energy dissipated in the resistance.
[33·9 m/s]

HIGHLIGHTS

1. One kilocalorie (kcal) is the amount of heat required to heat one kg of water through one °C.

2. According to Joule's Law of Electric Heating, the amount of heat produced is

$$H=\frac{I^2Rt}{J}$$

where J=4·186 joules/cal

=4·2 joules/cal (approx.)=4200 J/kcal

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3. General formula for heating is

$$\frac{I^2Rt}{J} \times \eta = (m+w)(t_2-t_1)$$

$$\frac{Wt}{4200} \times \eta = (m+w)(t_2-t_1)$$

$$\frac{V^2t}{4200\,R} \times \eta = (m+w)(t_2-t_1)$$

4. Quantity of electricity or charge is measured in terms of coulomb (C).

One coulomb of charge is the charge carried over by a current of one ampere flowing for one second. Larger unit of charge is Ampere-hour (Ah).

$$1 \text{ Ah} = 3600 \text{ coulombs}$$

5. Electric power is given in watts

$$\text{Volts} \times \text{amperes} = \text{watts}$$

6. Energy is measured in kWh

$$1 \text{ kWh} = 36 \times 10^5 \text{ joules} = 860 \text{ kcal}$$

7. One h.p. (metric) = 735·5 watt

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ELECTROSTATICS 5

1. Static Electricity 2. Absolute and Relative Permittivity 3. Laws of Electrostatics 4. Electric Field 5. Electrostatic Induction 6. Flux per Unit Charge 7. Electric Flux Density 8. Electric Field Intensity 9. Electric Flux Density and Electric Intensity 10. Electric Potential and Energy 11. Potential and Potential Difference 12. Potential at a Point 13. Potential Due to a Charged Sphere 14. Equi-potential Surface 15. Potential and Electric Intensity Inside a Conducting Sphere 16. Potential Gradient 17. Dielectric Strength of a Medium 18. Highlights.

5-1 Static Electricity

In the preceding chapters, we concerned ourselves exclusively with current electricity *i.e.* electricity in motion. Now, we will discuss the behaviour of static electricity and the laws governing it. In fact, electrostatics is that branch of science which deals with the phenomena associated with electricity *at rest.*

It has been already discussed that generally an atom is electrically neutral *i.e.* in a normal atom the sum total of positive charge of the protons is exactly equal to the sum total of negative charge of the electrons.

If, somehow, some negatively-charged electrons are removed from the atoms of a body, then it is left with a preponderance of positive charge. Then it is said to be positively charged. If, on the other hand, some electrons are added to it, then negative charge out-balances the positive charge and the body is said to be negatively charged.

In brief, we can say that positive electrification of a body results from a deficiency of electrons whereas negative electrification results from an excess of electrons.

The total deficiency or excess of electrons in a body is known as its charge.

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5-2 Absolute and Relative Permittivities of a Medium

(A.M.I.E. Winter 1983)

While discussing electrostatic phenomenon, a certain property of the medium called its *permittivity* plays a very important role. Every medium is supposed to possess two permittivities :

(*i*) absolute permittivity (ϵ) and

(*ii*) relative permittivity (ϵ_r).

For measuring relative permittivity, vacuum or free space is chosen as the reference medium. It is alloted an absolute permittivity of $8{\cdot}854 \times 10^{-12}$ farad/metre (F/m). Its symbol is ϵ_0. Obviously, the relative permittivity of vacuum with reference to itself is unity. Hence, for *free space*

$$\text{absolute permittivity, } \epsilon_0 = 8{\cdot}854 \times 10^{-12} \text{ F/m}$$

$$\text{relative permittivity, } \epsilon_r = 1$$

Now, take any other medium. If its relative permittivity, as compared to vacuum is ϵ_r, then its absolute permittivity is

$$\epsilon = \epsilon_0 \epsilon_r \text{ farad/metre}$$

If, for example, relative permittivity of mica is 5, then

$$\epsilon_r = 5 \qquad \text{(being a ratio, it has no units)}$$

$$\epsilon = \epsilon_0 \epsilon_r = 8{\cdot}854 \times 10^{-12} \times 5 \text{ F/m}$$

$$= 44{\cdot}27 \times 10^{-12} \text{ F/m}$$

5-3 Laws of Electrostatics *(A.M.I.E. Summer 1979)*

First Law. Like charges of electricity repel each other, whereas unlike charges attract each other.

Second Law. According to this law, the force exerted between two *point* charges (*i*) is directly proportional to the product of their strength (*ii*) is inversely proportional to the square of the distance between them and (*iii*) is inversely proportional to the absolute permittivity (ϵ) of the surrounding medium.

This law is known as Coulomb's Law and can be expressed mathematically as

$$F \propto \frac{Q_1 Q_1}{\epsilon d^2}$$

or

$$F = k \frac{Q_1 Q_2}{\epsilon d^2}$$

where k is the constant of proportionality whose value depends on the system of units employed. In the M.K.S.A. system of units, $k = 1/4\pi$.

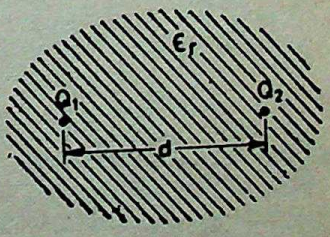

Fig. 5-1

$$\therefore \quad F = \frac{Q_1 Q_2}{4\pi \epsilon d^2}$$

If Q_1 and Q_2 are in coulombs, d in metres and ϵ in farad/metre, then F is in newtons.

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The above equation may be written as

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 \epsilon_r d^2} \text{ N} \qquad \text{—in a medium}$$

$$= \frac{Q_1 Q_2}{4\pi \epsilon_0 d^2} \text{ N} \qquad \text{—in air}$$

Now $$\frac{1}{4\pi \epsilon_0} = \frac{1}{4\pi \times 8{\cdot}854 \times 10^{-12}} = 9 \times 10^9 \text{ (approx)}$$

In that case, Coulomb's law for electric force becomes,

$$F = 9 \times 10^9 \frac{Q_1 Q_2}{\epsilon_r d^2} \text{ N} \qquad \text{—in a medium}$$

$$= 9 \times 10^9 \frac{Q_1 Q_2}{d^2} \text{ N} \qquad \text{—in air} \qquad \ldots(i)$$

If in Eq (*i*) above,

$$Q_1 = Q_2 = Q \text{ (say)}$$

$$d = 1 \text{ metre} ; \; F = 9 \times 10^9 \text{ N}$$

then $$Q^2 = 1 \quad \text{or} \quad Q = \pm 1 \text{ coulomb}$$

Hence, one coulomb of charge may be defined as :

that charge which when placed in air (strictly vacuum) from an equal and similar charge at a distance of one metre repels it with a force of 9×10^9 N.

Although coulomb is found to be a unit of convenient size in dealing with electric current, yet from the standpoint of electrostatics, it is an enormous unit. Hence, its submultiples like microcoulomb (μC) and micro-microcoulomb ($\mu\mu$C) are generally used.

$1\mu\text{C} = 10^{-6}$ C and $1\mu\mu\text{C} = 10^{-12}$ C

It may be noted here that *relative* permittivity of air is one, of water 81, of paper between 2 and 3, of glass between 5 and 10 and of mica between 2·5 and 6.

Example 5-1. *Three identical point charges Q coulombs each are placed at the vertices of an equilateral triangle 10 cm. apart. Calculate the force on each charge.*

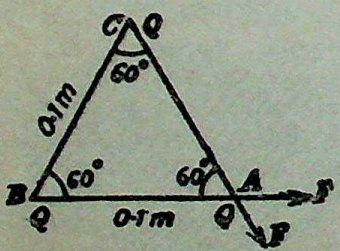

Fig. 5-2

Solution. Let us consider the force on the charge Q placed at point A (Fig. 5-2)

Force along BA due to charge at point B is $F = 9 \times 10^9 \frac{Q^2}{0{\cdot}1^2}$ N

Force along CA due to charge at point C is $F = 9 \times 10^9 \frac{Q^2}{0{\cdot}1^2}$ N

The angle between these two forces is 60°. The resultant force on charge Q at point A

$$= 2 \times 9 \times 10^9 \frac{Q^2}{0{\cdot}1^2} \times \cos 30° = \sqrt{3} \times 9 \times 10^{11} \, Q^2 \text{ newton}$$

The force exerted on other charges is also the same.

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Example 5-2. *Two charges Q coulombs each, are placed at two opposite corners of a square. What additional charge q placed at each of the other two corners, will reduce the resultant electric force on each of the charges Q to zero.*

Solution. Force on charge Q at point B due to charge Q at point D is

$$=9\times10^9\ Q^2/(\sqrt{2}d)^2=4{\cdot}5\times10^9\ Q^2/d^2\ \text{N}$$

It acts along DF (Fig. 5-3). Charges at points A and C are negative. Each of the two forces of attraction acting along BA and BC is

$$=9\times10^9\times q\ Q/d^2\ \text{N}$$

Their resultant acting along BD is

$$=2\times(9\times10^9\ q\ Q/d^2)\cos 45°$$

$$=\sqrt{2}\times9\times10^9\ q\ Q/d^2\ \text{N}$$

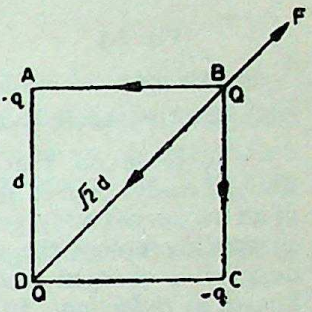

Fig. 5-3

The resultant electric force acting on charge Q at point B is zero when

$$4{\cdot}5\times10^9\ Q^2/d^2=\sqrt{2}\times9\times10^9\ qQ/d^2 \text{ or } \mathbf{q=Q/2\ \sqrt{2}\ coulomb.}$$

5-4 Electric Field

It is found that the medium around a charge is always under stress and that a force acts on a positive or negative charge when placed in that medium. If the charge is sufficiently large, then it may create such a huge stress as to cause the mechanical rupture of the medium, followed by the passage of an arc discharge.

The region in which the stress exists or in which electric forces act, is called an electric field or dielectric field or electrostatic field.

The stress is represented by imaginary *lines of force*. The direction of the line of force at any point is the direction along which a unit positive charge placed at that point would move if free to do so. It was suggested by Faraday that the electric field should be imagined to be divided into *tubes of force* containing a fixed number of lines of force. He assumed these tubes to be elastic and having the property of contracting longitudinally and repelling laterally. With the help of these properties, it becomes easy to explain (*i*) why unlike charges attract each other and try to come nearer to each other and (*ii*) why like charges repel each other (Fig. 5-4).

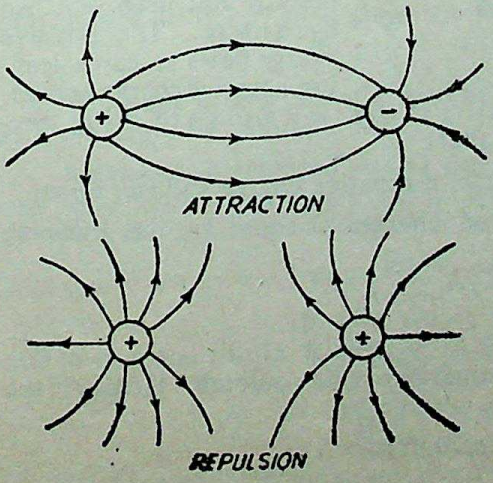

Fig. 5-4

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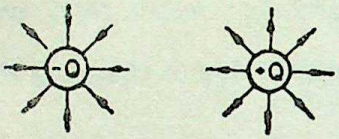

Fig. 5-5

However, it is more common to use the term *lines of force*. These lines are supposed to emanate from a positive charge and end on a negative charge (Fig. 5-5). These lines always leave or enter a conducting surface normally. Lines of force emanating from a charge of $+Q$ coulomb are Q/ϵ where ϵ is the absolute permittivity.

5-5. Electrostatic Induction

It is found that when an uncharged body is brought near a charged body, then it also acquires some charge. This phenomenon of an uncharged body getting charged merely by the nearness of a charged body is known as *induction*. In Fig. 5-6, a positively-charged body A is brought close to a perfectly insulated uncharged body B. It is found that the end of B nearer to A gets negatively charged whereas the farther end becomes positively charged. The negative and positive charges of B are known as *induced charges*. The negative charge of B is called *bound charge* because it must remain on B so long as positive charge of A remains there. However, the positive charge on the farther end of B is called free charge. In Fig. 5-7, the body B has been earthed by a wire. The positive charge flows to earth leaving negative charge behind. If A is removed, then this negative charge will also go to earth, leaving B uncharged. It is found that :

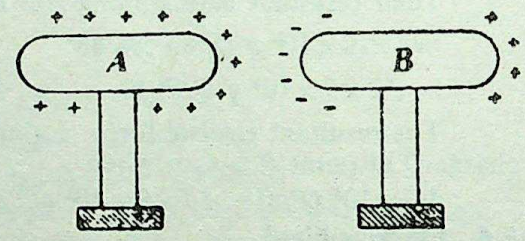

Fig. 5-6

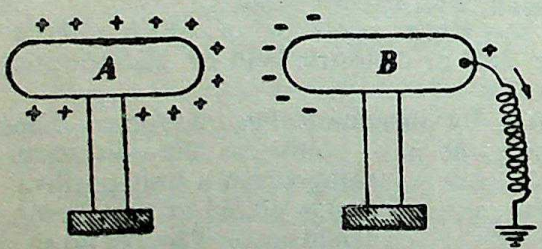

Fig. 5-7

(*i*) a positive charge induces a negative charge and *vice versa*.

(*ii*) each of the two induced charges is equal to the inducing charge.

5-6. Flux Per Unit Charge

In the rationalized M.K.S.A. System of Units, one *tube* of flux is supposed to emanate from a charge of one coulomb. Hence, if the charge is Q coulombs, then flux is given by

$$\Psi = Q \text{ coulombs}$$

In other words, flux is also measured in coulombs.

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5-7. Electric Flux Density (D) (*A.M.I.E. Summer 1986*)

Flux density is defined as the flux per unit area held perpendicular to the tubes of flux.

If Ψ coulomb is the flux passing normally through an area of A metre^2, then

$$D = \frac{\Psi}{A} \text{ C/m}^2$$

5-8. Field Strength or Field Intensity or Electric Intensity E. (*A.M.I.E. Summer 1986*)

Field strength or intensity of electric field at any point situated within an electrostatic field may be defined in any of the following three ways :

(*a*) It is equal to the lines of *force passing normally through a unit cross-section at that point*:

Suppose there is a point charge of Q coulomb. The number of lines of force produced by it is Q/ϵ. If these lines fall on an area of A m^2, then

$$E = \frac{Q/\epsilon}{A} = \frac{Q}{\epsilon A} \text{ V/m}$$

$$\therefore \quad E = \frac{Q}{\epsilon_0 \epsilon_r A} \text{ V/m} \quad \text{...in a medium}$$

$$= \frac{Q}{\epsilon_0 A} \text{ V/m} \quad \text{...in air}$$

(*b*) It is numerically equal to the *force experienced by a unit positive charge placed at the point in question.*

In this case, the unit of E is N/C.

For example, if a charge of q coulomb placed at a particular point within an electric field experiences a force of F newton, then electric field at that point is given by

$$E = F/q \quad \text{N/C}$$

The value of E within the field of a point charge can be found with the help of Coulomb's laws.

Suppose it is required to find the electric field at a point situated at a distance of d metres from a charge of Q coloumbs, then according to the second definition

$$F = \frac{Q \times 1}{4\pi\epsilon_0 \epsilon_r d^2} \text{ N/C}$$

$$E = \frac{Q}{4\pi\epsilon_0 \epsilon_r d^2} \text{ N/C} \quad \text{...in a medium}$$

$$= \frac{Q}{4\pi\epsilon_0 d^2} \text{ N/C} \quad \text{...in air}$$

(*c*) Electric intensity at any point in an electric field is *equal to the potential gradient at that point.*

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If, in an electric field, potential falls by an amount $-dV$ over a distance of dx, then

$$E = \frac{-dV}{dx} \text{ V/m}$$

It should be noted that E is a vector quantity, having both magnitude and direction.

The electric field at a point due to more than one charge can be found by taking the vector sum of fields due to individual charges.

Note. If a charge of Q coulomb is placed in an electric field of E V/m, then force acting on the charge is $F = EQ$ newton.

Example 5-3. *Three point charges of $+16 \times 10^{-9}$ C, $+64 \times 10^{-9}$ C and -48×10^{-9} C, are placed at the corners of a square of 4 cm sides. Calculate the electric field at the fourth corner.*

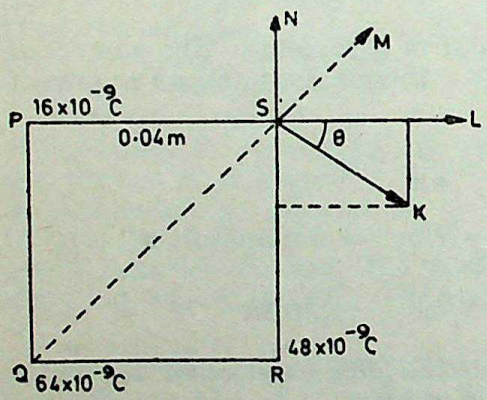

Fig. 5-8

Solution. The square is shown in Fig. 5-8. Value of electric field at S along SL due to charge at P is

$$= 9 \times 10^9 \times \frac{Q}{d^2} \text{ N/C}$$

$$= 9 \times 10^9 \times \frac{16 \times 10^{-9}}{0{\cdot}04^2}$$

$$= 9 \times 10^4 \text{ N/C}$$

Similarly, value of electric field at S along SM due to charge at Q is

$$= 9 \times 10^9 \times \frac{64 \times 10^{-9}}{(\sqrt{0{\cdot}0032})^2} = 18 \times 10^4 \text{ N/C}$$

Also value of electric field at S along SR due to charge at R is

$$= 9 \times 10^9 \times \frac{48 \times 10^{-9}}{0{\cdot}04^2} = 27 \times 10^4 \text{ N/C}$$

Resolving these electric fields into their X- and Y-components, we have

Total X-component along $SL = 9 \times 10^4 + 18 \times 10^4 \cos 45°$

$$= 9 \times 10^4 + \frac{18 \times 10^4}{\sqrt{2}}$$

$$= 21{\cdot}73 \times 10^4 \text{ N/C}$$

Total Y-component along $SR = 27 \times 10^4 - 18 \times 10^4 \sin 45°$

$$= 14{\cdot}27 \times 10^4 \text{ N/C}$$

Hence, resultant field along $SK = \sqrt{(21{\cdot}73 \times 10^4)^2 + (14{\cdot}27 \times 10^4)^2}$

$$= 26 \times 10^4 \text{ N/C}$$

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$$\tan\theta = \frac{Y\text{-component}}{X\text{-component}}$$

$$= 14{\cdot}27/21{\cdot}73 = 0{\cdot}656$$

$$\theta = 33{\cdot}3^\circ$$

5-9. Electric Flux Density or Electric Displacement D and E

$$E = \frac{Q}{\epsilon A} \qquad \text{...Art. 5-8}$$

$$D = \frac{\Psi}{A} = \frac{Q}{A} \qquad \text{...Art. 5-7}$$

$$\therefore \quad E = \frac{D}{\epsilon}$$

or

$$D = \epsilon E \ \text{C/m}^2$$

$$= \epsilon_0 \epsilon_r E \ \text{C/m}^2$$

In other words, the product of electric intensity E at any point within a dielectric medium and the absolute permittivity $\epsilon = \epsilon_0 \epsilon_r$ at the same point is called the "displacement" at that point.

Like electric intensity E, displacement D is also a vector quantity whose direction at every point is the same as that of E but whose magnitude is $\epsilon_0 \epsilon_r$ times E. As E is represented by lines of force, similarly D may also be represented by tubes called *tubes of displacement* or *tubes of flux*. The tangent to these tubes at any point gives the direction of D at that point and the number of tubes per unit area taken perpendicular to their direction is numerically equal to the displacement at that point. Hence, the number of tubes of displacement per unit area (D) is $\epsilon_0 \epsilon_r$ times the number of *lines* of force per unit area (E) at that point.

One useful property of D is that its surface integral over any closed surface equals the enclosed charge.

Let us find the value of D at a point distant r metres from a point charge of Q coulombs. Imagine a sphere of radius r metres surrounding the charge. Total flux is Q coulombs and it falls on a surface area of $4\pi r^2$ metres. Hence, electric flux density is

$$D = \frac{Q}{4\pi r^2} \ \text{C/m}^2$$

5-10. Electrical Potential and Energy

We know that a body raised above the ground level has a certain amount of mechanical potential energy which, by definition, is given by the amount of work done in raising it to that height. If, for example, a weight of 5 kg is raised against gravity through 10 m, then the potential energy of the body is $5 \times 10 = 50$ m-kg. The body falls because there is attraction due to gravity and always proceeds from a place of higher potential energy to one of lower potential energy. So, we speak of gravitational potential energy or

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briefly 'potential' at different points in the earth's gravitational field.

Now, consider an electric field. Imagine an isolated positive charge Q placed in air (Fig. 5-9). Like earth's gravitational field, it has its own electrostatic field which theoretically extends up to infinity. If the charge X is very far away from Q, say, at infinity then force on it is practically zero. As X is brought nearer to Q, a force of repulsion acts on it. Because similar charges repel each other, work or energy is required to bring it to a point like A in the electric field. Hence, when at point A, X has some amount of electric potential energy. Similar other points in the field will also have some potential energy.

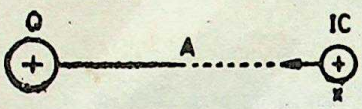

Fig. 5-9

In the gravitational field, usually 'sea level' is chosen as the place of 'zero' potential. In an electric field, infinity is chosen as the theoretical place of 'zero' potential although in practice earth is chosen as 'zero' potential, because earth is such a large conductor that its potential remains practically constant although it keeps on losing and gaining electricity every day.

5-11. Potential and Potential Difference (*A.M.I.E. Summer 1986*)

As explained above, since the force on a charge at infinity is zero, hence 'infinity' is chosen as the theoretical place of zero electric potential. Therefore, potential at a point in an electric field may be defined as *numerically equal to the work done in bringing positive charge of one coulomb from infinity to that point against the electric field.*

The unit of this potential will depend on the unit of charge taken and the work done.

If in shifting one coulomb from infinity to a certain point in the electric field the work done is one joule, then potential of that point is one volt.

Obviously, potential is work per unit charge.

$$\therefore \quad 1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

Similarly, potential difference (p.d.) of one volt exists between two points if one joule of work is done in shifting a charge of one coulomb from one point to the other.

5-12. Potential at a Point

Consider a positive point charge of Q coulombs placed in air. At a point x metres from it, the force on one coulomb positive charge is $Q/4\pi\epsilon_0 x^2$ (Fig. 5-10). Suppose, this one coulomb charge is moved towards Q through a small distance dx. Then work done

$$= \frac{Q}{4\pi\epsilon_0 x^2} \times (-dx)$$

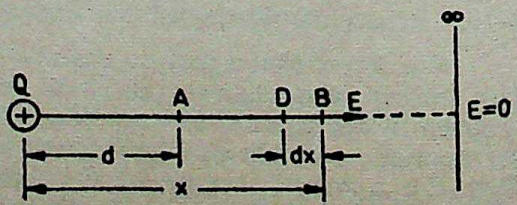

Fig. 5-10

The negative sign is taken because dx is taken along the negative direction of x.

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The total work done in bringing this coulomb of positive charge from infinity to any point d metres from Q is given by

$$W = \int_{x=\infty}^{x=d} -Q \cdot \frac{dx}{4\pi\epsilon_0 x^2} = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{d} \frac{dx}{x^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{d}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{d}-\left(-\frac{1}{\infty}\right)\right]$$

$$= \frac{Q}{4\pi\epsilon_0 d} \text{ joules}$$

By definition, this work in joules is numerically equal to the potential of that point in volts.

$$\therefore \quad V = \frac{Q}{4\pi\epsilon_0 d} = 9\times10^9 \frac{Q}{d} \text{ volts} \quad \text{...in air}$$

and $$V = \frac{Q}{4\pi\epsilon_0\epsilon_r d} = 9\times10^9 \frac{Q}{\epsilon_r d} \text{ volt} \quad \text{...in medium}$$

We find that as d increases, V decreases, till it becomes zero at infinity.

Note. If a charge of Q coulomb is shifted against a p.d. of V volts work done is W.D. $= QV$ joules.

5-13. Potential Due to a Charged Sphere

The above formula $V = Q/4\pi\epsilon_0\epsilon_r d$ applies only to a charge concentrated at a point.. The problem of finding potential at a point outside a charged sphere sounds difficult, because the charge on the sphere is distributed over its surface and so, is not concentrated at a point. But the problem is easily solved by noting that the lines of force of a charged sphere, like A in Fig. 5-11, spread out normally from its surface. If produced backwards, they meet at the centre of A. Hence, for finding the potentials at points outside the sphere, we can imagine the charge on the sphere as concentrated at its centre. If r

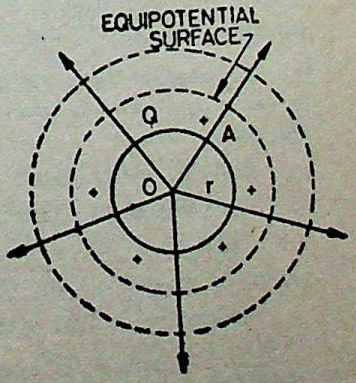

Fig. 5-11

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is the radius of the sphere in metres and Q its charge in coulombs then potential of its surface is $Q/4\pi\epsilon_0 r$ volts and electric intensity is $Q/4\pi\epsilon_0 r^2$. At any other point 'd' metres from the centre of the sphere, the corresponding values are $Q/4\pi\epsilon_0 d$ and $Q/4\pi\epsilon_0 d^2$ respectively. The variations of the potential and electric intensity with distance for a charged sphere are shown in Fig. 5-12.

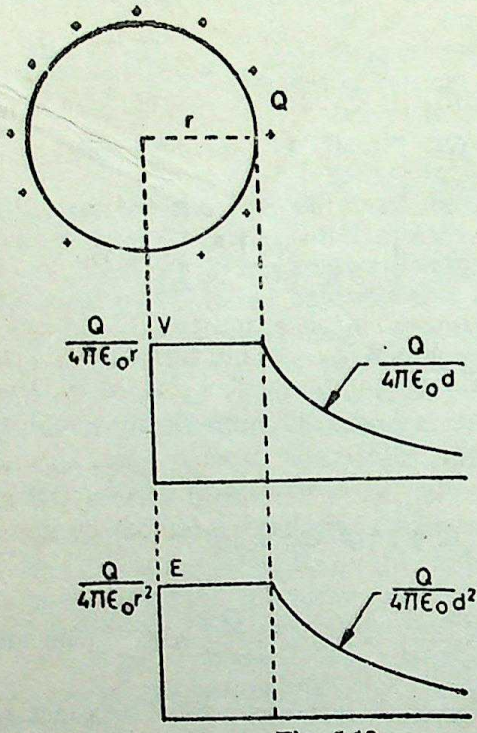

Fig. 5-12

5-14. Equipotential Surfaces

An equipotential surface is a surface in an electric field such that all points on it are at the same potential. For example, different spherical surfaces around a charged sphere are equipotential surfaces. One important property of an equipotential surface is that lines of force are always normal to it.

5-15. Potential and Electric Intensity Inside a Conducting Sphere

It has been experimentally found that when charge is given to a conducting body, say a sphere, then it resides entirely on its outer surface *i.e.* within a conducting body whether hollow or solid, the charge is zero. Hence (*i*) flux is zero (*ii*) field intensity is zero (*iii*) all points within the conductor are at the same potential as its surface (Fig. 5-13).

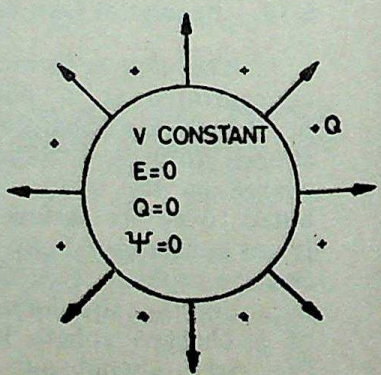

Fig. 5-13

Example 5-4. *A hollow sphere is charged to 12 μC of electricity. Find the potential* :

(*a*) *at its surface*

(*b*) *inside the sphere*

(*c*) *at a distance of 0·3 metre from the surface. The radius of the sphere is 0·1 metre.*

Solution. The potential at the surface of the sphere is $\frac{Q}{4\pi\epsilon_0\epsilon_r d}$ where ϵ_r is the relative permittivity of the surrounding medium. Potential anywhere inside the sphere is the same. For

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calculations, charge on the sphere is supposed to be concentrated at its centre.

(*a*) $$V = \frac{Q}{4\pi\epsilon_0 d} = 9\times10^9 \frac{Q}{d} \text{ volt}$$

$$= 9\times10^9 \times \frac{12\times10^{-6}}{0{\cdot}1} = \mathbf{108\times10^4 \text{ volt}}$$

(*b*) Inside also $V = \mathbf{108\times10^4\ V}$

(*c*) Distance of the point from the centre is 0·4 m

$\therefore$ Potential $$= 9\times10^9 \times \frac{12\times10^{-6}}{0{\cdot}4} = \mathbf{27\times10^4\ V}$$

Example 5-5. *Three concentric spheres of radii 4, 6 and 8 cm have charges of +8, −6 and +4 μμC respectively. What are the potentials and field strengths at points 2, 5, 7 and 10 cm from the centre ?*

Solution. As shown in Fig. 5-14, let the three spheres be marked *A*, *B* and *C*. It should be remembered that (*i*) the field intensity outside a sphere is the same as that obtained by considering the charge at its centre (*ii*) inside the sphere, the field strength is zero (*iii*) potential anywhere inside a sphere is the same as at its surface.

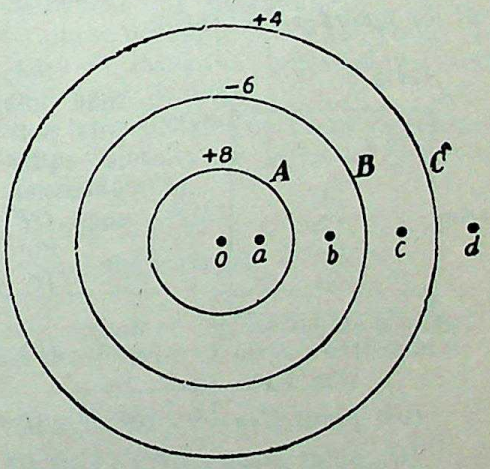

Fig. 5-14

(*i*) Consider point '*a*' at a distance of 2 cm from the centre *O*. Since it is inside all the spheres, the field strength at this point is zero.

Potential at '*a*'

$$= \Sigma \frac{Q}{4\pi\epsilon_0 d} = 9\times10^9 \Sigma \frac{Q}{d}$$

$$= 9\times10^9 \left(\frac{8\times10^{-12}}{0{\cdot}04} - \frac{6\times10^{-12}}{0{\cdot}06} + \frac{4\times10^{-12}}{0{\cdot}08} \right)$$

$$= \mathbf{1{\cdot}35\ V}$$

(*ii*) Since point '*b*' is outside sphere *A*, but inside *B* and *C*

$$\therefore \text{ Electric field} = \frac{Q}{4\pi\epsilon_0 d^2} = 9\times10^9 \frac{Q}{d^2} \text{ N/C}$$

$$= 9\times10^9 \times \frac{8\times10^{-12}}{0{\cdot}05^2} = \mathbf{28{\cdot}8\ N/C}$$

$$\text{Potential at '}b\text{'} = 9\times10^9 \left(\frac{8\times10^{-12}}{0{\cdot}05} - \frac{6\times10^{-12}}{0{\cdot}06} + \frac{4\times10^{-12}}{0{\cdot}08} \right)$$

$$= \mathbf{0{\cdot}99\ V}$$

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(*iii*) The field strength at point '*c*' distant 7 cm from centre *O*

$$=9\times10^9\times\left[\frac{8\times10^{-12}}{0.07^2}-\frac{6\times10^{-12}}{0.07}\right]$$

$$=3.67 \text{ N/C}$$

$$\text{Potential at '}c\text{'}=9\times10^9\left[\frac{8\times10^{-12}}{0.07}-\frac{6\times10^{-12}}{0.07}+\frac{4\times10^{-12}}{0.08}\right]$$

$$=0.77 \text{ V}$$

(*iv*) Field strength at point '*d*' distant 10 cm from point *O* is

$$=9\times10^9\left[\frac{8\times10^{-12}}{0.1^2}-\frac{6\times10^{-12}}{0.1^2}+\frac{4\times10^{-12}}{0.1^2}\right]$$

$$=5.4 \text{ N/C}$$

$$\text{Potential of '}d\text{' is }=9\times10^9\left(\frac{8\times10^{-12}}{0.1}-\frac{6\times10^{-12}}{0.1}+\frac{4\times10^{-12}}{0.1}\right)$$

$$=0.54 \text{ V}$$

Example 5-6. *What is the potential at the centre of a square having charges of +2, +1, −2 and +3 in terms of 10^{-8} C each if side of the square is 1·41 metre.* (*A.M.I.E. Summer 1975*)

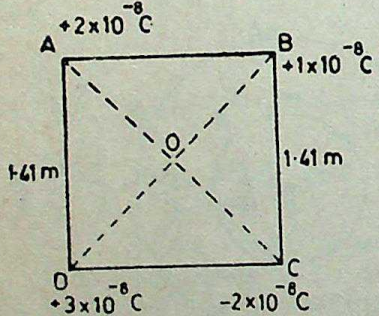

Fig. 5-15

Solution. It should be remembered that potential is a scaler quantity. The total potential at the centre *O* (Fig. 5-15) is equal to the *algebraic* sum of the potentials due to the four charges. As seen $OB=OD=OC=OA=2/2=1$ metre

Potential at point *O* due to charge at

(*i*) point $A=9\times10^9\times2\times10^{-8}/1$

$=180$ V

(*ii*) point $B=9\times10^9\times1\times10^{-8}/1$

$=90$ V

(*iii*) point $C=-9\times10^9\times2\times10^{-8}/1=-180$ V

(*iv*) point $D=9\times10^9\times3\times10^{-8}/1=270$ V

$\therefore$ Potential of point $O=180+90-180+270=$ **360 V**

5-16. Potential Gradient (g)

(*A.M.I.E. Winter 1983*)

It is defined as *the rate of change of potential with distance measured in the direction of electric force.*

i.e. $$g=\frac{dV}{dx}$$

Its unit is volt/metre although its derivative volt/cm or kV/mm is generally used in practice. Suppose, in an electric field of strength *E*, there are two points *dx* metre apart. The p.d. between them is

$$dV=E\,(-dx)=-E.dx$$

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$\therefore$ $$E=-\frac{dV}{dx}=-g \qquad ...(i)$$

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Hence, it means that electric intensity at a point is equal to the negative potential gradient at that point.

Now, electric intensity is measured in newton/coulomb (Art. 5-8) but equation (*i*) above shows that its unit may also be volt/metre, the same as that of potential gradient. It can be shown that N/C is the same as V/m.

$$1 \text{ volt} = 1 \frac{\text{joule}}{\text{coulomb}} \qquad \text{(Art. 5-11)}$$

$$= 1 \frac{\text{metre-newton}}{\text{coulomb}}$$

$$\therefore \quad 1 \frac{\text{volt}}{\text{metre}} = 1 \frac{\text{metre-newton}}{\text{coulomb-metre}} = 1 \frac{\text{newton}}{\text{coulomb}}$$

$$\therefore \quad 1 \text{ V/m} = 1 \text{ N/C.}$$

5-17. Dielectric Strength of a Medium (*A.M.I.E. Nov. 1971*)

It is measured in terms of the potential difference which when applied across 1 metre (or 1 mm) thickness of a dielectric medium or insulating medium will break down its insulation. It is also expressed in V/m. For example, when we say, the breakdown potential of air is 3×10^6 V/m, then we mean that the maximum p.d. which 1 metre thickness of air can withstand across it, is 3×10^6 volts. If the voltage exceeds this value, then air insulation breaks down and a spark is produced which punctures the medium.

It is obvious that dielectric strength of a medium represents the maximum value of the electric intensity or potential gradient that can be safely established in that medium.

TABLE No. 5-1

Dielectric Constant and Strength

Insulating material	*Dielectric constant or relative permittiivity*	*Dielectric strength in kV/mm*
Air	nearly 1	3·2
Glass	5—12	12—20
Miea	4—8	20—60
Rubber	2·5	—
Wood	2·5—6·8	—
Micanite	4·5—6	25—35
Paper	1·8 –2·6	—
Paraffin Wax	1·7—2·3	30
Porcelain	5—6·7	15
Quartz	4·5—4·7	8
Sulphur	3·6—4·1	—

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Example 5-7. *Find the radius of an isolated sphere capable of being charged to 1 million volt potential before sparking into the air. Given that breakdown voltage of air is 30,000 V/cm.*

Solution. Let r metre be the radius of the sphere

then $$V=\frac{Q}{4\pi\epsilon_0 r}=10^6 \text{ V} \qquad ...(i)$$

Breakdown voltage $=30{,}000$ V/cm
$=3{,}000{,}000$ V/m $=3\times10^6$ V/m

Since electric intensity equals breakdown voltage

$$E=\frac{Q}{4\pi\epsilon_0 r^2}=3\times10^6 \text{ V/m} \qquad ...(ii)$$

Dividing (i) by (ii), we get

$$r=\tfrac{1}{3} \text{ m}=\mathbf{0{\cdot}33 \text{ metre}}$$

Example 5-8. *It is found experimentally that an electric field of 3×10^6 V/m in air will cause electrical breakdown of the air. What is the greatest charge that can be placed on a sphere of 1 metre diameter ? What is the potential of the sphere for the charge ?*

Solution. Breakdown voltage $= 3\times10^6$ V/m

$$\therefore \quad E=\frac{Q}{4\pi\epsilon_0 r^2}=3\times10^6 \text{ V/m}$$

$$9\times10^9\frac{Q}{0{\cdot}5^2}=3\times10^6$$

$$\therefore \quad Q=\frac{3\times10^6\times0{\cdot}5^2}{9\times10^9}=\mathbf{8{\cdot}33\times10^{-5} \text{ C}}$$

Now, potential $V=\frac{Q}{4\pi\epsilon_0 r}=9\times10^9\times\frac{Q}{r}$

$$=9\times10^9\frac{8{\cdot}33\times10^{-5}}{0{\cdot}5}=15\times10^5 \text{ volt}$$

$$=\mathbf{1{,}500 \text{ kV}}$$

Example 5-9. *Two brass plates are arranged horizontally, one 2 cm above the other, and the lower plate is earthed. The plates are charged to a difference of potential of 6,000 volts. A drop of oil with an electric charge of $1{\cdot}6\times10^{-19}$ C is in equilibrium between the plates so that it neither rises nor falls. What is the mass of the drop ?*

Solution. The electric intensity is equal to the potential gradient between the plates.

$$g=6000/2=3000 \text{ volt/cm}=3\times10^5 \text{ V/m}$$
$$E=3\times10^5 \text{ V/m or N/C}$$

$$\therefore \quad \text{Force on drop} \quad =E\times Q=3\times10^5\times1{\cdot}6\times10^{-19}$$
$$=4{\cdot}8\times10^{-14} \text{ N}$$
$$=\text{wt of drop}=mg \text{ newton}$$

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$\therefore \quad m \times 9 \cdot 81 = 4 \cdot 8 \times 10^{-14}$

$$m = 4 \cdot 89 \times 10^{-15} \text{ kg.}$$

Example 5-10. *A parallel-plate capacitor has its plates 0·2 mm apart, with a solid dielectric between having $\epsilon_r = 3 \cdot 3$. The capacitor is charged so that surface charge density is 4×10^{-3} μC/cm^2. Find the electric flux density and the voltage between the plates.*

Solution.

Electric flux density = charge density

$\therefore \quad D = 4 \times 10^{-3}$ μC/cm^2 $= 4 \times 10^{-5}$ C/m^2

Now $\quad E = \dfrac{dV}{dx}$

where $\quad dV$ = p.d. between plates

dx = distance between plates

Here $\quad dx = 0 \cdot 2$ mm $= 2 \times 10^{-4}$ m

$$E = \frac{D}{\epsilon_0 \epsilon_r} = \frac{4 \times 10^{-5}}{8 \cdot 854 \times 10^{-12} \times 3 \cdot 3}$$

$$= 1 \cdot 37 \times 10^6 \text{ V/m}$$

$$dV = E.dx = 1 \cdot 37 \times 10^6 \times 2 \times 10^{-4} = 274 \text{ V}$$

Example 5-11. *A 0·23 cm thick sheet of fibre, with a relative permittivity of 5, is inserted between two parallel metal plates 0·25 cm apart. A p.d. of 1,500 V is applied between the plates. Determine the electric field intensity in the fibre and in the air film between the fibre and the metal plates. Will the air break down ?*

(A.M.I.E. Nov. 1970)

Solution. The arrangement is shown in Fig. 5-16.

Let E_1 and E_2 be the electric intensities or potential gradients in air and fibre sheet respectively.

Now, $\quad E_1 = V_1/x_1$

or $\quad V_1 = E_1 . x_1 = 0 \cdot 02 \times 10^{-2} E_1$

$$V_2 = E_2 . x_2 = 0 \cdot 23 \times 10^{-2} E_2$$

Also $\quad V = V_1 + V_2$

$$1500 = 10^{-2}(0 \cdot 02 E_1 + 0 \cdot 23 E_2)$$

or $\quad 15 \times 10^6 = 2E_1 + 23E_2 \quad \ldots(i)$

Now, value of electric flux density D is the same in the two media.

$\therefore \quad D = \epsilon_0 \epsilon_{r1} E_1 = \epsilon_0 \epsilon_{r2} E_2$

or $\quad E_1 = 5E_2 \quad \ldots(ii)$

From Eq. (*i*) and (*ii*), we get

$$33 E_2 = 15 \times 10^6$$

or $\quad E_2 = 0 \cdot 455 \times 10^6$ V/m

$$E_1 = 2 \cdot 275 \times 10^6 \text{ V/m}$$

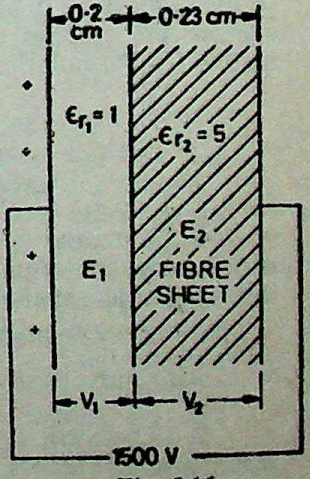

Fig. 5-16

Since dielectric strength of air is 3×10^6 V/m, it is obvious there will be no breakdown of the air film.

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HIGHLIGHTS

1. Coulomb's Law of electrostatic force between two point charges is

$$F = \frac{Q_1 Q_2}{4\pi\ \epsilon_0 \epsilon_r d^2} \text{ newton}$$

where $\epsilon_0 = 8{\cdot}854 \times 10^{-12}$ F/m

2. Electric flux is measured in coulombs and is equal to the charge

$$\therefore \quad \Psi = Q \text{ coulombs}$$

3. Electric intensity at a point within a field is equal to the force in newtons experienced by a positive charge of 1 C placed at that point. In other words, electric intensity is force per unit charge

$$E = F/Q \text{ newton/coulomb}$$

Other expressions for E are :

$$E = \frac{D}{\epsilon_0 \epsilon_r} \text{ V/m}$$

In the field due to a point charge Q its value is,

$$E = \frac{Q}{4\pi\epsilon_0 \epsilon_r d^2} \text{ V/m}$$

4. Electric displacement or flux density D is defined as

$$D = \Psi/A \text{ C/m}^2$$

It is related to E by the equations

$$D = \epsilon E \text{ C/m}^2$$
$$= \epsilon_0 \epsilon_r E \text{ C/m}^2$$

Also, $D = \sigma$ C/m^2 where σ is charge density.

5. Potential at any point within a field is numerically equal to the work done in shifting one coulomb from infinity to that point against the electric force. The potential at any point distant d metres from a point charge of Q coulombs is given by

$$V = \frac{Q}{4\pi\epsilon_0 \epsilon_r d} \text{ volts} \quad \text{...in a medium}$$
$$= \frac{Q}{4\pi\epsilon_0 d} \text{ volts} \quad \text{...in air}$$

6. Potential gradient (g) at any point within an electric field is defined as the rate of change of potential with distance measured in the direction of the electric force at that point

$$g = dV/dx$$

Also numerically, $E = g = -\frac{dV}{ax}$ V/m

7. Dielectric strength of a medium is given by the p.d. which when applied across one metre thickness of the medium will break down its insulation. It is measured in V/m.

8. The electric intensity (E), potential gradient (g) and dielectric strength are all numerically equal.

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CAPACITANCE 6

1. Capacitor 2. Capacitance 3. Capacitance of an Isolated Sphere 4. Spherical Capacitor 5. Parallel-plate Capacitor 6. Multiplate and Variable Capacitor 7. Types of Capacitors 8. Cylindrical Capacitor 9. Potential Gradient in a Cylindrical Capacitor 10. Capacitors in Series 11. Capacitors in Parallel 12. Insulation Resistance of a Cable Capacitor 13. Energy Stored in a Capacitor 14. Energy Stored per Unit Volume of the Dielectric 15. Force of Attraction Between Oppositely-charged Plates 16. Charging of a Capacitor 17. Time Constant 18. Discharging of a Capacitor 19. Highlights.

6-1. Capacitor

(*A.M.I.E. May 1969*)

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called *dielectric*. The conducting surfaces may be in the form of either circular (or rectangular) plates or of spherical or cylindrical shape. The purpose of a capacitor is to store electrical energy by electrostatic stress in the dielectric.

A parallel-plate capacitor is shown in Fig. 6-1. One plate is joined to the positive end of the supply and the other to the negative end or is earthed. It is experimentally found that in the presence of an earthed plate B, plate A is capable of withholding more charge than when B is not there. When such a capacitor is put across a battery, there is a momentary flow of electrons from A to B. As negatively charged electrons are withdrawn from A, it becomes positive and as these electrons collect on B, it becomes negative. Hence, p.d. is established between plates A and B. This transient flow of electrons gives rise to a charging current. The strength of the charging current is maximum when the two plates are uncharged, but it then decreases and finally ceases when p.d. across the plates becomes slowly and slowly equal and opposite to the battery e.m.f.

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In fact, the charging of capacitor is in many respects similar to the charging of a water tank T from a reservoir R (Fig. 6-2).

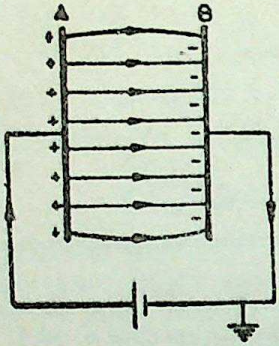

Fig. 6-1

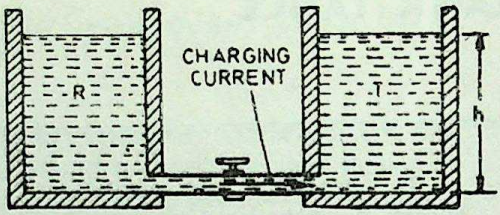

Fig. 6-2

On opening the valve, the water rushes through the connecting pipe from R to T. At first the rush would be great and then flow of water will go on decreasing as the level of water in T goes on increasing. Obviously, water flow will stop when the water levels in R and T become equal.

In the same way, when capacitor is fully charged *i.e.* when p.d. across its plates becomes the same in magnitude as the charging e.m.f. then no current flows. Two points should be mentioned here :—

(*i*) The battery itself does not create electricity on plates A and B, its function is merely to cause the transfer of electrons from A to B and hence create a p.d. between them. Its function is analogous to that of a pump.

(*ii*) When capacitor is connected across a battery, no continuous current can flow 'through' the capacitor. There is only a momentary shift of electrons from one plate to another. If a suitable lamp is included in the circuit as shown in Fig. 6-3, it will glow for a very short while *i.e.* so long as capacitor is being charged. After that, it does not glow. A capacitor blocks the passage of direct current.

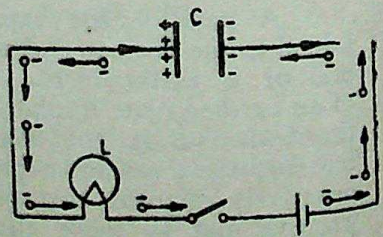

Fig. 6-3

It has been found that the dielectric medium between the capacitor plates is under a state of strain and energy is stored in it. If the p.d. across the plates is increased, the strain in the dielectric increases, till an electrical breakdown of the medium, accompanied by a spark across the plates, takes place. The maximum voltage per metre thickness which a medium can withstand without a rupture or breakdown is called its dielectric strength.

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6-2. Capacitance *(A.M.I.E. Summer 1975)*

The property of a capacitor to 'store electricity' may be called its capacitance.

The capacitance of a capacitor may be defined as

the amount of charge required to create a unit potential difference between its plates.

Suppose, we give Q coulombs of charge to one of the two plates of a capacitor and if a p.d. of V volts is established between the two, then its capacitance is

$$C = \frac{Q}{V} \quad \frac{\text{coulomb}}{\text{volt}}$$

By definition, the unit of capacitance is coulomb/volt which is also called *farad* (in honour of Michael Faraday)

$$\therefore \qquad 1 \text{ farad} = 1 \text{ coulomb/volt}$$

One farad is defined as the capacitance of a capacitor which requires a charge of one coulomb to establish a p.d. of one volt between its plates.

One farad is actually too large for practical purposes. Hence, much smaller units like microfarad (μF) and micro-microfarad ($\mu\mu$F) or picofarad (pF) are generally employed

$$1\mu\text{F} = 10^{-6} \text{ F}$$

$$1\mu\mu\text{F or pF} = 10^{-12} \text{ F}$$

6-3. Capacitance of an Isolated Sphere

Consider a charged sphere of radius r metres having a charge of Q coulombs placed in air as shown in Fig. 6-4.

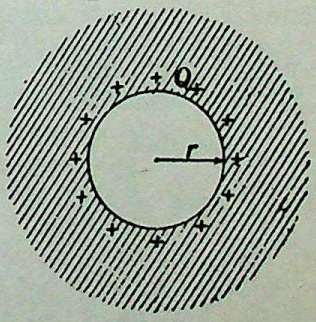

Fig. 6-4

It has been proved in Art. 5-13 that the free surface potential V of such a sphere with respect to infinity (in practice, earth) is given by

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\therefore \qquad \frac{Q}{V} = 4\pi\epsilon_0 r$$

By definition Q/V = capacitance C

$$\therefore \qquad C = 4\pi\epsilon_0 r \text{ F} \quad \text{...in air}$$

$$= 4\pi\epsilon_0\epsilon r \text{ F} \quad \text{...in a medium}$$

Note. It is sometimes felt surprising that an isolated sphere can act as a capacitor because, at first sight, it appears to have one plate only. The question arises as to which is the second plate or surface. But if we remember that the surface potential V is with respect to infinity (actually earth) then it is obvious that the other surface is earth. This capacitance $4\pi\epsilon_0 r$ exists between the surface of the sphere and earth.

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6-4. Spherical Capacitor

(A.M.I.E. May 1972)

Consider a spherical capacitor consisting of two concentric spheres of radii a and b metres as shown in Fig. 6-5. Suppose the inner sphere is given a charge of $+Q$ coulombs. It will induce a charge of $-Q$ coulombs on the inner surface of the outer sphere and a charge of $+Q$ coulombs on its outer surface which will go to earth. If the dielectric medium between the two spheres has a relative permittivity of ϵ_r, then the free surface potential of the inner sphere due to its own charge is $Q/4\pi\epsilon_0\epsilon_r a$ volts. The potential of the inner sphere due to $-Q$ charge on the inner surface of the outer sphere is $-Q/4\pi\epsilon_0\epsilon_r b$ (remembering that potential anywhere inside a sphere is the same as at its surface).

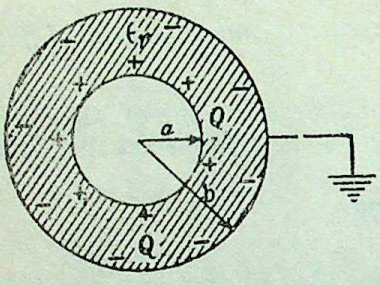

Fig. 6-5

$\therefore$ Total potential difference between the two surfaces is

$$V=\frac{Q}{4\pi\epsilon_0\epsilon_r a}-\frac{Q}{4\pi\epsilon_0\epsilon_r b}$$

$$=\frac{Q}{4\pi\epsilon_0\epsilon_r}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{Q}{4\pi\epsilon_0\epsilon_r}\left(\frac{b-a}{ab}\right)$$

$$\frac{Q}{V}=\frac{4\pi\epsilon_0\epsilon_r ab}{b-a}$$

$\therefore$ $$C=4\pi\epsilon_0\epsilon_r\frac{ab}{b-a}\ \text{F}$$

Example 6-1. *The radii of two spheres differ by 4 cm and the capacitance of this spherical capacitor is 53·33 pF. If outer sphere is earthed, calculate the radii assuming air as the dielectric.*

Solution. $$C=4\pi\epsilon_0\frac{ab}{b-a}\ \text{F}$$

Here $b=(a+0{\cdot}04)$ metre

$C=53{\cdot}33\ \text{pF}=53{\cdot}33\times10^{-12}\ \text{F}$

$\epsilon_0=8{\cdot}854\times10^{-12}\ \text{F/m}$

$\therefore$ $$53{\cdot}33\times10^{-12}=\frac{a(a+0{\cdot}04)}{0{\cdot}04}\times4\pi\times8{\cdot}854\times10^{-12}$$

$\therefore$ $$\frac{a(a+0{\cdot}04)}{0{\cdot}04}=\frac{53{\cdot}33\times10^{-12}}{4\pi\times8{\cdot}854\times10^{-12}}=0{\cdot}48$$

$\therefore$ $$a^2+0{\cdot}04a=0{\cdot}0192$$

or $$a^2+0{\cdot}04a-0{\cdot}0192=0$$

or $$(a+0{\cdot}16)(a-0{\cdot}12)=0$$

or $$a=0{\cdot}12\ \text{m}=12\ \text{cm}$$

$\therefore$ $$b=0{\cdot}16\ \text{m}=16\ \text{cm}.$$

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6-5. Parallel-plate Capacitor *(A.M.I.E. Summer 1983)*

(*i*) Uniform Dielectric Medium

A parallel-plate capacitor consisting of two plates M and N each of area A m² separated by a thickness d metres of a medium of relative permittivity ϵ_r is shown in Fig. 6-6. If a charge of $+Q$ coulombs is given to plate M, then flux between the two plates is Q coulomb and flux density in the dielectric medium is

$$D = \Psi/A = Q/A \text{ C/m}^2$$

Now $$E = \frac{D}{\epsilon} = \frac{D}{\epsilon_0 \epsilon_r} = \frac{Q/A}{\epsilon_0 \epsilon_r}$$

Also E = potential gradient

$$= V/d$$

$$\therefore \quad \frac{V}{d} = \frac{Q}{\epsilon_0 \epsilon_r A}$$

or $$\frac{Q}{V} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\therefore \quad C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ F...in a medium}$$

$$= \frac{\epsilon_0 A}{d} \text{ F ...in air}$$

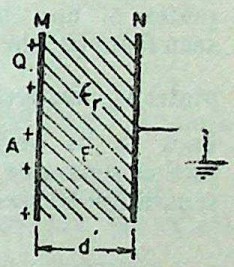

Fig. 6-6

(*ii*) Medium Partly Air

As shown in Fig. 6-7, the medium consists partly of air and partly of a parallel-sided dielectric slab of thickness t and relative permittivity ϵ_r. The electric flux density $D = Q/A$ is the same in both media. But electric intensities are different.

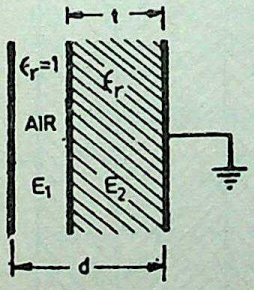

Fig. 6-7

In air $$E_1 = \frac{D}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

In the medium

$$E_2 = \frac{D}{\epsilon_0 \epsilon_r} = \frac{Q}{\epsilon_0 \epsilon_r A}$$

P.D. between plates

$$V = E_1.(d-t) + E_2$$

$$= \frac{D}{\epsilon_0}(d-t) + \frac{D}{\epsilon_0 \epsilon_r} t$$

$$= \frac{D}{\epsilon_0}\left(\frac{t}{\epsilon_r} + d - t\right)$$

$$= \frac{Q}{\epsilon_0 A}\left[d - \left(t - \frac{t}{\epsilon_r}\right)\right]$$

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$$\therefore \quad \frac{Q}{V}=\frac{\epsilon_0 A}{\left[d-\left(t-\frac{t}{\epsilon_r}\right)\right]}$$

or
$$C=\frac{\epsilon_0 A}{\left[d-\left(t-\frac{t}{\epsilon_r}\right)\right]} \text{ farad} \qquad \ldots(i)$$

If the medium were totally air, the capacitance would have been

$$C=\frac{\epsilon_0 A}{d} \text{ farad} \qquad \ldots(ii)$$

From (*i*) and (*ii*) it is obvious that when a dielectric slab of thickness t and relative permittivity ϵ_r is introduced between the plates of an air capacitor, its capacitance increases because as seen from (*i*), the denominator decreases. The distance between the plates is effectively reduced by $\left(t-\frac{t}{\epsilon_r}\right)$. To bring the capacitance back to its original value, the capacitor plates will have to be further separated by that much distance in air. Obviously, the new separation between the two plates would be

$$=d+\left(t-\frac{t}{\epsilon_r}\right)$$

(*iii*) **Composite Medium** (*A.M.I.E. Summer 1975*)

A parallel-plate capacitor having three slabs of different dielectric media is shown in Fig. 6-8. As before

$$V=V_1+V_2+V_3$$
$$=E_1d_1+E_2d_2+E_3d_3$$
$$=\frac{D}{\epsilon_0\epsilon_{r1}}d_1+\frac{D}{\epsilon_0\epsilon_{r2}}d_2+\frac{D}{\epsilon_0\epsilon_{r3}}d_3$$
$$=\frac{D}{\epsilon_0}\left(\frac{d_1}{\epsilon_{r1}}+\frac{d_2}{\epsilon_{r2}}+\frac{d_3}{\epsilon_{r3}}\right)$$
$$=\frac{Q}{\epsilon_0 A}\left(\frac{d_1}{\epsilon_{r1}}+\frac{d_2}{\epsilon_{r2}}+\frac{d_3}{\epsilon_{r3}}\right)$$
$$\therefore \frac{Q}{V}=C=\frac{\epsilon_0 A}{\left(\frac{d_1}{\epsilon_{r1}}+\frac{d_2}{\epsilon_{r2}}+\frac{d_2}{\epsilon_{r3}}\right)}$$

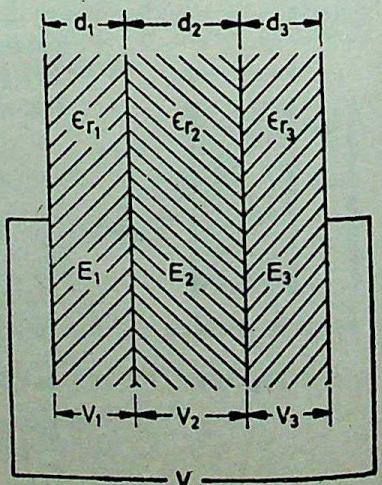

Fig. 6-8

In general,

$$C=\frac{\epsilon_0 A}{\Sigma d/\epsilon_r} \text{ F}$$

6-6. Multiplate and Variable Capacitors

Multiplate capacitors are shown in Fig. 6-9 and Fig. 6-10.

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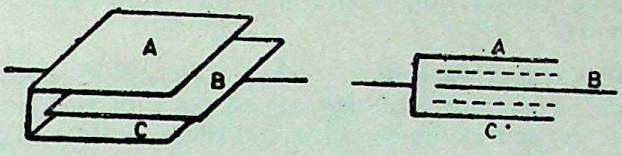
The arrangement of Fig. 6-9 is equivalent to two capacitors joined in parallel. Hence, its capacitance is double that of a single



(a) (b)

Fig. 6-9

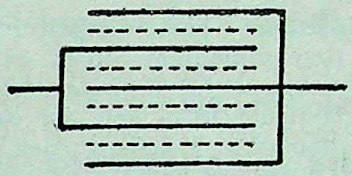


Fig. 6-10

capacitor. Similarly, the arrangement of Fig. 6-10 has four times the capacitance of a single capacitor.

If one set of plates is fixed and the other is capable of rotation, then capacitance of such a multiplate capacitor can be varied. Such variable-capacitance air-capacitors are widely used in radio work (Fig. 6-11). The set of fixed plates F is insulated from the other set R which can be rotated by turning the knob K. The common area between the two sets is varied by rotating K, hence the capacitance between the two is altered. Minimum capacitance is obtained when R is completely rotated out of F and maximum when R is completely rotated in *i.e.* when the two sets of plates completely overlap each other.

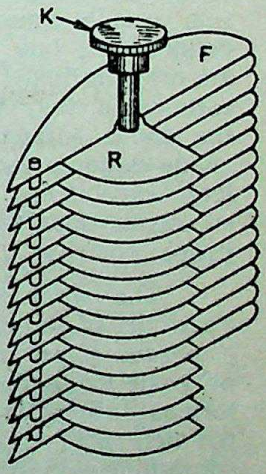


Fig. 6-11

The capacitance of such a capacitor is

$$= \frac{(n-1)\,\epsilon_0\epsilon_r\,A}{d} \text{ F}$$

where n is the total number of plates which means that $(n-1)$ is the number of capacitors.

6-7. Types of Capacitors

A few of the commonly-used capacitors are as follows :

(*i*) **Mica Capacitors.** Usually, they are multiplate type consisting of a series of plates, alternate plates being connected together and separated by a dielectric. Generally, the plates are of metal foil separated by thin mica sheets. The assembly is rigidly clamped together in a metal or ebonite case having two metal tugs or wire ends to which the two sets of plates are connected. The silver-mica

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type of capacitor consists of silver films deposited on thin mica dielectric sheets. Mica capacitors have very low power factors on alternating current and hence are very suitable for use at radio frequencies.

(*ii*) **Ceramic Capacitors.** These consist of discs of ceramic material whose opposite faces are coated with metallic silver. The ceramic disc acts as the dielectric and silver coatings as plates. Such a capacitor has very low power factor which decreases with increase in frequency. Hence, it is very suitable for short-wave work in radio.

(*iii*) **Paper Capacitors.** These consist of sheets of metal foils usually aluminium (but sometime) copper or tin interleaved with thin paper impregnated with oil or wax. The foils and paper are then rolled into a compact form and are housed in hermetically sealed tubes of cardboard or bakelized paper (for low voltage units) or in oil-filled metal boxes (for high-voltage units).

Such capacitors are suitable for use in audio-frequency (*A.F.*) stages of radio receivers as by-pass and coupling capacitors.

(*iv*) **Electrolytic Capacitors.** These are either wet type or dry type. The wet type consists of an aluminium anode which is centrally mounted in a metal cylinder filled with an electrolytic solution usually ammonium borate which acts as a cathode. Two connections, one from the anode and the other from the cathode are brought out. When current is passed between the anode and cathode, a very thin film of aluminium oxide (Al_2O_3) is formed on the anode surface. This film has insulating properties and the anode-cathode combination, thereafter, becomes a capacitor of very large capacitance because dielectric film is extremely thin. For successful operation, such capacitors must always be used with proper polarity.

The dry type capacitor consists of positive and negative electrodes of aluminium foil which are separated from each other by a porous paper or gauze saturated with either a very viscous liquid or paste containing the electrolyte. The whole arrangement is formed into a roll and housed in a waxed cardboard tube. Due to their high capacitance, such capacitors are generally used in smoothing circuits in radio work.

Example 6-2. *A capacitor consists of two similar square aluminium plates, each 10 cm × 10 cm mounted parallel and opposite each other. What is their capacitance in μμF when the distance between them is 1 cm and the dielectric air ? If the capacitor is given a charge of 500 μμC, what will be the difference of potential between plates ? How will this be affected if the space between the plates is filled with wax which has a relative permittivity of 4 ?*

Solution.

$$C = \frac{\epsilon_0 A}{d} F$$

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Here $\epsilon_0 = 8{\cdot}854 \times 10^{-12}$ F/m ; $d = 1$ cm $= 10^{-2}$ m

$A = 10 \times 10 = 100$ cm$^2 = 10^{-2}$ m^2

$$\therefore \quad C = \frac{8{\cdot}854 \times 10^{-12} \times 10^{-2}}{10^{-2}} = 8{\cdot}854 \times 10^{-12} \text{ F}$$

$$= \mathbf{8{\cdot}854\ \mu\mu F}$$

Now $C = \dfrac{Q}{V} \qquad \therefore \quad V = \dfrac{Q}{C}$

$$\therefore \quad V = \frac{500 \times 10^{-12}}{8{\cdot}854 \times 10^{-12}} = \mathbf{56{\cdot}5\ volts}$$

Now, when wax is introduced, their capacitance is increased four times because

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ F} = 4 \times 8{\cdot}854 = 35{\cdot}4\ \mu\mu F$$

The p.d. will, obviously, decrease to one-fourth value

$$\therefore \quad V = 56{\cdot}5/4 = \mathbf{14{\cdot}1\ volts}$$

Example 6-3 *A parallel-plate capacitor has plates 0·15 mm apart, a plate area of 0·1 m² and a dielectric of relative permittivity 3. Find electric flux density, electric field intensity and the p.d. between the plates if capacitor charge is 0·5 μC.* (*A.M.I.E. Summer 1986*)

Solution $C = \epsilon_0 \epsilon_r A/d = 8{\cdot}854 \times 10^{-12} \times 3 \times 0{\cdot}1/0{\cdot}15 \times 10$

$= 1{\cdot}77 \times 10^{-8}$ F

$D = Q/A = 0{\cdot}5 \times 10^{-6}/0{\cdot}1 = \mathbf{0{\cdot}5 \times 10^{-5}\ C/m^2}$

$Q = CV \quad \therefore \quad V = Q/C = 0{\cdot}5 \times 10^{-6}/1{\cdot}77 \times 10^{-8} = \mathbf{28V}$

$E = V/d = 28/0{\cdot}15 \times 10^{-3} = \mathbf{186{,}667\ V/m.}$

Example 6-4. *A parallel-plate capacitor has plates of area 2 m² spaced by three slabs of different dielectric materials. The relative permittivities are 2, 3, 6 and the thicknesses are 0·4, 0·6 and 1·2 mm respectively. Calculate the combined capacitance and the electric stress in each material when the applied voltage is 1000 V.* (*A.M.I.E. Summer 1975*)

Solution.

$$C = \frac{\epsilon_0 A}{\dfrac{d_1}{\epsilon_{r1}} + \dfrac{d_2}{\epsilon_{r2}} + \dfrac{d_3}{\epsilon_{r3}}} \text{ F}$$

$$= \frac{8{\cdot}854 \times 10^{-12} \times 2}{\dfrac{0{\cdot}4 \times 10^{-3}}{2} + \dfrac{0{\cdot}6 \times 10^{-3}}{3} + \dfrac{1{\cdot}2 \times 10^{-3}}{6}} \text{ F}$$

$$= \mathbf{0{\cdot}0295 \times 10^{-6}\ F}$$

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Charge on plates,

$$Q = CV = 0{\cdot}0295 \times 10^{-6} \times 1000 \text{ C}$$
$$= 29{\cdot}5 \times 10^{-6} \text{ C}$$

Electric flux density

$$D = \frac{29{\cdot}5 \times 10^{-6}}{2} = 14{\cdot}75 \times 10^{-6} \text{ C/m}^2$$

$$\therefore \quad E_1 = \frac{D}{\epsilon_0 \epsilon_1} = \frac{14{\cdot}75 \times 10^{-6}}{8{\cdot}854 \times 10^{-12} \times 2} = 833{\cdot}3 \text{ kV/m}$$

$$E_2 = \frac{14{\cdot}75 \times 10^{-6}}{8{\cdot}854 \times 10^{-12} \times 3} = 555{\cdot}4 \text{ kV/m}$$

$$E_3 = \frac{14{\cdot}75 \times 10^{-6}}{8{\cdot}854 \times 10^{-12} \times 6} = 277{\cdot}7 \text{ kV/m}$$

Example 6-5. *A parallel-plate capacitor has plates of 1500 cm² separated by 5 mm with air dielectric. If a layer of dielectric 2 mm thick and relative permittivity 3 is now introduced between the plates, what must be the separation between the plates to bring the capacitance to the original value ?*

Solution. By introducing a layer of a dielectric, the effective distance between the plates is reduced by $(t - t/\epsilon_r)$

$\therefore$ decrease in separation

$$= t - \frac{t}{\epsilon_r} = 2 - \frac{2}{3} = 1{\cdot}33 \text{ mm}$$

Hence, the plates have to be further separated by a distance of 1·33 mm. The total distance or the new separation between the plates should be $= 5 + 1{\cdot}33 = \mathbf{6{\cdot}33 \text{ mm}}$.

Example 6-6. *A capacitor is composed of two plates separated by a sheet of insulating material 3 mm thick and of relative permittivity 4. The distance between the plates is increased to allow the insertion of a second sheet 5 mm thick of relative permittivity ϵ_r. If the capacitance of the capacitor so formed is $\frac{1}{3}$ of the original capacitance, find ϵ_r.*

Solution. Let the plate area be A m².

In the first case,

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{4\epsilon_0 A}{3 \times 10^{-3}} \text{ F}$$

In the second case,

$$C_2 = \frac{\epsilon_0 A}{(d_1/\epsilon_{r1} + d_2/\epsilon_{r2})} \text{ F}$$

$$= \frac{\epsilon_0 A}{\frac{3 \times 10^{-3}}{4} + \frac{5 \times 10^{-3}}{\epsilon_{r2}}} \text{ F}$$

Since $C_2 = \frac{C_1}{3}$

$$\therefore \quad \frac{\epsilon_0 A}{\frac{3 \times 10^{-3}}{4} + \frac{5 \times 10^{-3}}{\epsilon_{r2}}} = \frac{4\epsilon_0 A}{3 \times 3 \times 10^{-3}} \qquad \therefore \quad \epsilon_{r2} = 3{\cdot}3$$

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Example 6-7. *Calculate the capacitance and energy stored in a parallel-plate capacitor which consists of two metal plates, each 60 cm² separated by a dielectric of 1·5 mm thickness and of $\epsilon_r = 3{\cdot}5$ if a p.d. of 1000 V is applied across it.*

Given : $\epsilon_0 = 8{\cdot}854 \times 10^{-12}$ *F/m.*

(A.M.I.E. Winter 1980)

Solution. $C = \dfrac{\epsilon_0 \epsilon_r A}{d}$ farad

Here, $\epsilon_r = 3{\cdot}5$, $A = 60 \text{ cm}^2 = 6 \times 10^{-3} \text{ m}^2$,

$d = 1{\cdot}5 \times 10^{-3}$ m

$$\therefore \quad C = \frac{8{\cdot}854 \times 10^{-12} \times 3{\cdot}5 \times 6 \times 10^{-3}}{1{\cdot}5 \times 10^{-3}}$$

$$= 124 \times 10^{-12} \text{ F}$$

$$= \mathbf{124 \text{ pF}}.$$

Energy stored $= \frac{1}{2} CV^2$

$$= \tfrac{1}{2} \times 124 \times 10^{-12} \times 1000^2$$

$$= 62 \times 10^{-6} \text{ J}$$

$$= \mathbf{62 \ \mu J}.$$

$\therefore$

Tutorial Problems—7

1. The plates of a 0·0005 μ F parallel-plate capacitor are spaced 5 mm apart in air. What is the area of each of the capacitor plates ?

If two dielectrics, each 2·5 mm thick and of relative permittivities 2 and 3 were placed between the plates, what would be the new capacitance ?

[0·283 m².; 0·0012 μF]

2. A parallel-plate capacitor has an effective plate area of each plate of 100 cm², the plates being separated by a dielectric 0·5 mm thick. Its capacitance is 442 μμF and it is charged to a p.d. of 10 kV. Calculate :

(*a*) the potential gradient in the dielectric
(*b*) the electric flux density in the dielectric
(*c*) the relative permittivity of the dielectric material

[(*a*) 2×10^7 V/m (*b*) 442 μC/m² (c) 2·5]

3. A p.d. of 10 kV is applied across a capacitor formed by two parall metal sheets, each of area 100 cm² separated by a dielectric 3 mm thick having a capacitance of 4×10^{-4} μF. Find :

(*a*) total flux
(*b*) potential gradient
(*c*) the relative permittivity of the dielectric meterial
(*d*) the electric flux density.

[(*a*) 4 μC (*b*) 3·3 kV/mm (*c*) 13·6 (*d*) 0·4 mC/m²]

4. Calculate the capacitance of a capacitor which has five plates spaced 1 mm apart in air. Each plate has an area of 20 cm². What would be the capacitance of this capacitor if it were immersed in oil of relative permittivity 2·2 ?

[70·7 pF ; 155·5 pF]

5. A p.d. of 15 kV is applied across the terminals of a capacitor consisting of two circular plates, each having an area of 200 cm² and separated by 1 mm thickness of dielectric. The capacitance of this capacitor is $4{\cdot}5 \times 10^{-4}$ μF. Calculate the dielectric constant (ϵ_r) and the electric flux density.

[337·5 μC/m² ; 2·54] *(A.M.I.E. Nov. 1967)*

6. A capacitor consists of two metal plates each 40 cm × 40 cm, spaced 6 mm apart. The space between the metal plates is filled with a glass plate 5 mm thick and a layer of paper 1 mm thick. The relative permittivities

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of glass and paper are 8 and 2 respectively. Calculate (i) the capacitance (ii) the potential gradient in each dielectric in kV/mm due to a p.d. of 10 kV between metal plates (neglect any fringing flux). (A.M.I.E Nov 1972)

[(i) $1{\cdot}257\times10^{-9}$ F (ii) 1·11 kV/mm, 4·44 kV/mm]

6-8. Cylindrical Capacitor

A.M.I.E. Summer 1981

A single-core cable or cylindrical capacitor consisting of two co-axial cylinders of radii a and b metres, is shown in Fig. 6-12. Let the charge per metre length of the cable on the outer surface of the inner cylinder be $+Q$ coulombs and on the inner surface of the outer cylinder be $-Q$ coulombs (by induction). For all practical purposes, the charge $+Q$ coulombs/metre on the surface of the inner cylinder can be supposed to be located along its axis. Let ϵ_r be the relative permittivity of the medium between the two cylinders. The outer cylinder is earthed.

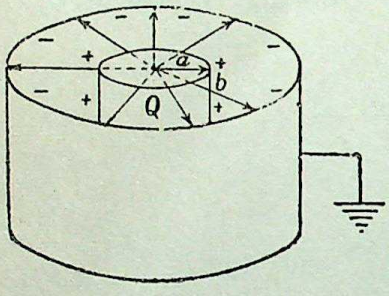

Fig. 6-12

Now, let us find the value of electric intensity at any point distant x metres from the axis of the inner cylinder. As shown in Fig. 6-13, consider an imaginary co-axial cylinder of radius x metres and length one metre between the two given cylinders. The electric field between the two cylinders is radial as shown. Total flux coming out radially from the curved surface of this imaginary cylinder is Q coulombs. Area of the curved surface

$$=2\pi x\times 1=2\pi x \text{ m}^2$$

Hence, the value of the electric flux density on the surface of the imaginary cylinder is

$$D=\frac{\text{Flux in coulombs}}{\text{Area in m}^2}$$

$$=\frac{\Psi}{A}=\frac{Q}{2\pi x}\ \text{C/m}^2$$

The value of electric intensity

$$E=\frac{D}{\epsilon_0\epsilon_r}\ \text{V/m}$$

$$=\frac{Q}{2\pi\,\epsilon_0\,\epsilon_r\,x}\ \text{V/m}$$

Now $dV=-E.dx$

or $$V=\int_{x=b}^{x=a}-E.dx=\int_b^a\frac{-Q\,.\,dx}{2\pi\,\epsilon_0\,\epsilon_r\,x}$$

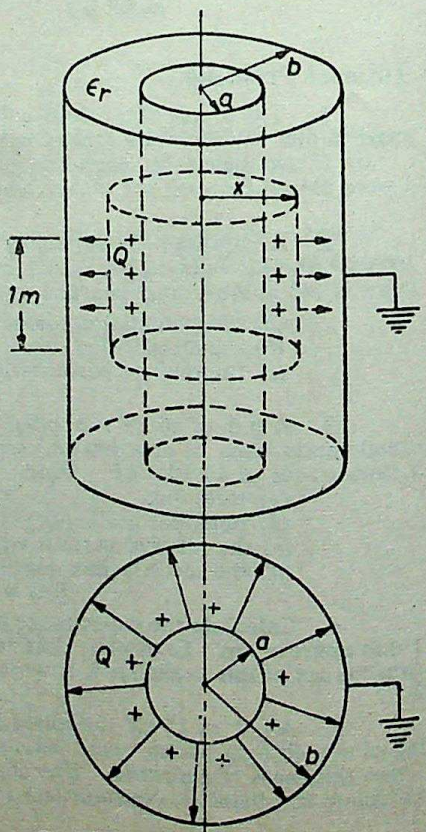

Fig. 6-13

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$$= \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \int_b^a \frac{dx}{x} = \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \Big| \log x \Big|_b^a$$

$$= \frac{-Q}{2\pi \epsilon_0 \epsilon_r} (\log_e a - \log_e b)$$

$$= \frac{-Q}{2\pi \epsilon_0 \epsilon_r} \log_e a/b = \frac{Q}{2\pi \epsilon_0 \epsilon_r} \log_e b/a$$

$$\frac{Q}{V} = \frac{2\pi \epsilon_0 \epsilon_r}{\log_e b/a}$$

$$\therefore \quad C = \frac{2\pi \epsilon_0 \epsilon_r}{2{\cdot}3 \log_{10} b/a} \text{ F/m} \qquad (\because \log_e b/a = 2{\cdot}3 \log_{10} b/a)$$

The capacitance of l metres length of this cable is

$$C = \frac{2\pi \epsilon_0 \epsilon_r l}{2{\cdot}3 \log_{10} b/a} \text{ F}$$

The capacitance of 1 km length of the cable in μF can be found by putting $l = 1$ km in the above expression.

$$C = \frac{2\pi \times 8{\cdot}854 \times 10^{-12} \times \epsilon_r \times 1000}{2{\cdot}3 \log_{10} b/a} \text{ F/km}$$

$$= \frac{0{\cdot}024\, \epsilon_r}{\log_{10} b/a} \mu\text{F/km}$$

6-9. Potential Gradient in a Cylindrical Capacitor

(A.M.I.E., Summer 1979)

It is seen from Art. 6-8 that in a cable capacitor

$$E = \frac{Q}{2\pi \epsilon_0 \epsilon_r x} \text{ V/m}$$

where x is the distance from cylinder axis to the point under consideration

Now $E = g$

$$\therefore \quad g = \frac{Q}{2\pi \epsilon_0 \epsilon_r x} \text{ V/m} \qquad ...(i)$$

From Art. 6-8, we find that

$$V = \frac{Q}{2\pi \epsilon_0 \epsilon_r} \log_e b/a$$

$$\therefore \quad Q = \frac{2\pi \epsilon_0 \epsilon_r V}{\log_e b/a}$$

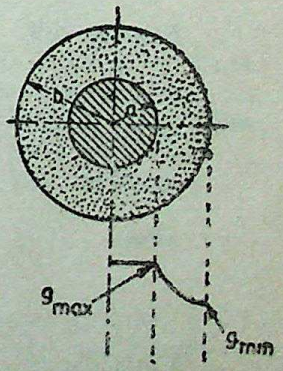

Fig. 6-14

Substituting the value of Q in (i) above, we get

$$g = \frac{2\pi \epsilon_0 \epsilon_r V}{\log_e b/a \times 2\pi \epsilon_0 \epsilon_r x} \text{ V/m}$$

or $$g = \frac{V}{x \log_e b/a} \text{ volt/metre}$$

or $$g = \frac{V}{2{\cdot}3\, x \log_{10} b/a} \text{ volt/metre}$$

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Obviously, potential gradient varies inversely as x.

Minimum value of $x=a$, hence maximum value of potential gradient is

$$g_{max}=\frac{V}{2{\cdot}3\ a\ \log_{10}{}^{b/a}}\ \text{V/m}$$

Similarly,

$$g_{min}=\frac{V}{2{\cdot}3\ b\ \log_{10}{}^{b/a}}\ \text{V/m}$$

Example 6-8. *A concentric cable has a core diameter of 2·36 cm. It is insulated with impregnated paper of thickness 1·65 cm and a relative permittivity of 3·5 and covered with lead sheath. If working voltage is 660 V d.c., calculate the capacitance of the cable in μF/km and maximum electric stress (potential gradient).*

(A.M.I.E. Winter, 1978)

Solution. Given :

$$a=2{\cdot}36/2=1{\cdot}18 \text{ cm} ; b=1{\cdot}18+1{\cdot}65=2{\cdot}83 \text{ cm}$$

$$b/a=2{\cdot}83/1{\cdot}18=2{\cdot}4 ; \epsilon_r=3{\cdot}5$$

$$C=\frac{0{\cdot}024\ \epsilon_r}{\log_{10}{}^{b/a}}\ \mu\text{F/km}$$

$$=0{\cdot}024\times 3{\cdot}5/\log_{10}{}^{2{\cdot}4}=0{\cdot}084/0{\cdot}38=\mathbf{0{\cdot}22\ \mu F/km}$$

$$g_{max}=\frac{V}{2{\cdot}3\ a\ \log_{10}{}^{b/a}}$$

$$=\frac{660}{2{\cdot}3\times 1{\cdot}18\times 10^{-2}\times 0{\cdot}38}=\mathbf{64\ kV/m}$$

Example 6-9. *Calculate (a) the capacitance and (b) the resistance of a cable 5 km long having an inner core of diameter 1 cm. and an impregnated insulation of thickness 0·7 cm and relative permittivity 4. Specific resistance of copper may be taken as 2μΩ-cm.*

Solution. (*a*) $$C=\frac{0{\cdot}024\ \epsilon_r}{\log_{10}{}^{b/a}}\ \mu\text{F/km}$$

Here $$a=0{\cdot}5 \text{ cm} ; b=0{\cdot}5+0{\cdot}7=1{\cdot}2 \text{ cm}$$

$$b/a=12/5=2{\cdot}4 ; \log_{10}{}^{2{\cdot}4}=0{\cdot}3802$$

$$C=0{\cdot}024\times 4/0{\cdot}3802\ \mu\text{F/km}$$

Capacitance for 5 km length

$$=5\times 0{\cdot}024\times 4/0{\cdot}3802=\mathbf{1{\cdot}263\ \mu F}$$

(*b*) Conductor resistance $R=\rho\frac{l}{A}$

Here $$\rho=2\times 10^{-6}\ \Omega/\text{cm}^3 ; l=5 \text{ km}=10^5 \text{ cm}$$

$$A=\pi\ d^2/4 \text{ cm}^2=\pi/4 \text{ cm}^2$$

$$R=\frac{2\times 10^{-6}\times 5\times 10^5}{\pi/4}=\mathbf{1{\cdot}27\ \Omega}$$

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6-10. Capacitors in Series *(A.M.I.E. May 1969)*

With reference to Fig. 6-15, let

C_1, C_2, C_3 = capacitances of three capacitors

V_1, V_2, V_3 = p.ds. across three capacitors

V = applied voltage across combination

C = combined or equivalent capacitance.

In series combination, charge on all capacitors is the same but p.d. across each is different.

$$\therefore \quad V = V_1 + V_2 + V_3$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Note. When two capacitors are connected in series, then voltages developed across the two are

$$V_1 = V\frac{C_2}{C_1+C_2} \quad \text{and} \quad V_2 = V\frac{C_1}{C_1+C_2}$$

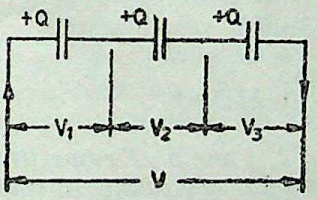

Fig. 6-15

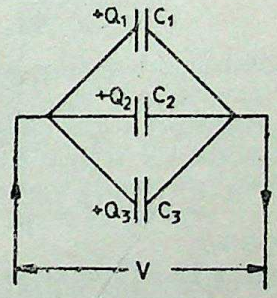

Fig. 6-16

6-11. Capacitors in Parallel *(A.M.I.E. May 1969)*

In this case, p.d. across each is the same but charge on each is different (Fig. 6-16).

$$\therefore \quad Q = Q_1 + Q_2 + Q_3$$

or $$CV = C_1V + C_2V + C_3V$$

or $$C = C_1 + C_2 + C_3$$

Example 6-10. *The total capacitance of two capacitors is 0·03 μF when joined in series and 0·16 μF when connected in parallel. Find the capacitance of each capacitor.*

Solution. Let C_1 and C_2 be the unknown capacitances.

Then $$C_1 + C_2 = 0{\cdot}16 \qquad \ldots(i) \text{ when in parallel}$$

$$\frac{C_1C_2}{C_1+C_2} = 0{\cdot}03 \qquad \ldots (ii) \text{ when in series}$$

Eliminating C_2 from (i) and (ii), we get

$$\frac{C_1(0{\cdot}16 - C_1)}{0{\cdot}16} = 0{\cdot}03$$

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or $C_1^2 - 0{\cdot}16C_1 + 0{\cdot}0048 = 0$

or $(C_1 - 0{\cdot}12)(C_1 - 0{\cdot}04) = 0$

$C_1 = 0{\cdot}12$ or $0{\cdot}04$

Hence $C_2 = 0{\cdot}04$ or $0{\cdot}12$

So, one capacitor is of **0·12 μF** and the other of **0·04 μF** capacitance.

Example 6-11. *Three capacitors are connected in series across 135-V supply. The voltages across them are 30, 45, and 60 and charge on each is 4500 microcoulombs. Find the capacitance of each capacitor and that of the combination.*

Solution.

$$C_1 = \frac{Q}{V_1} = \frac{4500\ \mu C}{30\ V} = \mathbf{150\ \mu F}$$

$$C_2 = \frac{4500\ \mu C}{45\ V} = \mathbf{100\ \mu F}$$

$$C_3 = \frac{4500\ \mu C}{60\ V} = \mathbf{75\ \mu F}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{150} + \frac{1}{100} + \frac{1}{75} = \frac{9}{300} = \frac{3}{100}$$

$$C = \frac{100}{3} = \mathbf{33{\cdot}3\ \mu F}$$

or

$$C = \frac{Q}{V} = \frac{4500}{135} = \mathbf{33{\cdot}3\ \mu F}$$

Example 6-12. *Three capacitors of 2, 3 and 6 μF respectively are connected in series across a 500-V d.c. supply. Calculate (a) charge on each capacitor (b) the p.d. across each capacitor and (c) the energy stored in 6 μF capacitor.*

Solution. Since the capacitors are connected in series, charge on each is the same.

Total capacitance is given by

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} \qquad \therefore\ C = \mathbf{1\ \mu F}$$

(*a*) $\therefore$ Charge $Q = CV = 1 \times 500 = \mathbf{500\ \mu C}$

(*b*) $$V_1 = \frac{Q}{C_1} = \frac{500}{2} = \mathbf{250\ V}$$

$$V_2 = \frac{500}{3} = \mathbf{166{\cdot}7\ V}$$

$$V_3 = \frac{500}{6} = \mathbf{83{\cdot}3\ V}$$

(*c*) Energy stored, $E = \frac{1}{2} C_3 V_3^2$ joules

$$= \frac{1}{2} \times 6 \times 10^{-6} \left(\frac{250}{3}\right)^2 = \mathbf{208{\cdot}3 \times 10^{-4}\ J}$$

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Example 6-13. *Find the charges on capacitors in Fig. 6-17 and the p.ds. across them.*

Solution. Equivalent capacitance between points A and B is $C_1+C_2=5+3=8\ \mu F$.

Capacitance of the whole combination

$$C=\frac{8\times 2}{8+2}=1{\cdot}6\ \mu F$$

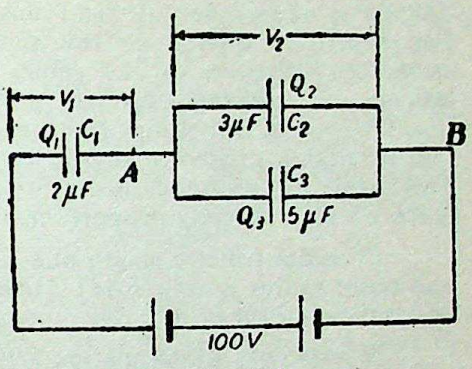

Fig. 6-17

Charge on the combination is

$$Q_1=CV=100\times 1{\cdot}6$$
$$=\mathbf{160}\ \mu C$$

$$V_1=\frac{Q_1}{C_1}=\frac{160}{2}=\mathbf{80\ V}$$

$$V_2=100-80=\mathbf{20\ V}$$

$$Q_2=C_2V_2=3\times 10^{-6}\times 20=60\times 10^{-6}\ C=\mathbf{60}\ \mu C$$

$$Q_3=C_3V_2=5\times 10^{-6}\times 20=\mathbf{100}\ \mu C$$

Example 6-14. *Two perfectly-insulated capacitors are connected in series. One is an air capacitor with a plate area of 100 cm², the plates being 1 mm apart; the other has a plate area of 10 cm², the plates being separated by a solid dielectric 0·1 mm apart with a relative permittivity of 5. Find the voltage across the combination if the potential gradient in the air capacitor is 20 kV/cm.*

(A.M.I.E. Nov. 1971)

Solution. Let C_1 and C_2 be the capacitances and V_1 and V_2 the potentials across the two capacitors connected in series as in Fig. 6-18.

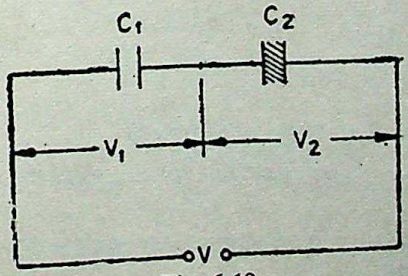

Fig. 6-18

Since the two capacitors are in series, charge across each is the same.

$$\therefore\quad Q=C_1V_1=C_2V_2$$

or $$C_1V_1=C_2V_2$$

or $$V_2=V_1\frac{C_1}{C_2}$$

Now, $$C_1=\frac{\epsilon_0\times 100\times 10^{-4}}{1\times 10^{-3}}$$

and $$C_2=\frac{5\epsilon_0\times 10\times 10^{-4}}{0{\cdot}1\times 10^{-3}}$$

$$\therefore\quad \frac{C_1}{C_2}=\frac{1}{5}$$

Now, $E_1=20$ kV/cm ; $d_1=1$ mm $=0{\cdot}1$ cm

$\therefore\quad V_1=E_1d_1=20\times 0{\cdot}1=2$ kV $=2000$ V

$\therefore\quad V_2=2000\times 1/5=400$ V $\quad\therefore\quad V=2000+400=\mathbf{2400\ V}$

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6-12. Insulation Resistance of a Cable Capacitor

(A.M.I.E. Nov. 1972)

In a cable capacitor, useful current flows along the axis of the core but there is always present some leakage of current. This leakage is radial *i.e.* at right angles to the flow of useful current. The resistance offered to this radial leakage of current is called *insulation resistance* of the cable. If cable length is greater, then leakage area is also greater. It means that more current will leak. In other words, insulation resistance is decreased. Hence, we find that insulation resistance is inversely proportional to the cable length. This insulation resistance is not to be confused with conductor resistance which is directly proportional to the cable length.

Consider l metre length of a single-core cable of inner radius r_1 and outer radius r_2 (Fig. 6-19). Imagine an annular ring of radius 'r' and radial thickness 'dr'.

If resistivity of insulating material is ρ, then resistance of this narrow ring is

$$dR = \frac{\rho\, dr}{2\pi r \times l}$$

$\therefore$ insulation resistance of l metre length of cable is

$$\int_0^R dR = \int_{r_1}^{r_2} \frac{\rho\, dr}{2\pi\, rl}$$

or
$$R = \frac{\rho}{2\pi l} \left| \log_e r \right|_{r_1}^{r_2}$$

$$R = \frac{\rho}{2\pi l} \log_e r_2/r_1$$

$\therefore$
$$R = \frac{2{\cdot}3\,\rho}{2\pi\, l} \log_{10} r_2/r_1\ \Omega$$

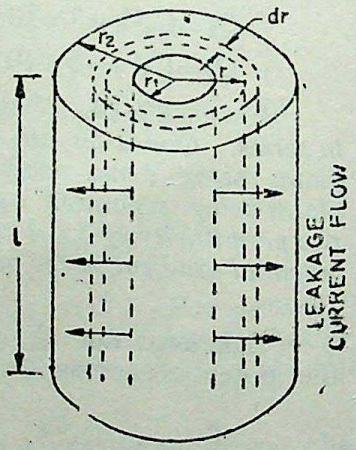

Fig. 6-19

It should be noted

(*i*) *that R is inversely proportional to the cable length*

(*ii*) *that R depends upon the ratio r_2/r_1 and not on the thickness of the insulator itself.*

Example 6-15. *Find the insulation resistance per metre of a single-core cable from the following data :—*

Core radius = 1·3 cm

Outer diameter of cable = 8 cm

Specific resistance of insulation material = 8×10^{14} ohm-metre.

Solution. Insulation resistance

$$R = \frac{2{\cdot}3\,\rho}{2\pi\, l} \log_{10} r_2/r_1 \text{ ohm}$$

Here $\rho = 8 \times 10^{14}\ \Omega$-m ; $l = 1$ m

$r_1 = 1{\cdot}3$ cm, $r_2 = 8/2 = 4$ cm

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$$\therefore \quad R = \frac{2.5 \times 8 \times 10^{14}}{2\pi \times 1} \log_{10} 4\,1.3 = 1.43 \times 10^{14}\ \Omega$$

6-13. Energy Stored in a Capacitor

(A.M.I.E. Summer 1983)

Charging of a capacitor always involves some expenditure of energy by the charging agency. This energy is stored up in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.

To begin with when the capacitor is uncharged, little work is done in transferring charge from one plate to another. But further instalments of charge have to be carried against the potential difference established between the plates. Let us find the energy spent in charging a capacitor of capacitance C to a voltage V.

(*a*) First Method

Suppose at any stage of charging, the p.d. across the plates is v and plate charge q. By definition, v is equal to the work done in shifting one coulomb of charge from one plate to another. If 'dq' is the charge next transferred, the work done is

$$dw = v.dq$$

Now $\quad q = Cv \qquad dq = C.dv$

$$\therefore \quad dw = C\,v\,d\,v$$

Total work done in giving V units of potential is

$$W = \int_0^V C\,v\,dv = C\left|\frac{v^2}{2}\right|_0^V$$

$$\therefore \quad W = \frac{1}{2} CV^2 \text{ joules}$$

provided C is in farads and V is in volts.

Also $\quad W = \frac{1}{2} QV$ joules $\qquad (\because\ C = Q/V)$

$$= \frac{Q^2}{2C} \text{ joules} \qquad (\because\ V = Q/C)$$

If Q is in coulombs and C is in farads, then energy stored is given in joules.

(*b*) Second Method

For constant capacity C, $V \propto Q$, hence the graph between V and Q is a straight line passing through the origin as shown in Fig. 6-20. The energy required to charge the capacitor with Q coulombs for a p.d. of V volts is given by the area of the triangle OAB which is equal to $\frac{1}{2}$(base × height).

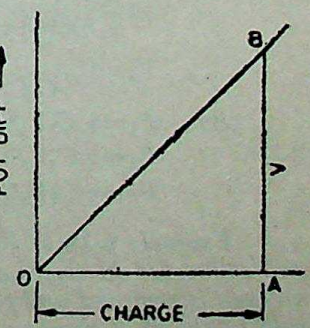

Fig. 6-20

$$\therefore \text{ Energy stored} = \tfrac{1}{2} QV \text{ joules}$$

$$= \tfrac{1}{2} CV^2 \text{ joules}$$

$$= Q^2/2C \text{ joules}$$

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Example 6-16. *A parallel-plate capacitor has 20 plates each of 0·5 m² area, the separation between the plates being 0·2 cm with a dielectric of relative permittivity 2·5.*

Calculate (i) the capacitance (ii) the value of the stored energy if it is charged to 1000 volts. *(A.M.I.E. Winter 1977)*

Solution. $C=\dfrac{(n-1)\,\epsilon_0\,\epsilon_r\,A}{d}$ —Art 6-6

$$=\frac{(20-1)\times 8{\cdot}854\times 10^{-12}\times 2{\cdot}5\times 0{\cdot}5}{2\times 10^{-3}}=\mathbf{105\times 10^{-9}\ F}$$

Energy stored

$$=\tfrac{1}{2}\,CV^2=\tfrac{1}{2}\times 105\times 10^{-9}\times 1000^2=\mathbf{0{\cdot}0525\ J}$$

Example 6-17. *Three capacitors 5, 10, and 12 μF capacitance are connected in series across 600-V dc mains. Determine*

(*i*) *equivalent capacitance of the combination*
(*ii*) *charge on each capacitor*
(*iii*) *potential across each capacitor*
(*iv*) *energy stored in each capacitor* *(A.M.I.E. Summer 1985)*

Solution. (*i*) $1/C=1/5+1/10+1/12$ —Art 6-10

$\therefore \quad C=(60/23)\ \mu F=2{\cdot}6\ \mu F$

(*ii*) charge on each capacitor is the same as that drawn by the combination from the supply.

$\therefore \quad Q_1=Q_2=Q_3=Q=CV=2{\cdot}6\times 600=\mathbf{1560\ \mu C}$

(*iii*) If V_1, V_2 and V_3 are the potentials across the three capacitors respectively, then

$$Q=C_1V_1=C_2V_2=C_3V_3=CV$$

$$\therefore \quad C_1V_1=CV \quad \text{or} \quad V_1=V\,.\,\frac{C}{C_1} \qquad \ldots(i)$$

$$\text{Now,}\ \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \quad \text{or} \quad C=\frac{C_1C_2C_3}{C_1C_2+C_2C_3+C_3C_1}$$

$$\therefore \quad V_1=V\frac{C_2C_3}{C_1C_2+C_2C_3+C_3C_1}=600\times\frac{10\times 12}{230}=\mathbf{313\ V}$$

$$V_2=V\frac{C_1C_3}{C_1C_2+C_2C_3+C_3C_1}=600\times\frac{5\times 12}{230}=\mathbf{157\ V}$$

$$V_3=V\frac{C_1C_2}{C_1C_2+C_2C_3+C_3C_1}=600\times\frac{5\times 10}{230}=\mathbf{130\ V}$$

(*iv*) $E_1=\frac{1}{2}\,C_1V_1^2=\frac{1}{2}\times 5\times 10^{-6}\times 313^2=\mathbf{0{\cdot}245\ J}$

$E_2=\frac{1}{2}\,C_2V_2^2=\frac{1}{2}\times 10\times 10^{-6}\times 157^2=\mathbf{0{\cdot}123\ J}$

$E_3=\frac{1}{2}\,C_3V_3^2=\frac{1}{2}\times 12\times 10^{-6}\times 130^2=\mathbf{0{\cdot}101\ J}$

Example 6-18(a). *Two capacitors of capacitance 8 μF and 2 μF are connected in series across a 100-V d.c. supply. Now, if the supply voltage is removed and capacitors are then connected in parallel, what will be the final charge on each capacitor ?* *(A.M.I.E. Summer 1976)*

Solution. Since the two capacitors are in series (Art. 6-10)

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$$V_1 = V\frac{C_2}{C_1+C_2} = 100\times\frac{2}{8+2} = 20 \text{ V}$$

$$V_2 = V\frac{C_1}{C_1+C_2} = 100\times\frac{8}{10} = 80 \text{ V}$$

$$Q_1 = C_1V_1 = 8\times 20 = 160\ \mu\text{C}$$

$$Q_2 = C_2V_2 = 2\times 80 = 160\ \mu\text{C}$$

When the two capacitors are connected in parallel, they attain a common potential, say, V, which is given by

$$V = \frac{\text{total charge}}{\text{total capacitance}} = \frac{2\times 160}{8+2} = 32 \text{ V}$$

$$\therefore \quad Q_1 = C_1V = 8\times 32 = \mathbf{256}\ \mu\text{C}$$

$$Q_2 = C_2V = 2\times 32 = \mathbf{64}\ \mu\text{C}$$

Example 6-18 (*b*) *Two capacitors A and B are placed* (*i*) *in series* (*ii*) *in parallel. Capacitor* $C_A = 100\ \mu F$ *and* $C_B = 50\ \mu F$. *Find the maximum stored energy in the circuit when 240 V, 50 Hz supply is applied.* (*A.M.I.E. Winter 1985*)

Solution. $V_{max} = \sqrt{2}.V = 240\sqrt{2}$ V —Art. 16—13

(*i*) $C = C_A.C_B/(C_A+C_B) = 100\times 50/150 = 100/3\ \mu\text{F}$

$$E_{max} = \tfrac{1}{2}CV_{max}^2 = \tfrac{1}{2}(100/3)(240\sqrt{2})^2 = \mathbf{1{\cdot}92\ J}$$

(*ii*) $C = C_A + C_B = 150\ \mu\text{F}$, $E_{max} = \tfrac{1}{2}\times 150\times(240\sqrt{2})^2 = \mathbf{8{\cdot}64\ J}$.

Example 6-19. *A steady voltage of 5,000 V is applied across two circular parallel metal plates each having an area of one square metre and 18 cm apart. Between the plates are two layers of dielectric of thicknesses* $t_1 = 6$ *cm and* $t_2 = 12$ *cm and relative permittivities of* $\epsilon_{r1} = 3$ *and* $\epsilon_{r2} = 4$ *respectively. Calculate the voltage gradient in each dielectric and the total energy stored.*

Solution.

$$C = \frac{\epsilon_0 A}{\left(\frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}}\right)}$$

$$= \frac{8{\cdot}854\times 10^{-12}\times 1}{\left(\frac{0{\cdot}06}{3} + \frac{0{\cdot}12}{4}\right)} = 1{\cdot}77\times 10^{-10}\text{ F}$$

Total energy stored $= \tfrac{1}{2}\times 1{\cdot}77\times 10^{-10}\times 5{,}000^2$ J

$$= \mathbf{22{\cdot}1\times 10^{-4}\ J}$$

Now, $D = \epsilon_0\epsilon_{r1}E_1 = \epsilon_0\epsilon_{r2}E_2$

$\therefore$ $3E_1 = 4E_2$ ...(*i*)

Also $V = V_1 + V_2 = E_1t_1 + E_2t_2$

$\therefore$ $5\times 10^3 = 0{\cdot}06\,E_1 + 0{\cdot}12\,E_2$

or $6E_1 + 12E_2 = 5\times 10^5$

$3E_1 + 6E_2 = 2{\cdot}5\times 10^5$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\mathbf{E_2 = 2{\cdot}5\times 10^4\ V/m\ ;\ E_1 = 3{\cdot}33\times 10^4\ V/m}$$

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6-14. Energy Stored Per Unit Volume of the Dielectric

It has been shown in Art. 6-13 that energy stored in a capacitor is

$$E=\tfrac{1}{2}\,C\,V^2$$

In the case of a parallel-plate capacitor

$$C=\epsilon A/d$$

$$\therefore \quad E=\tfrac{1}{2}\times\frac{\epsilon A}{d}\cdot V^2=\tfrac{1}{2}\,\epsilon A.d\left(\frac{V}{d}\right)^2$$

$$=\tfrac{1}{2}\,\epsilon E^2\times Ad$$

Now $A\times d$ = volume of the dielectric medium.

$$\therefore \text{ Energy/volume } =\frac{1}{2}\,\epsilon E^2=\frac{D^2}{2\epsilon}\ \text{J/m}^3$$

The above expression represents the energy required for creating a unit volume of an electric field having a flux density of D C/m².

6-15. Force of Attraction Between Oppositely-charged Plates

(A.M.I.E. Winter 1974)

Fig. 6-21 shows a parallel-plate capacitor having a plate area of A m² and plate separation of d metres. Let the force of attraction between the two oppositely-charged plates M and N be F newton. Let plate N be pulled further apart against force F through a small distance dx. Work done is $F\times dx$. This energy goes to supply the necessary energy for the additional volume of the magnetic field produced.

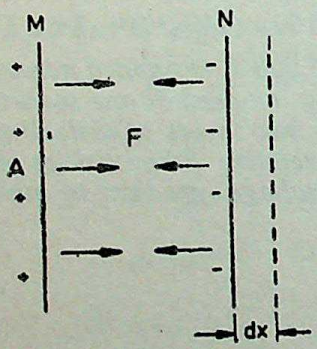

Fig. 6-21

Additional volume of electric field produced is

$$=A\times dx \text{ metre}^3$$

$$\text{Energy required}=A\,dx\times\frac{D^2}{2\epsilon}$$

$$\therefore \quad F\times dx=\frac{D^2}{2\epsilon}A\,dx \quad \text{or} \quad F=\frac{D^2A}{2\epsilon}\ \text{newton}$$

Force per unit area of either plate is

$$=\frac{D^2}{2\epsilon}\ \text{N/m}^2=\tfrac{1}{2}\,\epsilon E^2\ \text{N/m}^2$$

Example 6-20. *A parallel-plate capacitor having plates of 100 cm² area, is immersed in oil of relative permittivity 10. The plates are charged to a potential difference of 50 kV and the distance between them is 2 cm. Permittivity of free space* $=8{\cdot}854\times10^{-12}$ *F/m. Determine the force between the plates.* *(A.M.I.E. Winter 1974)*

Solution. $F=\tfrac{1}{2}\,\epsilon_0\epsilon_r AE^2$ newton

Here, $E=50\times10^2/2\times10^{-2}=25\times10^5$ V/m ; $\epsilon_r=10$

$A=100\ \text{cm}^2=10^{-2}\ \text{m}^2$; $\epsilon_0=8{\cdot}854\times10^{-12}$ F/m

$$\therefore \quad F=\tfrac{1}{2}\times8{\cdot}854\times10^{-12}\times10\times10^{-2}\times(25\times10^5)^2=5{\cdot}53\ \text{N}$$

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Tutorial Problems—8

1. A submarine cable is 3,000 km long. The inner core is of 0·4 cm diameter. The outside diameter of the gutta-percha is 1 cm. If relative permittivity of gutta-percha is 4·05, find the capacitance of the cable. **[739 μF]**

2. A circuit comprises of a 2 μF capacitor in parallel with a 3 μF capacitor and a 5 μF capacitor is in series with this group. Calculate the effective capacitance of the circuit.

A direct voltage of 100 V is applied to the above circuit. Calculate the p.d. across each capacitor and the charge stored by each capacitor. Show how the capacitors could be connected to give an effective capacitance of 6·2 μF.
[2·5 μF ; 250 μC, 50 V ; 100 μC ; 150 μC]

3. Two initially uncharged capacitors of 1 and 2 μF are connected in series across a 100 V source. Calculate

(*a*) the capacitance of the two in series

(*b*) the voltage across each capacitance

(*c*) the energy stored in each.

[(*a*) $\frac{2}{3}$ μF (*b*) 66·7 V ; 33·3 V (*a*) 2·22 mJ ; 1·11 mJ]

4. Two perfectly-insulated capacitors are connected in series ; one is an air capacitor with a plate area 100 cm², the plates being 1 mm apart. The other has a plate area of 10 cm², the plates being separated by a dielectric 0·1 mm thick with a relative permittivity of 5. Find the p.d. across the combination if the potential gradient in the air capacitor is 20 kV/ cm. **[2,400 V]**

5. Four perfect capacitors are connected as shown to the three terminals *A*, *B* and *C* (Fig. 6-22).

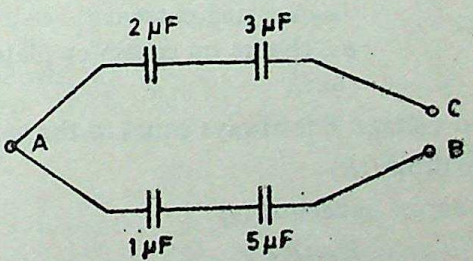

Fig. 6-22

(*a*) Calculate the value of capacitance that would be measured across terminals *BC*.

(*b*) Terminals *B* and *C* are now connected together. What capacitance would be measured across *AB* ?

(*c*) Find the energy stored in the whole circuit in the second condition when a 100 V battery is connected across *AB*.

[(*a*) $\frac{30}{61}$ μF (*b*) $\frac{61}{30}$ μF (*c*) 10·2 mJ]

6. Two parallel plates spaced 1·0 cm apart in air, each have an area of 1·0 m². If the p.d. between them is 1,000 V, calculate

(a) the energy stored in joules

(*b*) the force between them

If the dielectric were mica with $\epsilon_r = 7$ instead of air, calculate the new values of energy and force. **[(*a*) 442 μJ (*b*) 0·309 N]**

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7. A parallel-plate capacitor is immersed in alcohol of $\epsilon_r=25$. The plates are charged to a p.d. of 20 kV and the distance between the plates is 15 mm. Determine per m^2 of plate area the force between the plates and the energy stored. [197·9 N/m^2 ; 2·95 J) (*A.M.I.E. May 1969*)

8. Two capacitors of capacitance 2 μF and 4 μF respectively are connected in series. A potential difference of 900 volts is applied between the extreme terminals. Find the p.d. across each capacitor. [600 V, 300 V] (*A.M.I.E. Winter 1983*)

(*A.M.I.E. Winter 1986*)

6-16. Charging of a Capacitor

In Fig. 6-23 is shown an arrangement by which a capacitor C may be charged through a high resistance R from a battery of V volts. The voltage across C can be measured by a suitable voltmeter. When switch S is connected to 'a', C is charged, but when it is connected to 'b', C is short-circuited through R and is thus discharged. The voltage across C does not rise to V instantaneously but builds up slowly. Charging current is maximum at the start *i.e.* when C is uncharged, then it gradually decreases and finally ceases when p.d. across capacitor plates becomes equal and opposite to that of the battery. At any instant, let

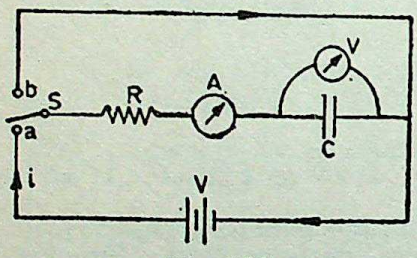

Fig. 6-23

v=p.d. across C

i=charging current

q=charge on capacitor plates

$=Cv$

The applied voltage V is always equal to the sum of

(*i*) resistive drop (iR)

(*ii*) voltage across capacitor (v)

$$\therefore \quad V=iR+v \qquad \ldots(i)$$

Now $$i=\frac{dq}{dt}=\frac{d}{dt}(Cv)=C\frac{dv}{dt}$$

$$\therefore \quad V=v+CR\frac{dv}{dt} \qquad \ldots(ii)$$

$$\therefore \quad -\frac{dv}{V-v}=-\frac{dt}{CR}$$

Integrating both sides, we get

$$-\int\frac{dv}{V-v}=-\frac{t}{CR}\int dt$$

$$\therefore \quad \log_e(V-v)=-\frac{t}{RC}+K \qquad \ldots(iii)$$

where K is the constant of integration whose value can be found from initial known conditions. We know that at the start of charging when $t=0$, $v=0$.

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Substituting these values in (*ii*), get $\log_e V = K$

Hence Eq. (*iii*) becomes

$$\log_e (V-v) = \frac{-t}{CR} + \log_e V$$

$$\log_e \frac{V-v}{V} = \frac{-t}{CR} = -\frac{t}{\lambda}$$

where $\lambda = CR =$ time constant

$$\therefore \quad \frac{V-v}{V} = e^{-t/\lambda} \quad \text{or} \quad v = V\left(1-e^{-t/\lambda}\right) \qquad \ldots(iv)$$

This gives variation with time of voltage across the capacitor plates and is shown in Fig. 6-24.

Since $v = \frac{q}{C}$ and $V = \frac{Q}{C}$

$\therefore$ Equation (*iv*) becomes

$$\frac{q}{C} = \frac{Q}{C}\left(1-e^{-t/\lambda}\right)$$

$$\therefore \quad q = Q\left(1-e^{-t/\lambda}\right) \qquad \ldots(v)$$

We find that the increase of charge, like growth of potential, follows an exponential law in which the steady value is reached after infinite time.

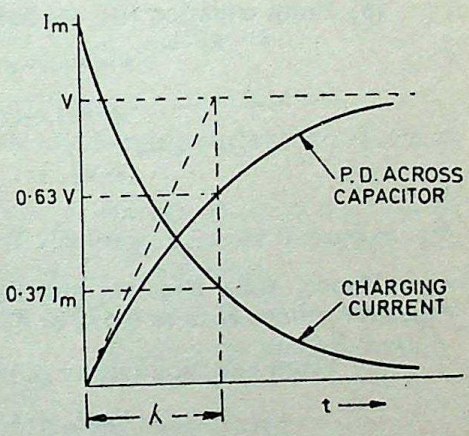

Fig. 6-24

Now $\quad i = \frac{dq}{dt}$

$\therefore$ from equation (*v*), differentiating both sides, we get

$$\frac{dq}{dt} = i = Q\frac{d}{dt}\left(1-e^{-t/\lambda}\right)$$

$$= Q\left(+\frac{1}{\lambda}e^{-t/\lambda}\right) = \frac{Q}{\lambda}\, e^{-t/\lambda}$$

$$= \frac{CV}{CR}\, e^{-t/\lambda} \qquad (\because \; Q = CV \text{ and } \lambda = CR)$$

$$\therefore \quad i = \frac{V}{R}\, e^{-t/\lambda} \quad \text{or} \quad i = I_m\, e^{-t/\lambda} \qquad \ldots(vi)$$

where $I_m =$ maximum current $= V/R$

As charging continues, charging current decreases according to equation (*vi*) as shown in Fig. 6-24.

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6-17. Time Constant *(A.M.I.E. Nov. 1970)*

(*a*) Just at the start, p.d. across capacitor is zero, hence from Eq (*ii*), putting $v=0$, we get

$$V=CR\ \frac{dv}{dt}$$

$\therefore$ Initial rate of rise of voltage

$$\frac{dv}{dt}=\frac{V}{CR}\ \text{V/s}$$

If this rate of rise were maintained, then time taken to reach voltage V would have been $=V\div V/CR=CR$. This time is known as *time constant* (λ) of the circuit.

(*b*) From equation (*iv*), we find that if $t=\lambda$, then

$$v=V(1-e^{-t/\lambda})$$

$$=V(1-e^{-\lambda/\lambda})=V(1-e^{-1})$$

$$=V\left(1-\frac{1}{e}\right)=V\left(1-\frac{1}{2{\cdot}718}\right)$$

$$v=0{\cdot}632\ \text{V}$$

Hence, time constant may be defined as *the time during which capacitor voltage rises to 0·632 or 63·2% of its final steady value.*

(*c*) From equation (*vi*) by putting $t=\lambda$, we get

$$i=I_m\ e^{-\lambda/\lambda}=I_m\ e^{-1}$$

$$i=\frac{I_m}{2{\cdot}718}=0{\cdot}37\ I_m\ ;\quad i=0{\cdot}37\ I_m$$

Hence, time constant of a circuit is also *the time during which the charging current falls to 0·37 or 37% of its initial maximum value.*

6-18. Discharging of a Capacitor

In Fig. 6-23 when S is shifted to '*b*', C is discharged through R. To begin with, discharge current is maximum but then decreases till it ceases when capacitor is fully discharged. As battery is cut out of the circuit, therefore, by putting $V=0$ in equation (*ii*), we get

$$0=CR\ \frac{dv}{dt}+v$$

or

$$v=-CR\ \frac{dv}{dt}$$

$$\frac{dv}{v}=-\frac{dt}{CR}$$

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or $$\int \frac{dv}{v} = -\frac{1}{CR}\int dt$$

$$\therefore \quad \log_e v = -\frac{t}{CR} + K$$

At the start of discharge, when $t=0$, $v=V$

$$\therefore \quad \log_e V = 0 + K$$

or $$\log_e V = K$$

Putting this value above, we get

$$\log_e v = -\frac{t}{\lambda} + \log_e V$$

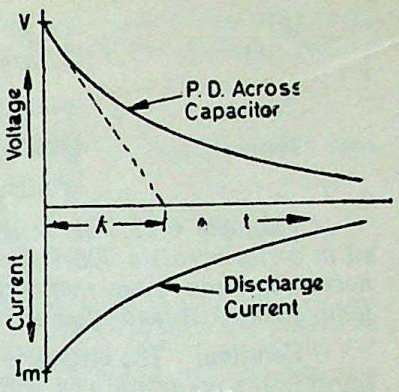

Fig. 6-25

or $$\log^{v/V} = -t/\lambda$$

or $$\frac{v}{V} = e^{-t/\lambda} \; ; \quad \text{or} \quad v = V\, e^{-t/\lambda}$$

Similarly $$q = Q\, e^{-t/\lambda} \quad \text{and} \quad i = -I_m\, e^{-t/\lambda}$$

The fall of potential and the discharging current are shown in Fig. 6-25.

Example 6-21. *A capacitor is charged from a d.c. source through a resistor of 0·5 megohm. If the p.d. across it reaches 75% of its initial value in half a second, find its capacitance.*

Solution. Using the relation $v = V(1-e^{-t/\lambda})$

we have $v = 0{\cdot}75\, V$ — V being the applied voltage

$t = 0{\cdot}5$ second

$$\therefore \quad 0{\cdot}75V = V(1-e^{-1/2\lambda}) \quad \text{or} \quad e^{-1/2\lambda} = 0{\cdot}25$$

$$\therefore \quad \frac{-1}{2\lambda} = \log_e 0{\cdot}25 \quad \text{or} \quad \lambda = 0{\cdot}3612 \text{ second (approx)}$$

Now $\lambda = CR$

When C is in farads and R in ohms, then λ is in seconds.

$$\therefore \quad C \times 0{\cdot}5 \times 10^6 = 0{\cdot}3612$$

$$C = 0{\cdot}3612/500{,}000 \text{ farad}$$

$$= 0{\cdot}7224 \times 10^{-6} \text{ farad} = \mathbf{0{\cdot}7224\ \mu F}$$

Example 6-22. *A 16 μF capacitor is connected in series with a 0·25 MΩ resistor across a 400-V d.c. supply. Calculate (a) the time taken for the p.d. across the capacitor to grow to 320 V (b) the charge across the capacitor 4 seconds after it is connected to the supply.*

Solution. (*a*) $v = V(1-e^{-t/\lambda})$

$v = 320$ V ; $V = 400$ volts

$\lambda = CR = 16 \times 10^{-6} \times 0{\cdot}25 \times 10^6 = 4$ seconds

$$\therefore \quad 320 = 400\,(1-e^{-t/4})$$

$$e^{-t/4} = 0{\cdot}2 \quad \therefore \quad t = \mathbf{6{\cdot}43 \text{ second}}$$

or

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(*b*) $q=Q\,(1-e^{-t/\lambda})$

$t=4$ second ; $\lambda=CR=4$ second

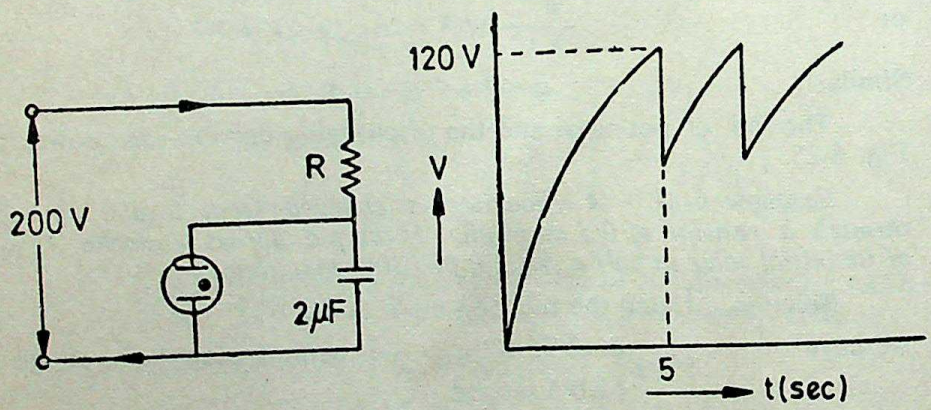
$\therefore$ $q=Q\,(1-e^{-1})=0{\cdot}632\ Q$

Now $Q=CV$

$\therefore$ $q=0{\cdot}632\times16\times10^{-6}\times400=\mathbf{4045\times10^{-6}\ C}$

Example 6-23. *A resistor R and a 2 μF capacitor are connected in series across a 200-V d.c. supply. Across the capacitor is connected a neon lamp that strikes at 120 V. Calculate R to make the lamp strike 5 second after the switch is closed.* (*A.M.I.E. Nov. 1970*)

Solution. The circuit is shown in Fig. 6-26 (*a*). The neon lamp lights up as soon as voltage across the capacitor rises to 120 V. Now, the equation for the rise of voltage across the capacitor is



(*a*) (*b*)

Fig. 6-26

$v=V\,(1-e^{-t/\lambda})$ where $\lambda=CR$

Here $t=5$ second ; $V=200$ V, $v=120$ V

$\therefore$ $120=200(1-e^{-5/\lambda})$

or $e^{-5/\lambda}=0{\cdot}4$ or $e^{5/\lambda}=2{\cdot}5$

or $5/\lambda=\log_e 2{\cdot}5=2{\cdot}3\times\log_{10}2{\cdot}5=2{\cdot}3\times0{\cdot}3979$

$\therefore$ $\lambda=5{\cdot}464$ second

$\therefore$ $R=5{\cdot}464/2\times10^{-6}=2{\cdot}73\times10^{6}\ \Omega=\mathbf{2{\cdot}73\ M\Omega}$

Example 6-24. *If a capacitor of 1 μF and resistance of 82 kΩ are connected in series with an e.m.f. of 100 V, calculate the magnitude of energy and the time in which the energy stored in the capacitor will reach half of its equilibrium value.*

(*A.M.I.E. Summer 1975*)

Solution. Equilibrium value of stored energy

$=\frac{1}{2}CV^2=\frac{1}{2}\times1\times10^{-6}\times100^2=0{\cdot}005$ J

Half of this energy $=\mathbf{0{\cdot}0025\ J}$

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Since stored energy varies as V^2, half of the equilibrium energy would be stored in that time during which capacitor voltage rises to $V/\sqrt{2}$ or $100/\sqrt{2}$ or 70·7 V

Now, $v=V(1-e^{-t/\lambda})$

Now, $v=70{\cdot}7$ V ; $V=100$ V ; $\lambda=CR=0{\cdot}082$ ions

$\therefore \quad 70{\cdot}7=100\,(1-e^{-t/0.082})$

$\therefore \quad t=\mathbf{0{\cdot}1}$ **second.**

Tutorial Problems—9

1. A capacitor of 100 μF is connected in series with an 8 kΩ resistor. Estimate the time constant of the circuit. If the combination is connected suddenly to a 100-V d.c. supply, find (*i*) the initial rate of rise of the p.d. across the capacitor (*ii*) the initial charging current and (*iii*) the ultimate charge on the capacitor. **[0·8 second (*i*) 125 V/s (*ii*) 12·5 mA (*iii*) 0·01C]**

2. A capacitor of 2 μF capacitance is joined in series with a 2-MΩ resistance to a d.c. supply of 100 V. Calculate the current flowing and the energy stored in the capacitor at the end of an interval of 4 seconds from the start. **[18·4 μA ; 4 mJ]**

3. A capacitor of 100 μF is charged through a 500 Ω resistance from a d.c. supply of 1000 V. What period after switching on will elapse before the capacitor acquires 50% of its final charge. What will be the maximum rate of change of current ? **[34·6 ms ; 40 A/s decreasing]**

HIGHLIGHTS

1. Capacitance of a capacitor is given by the ratio of its charge to the p.d. between its plates *i.e.*

$$C=Q/V \text{ farad}$$

2. Capacitance of a spherical capacitor is

$$C=4\pi\epsilon_0\epsilon_r.\frac{ab}{b-a}\text{ F}$$

3. Capacitance of a parallel-plate capacitor is given by

$$C=\frac{\epsilon_0\epsilon_r A}{d} \quad \text{—in a medium}$$

$$=\frac{\epsilon_0 A}{d} \quad \text{—in air}$$

4. For a composite medium, the capacitance is given by

$$C=\frac{\epsilon_0 A}{\Sigma d_i/\epsilon_r}\text{ farad}$$

5. Capacitance of a multiplate parallel-plate capacitor having n plates is given by

$$C=\frac{(n-1)\epsilon_0\epsilon_r A}{d}\text{ farad}$$

6. Capacitance of a cable capacitor is given by

$$C=\frac{2\pi\,\epsilon_0\,\epsilon_r\,l}{2{\cdot}3\,\log_{10}{}^{b/a}}$$

Also $$C=\frac{0{\cdot}024\epsilon_r}{\log_{10}{}^{b/a}}\ \mu\text{F/km}$$

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7. Electric stress or potential gradient in a cable capacitor is given by

$$g=\frac{V}{2{\cdot}3x\ \log_{10}{}^{b/a}} \text{ volt/metre}$$

8. Capacitance of combined capacitors is given by

$C=C_1+C_2$ ...when in parallel

$C=\frac{C_1C_2}{C_1+C_2}$ ...when in series

or, in general $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+$...when in series

9. Insulation resistance of a cable capacitor is given by

$$C=\frac{2{\cdot}3}{2\pi l}\rho\ \log_{10} r_2/r_1\ \Omega$$

10. Energy stored in a capacitor is

$$E=\tfrac{1}{2}CV^2 \text{ joules}$$

11. Energy stored per unit volume of a dielectric is

$$E=\frac{\epsilon E^2}{2}=\frac{D^2}{2\epsilon} \text{ J/m}^3$$

12. Force of attraction between two oppositely-charged plates is

$$F=\frac{D^2A}{2\epsilon} \text{ newton}$$

13 Time constant of an R—C circuit is defined as

(*i*) the time during which capacitor voltage rises to 63·2% of its final steady value.

Or

(*ii*) the time during which the charging current falls to 37% of its initial maximum value.

14. During the charging of a capacitor

(*a*) Voltage across it rises according to the equation

$$v=V(1-e^{-t/\lambda})$$

(*b*) Charge increases according to the relation

$$q=Q\ (1-e^{-t/\lambda})$$

(*c*) The charging current decreases according to the equation

$$i=I_m\ e^{-t/\lambda}$$

15. During the discharging of a capacitor

(*i*) Voltage across it decreases according to the equation

$$v=Ve^{-t/\lambda}$$

(*ii*) Charge decreases according to the relation

$$q=Q\ e^{-t/\lambda}$$

(*iii*) Current decreases according to the equation

$$i=-I_m\ e^{-t/\lambda}$$

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MAGNETISM AND ELECTROMAGNETISM 7

1. Magnetic Field 2. Pole Strength 3. Laws of Magnetic Force 4. Field Strength 5. Magnetic Potential 6. Magnetic Flux 7. Flux Density 8. Magnetic Induction 9. Absolute Permeability (μ) and Relative Permeability (μ_r) 10. Intensity of Magnetisation 11. Susceptibility 12. Relation Between B, H, I, and K 13. Weber and Ewing's Molecular Theory of Magnetism 14. Magnetic Effect of Electric Current 15. Direction of Magnetic Field and Current 16. Force on a Conductor Lying in a Magnetic Field 17. Work Law and Its Applications 18. Magnetising Force on a Long Straight Conductor 19. Magnetising Force of a Long Solenoid 20. Force Between two Parallel Conductors 21. Magnitude of Mutual Force 22. Definition of Ampere. 23. Magnetic Circuit 24. Definitions 25. Composite Magnetic Circuit 26. How to Find Ampere Turns 27. Comparison Between Magnetic and Electric Circuits 28. Leakage Flux and Hopkinson's Leakage Coefficient 29. Magnetisation Curves 32. Highlights.

7-1. Magnetic Field

(A.M.I.E. Winter 1979)

The space or region around a magnet which is permeated by the lines of force and within which conductors carrying electric current are perceptibly influenced, is conventionally called the magnetic field of force or simply a magnetic field. It is assumed that lines of force emanate from a *N*-pole, pass through the surrounding medium, re-enter the *S*-pole and complete their path from *S* to *N* pole through the body of the magnet. Since every line of force must have a complete circuit, it is impossible to get a magnet having only one pole. These lines of force complete their paths independently and never cut or cross or merge into each other.

7-2. Pole Strength

It is well-known that a magnet can be made by stroking an iron piece with a permanent magnet or by placing an iron bar inside a long coil of insulated wire (*i.e.* a solenoid) carrying a strong current. It is found that the magnet made by the electrical method (known as electromagnet) is capable of raising a heavier load than

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the magnet made by stroking, showing that the poles are stronger in the first case.

In order to deal with pole strength, we need a unit of the strength or a '*unit pole*' just as we need the definition of the mass of one kg for measuring the masses of different objects. As discussed in the next article, a unit pole-strength is defined as the strength of that pole which when placed in vacuum at a distance of 1 metre from a similar and equal pole repels it with a force of $1/4\pi\mu_0$ newton where

$$\mu_0 = \text{permeability of free space} = 4\pi \times 10^{-7} \text{ H/m}$$

7-3. Laws of Magnetic Force

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two isolated point poles. It may be noted here that, in view of the fact that magnetic poles always exist in pairs, it is impossible in practice, to get an isolated pole. The concept of an isolated pole is purely theoretical. However, poles of a thin but long magnet may be assumed to be isolated point poles for all practical purposes (Fig. 7-1). By using a torsion balance, he found that the force between two magnetic poles placed in a medium is,

(*i*) directly proportional to their pole strengths

(*ii*) inversely proportional to the square of the distance between them.

(*iii*) inversely proportional to the absolute permeability of the surrounding medium.

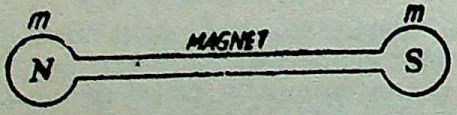

Fig. 7-1

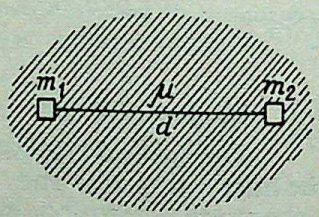

Fig. 7-2

For example, if m_1 and m_2 represent the magnetic strength of the two poles (their unit as yet being undefined) and d the distance in metres between them (Fig. 7-2) then the force F is given by the expression :

$$F \propto \frac{m_1 m_2}{\mu d^2}$$

$$= k\frac{m_1 m_2}{\mu d^2}$$

where k is a constant of proportionality whose value depends on the system of units employed. In the M.K.S.A. as well as SI system, value of $k = 1/4\pi$

$$\therefore \quad F = \frac{m_1 m_2}{4\pi\mu d^2} = \frac{m_1 m_2}{4\pi\mu_0 \mu_r d^2}$$

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If the two poles are placed in vacuum or air, then

$$F = \frac{m_1 m_2}{4\pi \mu_0 d^2}$$

If, in the above equation,

$$m_1 = m_2 = m \text{ (say)} ; \quad r = 1 \text{ metre}$$

$$F = \frac{1}{4\pi\mu_0} \text{ newton}$$

Then $m^2 = 1$ or $m = \pm 1$ weber*

Hence, a unit magnetic pole may be defined as that pole which when placed in air at a distance of one metre from a similar and equal pole repels it with a force of $1/4\pi\mu_0$ newtons. †

7-4. Field Intensity or Strength or Magnetising Force (H)

(A.M.I.E. May 1974)

Field strength at any point within a magnetic field is measured by the force experienced by a *N*-pole of one weber placed at that point. Hence, unit of *H* is N/Wb.‡

Suppose it is required to find the field intensity at a point *A* distant *d* metres from a pole of strength *m* webers lying in air. For that purpose, imagine a pole of one weber placed at that point *A*. The force experienced by this pole is

$$F = \frac{m \times 1}{4\pi \mu_0 d^2} \text{ newton}$$

$$\therefore \quad H = \frac{m}{4\pi \mu_0 d^2} \text{ N/Wb (or AT/m)}$$

Also, if a pole of *m* Wb is placed in a uniform field of strength *H* N/Wb, then force experienced by the pole is $= mH$ newtons.

It should be noted that field strength is a vector quantity having both magnitude and direction.

7-5. Magnetic Potential (M)

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a *N*-pole of one weber from infinity to that point against the force of magnetic field. Its value is

$$M = \frac{m}{4\pi \mu_0 d} \text{ J/Wb}$$

It is a scalar quantity.

7-6. Magnetic Flux (Φ)

In the M.K.S.A. as well as SI system of units, a unit *N*-pole is supposed to radiate out a flux of one weber. Its symbol is Φ. Therefore, the flux coming out of a *N*-pole of *m* weber is given by

$$\Phi = m \text{ Wb}$$

---

*To commemorate the memory of German physicist Wilhelm Edward Weber (1804—1891).

†A unit magnetic pole is also defined as that magnetic pole which when placed at a distance of one metre from a very long straight conductor carrying a current of one ampere experiences a force of $1/2\pi$ newtons.

‡It should be noted that N/Wb is the same thing as ampere-turn/m.

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7-7. Flux Density (B) *(A.M.I.E. Winter 1986)*

Flux density at a point is given by the magnetic flux passing per unit cross-section at that point.

It is usually designated by the capital letter B and is measured in weber/metre^2.

If Φ is the total flux passing through an area of A m^2, then

$$B = \Phi/A \quad \text{Wb/m}^2$$

7-8. Magnetic Induction

A magnet can impart magnetism to a magnetic substance without actually coming in physical contact with it. For example, if an unmagnetised piece of soft iron be placed in the field of a magnet, it also becomes magnetised. The iron piece is under induction, the magnet is the inducing body and this phenomenon is known as *magnetic induction*. When magnetism is developed by induction, at least two poles are produced in the body.

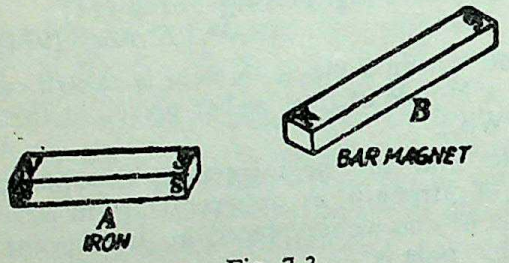

Fig. 7-3

The two poles, one of the body and the other of the inducing magnet which are nearest to each other, are unlike poles (Fig. 7-3). Magnetic induction can be easily explained on Molecular Theory of Magnetism (Art. 7-13).

7-9. Absolute Permeability (μ) and Relative Permeability (μ_r) *(A.M.I.E. Winter 1983)*

In Fig. 7-4 is shown a bar of a magnetic material, say, iron placed in a uniform field of strength H N/Wb (or AT/m). The iron bar is magnetised by induction. Suppose it develops a polarity of m webers. Then its own flux is m webers. These lines of flux emanate from its N-pole, re-enter its S-pole and continue from S to N-pole within the magnet. These lines are seen to be in opposition to the lines of force of the main field H outside the magnet but in the same direction within it. The resultant field is shown in Fig. 7-5.

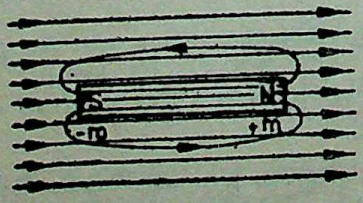

Fig. 7-4

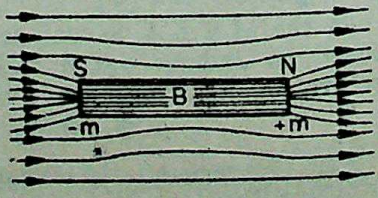

Fig. 7-5

If Φ = total flux through the bar in Wb

A = pole area of the bar in m^2

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The flux density established through the bar is

$$B=\Phi/A \quad \text{Wb/m}^2$$

The absolute permeability of a medium is given by the ratio

$$\mu=\frac{B}{H}=\frac{\text{flux density}}{\text{magnetising force}}$$

or $$B=\mu H$$

Now, suppose a magnetising force of H is set up first in vacuum (or air) and then in some other medium. The flux density established by it is

$$B_0=\mu_0 H \quad \text{—in vacuum}$$

and $$B=\mu_0 \mu_r H \quad \text{—in that medium}$$

The relative permeability of that medium is given by

$$\mu_r=\frac{\text{flux density in the medium}}{\text{flux density in vacuum}} \text{ for same } H$$

$$=\frac{B}{B_0} \quad \text{—for same } H$$

Being a mere ratio, it has no units.

7-10. Intensity of Magnetisation (J or I)

It is defined as the pole strength developed per unit area of the bar. Also, it is the magnetic moment developed per unit volume of the bar.

Let m=pole strength induced in the bar in Wb

A=face or pole area of the bar in m²

Then $$I=\frac{m}{A} \quad \text{Wb/m}^2$$

Now, a pole of m webers, produces a flux of m webers. Hence, intensity of magnetisation of a substance may also be defined as *the flux density produced in it due to its own magnetism.* It is known as induction flux density B_i.

If l is the magnetic length of the bar, then the product $(l \times m)$ is known as its magnetic moment M.

$$\therefore \quad I=\frac{m}{A}=\frac{m \times l}{A \times l}=\frac{M}{V} \text{ magnetic moment/volume.}$$

7-11. Susceptibility (K)

Susceptibility is defined as the ratio of intensity of magnetisation I to the magnetising force H. *(A.M.I.E. Winter 1983)*

$$\therefore \quad K=I/H \text{ henry/metre}$$

7-12. Relation Between B, H, I and K

It is obvious from the above discussion that flux density B in a material is given by *(A.M.I.E. Winter 1983)*

$$B=B_0+B_i=B_0+m/A$$

$$\therefore \quad B=\mu_0 H+I$$

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Now, absolute permeability

$$\mu = \frac{B}{H} = \frac{\mu_0 H + I}{H} = \mu_0 + \frac{I}{H}$$

$$\therefore \quad \mu = \mu_0 + K$$

Also $\quad \mu = \mu_0 \, \mu_r$

$$\therefore \quad \mu_0 \mu_r = \mu_0 + K$$

$$\therefore \quad \mu_r = 1 + \frac{K}{\mu_0}$$

For para-magnetic substances K is positive, for non-magnetic substances, is zero and for diamagnetic substances, it is negative. For para-magnetic substances $\mu_r > 1$, for non-magnetic substances, $\mu_r = 1$ and for diamagnetic substances, $\mu_r < 1$.

7-13. Weber and Ewing's Molecular Theory

This theory was first advanced by Weber in 1852 and was later on, further developed by Ewing in 1890. The basic assumption of this theory is *that molecules of all substances are inherently magnets in themselves, each having a N- and S-pole.* In an unmagnetised state it is supposed that these small molecular magnets lie in all sorts of haphazard manner forming more or less closed loops

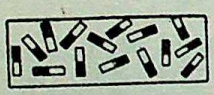
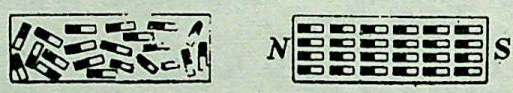


Fig. 7-6



(a) (b)

Fig. 7-7

(Fig. 7-6). According to the laws of attraction and repulsion, these closed magnetic circuits are satisfied internally, hence there is no resultant external magnetism exhibited by the iron bar. But, when such an iron bar is placed in a magnetic field or under the influence of a magnetising force, then these molecular magnets start turning round their axes and orientate themselves more or less along straight lines parallel to the direction of the magnetising force. This linear arrangement of the molecular magnets results in N-polarity at one end of the bar and S-polarity at the other (Fig. 7-7). As the small magnets turn more nearly in the direction of the magnetising force, it requires more and more of this force to produce a given turning moment, thus accounting for the magnetic saturation. On this theory, the hysteresis loss is supposed to be due to molecular friction of these turning magnets.

Because of the limited knowledge of molecular structure available at the time of Weber, it was not possible to explain firstly as to why the molecules themselves are magnets and secondly, why it is impossible to magnetise certain substances like wood etc. The first objection was explained by Ampere who maintained that orbital movement of the electrons round the nucleus of an atom constituted a flow of current which, due to its associated magnetic effect, made the molecule a magnet.

Later on, it became difficult to explain the phenomenon of diamagnetism, erratic behaviour of ferromagnetic (intensely mag-

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netisable) substances like iron, steel etc. and the paramagnetic (weakly magnetisable) substances like cobalt, nickel etc. Moreover, it was asked : if molecules of all substances are magnets, then why does not wood or air etc. become magnetised ?

All this has been explained satisfactorily by the atom-domain theory which has superseded the molecular theory. It is beyond the scope of this book to go into the details of this theory. The interested reader is advised to refer to some standard book on magnetism. However, it may just be mentioned that this theory takes into account not only the planetary motion of the electron, but its rotation about its own axis as well. This latter rotation is called 'electron spin'. The gyroscopic behaviour of an electron gives rise to a magnetic moment which may be either positive or negative. A substance is ferromagnetic or diamagnetic accordingly as there is an excess of unbalanced positive spins or negative spins. Substances like wood or air are non-magnetisable because in their case, the positive and negative electron spins are equal hence they cancel each other out.

ELECTROMAGNETISM

7-14. Magnetic Effects of Electric Current

In 1819, about two decades after the invention of voltaic cell, Oersted of Copenhagen discovered that an electric current is always accompanied by a magnetic field. He verified the existence of magnetic effects by the following simple experiments.

When Oersted placed a current-carrying conductor *AB* near and parallel to a magnetic needle, as in Fig. 7-8, he found that the needle was deflected in a certain direction. When the wire *AB* was

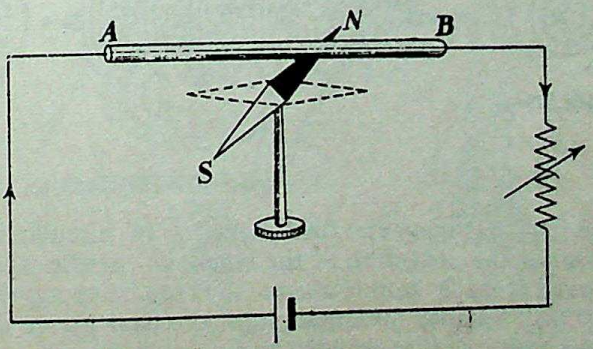

Fig. 7-8

placed beneath the needle then it was deflected in a direction opposite to the first direction. This deflection is a convincing

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proof of the existence of a magnetic field around a conductor carrying electric current. It was further found that if the current strength is decreased by introducing a resistance in the circuit, the amount of deflection produced is also decreased. It shows that the field strength decreases with current.

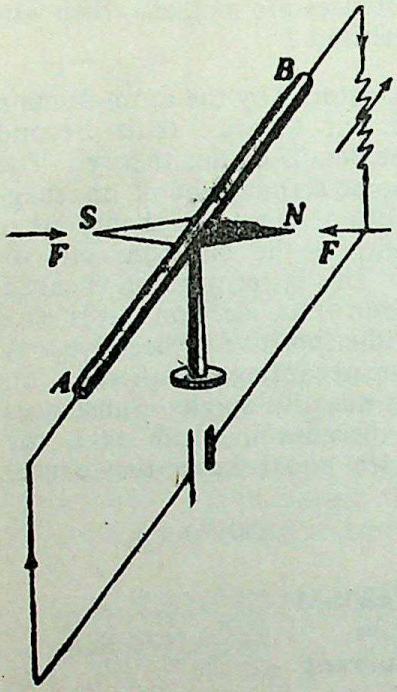

Fig. 7-9

He also found that when wire *AB* is held at right angles over the needle (Fig. 7-9), then latter is not deflected at all. It shows that magnetic field of the current is at right angles to it, hence in this case parallel to the needle, which explains the absence of deflection.

Further investigation showed that the magnetic field consists of lines of force which form complete circles round the conductor. These circles have their centres at the centre of the current-carrying conductor and their planes perpendicular to it as in Fig. 7-10 (*b*). It should be noted from Fig. 7-10 (*a*) and (*c*) that the circular lines of force become further and further apart as the distance from each to the conductor increases. It is a graphic way of illustrating the fact that the field strength decreases as the distance from the conductor increases.

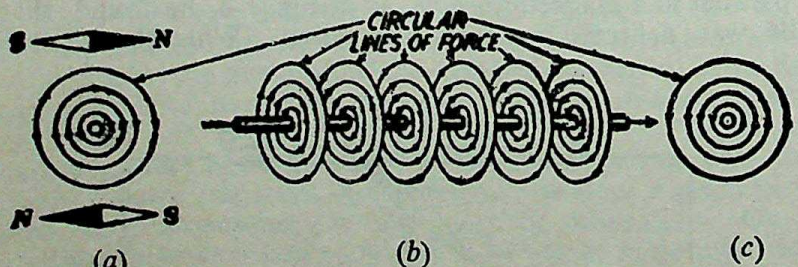

Fig. 7-10

The fact that magnetic field consists of circular lines explains the reversal of the deflection of the magnetic needle when conductor *AB* is moved from a point above the needle to a point beneath it [Fig. 7-10 (*a*)] because the direction of the field above the conductor must be opposite to that beneath the conductor.

7-15. Direction of Magnetic Field and Current

There exists a definite relation between the direction of the current flowing in a conductor and the direction of the magnetic

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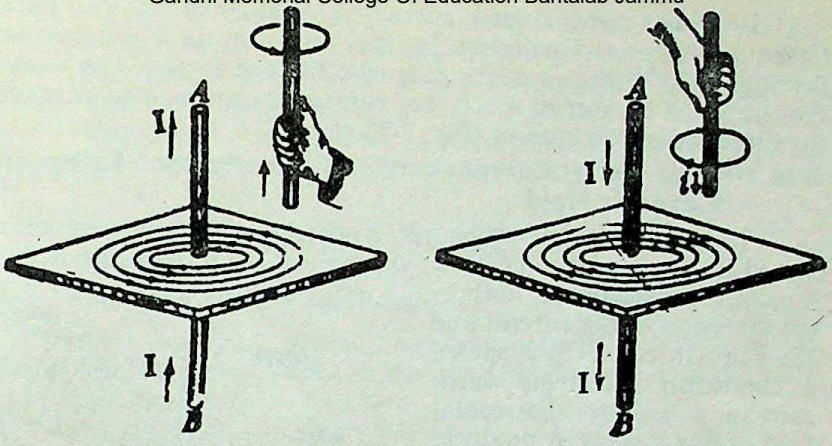
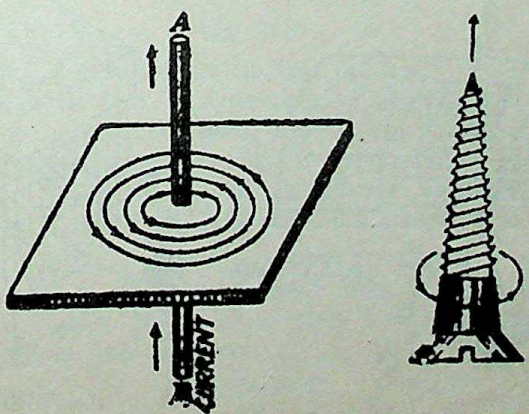


(*a*) Fig. 7-11 (*b*)

field surrounding it. This relationship may be obtained with the help of the following simple rules :

(1) **Right-hand Rule.** In Fig. 7-11 is shown a long straight conductor *AB* carrying a steady current and passing through a piece of card-board. If we plot the magnetic field with the help of a compass needle, we obtain the pattern shown in Fig. 7-11 (*a*) when current flows upwards from *B* to *A*. The lines of force are circular in shape and their direction is the direction in which an isolated *N* pole would move if free to do so. In Fig. 7-11 (*b*) is shown the field when current flows downwards from *A* to *B*. The direction of the field in each case can be found by the right-hand rule thus :

Grasp the wire or conductor in the *right hand* with the thumb pointing in the direction of the current. The fingers then curl round in the direction of the lines of force.



(*a*) Fig. 7-12 (*b*)

(2) **Maxwell's Corkscrew Rule.** The direction of the current and that of the magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated (Fig. 7-12).

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Imagine a right-handed corkscrew held with its axis parallel to the conductor and pointing in the direction in which current is flowing. The direction of the magnetic lines of force is the same as that in which corkscrew must be rotated to cause it to advance in the direction of the current [Fig. 7-12 (*b*)].

7-16. Force on a Current-carrying Conductor Lying in a Magnetic Field

It is found that whenever a current-carrying conductor is placed in a magnetic field, it experiences a force which acts in a direction perpendicular both to the direction of the current and the field. In Fig. 7-13 is shown a conductor XY lying vertically in a uniform horizontal field of flux density B produced by two solenoids A and B. If l is the length of the conductor lying within this field and I ampere the current carried by it, then the magnitude of the force exerted on the conductor is

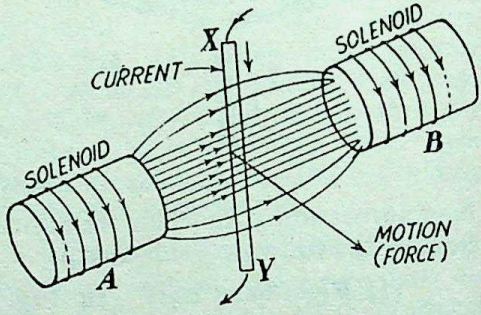

Fig. 7-13

$$F = BIl \text{ newton}$$
$$= \mu_0 \mu_r HIl \text{ newton}$$

The direction of this force may be easily found by Fleming's left-hand rule given below and illustrated in Fig. 7-14 (*a*).

Hold out your left hand with fore-finger, second finger and thumb at right angles to one another. If the fore-finger represents the direction of the field, and the second finger that of the current, then thumb gives the direction of the motion.

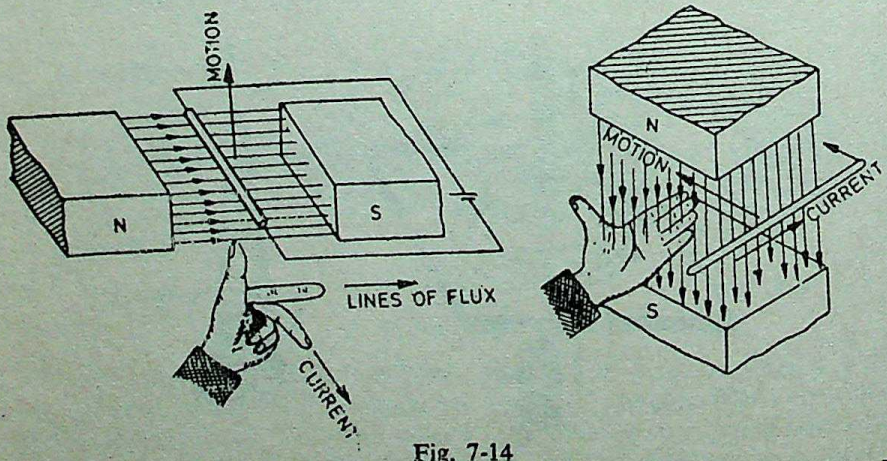

Fig. 7-14

An other way of applying left-hand rule is shown in Fig. 7-14 (*b*). In this case, the hand is so placed that lines of flux leaving N-pole enter the palm perpendicularly and the four fingers are in the direction of flow of the current in the conductor, then thumb gives the direction of the force.

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It should be noted that no force is exerted on a conductor when it lies parallel to the magnetic field. In general, if the conductor lies at an angle θ with a field of flux density B, then B can be resolved into two components, $B \cos \theta$ parallel to and $B \sin \theta$ perpendicular to the conductor. The former produces no effect whereas the latter is responsible for the motion observed. In that case,

$$F = BIl \sin \theta \text{ newton}$$

7-17. Work Law and its Applications

It a unit *N*-pole is moved in a magnetic field, it experiences a magnetic force which depends on the field strength. If the unit

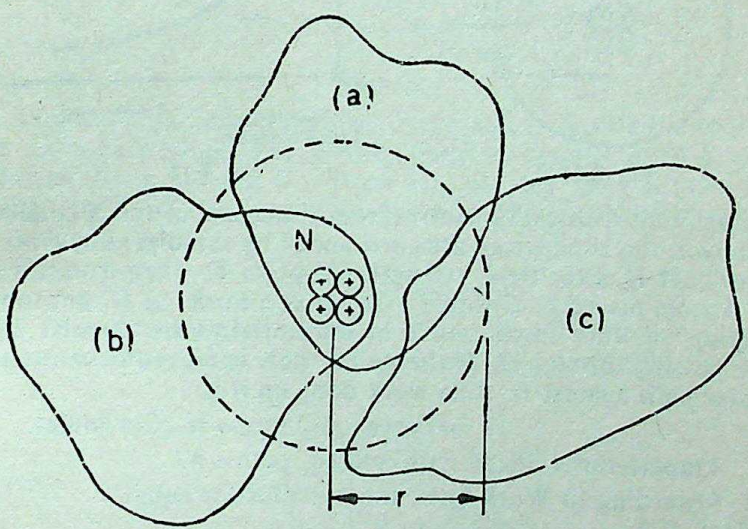

Fig. 7-15

pole is moved in opposition to the lines of force, then work is done against the magnetic field. Conversely, if the unit pole is moved in the direction of the magnetic field, then work is done by the magnetic force on whatever is restraining the movement of the pole. Consider *N* straight conductors, each carrying *I* amperes as shown in Fig. 7-15. The product '*NI*' is called the 'ampere-turns'. According to Work Law :—

The net work done on or by a unit N-pole in moving once round any single closed path in a magnetic field is numerically equal to the ampere-turns linked with that path.

Mathematically, $\oint H_r \, dr = NI$

where H_r is the magnetising force at a distance of r metres.

Explanation. In Fig. 7-15, paths *a* and *b* are both closed paths which link *N* conductors once. Hence, work done in moving a unit *N*-pole once round either of these two paths is '*NI*'. Even though path 'c' is also a closed path, it does not link any current-

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carrying conductor, hence work done in this case would be zero. Work law will now be used to find the magnetising force (H) due to (a) a long straight current-carrying conductor (b) a long solenoid.

7-18. Magnetising Force of a Long Straight Conductor

Fig. 7-16 shows a bundle of N long straight conductors each carrying I amperes. It is required to find the value of the magnetising

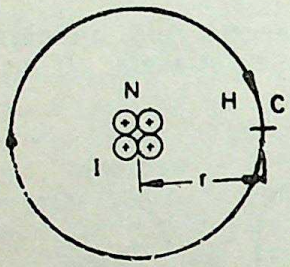

Fig. 7-16

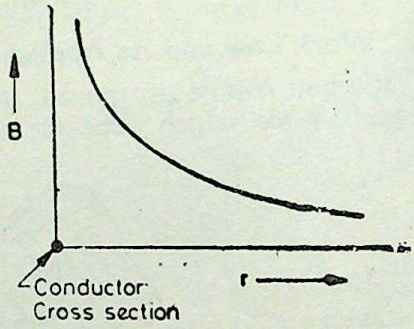

Fig. 7-17

force at point C distant r metres from the centre of the N conductors. As shown, the conductors are surrounded by circular lines of force or flux. If H is the field strength at point C, then it means that a unit N-pole placed at C would experience a force of H newton. The direction of this force would be tangential to the circular lines of force passing through C. If the unit N-pole is moved once round this circular path against H, then work done on it is

$$= \text{Force} \times \text{distance} = H \times 2\pi r \text{ joules}$$

Ampere-turns linked with closed path $= NI$

According to Work Law, the two must be equal.

$$\therefore \quad H \times 2\pi r = NI$$

or
$$H = \frac{NI}{2\pi r} \text{ AT/m}$$

$$B = \frac{\mu NI}{2\pi r} \text{ Wb/m}^2$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \text{ Wb/m}^2 \quad \text{...in a medium}$$

$$= \frac{\mu_0 NI}{2\pi r} \text{ Wb/m}^2 \quad \text{...in air}$$

Variation of the flux density around a long cylindrical conductor is shown in Fig. 7-17. Obviously, B varies inversely as the distance from the centre of the conductor.

7-19. Magnetising Force of a Long Solenoid

With reference to Fig. 7-18, let the axial field of the solenoid be H. Let it be assumed further that

(*i*) value of H remains constant throughout the length l of the solenoid and

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(*ii*) the value of H outside the solenoid is negligible.

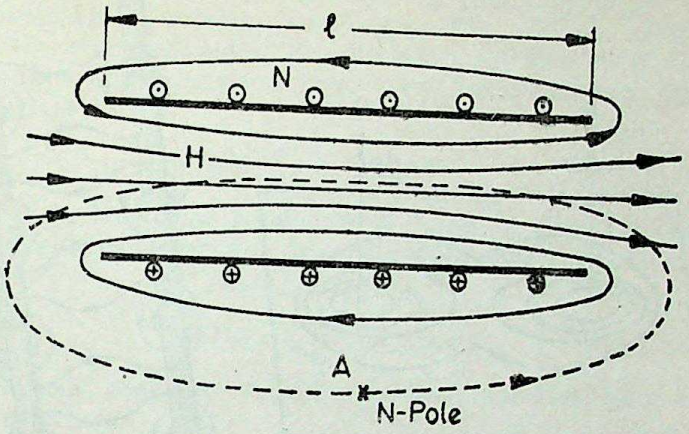

Fig. 7-18

Suppose a unit *N*-pole is placed at point *A* outside the solenoid and is taken once round the dotted path in opposition to *H*. Remembering that a force of *H* newton acts on this unit *N*-pole only over the length *l* of the journey (it being negligible elsewhere), work done in one round is

$$= H \times l \text{ joules}$$

If N is the number of turns of the solenoid and I its current then ampere-turns linked with this path are NI. As per Work Law,

$$H \times l = NI$$

or $$H = NI/l \text{ AT/m}$$

Also $$B = \mu H = \mu NI/l$$

$$= \mu_0 \mu_r NI/l \quad \text{Wb/m}^2 \quad \text{...in a medium}$$

$$= \mu_0 NI/l \quad \text{Wb/m}^2 \quad \text{...in air}$$

7-20. Force Between Two Parallel Conductors

(*A.M.I.E. Summer 1975*)

(*i*) **Currents in the Same Direction.** In Fig. 7-19 are shown two parallel conductors P and Q carrying currents I_1 and I_2 amperes in the same direction *i.e.* upwards. The field strength in the space between the two conductors is decreased due to the two fields there being in opposition to each other. Hence, the resultant field is as shown in the figure. Obviously, the two conductors are attracted towards each other.

(*ii*) **Currents in Opposite Directions.** If, as shown in Fig. 7-20, the parallel conductors carry currents in the opposite directions, then field strength is increased in the space between the two conductors due to the two fields being in the same direction there. Because of the lateral repulsion of the lines of force, the two conductors experience a mutual force of repulsion as shown.

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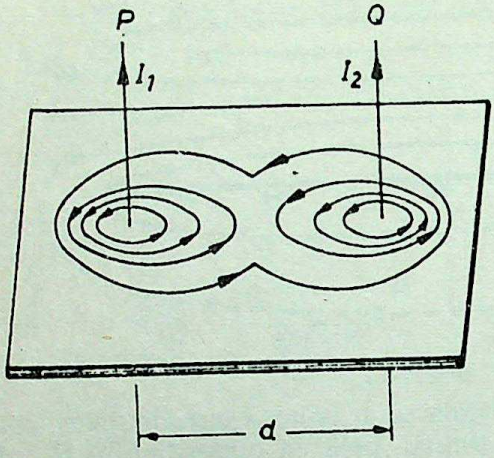

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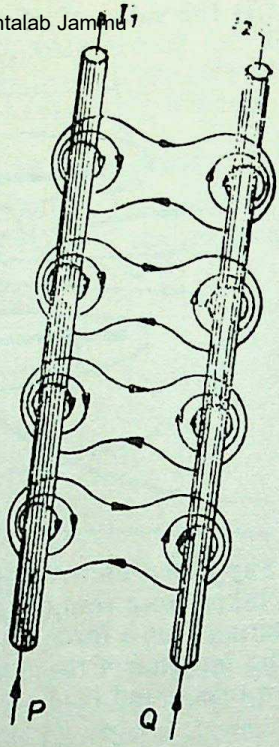
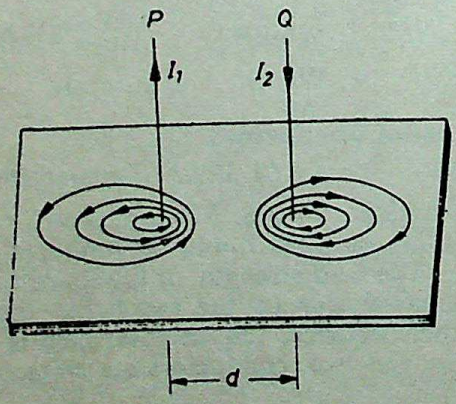


(b)

Fig. 7-19



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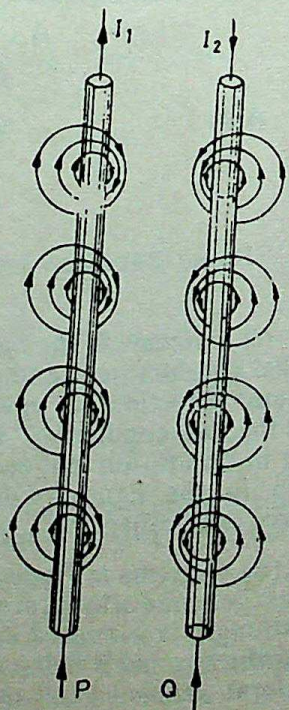

(b)

Fig. 7-20

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7-21. Magnitude of Mutual Force

It is obvious that each of the two parallel conductors lies in the magnetic field of the other conductor. For example, conductor Q lies in the magnetic field of P and P lies in the field of Q. If 'd' metres is the distance between them, then flux density at Q due to P is (Art. 7-18)

$$B = \frac{\mu_0 I_1}{2\pi d} \text{ Wb/m}^2$$

If l is the length of conductor Q lying in this flux density, then force (either of attraction or repulsion) as given in Art. 7-16 is

$$F = B I_2 l \text{ newton}$$

or

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ newton}$$

Obviously, conductor P will experience an equal force in the opposite direction.

The above facts are known as Laws of Parallel Currents and may be stated as follows :

(*i*) Two parallel conductors attract each other if currents through them flow in the *same* direction and repel each other if the currents through them flow in *opposite* directions.

(*ii*) The force between two such parallel conductors is proportional to the product of the current strengths and to the lengths of the conductors considered and varies inversely as the distance between them.

7-22. Definition of Ampere

It has been proved in Art. 7-21 above that the force between two infinitely long parallel current-carrying conductors is given by the expression

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

Since $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$\therefore \quad F = \frac{4\pi \times 10^{-7} I_1 I_2 l}{2\pi d} \text{ N}$$

$$= 2 \times 10^{-7} \frac{I_1 I_2 l}{d} \text{ N}$$

The force per metre run of the conductors is

$$F = 2 \times 10^{-7} \frac{I_1 I_2}{d} \text{ N/m}$$

If $I_1 = I_2 = 1$ ampere (say) and $d = 1$ metre, then

$$F = 2 \times 10^{-7} \text{ N}$$

Hence, we can define one ampere current as *that current which when flowing in each of the two infinitely long parallel conductors situated in vacuum and separated 1 metre between centres, produces on each conductor a force of* 2×10^{-7} *newton per metre length.*

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Example 7-1. *Force between two wires carrying currents in opposite directions is 20·4 kg/m when they are placed parallel with axes 5 cm apart. Calculate the current in one conductor when the current flowing through the second conductor is 5000 A. Mention whether it is a force of attraction or repulsion.* (*A M.I.E. Summer 1975*)

Solution. As shown in Art. 7-21

$$F = 2 \times 10^{-7} I_1 I_2 \, l/d$$

Here, $F = 20{\cdot}4$ kg-wt $= 20{\cdot}4 \times 9{\cdot}8$ N

$l = 1$ m ; $I_1 = 5000$ A ; $d = 0{\cdot}05$ m

$\therefore \quad 20{\cdot}4 \times 9{\cdot}8 = 2 \times 10^{-7} \times 5000 \times I_2 \times 1/0{\cdot}05$

$\therefore \quad I_2 = \mathbf{10{,}000\ A}$

Example 7-2. *If a pair of straight parallel bus-bars of circular cross-section, spaced 23 cm between centres, each carry a current of 70,000 A, calculate the force in newtons per metre run which the conductors have to withstand.* (*A.M.I.E. Nov. 1972*)

Solution. $$F = \frac{\mu_0 I_1 I_2 \, l}{2\pi d} = 2 \times 10^{-7} \frac{I_1 I_2 \, l}{d} \text{ newton}$$

Here, $I_1 = I_2 = 70{,}000$ A ; $d = 0{\cdot}23$ m ; $l = 1$ m

$F = 2 \times 10^{-7} \times 70{,}000 \times 70{,}000/0{\cdot}23 = \mathbf{4{,}260\ N/m}$

Note. It is a force of repulsion since currents flow in opposite directions in the bus-bars.

7-23. Magnetic Circuit

It may be defined as the route or path which is followed by magnetic flux. The laws of magnetic circuit are quite similar to (but not the same as) those of the electric circuit.

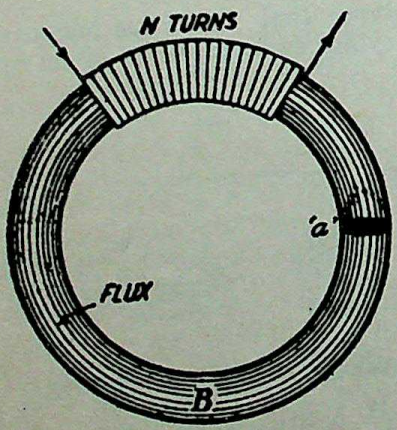

Fig. 7-21

Consider a solenoid or a toroidal iron ring having a magnetic path of l metre, area of cross-section A m^2 and a coil of N turns carrying I amperes wound anywhere on it as in Fig. 7-21.

Then, as seen from Art. 7-19, field-strength inside the solenoid is

$$H = \frac{NI}{l} \text{ AT/m}$$

Now $B = \mu_0 \mu_r H$

$$\therefore \quad B = \frac{\mu_0 \mu_r \, NI}{l} \text{ Wb/m}^2$$

Total flux produced

$$\Phi = B \times A = \frac{\mu_0 \mu_r A N I}{l} \text{ Wb}$$

$$\Phi = \frac{NI}{l/\mu_0 \mu_r A} = \frac{NI}{S}$$

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The numerator '*NI*' which produces magnetisation in the magnetic circuit is known as magnetomotive force (m.m.f.). Obviously, its unit is ampere-turn

It is analogous to e.m.f. in an electric circuit.

The denominator $\frac{l}{\mu A}$ or $\frac{l}{\mu_0 \mu_r A}$ is called the *reluctance* of the circuit and is analogous to resistance in electric circuit.

$$\therefore \quad \text{flux} = \frac{\text{mmf}}{\text{reluctance}}$$

Sometimes, the above equation is called the "Ohm's Law of Magnetic Circuit" because it resembles a similar expression in electric circuit *i.e.*

$$\text{current} = \frac{\text{emf}}{\text{resistance}}$$

7-24. Definitions

(*i*) **Magnetomotive force (m.m.f.)** (*A.M.I.E. Winter 1986*)

It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit. It is given by the product '*NI*'.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one point to another, similarly, m.m.f. between two points is measured by work done in joules in carrying a unit magnetic pole from one point to another.

(*ii*) **Ampere-turns.** It is the unit of magnetomotive force (m.m.f.).

(*iii*) **Reluctance** (*A.M.I.E. Winter 1986*)

It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the resistance offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even in form. Its unit is AT/Wb*

$$\text{Reluctance} = \frac{l}{\mu_0 \mu_r A} \quad \text{or} \quad \frac{l}{\mu A}$$

$$\text{Resistance} = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

In other words the reluctance of a magnetic circuit is the number of ampere-turns required per weber of magnetic flux in the

*From the relation $\phi = \frac{\text{mmf}}{\text{reluctance}}$ it is obvious that reluctance = mmf/ϕ. Since m.m.f. is in ampere-turns and flux in webers, unit of reluctance is ampere-turn/Wb.

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circuit. Since 1 AT/Wb = 1//henry, hence unit of reluctance is also 'reciprocal' henry.

(*iv*) **Flux** (*A.M.I.E. Winter 1986*)

It is equal to the total number of lines of induction existing in a magnetic circuit and is analogous to current in an electric circuit. It is measured in webers.

(*v*) **Permeance.** It is reciprocal of reluctance and implies the ease or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry,

(*vi*) **Reluctivity.** It is specific reluctance and corresponds to resistivity which is 'specific resistance'.

7-25. Composite Magnetic Circuit

In Fig. 7-22 is shown a composite magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ($\mu_r = 1$). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

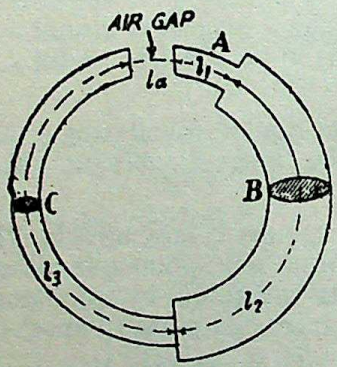

Fig. 7-22

$$\therefore \text{ total reluctance} = \Sigma \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_3}{\mu_0 \mu_{r3} A_3} + \frac{l_a}{\mu_0 A_a}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\Sigma l / \mu_0 \mu_r A}$$

7-26. How to Find Ampere-turns ?

It has already been shown in Art. 7-19 that $H = NI/l$

$$\therefore \quad NI = H \times l$$

$\because$ ampere-turns or $\text{AT} = H \times l$

Hence, following procedure should be adopted for calculating the total ampere-turns of a composite magnetic path.

(*i*) Find H for each portion of the composite circuit.
For air, $H = B/\mu_0$, otherwise $H = B/\mu_0 \mu_r$

(*ii*) Find ampere-turns for each path separately by using the relation $\text{AT} = H \times l$.

(*iii*) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

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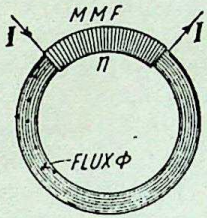
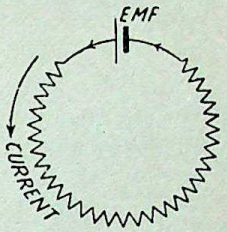
7-27. Comparison Between Magnetic and Electric Circuits

(A.M.I.E. Summer 1975)

SIMILARITIES

Magnetic Circuit	Electric Circuit
 Fig. 7-23	 Fig. 7-24
1. Flux $= \frac{\text{m.m.f.}}{\text{reluctance}}$	Current $= \frac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (in volts)
3. Flux (in webers)	Current I (in amperes)
4. Flux density B (in Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$	Resistance $R = \rho \frac{l}{A} = \frac{l}{\sigma A}$
6. Permeance (=1/reluctance)	Conductance (=1/resistance)
7. Reluctivity	Resistivity
8. Permeability (=1/reluctivity)	Conductivity (=1/resistivity)

DIFFERENCES

1. Strictly speaking, flux does not actually 'flow' in the sense in which an electric current flows.

2. If temperature is kept constant, then conductivity (or resistivity) of an electric circuit is constant and is independent of current strength, but the magnetic conductivity *i.e.* permeability depends on the total flux or flux density established through the material (Art. 7-29).

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3. Flow of current in an electric circuit involves continuous expenditure of energy, but in a magnetic circuit, energy is needed only for creating the flux initially but not for maintaining it.

7-28. Leakage Flux and Hopkinson's Leakage Coefficient L

(A.M.I.E. Nov. 1970)

Leakage flux is the flux which follows a path not intended for it. In Fig. 7-25 is shown an iron ring wound with a coil and having an air-gap. The flux in the air-gap is known as the useful flux because it is only this flux which can be utilised for various useful purposes.

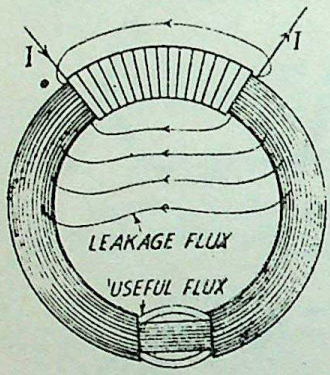

Fig. 7-25

It is found that it is impossible to confine all the flux to the iron path only, althought it is usually possible to confine most of the electric current to a definite path, say, a wire, by surrounding it with insulators. Unfortunately, there is no known insulator for magnetic flux. Air, which is splendid insulator of electricity, is unluckily a fairly good magnetic conductor. Hence, as shown, some of the flux leaks through air surrounding the iron ring. The presence of leakage flux can be detected by a compass. Even in the best designed dynamos, it is found that 15 to 20% of the total flux produced leaks away without being utilised usefully.

If Φ_t = total flux produced

Φ = useful flux available in the air gap

then leakage coefficient

$$\lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\Phi_t}{\Phi}$$

In electric machines like motors and generators magnetic leakage is undesirable, because, although it does not lower their power efficiency, yet it leads to their increased weight and cost of manufacture. Magnetic leakage can be minimised by placing the exciting coils or windings as closely as possible to the air-gap or to the points in the magnetic circuit where flux is to be utilized for useful purposes.

It is also seen from Fig. 7-25 that there is fringing or spreading of lines of flux at the edges of the air-gap. This fringing increases the effective area of the air-gap.

The value of λ for modern electric machines varies between 1·1 and 1·25.

7-29. Magnetisation Curves

(A.M.I.E. Winter 1979)

The approximate magnetisation or *B/H* curves of a few magnetic materials are shown in Fig. 7-26.

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These curves can be determined by the following methods provided the material is in the form of a ring :

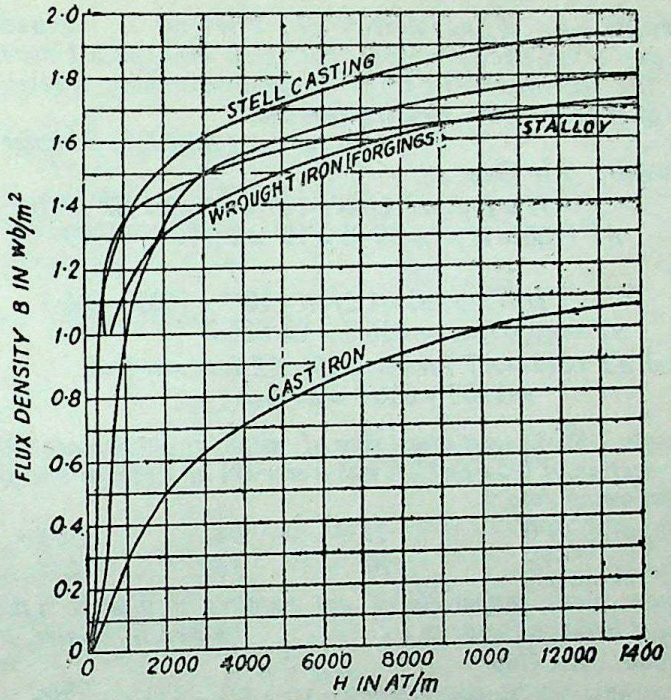

Fig. 7-26

(*a*) By means of a Ballistic Galyanometer and

(*b*) By means of Fluxmeter.

It is worth noting that the slope of a magnetisation curve is not constant. It is maximum at the beginning and minimum towards the end. The slope of a B/H curve represents μ or μ_r. Hence, it means that at low values of B, μ_r is high and at high values of B, μ_r is low. Since $S = l/\mu_0\mu_r A$, it means that reluctance offered by the material also varies with B because μ_r depends on B. At high flux densities, μ_r is less, so that S is more. At low flux densities, μ_r is large, hence S is small.

Example 7-3. *A mild steel ring having a cross-sectional area of 5 cm² and a mean circumference of 40 cm has a coil of 200 turns wound uniformly around it. Calculate (i) the reluctance of the ring (ii) the current required to produce a flux of 800 μWb in the ring. Assume relative permeability of mild steel to be 380 at the flux density developed in the core.* (*A.M.I.E. Nov. 1971*)

Solution.

$$S = \frac{l}{\mu_0\mu_r A} = \frac{0{\cdot}4}{4\pi \times 10^{-7} \times 380 \times 5 \times 10^{-4}}$$

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$$= 1{\cdot}675 \times 10^6 \text{ AT/Wb}$$

Now, $\Phi = NI/S$

$$800 \times 10^{-6} = 200 \times I / 1{\cdot}675 \times 10^6 \quad \therefore \quad I = 6{\cdot}7 \text{ A}$$

Example 7-4. *A flux density of 1·2 Wb/m² is required in the 2 mm air-gap of an electro-magnet having an iron path 1 metre long. Calculate the magnetomotive force required assuming a relative permeability of iron of 1500. Neglect leakage.* (*A.M.I.E. Summer 1978*)

Solution. **Air Gap**

$$H = B/\mu_o = 1{\cdot}2/4\pi \times 10^{-7} = 95{\cdot}45 \times 10^4 \text{ AT/m}$$

$$AT \text{ reqd} = H \times l = 95{\cdot}45 \times 10^4 \times 2 \times 10^{-3} = 1909$$

Iron Path

$$H = B/\mu_o \mu_r = 1{\cdot}2/4\pi \times 10^{-7} \times 1500 = 636{\cdot}3$$

$$AT \text{ reqd} = H \times l = 636{\cdot}3 \times 1 = 636{\cdot}3$$

Total AT reqd = AT for air-gap + AT for iron path

$$= 1909 + 636{\cdot}3 = \mathbf{2545{\cdot}3}$$

Example 7-5. *A cast-steel ring of mean circumference 50 cm has a cross-section of 0·52 cm². It has a saw-cut of 1 mm at one place. Given the following data :*

B(Wb/m²) :	1·0	1·25	1·46	1·60
μ_r	714	520	360	247

Calculate how many ampere-turns are required to produce a flux of 0·052 mWb if leakage factor is 1·2. (*A.M.I.E. Winter, 1978*)

Solution. **Air Gap**

$$\Phi = 0{\cdot}052 \text{ mWb} = 0{\cdot}052 \times 10^{-3} \text{ Wb} = 52 \times 10^{-6} \text{ Wb}$$

$$A = 0{\cdot}52 \text{ cm}^2 = 0{\cdot}52 \times 10^{-4} \text{ m}^2 = 52 \times 10^{-6} \text{ m}^2$$

$$B = \Phi/A = 52 \times 10^{-6}/52 \times 10^{-6} = 1 \text{ Wb/m}^2$$

$$H = B/\mu_0 = 1/4\,\pi \times 10^{-7} = 795 \times 10^3 \text{ AT/m}$$

$$l = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\text{AT required} = H \times l = 795 \times 10^3 \times 10^{-3} = 795 \text{ AT}$$

Cast-steel Path

$$B = 1{\cdot}2 \times B_{air} = 1{\cdot}2 \times 1 = 1{\cdot}2 \text{ Wb/m}^2.$$

If we draw a graph between flux density and relative permeability (μ_r) from the given data, then value of μ_r for $B = 1{\cdot}2$ Wb/m² is = 560

$$\therefore \quad H = B/\mu_0\mu_r = 1{\cdot}2/4\pi \times 10^{-7} \times 560 = 170 \text{ AT/m}$$

$$l = 50 \text{ cm} = 0{\cdot}5 \text{ m}.$$

$$\text{AT reqd} = 170 \times 0{\cdot}5 = 85$$

$$\text{Total AT reqd} = 795 + 85 = \mathbf{880}$$

Example 7-6. *A ring has a mean diameter of 15 cm, a cross-section of 1·5 cm² and has a radial gap of 0·5 mm cut in it. It is uniformly wound with 1500 turns of insulated wire and a current of 1 A produces a flux of 0·1 mWb across the gap. Calculate the relative permeability of iron on the assumption that there is no magnetic leakage.*

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Solution. Air Gap

$\Phi = 10^{-4}$ Wb ; $A = 1{\cdot}5 \times 10^{-4}$ m²

$B = 10^{-4}/1{\cdot}5 \times 10^{-4} = 1/1{\cdot}5$ Wb/m²

$H = B/\mu_0 = 1/1{\cdot}5 \times 4\pi \times 10^{-7}$ AT/m

$= 5{\cdot}306 \times 10^5$ AT/m

Air-gap length $= 0{\cdot}5 \times 10^{-3}$ m

AT required

$= 5{\cdot}306 \times 10^5 \times 0{\cdot}5 \times 10^{-3} = 265{\cdot}3$

Total AT provided $= 1500 \times 1 = 1500$

AT available for iron path

$= 1500 - 265{\cdot}3 = 1234{\cdot}7$

H for iron path $= \dfrac{1234{\cdot}7}{\pi \times 0{\cdot}15}$

$= 2{,}630$ AT/m

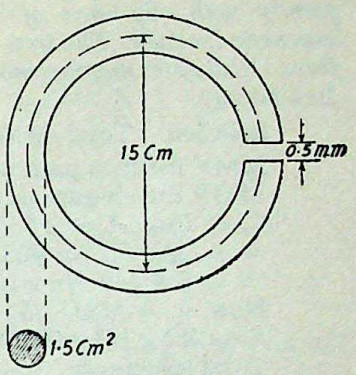

Fig. 7-27

Now $H = B/\mu_0 \mu_r$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1/1{\cdot}5}{2{,}630 \times 4\pi \times 10^{-7}} = \mathbf{202}$$

Example 7-7. *An iron ring has a mean diameter of 25 cm and a cross-sectional area of 4 cm². It is wound with a coil of 1200 turns. An air-gap of 1·5 mm width is cut in the ring. Determine the current required in the coil to produce a flux of 0·48 mWb in the air-gap if μ_r for iron under these conditions is 800. Neglect magnetic leakage and fringing. Given : $\mu_0 = 4\pi \times 10^{-7}$ H/m.*

(A.M.I.E. Winter 1980)

Solution. Air-Gap

$\Phi = 0{\cdot}48$ mWb $= 0{\cdot}48 \times 10^{-3}$ Wb

$= 4{\cdot}8 \times 10^{-4}$ Wb

$A = 4$ cm² $= 4 \times 10^{-4}$ m²

$B = 4{\cdot}8 \times 10^{-4}/4 \times 10^{-4} = 1{\cdot}2$ Wb/m²

$H = B/\mu_0 = 1{\cdot}2/4\pi \times 10^{-7}$

$= 95{\cdot}4 \times 10^4$ AT/m

AT reqd $= H \times l = 95{\cdot}4 \times 10^4 \times 1{\cdot}5 \times 10^{-3}$

$= 1431$

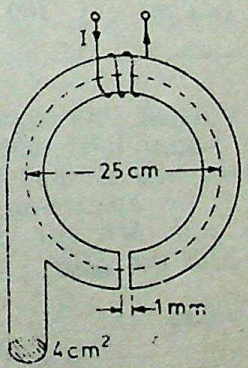

Fig. 7-28

Iron Path

Since leakage is negligible, flux and hence flux density remain the same.

$H = B/\mu_0 \mu_r = 1{\cdot}2/4\pi \times 10^{-7} \times 800$

$= 1192$ AT/m

$l = \pi D = \pi \times 0{\cdot}25 = 0{\cdot}785$ m

AT reqd $= 1192 \times 0{\cdot}785 = 936$

Total AT reqd $= 1431 + 936 = 2367$

$\therefore \quad 1200\, I = 2367, \qquad I = 1{\cdot}97$ A.

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Example 7-8. *A steel ring 30 cm mean diameter and of circular section 2 cm in diameter has an air-gap 1 mm long. It is wound uniformly with 600 turns of wire carrying a current of 2·5 A. Neglect magnetic leakage. The iron path takes 40% of the total magnetomotive force. Estimate magnetomotive force, magnetic flux, reluctance and flux density.* (*A.M.I.E. Winter 1986*)

Solution. Total MMF $= 600 \times 2{\cdot}5 = 1500$ AT
MMF for iron path $=$ 40% of $1500 = 600$ AT
MMF for air-gap $= 1500 - 600 = 900$ AT
$S_a = l/\mu_0\mu_r A = 2{\cdot}5 \times 10^6$ AT/Wb
$\Phi = MMF/S = 900/2{\cdot}5 \times 10^6 = 3{\cdot}6 \times 10^{-4}$ Wb
$B = 3{\cdot}6 \times 10^{-4}/\pi \times 10^{-4} = 1{\cdot}145$ Wb/m^2
Now, $\Phi = MMF_a/S_a = MMF_i/S_i$
$S_i = 2{\cdot}5 \times 10^6 \times 600/900 = 1{\cdot}67 \times 10^6$ AT/Wb
Total reluctance, $S = S_a + S_i = 4{\cdot}17 \times 10^6$ AT/Wb.

Example 7-9. *A shell-type magnetic core made of silicon steel has the dimensions as shown in Fig. 7-29 below. The area of cross-section of the central limb is 10 cm^2 and that of each side limb is 8 cm^2. The central limb is wound with a coil of 400 turns. Estimate the ampere-turns required for (i) air-gap (ii) central limb and (iii) the rest of the magnetic circuit when the air-gap flux is 1·2 mWb. Assume no leakage. Also calculate the current required in the coil to produce the above flux. The data for the B-H curve for silicon steel are :*

B (Wb/m^2)	*0·5*	*0·75*	*1·0*	*1·2*	*1·4*
H (AT/m)	*100*	*150*	*240*	*470*	*900*

(*A.M.I.E. Summer 1979*)

Solution. As seen from Fig. 7-29, the magnetic path consists of the central limb with the air-gap *i.e.* path *EN* and two parallel paths between *E* and *N i.e.* paths *EFN* and *EGN*. The total m. m. f. *i.e.* ampere-turns required are the sum of those required for (*i*) path *MN* (*ii*) air-gap and (*iii*) either of the two parallel paths *EFN* and *EGN*.

(*i*) Air Gap

$\Phi = 1{\cdot}2$ mWb, $A = 10$ cm^2 $= 10 \times 10^{-4}$
$= 10^{-3}$ m^2

$$B = \frac{1{\cdot}2 \times 10^{-3}}{10^{-3}} = 1{\cdot}2 \text{ Wb/m}^2$$

$l = 0{\cdot}5$ mm $= 0{\cdot}5 \times 10^{-3}$ m
$H = B/\mu_0 = 1{\cdot}2/4\pi \times 10^{-7}$
$= 95{\cdot}45 \times 10^4$ AT/m
AT required $= H \times l = 95{\cdot}45 \times 10^4 \times 0{\cdot}5 \times 10^{-3} = 477$

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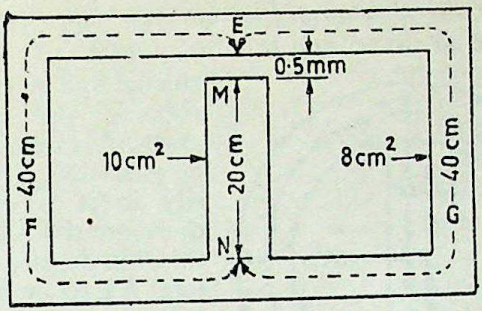

Fig. 7-29

(*ii*) **Central limb MN**

$B = 1{\cdot}2$ Wb/m² —as in the air-gap

Value of H corresponding to the above flux density as found from the given data is = 470 AT/m. Since MN = 20 cm = 0·2 m,

∴ AT required = $470 \times 0{\cdot}2 = 94$.

(*iii*) **Side limb EFN**

At point E, the flux is divided into two equal parts. Flux through either side limb is = 1·2/2 = 0·6 mWb.

∴ $B = 0{\cdot}6 \times 10^{-3}/8 \times 10^{-4} = 0{\cdot}75$ Wb/m²

From the given data, corresponding value of H = 150 AT/m

AT required = $150 \times 0{\cdot}4 = 60$.

Total AT required = 477+94+60 = **631**

Exciting current = 631/400 = **1·58 A**

Example 7-10. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and across the ring an ordinary steel bar 18 cm × 3 cm × 0·4 cm is fitted with negligible gap. Calculate the number of ampere-turns required to be applied to one-half of the ring to produce a flux density of 1 Wb/m² in the other half. Neglect leakage. The B-H characteristics are as below :*

For Cast Steel

B in Wb/m²	*1·0*	*1·1*	*1·2*
Amp-turns/m	*900*	*1020*	*1220*

For Ordinary Steel

B in Wb/m²	*1·2*	*1·4*	*1·45*
Amp-turns/m	*590*	*1200*	*1650*

Solution. The magnetic circuit is shown in Fig. 7-30.

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The m.m.f. (or AT) produced on the half A acts across the parallel magnetic circuits C and D. First, total AT across C is calculated and since these amp-turns are also applied across D, the flux density B in D can be estimated. Next, flux density in A is calculated and therefore the AT required for this flux density. In fact, the total AT (or m.m.f.) required is the sum of that required for A and that for any one of the two parallel paths C or D

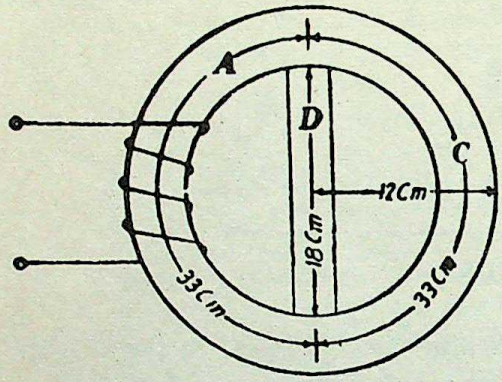

Fig. 7-30

Value of flux density in C $=1{\cdot}0$ Wb/m^2

Mean diameter of the ring $=(24+18)/2=21$ cm

$\therefore$ Mean circumference $=\pi\times 21=66$ cm

Length of path A or C $=66/2=33$ cm $=0{\cdot}33$ m

Value of AT/m for a flux density of $1{\cdot}0$ Wb/m^2 as seen from the given B-H characteristics $=900$ AT/m

$\therefore$ Total AT for path C $=900\times 0{\cdot}33=297$

The same ATs are applied across path D.

Length of path D $=18$ cm $=0{\cdot}18$ m

$\therefore$ AT/m for D $=297/0{\cdot}18=1650$

Value of B corresponding to this value of AT/m from the given table is $=1{\cdot}45$ Wb/m^2

Flux through path $C=B\times A=1{\cdot}0\times 9\times 10^{-4}=9\times 10^{-4}$ Wb

Flux through path $D=1{\cdot}45\times(3\times 0{\cdot}4\times 10^{-4})=1{\cdot}74\times 10^{-4}$ Wb

$\therefore$ Total flux through A

$=9\times 10^{-4}+1{\cdot}74\times 10^{-4}=10{\cdot}74\times 10^{-4}$ Wb

Flux density through A

$=10{\cdot}74\times 10^{-4}/9\times 10^{-4}=1{\cdot}193$ Wb/m^2

No. of AT/m required to produce this flux density as found from the given data $=1200$.

$\therefore$ Ampere-turns required on limb $A=1200\times 0{\cdot}33=396$

Total AT required $=396+297=693$.

Example 7-11. *Determine the exciting current required to establish a flux of 1·6 mWb in each air gap in the magnetic circuit shown in Fig. 7-31. Dimensions given are in mm. Cross-section of A, B, C is square 40 mm side and that of iron armature D is 30 mm × 40 mm. The relative permeabilities of A, B and C are 900 and that of D is 750. There are 1500 turns in each of the exciting coils connected in series. Neglect fringing and leakage.* (*A.M.I.E. May 1969*)

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Solution. The mean flux path is shown by the dotted line in Fig. 7-31. Let us find AT required for each path separately.

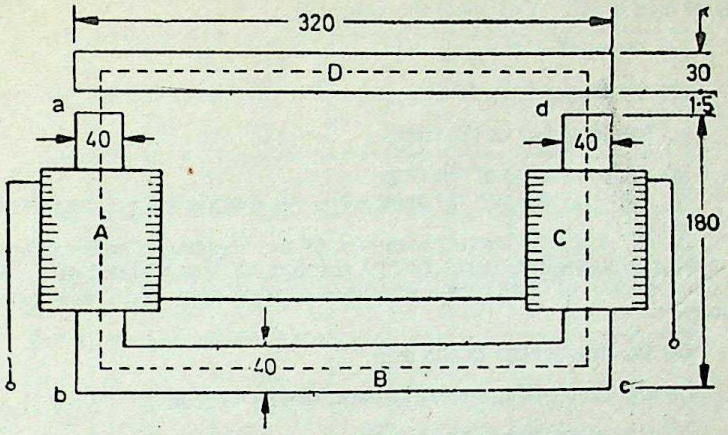

Fig. 7-31

Air Gaps

$$\Phi = 1{\cdot}6 \times 10^{-3} \text{ Wb} \; ; \; A = (40 \times 40) \times 10^{-6} = 16 \times 10^{-4} \text{ m}^2$$

$$B = 1{\cdot}6 \times 10^{-3} / 16 \times 10^{-4} = 1 \text{ Wb/m}^2$$

$$H = B/\mu_0 = 1/4\pi \times 10^{-7} = 79{\cdot}54 \times 10^4 \text{ AT/m}$$

Length of the two air-gaps is $= 2 \times 1{\cdot}5 = 3$ mm

$$\text{AT reqd} = 79{\cdot}54 \times 10^4 \times 3 \times 10^{-3} = 2{,}386$$

Paths A, B and C

$$B = 1 \text{ Wb/m}^2$$

$$H = 1/4\,\pi \times 10^{-7} \times 900 = 884 \text{ AT/m}$$

Mean length of the path '*abcd*' $= 160 + 280 + 160 = 600$ mm

$$\text{AT reqd} = 884 \times 600 \times 10^{-3} = 530$$

Iron Armature D

$$\Phi = 1{\cdot}6 \text{ mWb} \; ; \; A = (30 \times 40) \times 10^{-6} = 12 \times 10^{-4} \text{ m}^2$$

$$B = 1{\cdot}6 \times 10^{-3} / 12 \times 10^{-4} = 4/3 \text{ Wb/m}^2$$

$$H = 4/3 \times 4\pi \times 10^{-7} \times 750 = 1415 \text{ AT/m}$$

Mean flux length $= 320 - (2 \times 20) = 280$ mm

$$\text{AT reqd} = 1415 \times 280 \times 10^{-3} = 396$$

Total AT required for the whole circuit

$$= 2{,}386 + 530 + 396 = 3{,}312$$

Total No. of turns $= 3{,}000$

Exciting current $= 3{,}312/3000 = \mathbf{1{\cdot}104}$ **A**

Tutorial Problems—10

1. Two straight wires each 1 metre long are parallel and 0·02 m apart. Calculate the force between them if the current in each wire is 40 A. [16×10^{-3} N] (*A.M.I.E. Summer 1975*)

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2. A coil of 500 turns and resistance 20 Ω is wound uniformly on an iron ring of mean circumference 50 cm and cross-sectional area 4 cm². It is connected to a 24-V d.c. supply. Under these conditions, the relative permeability of iron is 800. Calculate the values of :

(*a*) the mmf of the coil

(*b*) the magnetising force

(*c*) the total flux in the iron

(*d*) the reluctance of the ring.

[(a) 600 AT (*b*) 1200 AT/m (*c*) 0·483 mWb (*d*) $1{\cdot}24\times10^6$ AT/Wb]

3. A magnetic circuit consists of an iron ring of mean circumference 80 cm with cross-sectional area 12 cm² throughout. A current of 2 A in the magnetising coil of 200 turns produces a total flux of 1·2 mWb in the iron. Calculate :

(a) the flux density in the iron

(*b*) the absolute and relative permeability of iron

(*c*) the reluctance of the circuit.

[(a) 1 Wb/m² (*b*) 0·002 ; 1590 (*c*) $3{\cdot}33\times10^5$ AT/Wb]

4. A series magnetic circuit has an iron path of length 50 cm and an air-gap of length 1 mm. The cross-sectional area of the iron is 6 cm² and the exciting coil has 400 turns. Determine the current required to produce a flux of 0·9 mWb in the circuit. The following points are taken from the magnetisation curve for the iron

Flux density (Wb/m²) :	1·2	1·35	1·45	1·55
Magnetising force (AT/m) :	500	1,000	2,000	4,000

[6·35 A]

5. An iron ring 0·15 m diameter and 10 cm² cross-section with a saw-cut 2 mm wide is wound with 200 turns of wire. The gap flux density is 1 Wb/m². The relative permeability of iron is 500. Find the exciting current and inductance. Neglect leakage and fringing.

[11·7 A ; 17·1 mH] (*A.M.I.E. Nov. 1969*)

6. An iron ring 100 cm mean circumference and of circular cross-section 5 cm² has a saw-cut 2 mm length. It is wound with 500 turns of wire. If 0·5 mWb flux exists across the air-gap, what will be the value of exciting current ? Take leakage coefficient = 1·26 and relative permeability of iron = 500. **[3·6 A]** (*A.M.I.E. Summer 1976*)

7. Calculate the mmf required to produce a flux of 0·8 mWb in each air-gap in the symmetrical magnetic circuit shown in Fig. 7-32. All dimensions are in mm and the iron is 5 cm thick throughout. The material is cast steel with data shown in the graph of Fig. 7-33. **[1616 A]**

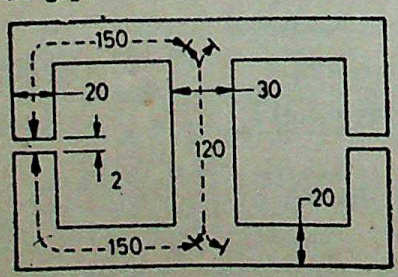

Fig. 7-32

8. Calculate the approximate value of current needed to produce a flux of 2 mWb in the air gap of the magnetic circuit shown in Fig. 7-34. The material is cast steel (for data refer to Fig. 7-33) and is 10 cm. thick. The exciting coil has 1000 turns and is wound on the right hand limb. Neglect leakage and fringing. All dimensions are in cm. **[1·92 A]**

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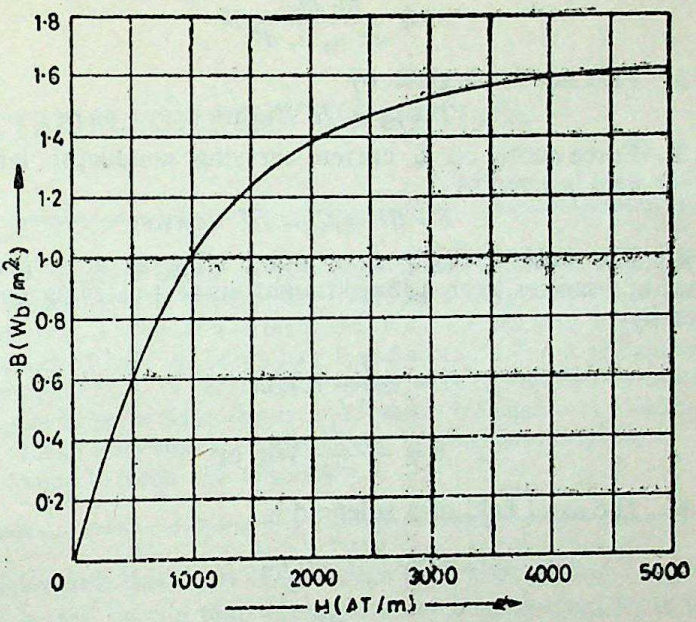

Fig. 7-33

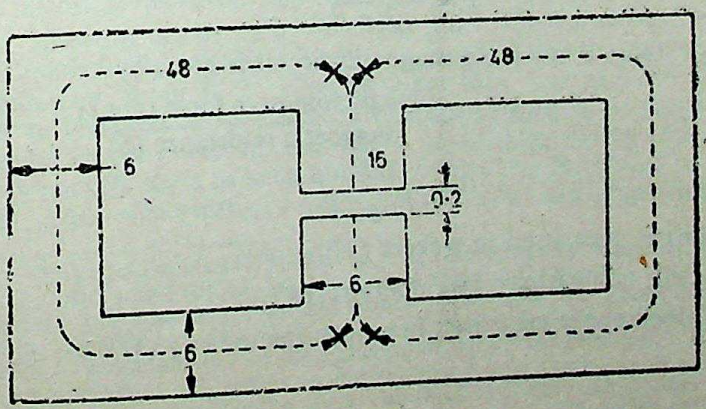

Fig. 7-34

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HIGHLIGHTS

1. The force between two magnetic poles is given by

$$F = \frac{m_1 m_2}{4\pi \mu_0 \mu_r d^2} \text{ N}$$

2. Flux density is given by

$$B = \mu_0 \mu_r H \text{ Wb/m}^2$$

3. Force acting on a current-carrying conductor lying in a magnetic field is given by

$$F = BIl = \mu_0 \mu_r HIl \text{ newton}$$

4. The field strength at a point lying at a perpendicular distance of r metres from a long straight current-carrying conductor is given by

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

and

$$B = \frac{\mu_0 I}{2\pi r} \text{ Wb/m}^2 \quad \text{—in air}$$

5. The axial field of a solenoid is

$$H = \frac{NI}{l} \text{ AT/m}$$

and

$$B = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \quad \text{—in air}$$

6. According to Work Law, the net work done on or by a unit pole in moving round any single closed path is numerically equal to the ampere-turns linked with the path.

7. Force between two long current-carrying conductors is given by

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ N}$$

8. Ohm's law for a magnetic circuit is given by

$$\text{Flux } (\Phi) = \frac{\text{magnetomotive force (mmf)}}{\text{magnetic reluctance (S)}}$$

$$\therefore \quad \Phi = \frac{NI}{l/\mu_0 \mu_r A}$$

Flux is measured in webers (Wb)

MMF is measured in ampere-turns (AT)

Reluctance is measured in amp-turns/weber (AT/Wb)

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ELECTROMAGNETIC INDUCTION 8

1. Relation Between Magnetism and Electricity. 2. Production of Induced E.M.F. and Current. 3. Faraday's Laws of Electromagnetic Induction. 4. Direction of Induced E.M.F. and Current. 5. Fleming's Right-hand Rule 6. Lenz's Law 7. Induced E.M.F. 8. Dynamically Induced E.M.F. 9. Statically Induced E.M.F. 10. Self-inductance 11. Coefficient of Self-inductance 12. Mutual Inductance 13. Coefficient of Mutual Inductance 14. Coefficient of Magnetic Coupling 15. Inductances in Series 16. Highlights.

8-1. Relation Between Magnetism and Electricity

It is well known that whenever an electric current flows through a conductor, a magnetic field is immediately brought into existence in the space surrounding the conductor. We can say that when electrons are in motion, they produce a magnetic field. The converse of this is also true *i.e.* when a magnetic field embracing a conduct or moves relative to the conductor, it produces a flow of electrons. This phenomenon whereby an e.m.f. and hence current (*i.e.* flow of electrons) is induced in any conductor that is cut across or is cut by a magnetic flux is known as *electromagnetic induction.* The historical background of this phenomenon is this :

After the discovery by Oersted that electric current produces a magnetic field, scientists began to search for the reverse phenomenon from about 1821. The problem they put to themselves was how to 'convert magnetism into electricity'. It is recorded that Michael Faraday[1] was in the habit of walking about with magnets in his pockets so as to keep himself constantly reminded of the problem. After nine years of continuous research and experimentation,

[1]Michael Faraday (1791—1867), an English physicist and chemist was the son of a blacksmith and till 1813 was an apprenticed book-binder. Later on, he got the job of an assistant in the laboratory of the Royal Institution of Great Britain. By dint of his hard labour, he rose to the position of Director in 1825. In 1833 he was appointed for life Fullerian professor of chemistry. In addition to his notable contributions to chemistry, he did outstanding work in electricity. He was the first to discover the phenomenon of electromagnetism. He discovered the laws of electrolysis known after his name, studied the effect of magnetism on polarised light and diamagnetism etc. and carried out much investigation connected with the electrostatic phenomenon.

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he succeeded in producing electricity by 'converting' magnetism. In 1831, he formulated the basic laws underlying the phenomenon of electromagnetic induction (known after his name), upon which is based the operation of most of the commercial apparatus like motors, generators and transformers etc.

8-2. Production of Induced E.M.F. and Current

In Fig. 8-1 is shown an insulated coil whose terminals are connected to a sensitive galvanometer *G*. It is placed close to a stationary bar magnet initially at position *AB* (shown dotted). As it is, some lines of flux from the *N*-pole of the magnet are linked with or thread through the coil but, as yet, there is no deflection of the galvanometer. Now, suppose, that the magnet is *suddenly* brought closer to the coil in position *CD* (Fig. 8-1). Then it is found that there is a jerk or a sudden but momentary deflection in the galvanometer and that this *lasts so long as the magnet is in motion relative to the coil, not otherwise*. This deflection is reduced to zero when the magnet becomes again stationary at its new position *CD*. It should be noted that due to the approach of the magnet, flux linked with coil is increased.

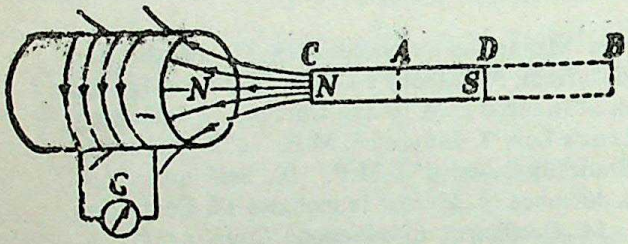

Fig. 8-1

Next, the magnet is *suddenly* withdrawn away from the coil as shown in Fig. 8-2. It is found that again there is a *momentary* deflection in the galvanometer and it persists so long as the magnet is in motion, not when it becomes stationary. It is important to note that this deflection is in a direction opposite to that of Fig. 8-1. Obviously, due to the withdrawal of the magnet, flux linked with the coil is decreased.

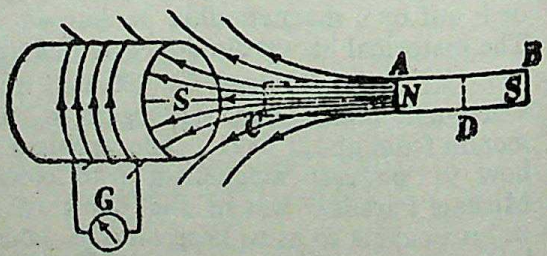

Fig. 8-2

The deflection of the galvanometer indicates the production of e.m.f. in the coil. The only cause of the production can be the sudden approach or withdrawal of the magnet from the coil. It is found that the actual cause of this e.m.f. is the change of the flux linking with the coil. This e.m.f. exists so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f. in a stationary conductor. In fact, the same results can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

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The direction of current set up by the induced e.m.f. is as shown in the two figures given above.

The production of this electromagnetically-induced e.m.f. is further illustrated by considering a conductor *AB* lying within a magnetic field and connected to a galvanometer as shown in Fig. 8-3. It is found that whenever this conductor is moved up or down, a *momentary* deflection is produced in the galvanometer. It means that some transient e.m.f. is induced in *AB*. The magnitude of this induced e.m.f. (and hence the amount of deflection in the galvanometer) *depends on the quickness of the movement of AB.*

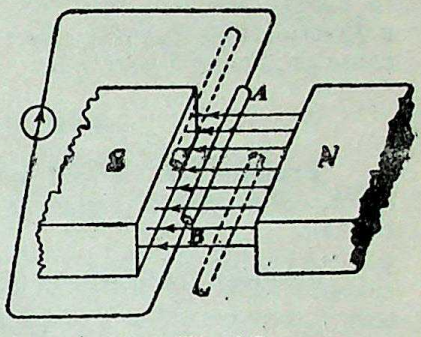

Fig. 8-3

From this experiment we conclude that whenever a conductor cuts or *shears* the magnetic lines of flux, an e.m.f. is always induced in it.

It is also found that if the conductor is moved parallel to the direction of the lines of flux (so that it cuts none of these lines), then no e.m.f. is induced.

8-3. Faraday's Laws of Electromagnetic Induction

(A.M.I.E. Winter 1986)

Faraday summed up the above facts into two laws known as Faraday's laws of Electromagnetic Induction.

First Law. It states :—

'Whenever the magnetic flux linked with a circuit changes, an e.m.f. is always induced in it.' *or*

'Whenever a conductor cuts across magnetic lines of flux, an e.m.f. is induced in that conductor.

Second Law. It states :—

'The magnitude of the induced e.m.f. is equal to the rate of change of *flux linkages*.'

Explanation. Suppose a coil has N turns and flux through it changes from an initial value of Φ_1 Wb to the final value of Φ_2 Wb in time t seconds. Then remembering that by flux linkages is meant the product of number of turns by the flux linked with coil, we have

Initial flux linkages $= N\Phi_1$

Final flux linkages $= N\Phi_2$

$\therefore$ Induced e.m.f. $e = \dfrac{N\Phi_2 - N\Phi_1}{t}$ volt

or $$e = N\,\frac{\Phi_2 - \Phi_1}{t} \text{ volt}$$

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Putting the above expression in its differential form, we get

$$e=\frac{d}{dt}(N\Phi) \qquad \text{or} \qquad e=N\frac{d\Phi}{dt} \text{ volt}$$

Usually, a minus sign is given to the right-hand side expression to signify the fact that the induced e.m.f. sets up current in such a direction that magnetic effect produced by it, opposes the very cause producing it (Art. 8-6).

$$\therefore \qquad e=-N\frac{d\Phi}{dt} \text{ volt}$$

Example 8-1. *A coil of 500 turns is linked by a flux of 0·4 mWb. If the flux is reversed in 0·01 second, find the e.m.f. induced in the coil.*

Solution. Induced e.m.f. is

$$e=N\frac{d\Phi}{dt} \text{ volt}$$

$N=500$, flux$=0{\cdot}4$ mWb

$dt=0{\cdot}01$ second ; $e=?$

$d\Phi=0{\cdot}4-(-0{\cdot}4)=0{\cdot}8$ mWb$=8\times10^{-4}$ Wb

$$e=500\times\frac{8\times10^{-4}}{0{\cdot}01}=\mathbf{40 \text{ volt}}$$

Example 8-2. *The field coils of a 6-pole d.c. generator each having 500 turns, are connected in series. When the field is excited, there is a magnetic flux of 0·02 Wb/pole. If the field circuit is opened in 0·02 second and the residual magnetism is 0·002 Wb/pole, calculate the average voltage which is induced across the field terminals. In which direction is this voltage directed relative to the direction of the current ?*

Solution. Total number of the turns,

$$N=6\times500=3000$$

Total initial flux $=6\times0{\cdot}02=0{\cdot}12$ Wb

Total residual flux $=6\times0{\cdot}002=0{\cdot}012$ Wb

Change in flux, $d\Phi=0{\cdot}12-0{\cdot}012=0{\cdot}108$ Wb

Time of opening the circuit,

$$dt=0{\cdot}02 \text{ second}$$

$$\therefore \quad \text{induced e.m.f.}=N\frac{d\Phi}{dt}=3000\times\frac{0{\cdot}108}{0{\cdot}02}=\mathbf{16{,}200 \text{ V}}$$

The direction of this induced e.m.f. is the same as that of the *original* direction of the exciting current.

Example 8-3. *A coil of resistance 100 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it. Find the average e.m.f. and the current if the coil is moved in 1/10th second from the given field to a field strength of 0·2 mWb.*

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Solution.

Induced e.m.f. $e = N \cdot \frac{d\Phi}{dt}$ volt

Here, $d\Phi = 1 - 0{\cdot}2 = 0{\cdot}8$ mWb $= 0{\cdot}8 \times 10^{-3}$ Wb

$dt = 1/10$ second $= 0{\cdot}1$ s ; $N = 100$

$\therefore$ $e = 100 \times 0{\cdot}8 \times 10^{-3}/0{\cdot}1 = \mathbf{0{\cdot}8\ V}$

Total circuit resistance

$$= 100 + 400 = 500\ \Omega$$

$\therefore$ Induced current $= 0{\cdot}8/500 = 1{\cdot}6 \times 10^{-3}$ A $= \mathbf{1{\cdot}6\ mA}$

8-4. Direction of Induced E.M.F. and Current

There exists a definite relationship between the direction of the induced current, the direction of the lines of flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either Fleming's Right hand Rule or Lenz's Law.

8-5. Fleming's Right-hand Rule *(A.M.I.E. Summer 1979)*

Hold out the right hand with the first finger, second finger and thumb at right angles to each other [Fig. 8-4 (*a*)].

If Forefinger represents the direction of the lines of Flux, the thuMb points in the direction of Motion, then second finger points in the direction of the induced Current.

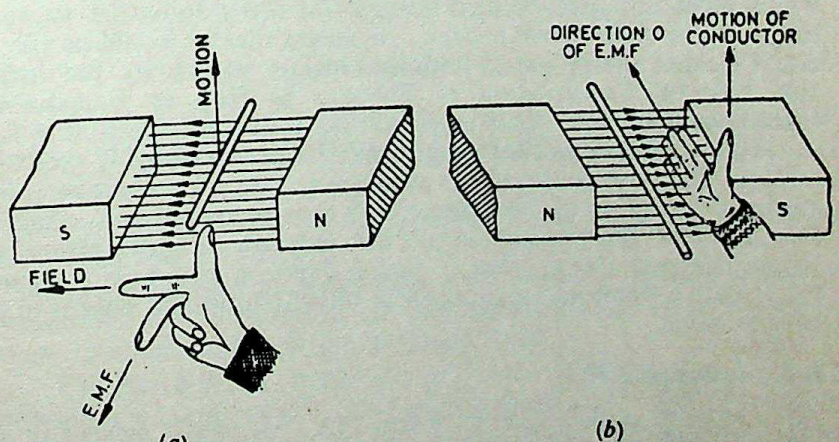

Fig. 8-4

Another way of applying right-hand rule is shown in Fig. 8-4 (*b*). According to this method, when the right hand is so placed in a magnetic field along the conductor that flux coming from *N*-pole *enters* the palm perpendicularly and the thumb points in the direction in which the conductor moves, the other four fingers give the direction of the induced e.m.f.

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8-6. Lenz's Law[1] (*A.M.I.E. May 1974*)

The direction of the induced current may also be found by this law which was formulated by Lenz in 1835. This law states, in effect, that electromagnetically-induced current always flows in such a direction that the action of the magnetic field set up by it tends to oppose the very cause which produces it.

This statement will be clarified with reference to Fig. 8-1 and 8-2. It is found that when *N*-pole of the bar magnet approaches the coil, the induced current set up by the induced e.m.f. flows in the *anti-clockwise direction in the coil as seen from the magnet side.* The result is that that face of the coil becomes a *N*-pole and so tends to retard the onward approach of the *N*-pole of the magnet (like poles repel each other). The mechanical energy spent in overcoming this repulsive force is converted into electrical energy which appears in the coil.

When the magnet is withdrawn, as in Fig. 8-2, the induced current flows in the *clockwise* direction, thus making the face of the coil (facing the magnet) a *S*-pole. Therefore, the *N*-pole of the magnet has to be withdrawn against the attractive force of the *S*-pole of the coil. Again, the mechanical energy required to overcome this force of attraction is converted into electric energy.

It can be shown that Lenz's law is a direct consequence of Law of Conservation of Energy. Imagine for a moment that when *N*-pole of the magnet (see Fig. 8-1) approaches the coil, induced current flows in such a direction as to make the coil face a *S*-pole. Then due to inherent attraction between unlike poles, the magnet would be automatically pulled towards the coil without the expenditure of any mechanical energy. It means that we would be able to create electric energy out of nothing which is denied by the inviolable Law of Conservation of Energy. In fact, to maintain the sanctity of this law, it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause which produces it. In the present case, it is the relative motion of the magnet with respect to the coil which is the cause of the production of the induced current. Hence, the induced current always flows in such a direction as to tend to oppose this relative motion *i.e.* approach or withdrawal of magnet in the present case.

8-7. Induced E.M.F.

Induced e.m.f. can be either (*i*) *dynamically induced* or (*ii*) *statically induced*. In the first case, usually, the field is stationary

---

1. After the Russian-born geologist and physicist Heinrich Friedrich Emil Lenz (1808—1865). He did a lot of investigation on electromagnetism himself and carefully studied the experiments conducted by Faraday. The familiar statement afterwards known as Lenz's law that "the electrodynamic action of an induced current opposes equally the mechanical action inducing it" was contained in one of his papers "On the Direction of Galvanic Currents which are Excited through Electro-dynamic Action" read on 29th Nov. 1833.

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and conductors cut across it (as in d.c. generators). But, in the second case, usually the conductor or the coil remains stationary and flux linked with it is changed by simply increasing or decreasing the current producing this flux (as in transformers).

8-8. Dynamically Induced E.M.F. *(A.M.I.E. Summer 1980)*

In Fig. 8-5, a conductor A is shown in cross-section lying within a uniform magnetic field of flux density B Wb/m². The arrow attached with A shows its direction of motion. Consider the condition shown in Fig. 8-5 (a) when A cuts across at right angles to the flux. Suppose 'l' is its length lying within the field and let it move a distance dx in time dt. Then area swept by it is $= l\,dx$. Hence, flux cut $= l.dx \times B$.

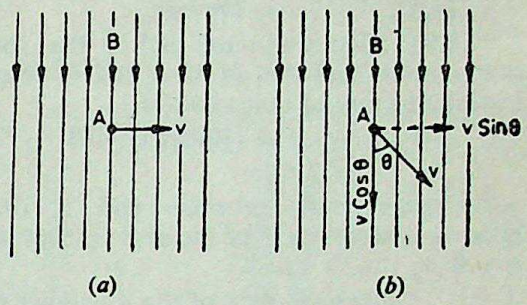

Fig. 8-5

$\therefore$ change in flux, $d\Phi = B\,l\,dx$

Time taken $= dt$

Hence, according to Faraday's Laws (Art. 8-3) the e.m.f. induced in it (known as dynamically induced e.m.f.) is

$$= \text{rate of change of flux linkages}$$

$$= \frac{d\Phi}{dt} = \frac{Bldx}{dt} = Bl\,\frac{dx}{dt} = Blv \text{ volt}$$

where $\frac{dx}{dt} = \text{velocity } v$

If the conductor A moves at an angle θ with the direction of the lines of flux [Fig. 8-5 (b)] then the induced e.m.f. is

$$e = Blv \sin\theta \text{ volt}$$

The direction of the induced e.m.f. is given by Fleming's Right-hand rule (Art. 8-5).

It should be noted that generators work on the production of dynamically induced e.m.f. in the conductors housed in a revolving armature lying within a strong magnetic field.

Example 8-4. *A conductor of length 100 cm moves at right angles to a uniform magnetic field of flux density 1·5 Wb/m² with a velocity of 50 metres/second. Calculate the e.m.f. induced in it. Find also the value of induced e.m.f. when the conductor moves at an angle of 30° to the direction of the field.* **(A.M.I.E. Winter 1986)**

Solution. Here $B = 1{\cdot}5$ Wb/m² $\quad l = 100$ cm $= 1$ m

$v = 50$ m/s $\quad e = ?$

Now $e = Blv$ volt $= 1{\cdot}5 \times 1 \times 50 = \mathbf{75\ V}$

In the second case, $\theta = 30°$, and $\sin 30° = 0{\cdot}5$

$e = 75 \times 0{\cdot}5 = \mathbf{37{\cdot}5\ V}$

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Example 8-5. *A conductor 50 cm long rotates about one end at 1000 r.p.m. in a plane perpendicular to a magnetic field of strength 0·5 Wb/m². Find the emf induced in it.*

(A.M.I.E. Winter 1979)

Solution. As shown in Fig. 8-5(*c*), as the rod *MN* rotates about an axis passing through its end *M*, it cuts the flux perpendicularly.

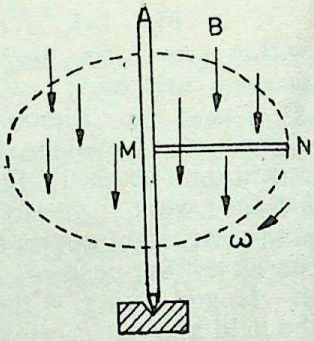

Fig. 8-5 (*c*)

Now, $e = Blv$ volt

The velocity of outer end of the rod is maximum and that of inner end is zero. Linear velocity of outer end is

$$v = \omega r = (1000/60) \times 0{\cdot}3$$
$$= 5 \text{ m/s}$$

Velocity of the inner end is zero because it forms part of the axis so that its $r = 0$

$\therefore$ mean velocity of the revolving rod

$$= (5+0)/2 = 2{\cdot}5 \text{ m/s}$$

$\therefore$ $$e = 0{\cdot}5 \times 0{\cdot}3 \times 2{\cdot}5 = \mathbf{0{\cdot}375\ V.}$$

Example 8-6. *A conductor 10 cm long and carrying a current of 50 A lies perpendicular to a field of strength 1000 AT/m. Calculate*

(i) the force acting on the conductor

(ii) the mechanical power to move this conductor against this force with a speed of 1 m/s and

(iii) e.m. f. induced in the conductor. *(A.M.I.E. June 1974)*

Solution. (*i*) $F = BIl$ newton

Now, $H = 1000$ AT/m

$\therefore$ $B = \mu_0 H = 4\pi \times 10^{-7} \times 10^3 = 4\pi \times 10^{-4}$ Wb/m²

$F = 4\pi \times 10^{-4} \times 50 \times 0{\cdot}1 = \mathbf{6{\cdot}28 \times 10^{-3}\ N}$

(*ii*) $P = F \times v = 6{\cdot}28 \times 10^{-3} \times 1 = \mathbf{6{\cdot}28 \times 10^{-3}\ W}$

(*iii*) $e = Blv = 4\pi \times 10^{-4} \times 0{\cdot}1 \times 1 = \mathbf{4\pi \times 10^{-5}\ V}$

Note. Incidentally, electric power produced

$$= eI = 4\pi \times 10^{-5} \times 50 = 6{\cdot}28 \times 10^{-3} \text{ W}$$

the same as mechanical power input.

Example 8-7. *In a 4-pole dynamo, the flux/pole is 15 mWb. Calculate the average e.m. f. induced in one of the armature conductors if armature is driven at 600 r.p.m.*

Solution. It should be noted that each time a conductor passes under a pole (whether *N* or *S*) it cuts a flux of 15 mWb. Hence, total flux cut in one revolution is $15 \times 4 = 60$ mWb. Since conductor is rotating at $600/60 = 10$ r.p.s., time taken for one revolution is 1/10 second = 0·1 second.

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$\therefore$ average e.m.f. generated $= N \frac{d\Phi}{dt}$ volt

Here $N = 1$; $d\Phi = 60$ mWb $= 6 \times 10^{-2}$ Wb

$dt = 0{\cdot}1$ second

$\therefore$ $e = 1 \times 6 \times 10^{-2} / 0{\cdot}1 = 0{\cdot}6$ V

8-9. Statically-induced E.M.F.

(A.M.I.E. Summer 1980)

It can be further sub-divided into (*a*) *mutually-induced e.m. f. and* (*b*) *self-induced e.m. f.*

(*a*) Mutually-induced e.m.f.

(*i*) Let us first take the case of two parallel conductors *A* and *B* lying close to but not touching each other (Fig. 8-6). Conductor *A* is connected to a battery through a key *K*, whereas *B* is connected across a voltmeter. It is found that at the instant of closing or opening *K*, there is a momentary deflection in voltmeter. As is explained below, it is due to the production of mutually-induced e.m.f. in *B* which is produced because of change in flux linked with it.

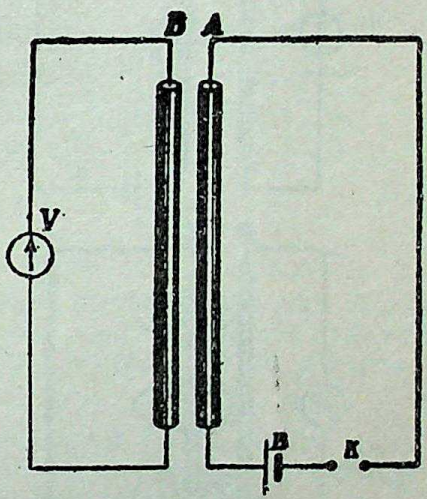

Fig. 8-6

Explanation. It has been shown before that every current-carrying conductor is enshrouded by circular magnetic lines of force. But it should be remembered that this field is not established to its full strength *instantaneously* on closing the key *K*. It needs both time and energy (supplied by the battery) to bring this field to its full value, because current always takes some time (though very small) to come to its maximum steady value (see Art. 9-8 and Fig. 9-10). The gradual increase of current strength and hence of field strength around *A* is shown in Fig. 8-7.

In Fig. 8-7 (*a*), *K* has been closed and only a few lines of flux encircle *A*. Fig. 8-7 (*b*), (*c*) and (*d*) show in a rough diagrammatic way the increasing number of lines of flux that might be produced at successive instants as the current in *A* increases.. In Fig. 8-7 (*d*) it is assumed that both current and flux around *A* have reached their maximum steady values and will continue at these values till *K* is opened.

As the flux increases, these lines of flux may be thought of as emanating *i.e.* expanding outwardly from the axis of the conductor *A* just as ripples emanate from the point where a stone piece is drop-

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ped in a pond. These circular lines of flux expand very rapidly and, moreover, expand at right angles to the conductor.

It will be seen from Fig. 8-7 (*c*) that some flux produced by current in *A* has cut conductor *B*. The flux that cuts *B* induces an e.m.f. in *B* which is called *mutually-induced e.m.f.* The induced e.m.f. is so called because it is produced in a conductor due to the changes of flux in a neighbouring conductor. In Fig. 8-7 (*d*) maximum flux has been established. Hence, *B* will not be cut further

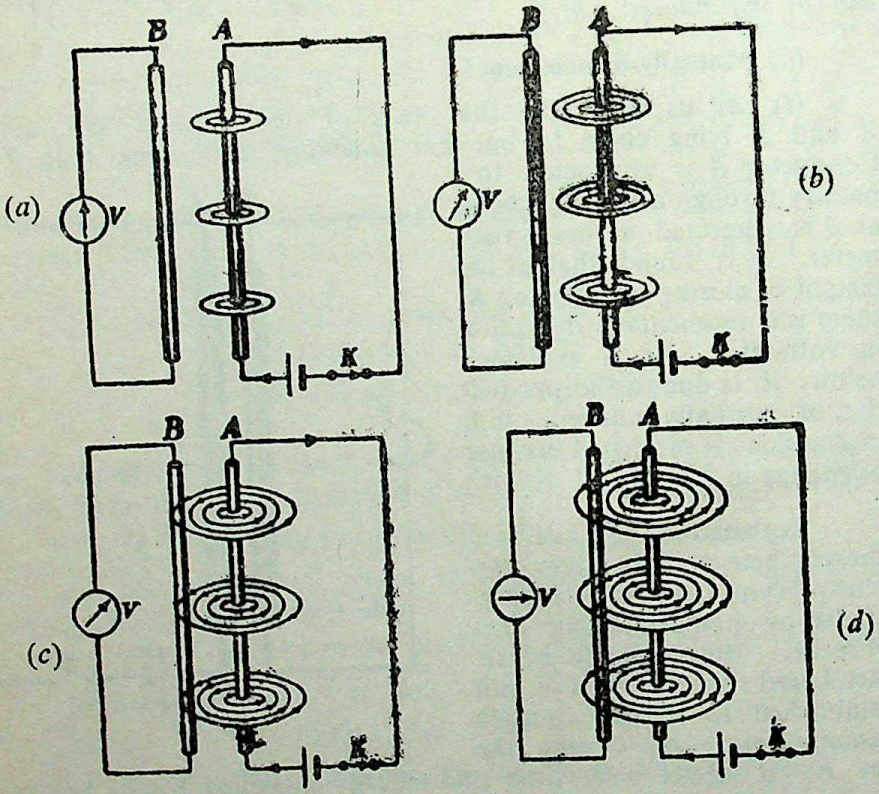

Fig. 8-7

by more flux with the result that no further e.m.f. will be induced in it, because e.m.f. is induced only when the flux linked with a coil or a conductor is changing.

When *K* is opened, the current in *A* will rapidly but not *instantaneously* decrease to zero value. Hence, flux will also decrease to zero. The circular lines of flux will, therefore, contract, return to the axis of *A* and vanish. These lines while returning to *A* will cut *B* in a direction opposite to that in which they cut it while expanding outwards from *A*. Hence, they will again induce an e.m.f. in *B*, through this time in the opposite direction.

So, we conclude that :—

(1) *A momentary e.m.f. is induced in B whenever K is closed or opened. This is shown by the momentary deflection of the voltmeter.*

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(2) *No e.m.f. is induced in B when current through A is constant.*

(3) *The direction of the induced e.m. f. in B is opposite to that in A.*

(*ii*) Let us now consider two coils *A* and *B* lying close to each other [Fig. 8-8 (*a*)],

Coil *A* is joined to a battery, a switch and a variable resistance *R* whereas coil *B* is connected to a sensitive voltmeter *V*. When current through *A* is established by closing the key, its magnetic flux is set up which partly links with or threads through the coil

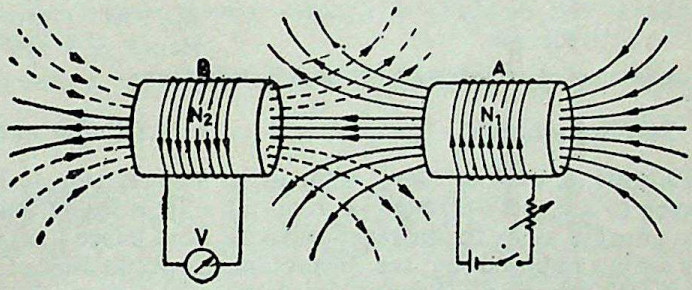

Fig. 8-8 (*a*)

B. As current through *A* is changed, the flux linked with *B* is also changed. Hence, mutually induced e.m.f. is produced in *B* whose magnitude is given by Faraday's Laws (Art. 8-3) and direction by Lenz's Law (Art. 8-6).

If, now, battery is connected to *B* and voltmeter across *A* [Fig. 8-8 (*b*)] then the situation is reversed and now a change of current in *B* will produce mutually-induced e.m.f. in *A*.

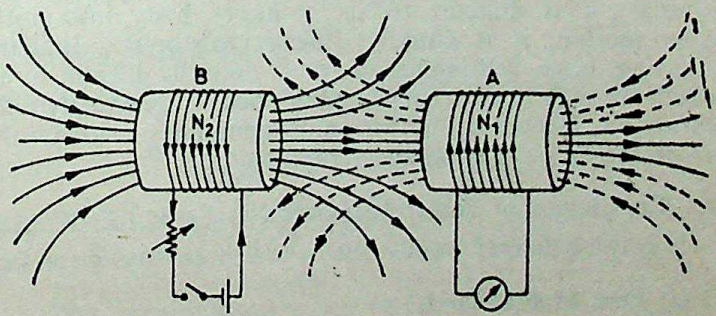

Fig. 8-8 (*b*)

It is obvious that in either of the two examples considered above, there is no movement of any conductor, the flux variations being brought about by variations in current strength only. *Such an e.m. f. induced in one coil by the influence of the other coil is called (statically but) mutually-induced e.m. f.*

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(*b*) **Self-induced e.m.f.** This is the e.m.f. induced in a coil due to the *change of its own flux linked with it*. If current through the coil (Fig. 8-9) is changed, then the flux linked with its own turns will also change which will produce in it what is called *self-induced* e.m.f. The direction of this induced e.m.f. would be such as to oppose any change of flux which is, in fact, the very cause of its production. Hence, it is also known as the opposing or counter e.m.f. of self-induction.

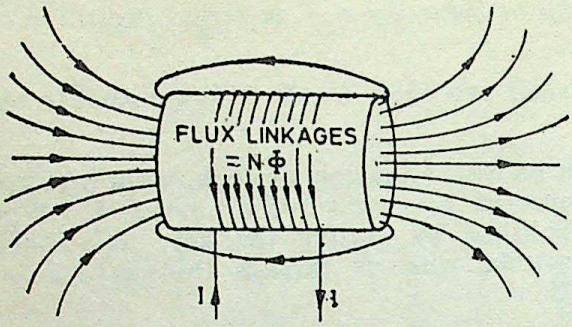

Fig. 8-9

8-10. Self-inductance *(A.M.I.E. Summer 1981)*

Imagine a coil of wire, similar to one shown in Fig. 8-9, connected to a battery through a rheostat. It is found that whenever an effort is made to increase current (and hence flux) through it, it is always opposed by the instantaneous production of counter e.m.f. of self-induction. Energy required to overcome this opposition is supplied by the battery. As will be further explained later on, this energy is stored in the additional flux produced.

If, now, an effort is made to decrease the current (and hence the flux) then again it is delayed due to the production of self-induced e.m.f., this time in the opposite direction. This property of the coil due to which it opposes any increase or decrease of current or flux through it, is known as *self-inductance*. It is quantitatively measured in terms of coefficient of self-inductance *L*. This property is analogous to inertia in a material body. We know by experience that initially it is difficult to set a heavy body into motion, but once in motion, it is equally difficult to stop it. Similarly, in a coil having large self-induction, it is initially difficult to establish a current through it, but once established, it is equally difficult to withdraw it. Hence, self-induction, is sometimes analogously called *electrical inertia or electromagnetic inertia*.

8-11. Coefficient of Self-inductance (1) *(A.M.I.E. Winter 1979)*

It may be defined in any one of the three ways given below :

(*i*) **First Method for L**

The coefficient of self-induction of a coil is defined as

"the weber-turns per ampere in the coil"

By 'weber-turns' is meant the product of flux in webers and the number of turns with which the flux is linked. In other words, it is the flux linkages of the coil.

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Consider a solenoid having N turns and carrying a current of I amperes. If the flux produced is Φ webers, then weber-turns are $N\Phi$. Hence, weber-turns per ampere are $N\Phi/I$.

$\therefore$ By definition, $L = \dfrac{N\Phi}{I}$

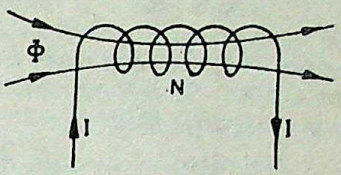

Fig. 8-10

The unit of self-induction is henry[1].

If in the above relation,

$N\Phi = 1$ Wb-turn, $I = 1$ ampere

then $L = 1$ henry (H).

Hence, a coil is said to have a self-inductance of one henry if a current of one ampere flowing through it produces flux linkages of one wb-turn in it.

Therefore, the above relation becomes

$$L = \frac{N\Phi}{I} \text{ henry}$$

Example 8-8. *The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω. When the exciting voltage is 230 V, the magnetic flux linking the coil is 0·005 Wb. Calculate the self-inductance of the coil and the energy stored in the magnetic field.*

Solution. Formula used : $L = \dfrac{N\Phi}{I}$ henry

Current through coil $= 230/50 = 4{\cdot}6$ A

$$\Phi = 0{\cdot}005 \text{ Wb} ; \quad N = 960$$

$$\therefore \quad L = \frac{960 \times 0{\cdot}005}{4{\cdot}6} = 1{\cdot}0435 \text{ H}$$

$$\text{Energy stored} = \frac{1}{2} LI^2 = \frac{1}{2} \times 1{\cdot}0435 \times 4{\cdot}6^2 = \mathbf{11{\cdot}84\ J}$$

(*ii*) Second Method for L

We have seen in Art. 7-23 that flux produced in a solenoid

$$\Phi = \frac{NI}{l/\mu_0 \mu_r A} \qquad \therefore \quad \frac{\Phi}{I} = \frac{N}{l/\mu_0 \mu_r A}$$

$$\text{Now} \qquad L = N \cdot \frac{\Phi}{I} = N \cdot \frac{N}{l/\mu_0 \mu_r A} \text{ H}$$

$$\therefore \qquad L = \frac{N^2}{l/\mu_0 \mu_r A} = \frac{N^2}{S} \text{ H}$$

$$\text{or} \qquad L = \frac{\mu_0 \mu_r A N^2}{l} \text{ H}$$

[1]After the American scientist Joseph Henry (1707—1878), a contemporary of Faraday. He discovered electromagnetic induction independent of Faraday. In 1831, he discovered self-inductance. He was the first to insulate wires. He invented the bobbin winding for electromagnets and electromagnetic relays which are extensively used for the control of electromagnetic machinery.

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It gives the value of self-induction in terms of the dimensions of the solenoid.

Example 8-9. *A magnetic field is produced by a coil of 300 turns which is wound on a closed iron ring. The ring has a cross-section of 20 cm² and mean length of 120 cm. The permeability of the iron is 800. If the current in the coil is 10 A, find the energy stored in the magnetic field.* (*A.M.I.E. Summer 1976*)

Solution. $L=\mu_0\mu_r AN^2/l$ henry

$=4\pi\times10^{-7}\times800\times20\times10^{-4}\times300^2/1{\cdot}2=0{\cdot}151$ H

$E=\frac{1}{2}LI^2=\frac{1}{2}\times0{\cdot}151\times10^2=7{\cdot}55$ J

(*iii*) **Third Method for L**

As seen from above,

$$L=\frac{N\Phi}{I}$$

$$\therefore \quad N\Phi=LI$$

or $$-N\Phi=-LI$$

Differentiating both sides, we get

$$-\frac{d}{dt}(N\Phi)=-\frac{d}{dt}(LI)$$

$$\therefore \quad -N\cdot\frac{d\Phi}{dt}=-L\frac{dI}{dt} \quad \text{(assuming } L \text{ to be constant)}$$

As seen from Art. 8-3

$$-N\cdot\frac{d\Phi}{dt}=\text{self-induced e.m.f. } e_L$$

$$\therefore \quad e_L=-L\frac{dI}{dt} \quad \text{or} \quad L=\frac{-e_L}{dI/dt} \text{ H}$$

If $\frac{dI}{dt}=1$ ampere/second and $e_L=1$ volt

then $L=1$ H

Hence, a coil has a self-inductance of one henry if one volt is induced in it when current through it changes at the rate of one ampere/second.

Example 8-10. *An air-cored toroidal coil has 450 turns and a mean diameter of 30 cm and a cross-sectional area of 5 cm². Calculate (i) the inductance of the coil and (ii) the average induced e.m.f. if a current of 4 A is reversed in 60 milli-seconds.* (*A.M.I.E. Summer 1981*)

Solution. (*a*) $A=5\times10^{-4}$ m² ; $L=0{\cdot}3\pi$ metre

$$L=\frac{N^2}{l/\mu_0\mu_r A}=\frac{\mu_0\mu_r AN^2}{l}$$

$=4\pi\times10^{-7}\times5\times10^{-4}\times450^2/0{\cdot}3\pi$ henry

$=135\times10^{-6}$ H $=135$ μH.

(*b*) $dI=4-(-4)=8$ A

Induced e.m.f. $=LdI/dt=135\times10^{-6}\times8/60\times10^{-3}=18$ mV.

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Example 8-11. *An iron rod 2 cm in diameter and 20 cm long is bent into a closed ring and is wound with 3000 turns of wire. It is found that when a current of 0·5 A is passed through this coil, the flux density in the coil is 0·5 Wb/m². Assuming that all the flux links with every turn of the coil, what is (a) the B/H ratio for the iron (b) the inductance of the coil ? What voltage would be developed across the coil if the current through the coil is interrupted and the flux in the iron falls to 10% of its former value in 0·001 second ?*

Solution. $H = NI/l$ AT/m

$= 3000 \times 0{\cdot}5/0{\cdot}2 = 7{,}500$ AT/m

$B = 0{\cdot}5$ Wb/m²

(*a*) Now $\frac{B}{H} = \frac{0{\cdot}5}{7500}$ H/m $= 6{\cdot}67 \times 10^{-5}$ H/m

Also $\mu_r = B/\mu_0 H = 6{\cdot}67 \times 10^{-5}/4\pi \times 10^{-7} = 53$

(*b*) $L = N\Phi/I$ H

$= \frac{3000 \times \pi \times 0{\cdot}02^2 \times 0{\cdot}5}{4 \times 0{\cdot}5} = 0{\cdot}94$ H

$e_L = N \frac{d\Phi}{dt}$ volt

$d\Phi = 90\%$ of original flux

$= \frac{0{\cdot}9 \times \pi (0{\cdot}02)^2 \times 0{\cdot}5}{4} = 0{\cdot}45\pi \times 10^{-4}$ Wb

$dt = 0{\cdot}001$ second

$e = 3000 \times 0{\cdot}5\pi \times 10^{-4}/0{\cdot}001 = 424$ V

8-12. Mutual Inductance

A.M.I.E. Summer 1981

In Art. 8-9 (Fig. 8-8) we have seen that any change of current in coil *A* is always accompanied by the production of mutually-induced e.m.f. in coil *B*. Mutual inductance may, therefore, be defined as the ability of one coil (or circuit) to produce an e.m.f. in a nearby coil by induction when the current in the first coil changes. This action being reciprocal, the second coil can also induce an e.m.f. in the first when current in second coil changes. This ability of reciprocal induction is measured in terms of the co-efficient of mutual induction *M*.

8-13. Coefficient of Mutual Inductance (M)

(A.M.I.E. Winter 1979)

It can also be defined in three ways as given below :

(*i*) First Method for M

(A.M.I.E. Dec 1974)

Let there be two magnetically-coupled coils having N_1 and N_2 turns respectively (Fig. 8-11).

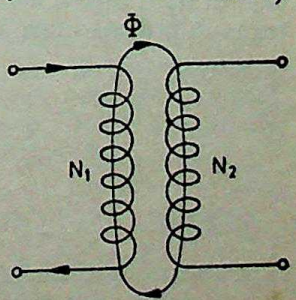

Fig. 8-11

Coefficient of mutual inductance between the two coils is defined as "*the weber-turns in one coil due to one ampere current in the other.*"

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Let a current of I_1 amperes when flowing in the first coil produce a flux of Φ_1 weber in it. *It is supposed that whole of this flux links with the turns of the second coil.*[1] Then flux linkages *i.e.* weber-turns in the *second* coil for unit current in the *first* coil are $N_2 \Phi_1/I_1$. Hence, by definition,

$$M = \frac{N_2 \Phi_1}{I_1}$$

If weber-turns in *second* coil due to one ampere current in the *first* coil *i.e.* $\frac{N_2 \Phi_1}{I_1} = 1$ then, as seen from above, $M = 1$ H

Hence, two coils are said to have a mutual inductance of 1 henry if one ampere current flowing in one coil produces flux-linkages of one Wb-turn in the other.

In general, $M = N_2 \Phi_2/I_1$

Example 8-12. *Two identical coils X and Y of 1,000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5A flowing in X produces a flux of 0·05 mWb in it, find the mutual inductance between X and Y.*

Solution. Formula used :

$$M = \frac{N_2 \Phi_2}{I_1} \text{ henry}$$

Flux produced in $X = 0{\cdot}05$ mWb $= 0{\cdot}05 \times 10^{-3}$ Wb

Flux linked with $Y = 0{\cdot}05 \times 10^{-3} \times 0{\cdot}8 = 0{\cdot}04 \times 10^{-3}$ Wb

$$\therefore \quad M = \frac{1000 \times 0{\cdot}04 \times 10^{-3}}{5} = \mathbf{8 \times 10^{-3}\ H}$$

(*ii*) Second Method for M *(A.M.I.E. Dec. 1974)*

We will now deduce an expression for the coefficient of mutual inductance in terms of the dimensions of the two coils.

$$\text{Flux in the first coil } \Phi_1 = \frac{N_1 I_1}{l/\mu_0 \mu_r A} \text{ Wb}$$

$$\text{Flux/ampere} = \frac{\Phi_1}{I_1} = \frac{N_1}{l/\mu_0 \mu_r A}$$

Assuming that whole of this flux (it usually is some percentage of it) is linked with the other coil having N_2 turns, then weber-turns in it due to flux/ampere in the first coil is

$$M = \frac{N_2 \Phi_1}{I_1} = \frac{N_2 N_1}{l/\mu_0 \mu_r A} \text{ H}$$

$$\therefore \quad M = \frac{\mu_0 \mu_r A N_1 N_2}{l} \text{ H}$$

Also

$$M = \frac{N_1 N_2}{l/\mu_0 \mu_r A} = \frac{N_1 N_2}{\text{reluctance}} = \frac{N_1 N_2}{S} \text{ H}$$

---

1. In other words, $\phi_2 = \phi_1$. In case it is not so, then only that part of the flux of the first coil is used which is linked with the second coil.

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(*iii*) **Third Method for M**

As seen from above

$$M=\frac{N_2\Phi_2}{I_1}$$

$$=\frac{N_2\Phi_1}{I_1} \quad (\because \quad \Phi_2=\Phi_1)$$

$$\therefore \qquad N_2\Phi_1=MI_1$$

or $$-N_2\,\Phi_1=-MI_1$$

Differentiating both sides, we get

$$-\frac{d}{dt}(N_2\,\Phi_1)=-M\,.\,\frac{dI_1}{dt}$$ (assuming M to be constant)

Now $-\frac{d}{dt}(N_2\,\Phi_1)$ = mutually-induced e.m.f. e_M in the second coil.

$$\therefore \qquad e_M=-M\,\frac{dI_1}{dt}$$

If $dI_1/dt=1$ ampere/second and $e_M=1$ volt, then $M=1$ H

Hence, two coils are said to have a mutual inductance of one henry if current changing at the rate of 1 ampere/second in one coil induces an e.m.f. of one volt in the other.

Example 8-13. *Two coils having 150 and 200 turns respectively are wound side by side on a closed iron circuit of section 150 cm² and mean length of 300 cm. Determine the mutual inductance between the coils and the induced e.m.f. in the second coil if the current changes from zero to 10 A in the first coil in 0·02 second. Relative permeability of iron=2000.* (*A.M.I.E. Dec. 1974*)

Solution. $M=\mu_0\,\mu_r\,A\,N_1\,N_2/l$ henry

$\mu_r=2000$; $A=150$ cm$^2=15\times10^{-3}$ m^2

$N_1=150$; $N_2=200$; $l=300$ cm $=3$ m

$$\therefore \qquad M=4\pi\times10^{-7}\times2000\times15\times10^{-3}\times150\times200/3$$

$$=\mathbf{0{\cdot}377\ H}$$

$e_M=M\,dI_1/dt$

Here, $dI_1=10-0=10$ A ; $dt=0{\cdot}02$ second

$$\therefore \qquad e_M=0{\cdot}377\times10/0{\cdot}02=\mathbf{188{\cdot}5\ V}$$

Example 8-14. *Two coils, A of 5,000 turns, and B of 3,000 turns lie in parallel planes. A current of 6 A in coil A produces a flux of 0·1 mWb. If 60% of the flux produced by coil A links with the turns of coil B, calculate the e.m.f. induced in coil B when the current in coil A changes from 5 A to −5A in 0·01 second.*

Solution.

Flux/ampere in coil $A=\Phi_1/I_1=0{\cdot}1\times10^{-3}/6=1{\cdot}667\times10^{-5}$ Wb

Flux linked with coil $B=0{\cdot}6\times1{\cdot}667\times10^{-5}=10^{-5}$ Wb

Now $$M=\frac{N_2\Phi_2}{I_1}=3000\times10^{-5}=0{\cdot}03\ \text{H}$$

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Mutually-induced e.m.f. in coil B

$$e_M = M \frac{dI_1}{dt} = 0{\cdot}03 \times 10/0{\cdot}01 = 30 \text{ V}$$

Example 8-15. *The self-inductance of a coil of 500 turns is 0·25 H. If 60% of the flux is linked with a second coil of 10,000 turns, calculate (i) the mutual inductance of the two coils (ii) e.m. f. induced in the second coil when current in the first coil changes at the rate of 100 A/second.*

Solution. $$L_1 = \frac{N_1 \Phi_1}{I_1} = 0{\cdot}25$$

$\therefore$ Flux/ampere in first coil

$$= \frac{\Phi_1}{I_1} = \frac{0{\cdot}25}{N_1}$$
$$= 0{\cdot}25/500 = 5 \times 10^{-4} \text{ Wb}$$

Flux linked with second coil

$$= 0{\cdot}6 \times 5 \times 10^{-4} = 3 \times 10^{-4} \text{ Wb}$$

(*i*) $\therefore$ $$M = N_2 \frac{\Phi_1}{I_1} = 10{,}000 \times 3 \times 10^{-4} = 3 \text{ H}$$

$$e_M = M\, dI_1/dt = 3 \times 100 = 300 \text{ V}$$

8-14. Coefficient of Magnetic Coupling (*A.M.I E. Dec. 1974*)

Two coils are said to be magnetically coupled if full or part of the flux produced by one links with the other. Let L_1 and L_2 be the self-inductances of the two coils and M their mutual inductance, then

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

When all the flux produced by one coil links with the other, then mutual inductance between the two is maximum and is given by

$$M = \sqrt{L_1 L_2}$$

In that case, $k = 1$

When there is no common flux between the two coils, they are said to be magnetically isolated. Since, in that case, $M = 0$, $k = 0$ also.

Hence, coefficient of coupling may be defined as the ratio of *actual* mutual inductance present between the two coils to the maximum possible value.

Example 8-16. *The windings of a transformer have an inductance $L_1 = 6$ H, $L_2 = 0{\cdot}06$ H and a coefficient of coupling $k = 0{\cdot}9$. Find the e.m. f. induced in both windings when the primary current increases at the rate of 1000 A/s.*

Solution. It is obvious that self-induced e.m.f. e_L is produced in primary but mutually induced e.m.f. e_M is produced in secondary.

$$e_L = L.dI_1/dt = 6 \times 1000 = 6000 \text{ V}$$

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Now, $M = k\sqrt{L_1 L_2}$ —Art 8-14

$= 0{\cdot}9\ \sqrt{6 \times 0{\cdot}06} = 0{\cdot}54$

$\therefore$ $e_M = M.dI_1/dt = 0{\cdot}54 \times 1000 = 540$ V

Example 8-17. *Two long single-layer solenoids have the same length and the same number of turns but are placed co-axially one within the other. The diameter of the inner coil is 8 cm and that of the outer coil 10 cm. Calculate the coefficient of coupling between the coils.* (*A.M.I.E. May 1970*)

Solution. Coefficient of coupling

$$k = M/\sqrt{L_1 L_2}$$

Since N and l are the same for the two solenoids,

$$L_1 = \mu A_1 N^2/l \quad \text{and} \quad L_2 = \mu A_2 N^2/l$$

$$\therefore \qquad \sqrt{L_1 L_2} = \frac{\mu N^2}{l}\sqrt{A_1 A_2} \qquad \ldots(i)$$

Now, let us find the value of M between the solenoids by using the relation

$$M = N_2\Phi_2/I_1 \qquad \text{(Art. 8-13)}$$

Flux density produced by the outer solenoid is

$$B = \mu H = \mu N I_1/l$$

Flux linked with the inner solenoid of cross-section A_2 is

$$\Phi_2 = BA_2 = \mu N I_1 A_2/l$$

$$M = \frac{N_2\Phi_2}{I_1} = \frac{N\Phi_2}{I_2}$$

$$= \mu N^2 A_2/l$$

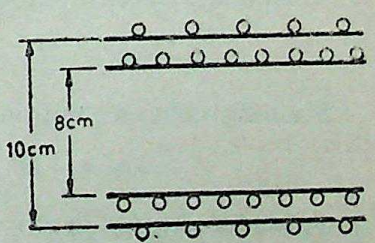

Fig. 8-12

From (*i*) and (*ii*) above, we get

$$k = \frac{M}{\sqrt{L_1 L_2}} = \sqrt{\frac{A_2}{A_1}} = \frac{d_2}{d_1} = \frac{8}{10}$$

or $k = 0{\cdot}8$ or 80%

Example 8-18. *A solenoid 70 cm in length and of 2100 turns has a radius of 4·5 cm. A second coil of 750 turns is wound upon the middle part of the solenoid. Find the self-inductance of the solenoid and the mutual inductance of the two coils.* (*A.M.I.E. Summer 1983*)

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$A = \pi r^2 = \pi(4{\cdot}5 \times 10^{-2})^2 = 20{\cdot}25\pi \times 10^{-4}$ m^2
$\mu_r = 1$ (assumed), $l = 70$ cm $= 0{\cdot}7$ m
$M = 4\pi \times 10^{-7} \times 20{\cdot}25\pi \times 10^{-4} \times 2100 \times 750/0{\cdot}7$ H
$= \mathbf{18}$ **mH**

8-15. Inductances in Series

(*i*) When the coils are so joined in series such that their fluxes (or m.m.fs) are additive *i.e.* in the same direction (Fig. 8-13).

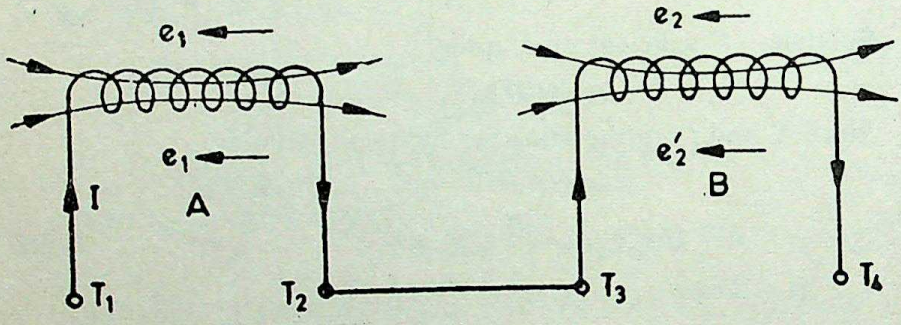

Fig. 8-13

Let M = coefficient of mutual inductance
L_1 = coefficient of self-inductance of 1st coil
L_2 = coefficient of self-inductance of 2nd coil

Then, self-induced e.m.f. in A is

$$= e_1 = -L_1 \cdot \frac{dI}{dt}$$

Mutually-induced e.m.f. in A due to change of current in B

$$= e_1' = -M \cdot \frac{dI}{dt}$$

Self-induced e.m.f. in B is

$$= e_2 = -L_2 \cdot \frac{dI}{dt}$$

Mutually-induced e.m.f. in B due to change of current in A is

$$= e_2' = -M \cdot \frac{dI}{dt}$$

(All have −ve sign, because both self and mutually-induced e.m.f.s are in opposition to the applied e.m.f.)

Total induced e.m.f. in the combination

$$= -\frac{dI}{dt}(L_1 + L_2 + 2M) \qquad \ldots(i)$$

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If L is the equivalent inductance, then total induced e.m.f. in that single coil would have been

$$= -L\frac{dI}{dt} \qquad \ldots(ii)$$

Equating (*i*) and (*ii*), we have

$$L = L_1 + L_2 + 2M$$

ii) When the coils are so joined that their fluxes are in opposi ections (Fig. 8-14).

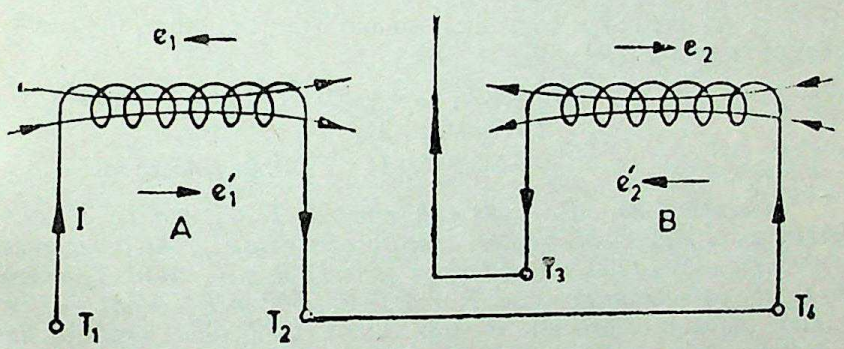

Fig. 8-14

As before $e_1 = -L_1\frac{dI}{dt}$

$e_1' = +M\frac{dI}{dt}$ (mark the direction)

$e_2 = -L_2\frac{dI}{dt}$

$e_2' = +M\frac{dI}{dt}$

Total induced e.m.f.

$$= -\frac{dI}{dt}(L_1 + L_2 - 2M)$$

$\therefore$ Equivalent inductance

$$L = L_1 + L_2 - 2M$$

In general, we have

$L = L_1 + L_2 + 2M$ ...if m.m.fs. are additive

$\therefore$ $L = L_1 + L_2 - 2M$ ...if m.m.fs. are subtractive

Example 8-19. *A wooden ring has a mean length of magnetic path of 100 cm and a cross-section of 10 cm². A coil of 2000 turns of wire is wound over it uniformly. Closely wound over this is a second coil of 4000 turns. The terminals of each coil are brought out separately.*

(*a*) *What is the inductance of first coil alone ?*

(*b*) *What is the inductance of second coil alone ?*

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(c) *If the two coils were connected electrically in series so that their m.m. fs. are additive, what would be the inductance of the combination?*

Solution. $N_1=2000$; $N_2=4000$

$l=1$ m ; $A=10^{-3}$ m^2

$\mu_0=4\pi\times10^{-7}$ H/m ; $\mu_r=1$

(a) $L_1=\mu_0\,\mu_r\,AN_1^2/l$ H

$=10^{-3}\times4\pi\times10^{-7}\times2000^2=5{\cdot}03$ mH

(b) $L_2=10^{-3}\times4\pi\times10^{-7}\times4000^2=20{\cdot}11$ mH

(c) As one coil is completely wound over the other, the coefficient of coupling $k=1$

$\therefore$ $M=\sqrt{L_1L_2}=\sqrt{5{\cdot}03\times20{\cdot}11}=10{\cdot}06$ mH

$L=L_1+L_2+2M$

$=5{\cdot}03+20{\cdot}11+2\times10{\cdot}06=45{\cdot}26$ mH

Example 8-20. *Two coils with terminals T_1, T_2, and T_3, T_4, respectively are placed side by side. Measured separately, the inductance of the first coil is 1200 mH and of the second 800 mH. With T_2 joined to T_3, the inductance between T_1 and T_4 is 2500 mH. What is the mutual inductance between the two coils? And what would be the inductance between T_1 and T_3 with T_2 joined to T_4?*

Solution. Formula used : $L=L_1+L_2\pm2M$

$L_1=1200$ mH, $L_2=800$ mH

When joined additively,

$L=L_1+L_2+2M$

or $2500=1200+800+2M$

$\therefore$ $M=250$ mH

When T_2 is joined to T_4, fluxes are in opposition

$L=1200+800-2\times250=1500$ mH

Example 8-21. *Two coils are placed side by side. Their combined inductance when connected in series is 1 H or 0·2 H depending on the relative directions of the currents in the coils. Calculate the mutual inductance between the two coils. If the self-inductance of one of the coils when isolated is 0·2 H, find the self-inductance of the other coil.*

Solution. Let L_1 and L_2 be the coefficients of self-inductance of the two coils and M their coefficient of mutual inductance. Obviously, when the two coils are connected in series such that their fluxes are in opposite directions, their combined inductance is 0·2 H and when their fluxes are in the same direction, their combined inductance is 1 H.

$\therefore$ $1=L_1+L_2+2M$ ...(*i*)

and $0{\cdot}2=L_1+L_2-2M$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$1{\cdot}2=2(L_1+L_2)$

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Now $L_1 = 0{\cdot}2$ H

$\therefore \quad 1{\cdot}2 = 2(0{\cdot}2 + L_2)$

$\therefore \quad 2L_2 = 0{\cdot}8$ H $\quad \therefore \quad L_2 = 0{\cdot}4$ H

Putting this value of L_2 in (i) above, we get

$$1 = 0{\cdot}2 + 0{\cdot}4 + 2M$$

$\therefore \quad 2M = 0{\cdot}4$ or $M = 0{\cdot}2$ H

Tutorial Problems—11

1. A d.c. motor field pole is wound with 800 turns and carries a flux of 20 mWb when excited. The exciting current is then switched off and the flux is reduced to 0·2 mWb in 50 milliseconds. Calculate the average e.m.f. induced in the coil. [316·8 V]

2. An iron-cored toroidal coil has 100 turns, a cross-sectional area of 10 cm² and a mean length of 314 cm. If the relative permittivity of iron is 1000, calculate the inductance of the coil. [4 mH]

3. If a coil of 150 turns is linked with a flux of 0·01 Wb when carrying a current of 10A, calculate the inductance of the coil. If this current is uniformly reversed in 0·1 second, calculate the induced e.m.f. [0·15 H ; 30 V]

4. Two coils A and B are coupled together, coil A having 400 turns and coil B 600 turns. When 5 A flows in coil A, a flux of 5 mWb links with coil B. Calculate the self-inductance of coil A and the mutual inductance between the two coils. [0·4 H ; 0·6 H]

5. A wooden ring has a mean diameter of 40 cm and a cross-sectional area of 3 cm² and is uniformly wound with two coils A and B. Coil A has 400 turns and coil B 500 turns. Calculate the mutual inductance between the two coils. Neglect any magnetic leakage.

If the current in coil A is 2A and this is reversed in 5 milliseconds, calculate the value of the e.m.f. induced in coil B. [60 μH ; 48 mV]

6. A sample of cast steel in the shape of a ring has a mean diameter of 0·15 m and a cross-sectional area of $0{\cdot}182 \times 10^{-3}$ m². Determine μ_r of the sample when a coil of 330 turns carrying a current of 2·4 A produces a flux of 0·22 mWb in the ring. What is the inductance of the coil under this condition ? [572 ; 30·3 mH]

7. A non-magnetic ring having a mean diameter of 30 cm and a cross-sectional area of 4 cm² is uniformly wound with two coils A and B, one over the other. A has 90 turns and B has 240 turns. Calculate the mutual inductance between the coils.

Also calculate the e.m.f. induced in B when a current of 6 A in coil A is reversed in 0·02 second. Indicate the direction of this e.m.f. with reference to the initial direction of the current. [11·52 μH ; 6·9 mV ; same] (*A.M.I.E. May 1972*)

8. A coil of 50 turns having a mean diameter of 3 cm is placed coaxially at the centre of a solenoid 60 cm long wound with 2500 turns and carrying a current of 2A.

Determine the mutual inductance of the arrangement. [0·185 mH] (*A.M.I.E. May 1965*)

9. The total inductance of two coils, A and B, when connected in series is 0·5H or 0·2H depending on the relative directions of the current in the coils. Coil A when isolated from coil B, has a self-inductance of 0·2 H. Calculate :

(a) the mutual inductance between the two coils

(b) the self-inductance of coil B

(c) the coupling factor between the coils

(d) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A/s in the series circuit.

[(a) 0·075 H (b) 0·15 H (c) 0·433 H (d) 275 or 125 V]

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HIGHLIGHTS

1. Faraday's Laws of Electromagnetic induction are :

(*i*) Whenever the magnetic flux linked with a circuit changes, an e.m.f. is induced in it.

or

Whenever a conductor cuts across magnetic flux, an e.m.f. is induced in that conductor.

(*ii*) The magnitude of the induced e.m.f. is equal to the rate of change of flux linkages.

$$e = \frac{d}{dt}(N\Phi) \text{ volts}$$

$$= N\frac{d\Phi}{dt} \text{ volts}$$

2. The direction of the induced e.m.f. and hence current may be found either by applying Fleming's Right-hand rule or Lenz's Law which states that the electromagnetically induced current always flows in such a direction that the action of the magnetic field set up by it is such as to oppose the very cause which produces it.

3. The induced e.m.f. may be subdivided as follows :

induced e.m.f.

- Dynamically induced e.m.f. $e = Blv$ volts
- Statically induced e.m.f.
  - Self-induced e.m.f. $e_L = -L\frac{dI}{dt}$
  - Mutually-induced e.m.f. $e_M = -M\frac{dI}{dt}$

4. The value of the coefficient of self-inductance of a solenoid is given by

$$L = \frac{N\Phi}{I} \text{ H } \textit{i.e.} \text{ weber-turns/ampere}$$

$$= \mu_0\mu_r A N^2/l \text{ H}$$

$$= \frac{e_L}{dI/dt} \text{ H } \textit{i.e.}$$ numerically equal to the induced e.m.f. per unit rate of change of current.

5. Similarly, the coefficient of mutual inductance of two coils is given by

$$M = \frac{N_2\Phi_2}{I_1} \text{ H } \textit{i.e.}$$ flux linkages in one coil per ampere current in the other.

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$= \mu_0 \mu_r \, AN_1N_2/l$ H

$= \frac{e_M}{dI_1/dt}$ *i.e.* numerically equal to the induced e.m.f. in one coil per unit rate of change of current in the other.

6. Coefficient of coupling between two magnetically coupled coils is given by

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

7. The equivalent inductance of two series-connected inductances is

$L = L_1 + L_2 + 2M$ ...when two fluxes assist each other

$= L_1 + L_2 - 2M$ ...when two fluxes oppose each other.

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MAGNETIC HYSTERESIS

9

1. Magnetic Hysteresis 2. Area of Hysteresis Loop 3. Steinmetz Law 4. Energy Stored in a Magnetic Field 5. Energy Stored per Unit Volume 6. Lifting Power of a Magnet 7. Rise of Current in an Inductive Circuit 8. Decay of Current in an Inductive Circuit 9. Highlights.

9-1. Magnetic Hysteresis *(A.M.I.E. Winter 1983)*

It may be defined as the lagging of magnetisation or induction flux density (B) behind the magnetising force (H). Alternately, it may be defined as the quality of a magnetic substance due to which energy is dissipated in it on the reversal of its magnetism.

Let us take an unmagnetised bar of iron AB and magnetise it by placing it within the field of a solenoid (Fig. 9-1). The field H produced by the solenoid is called the magnetising field or force. The value of this magnetising force H can be increased or decreased by increasing or decreasing current through it. Let H be increased in steps from zero up to a certain maximum value and the corresponding values of induction flux density (B) be noted. If we plot the relation between H and B, a curve like OA, as shown in Fig. 9-1, is obtained. The material becomes magnetically saturated for $H=OM$ and has, at that time, a maximum flux density of B_m established through it.

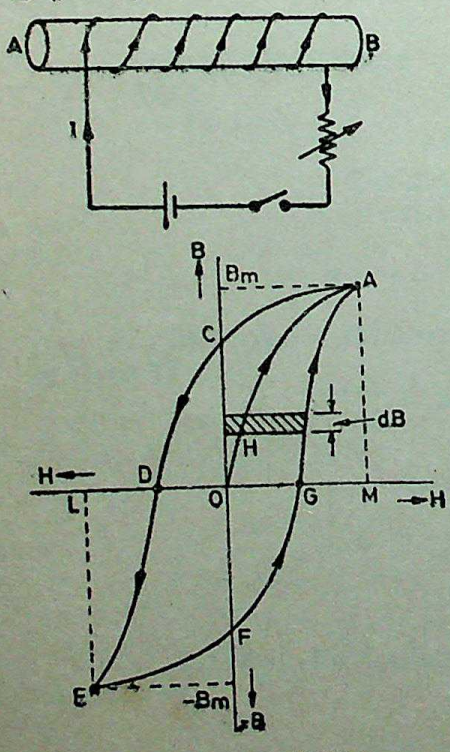

Fig. 9-1

If H is now decreased gradually (by decreasing solenoid current) flux density B will not decrease along AO (as might be expected) but will decrease less rapidly along AC. When H is zero B is not, but has a definite value $=OC$. It means that on removing the magnetising force H, the

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iron bar is not completely demagnetised. This value of B $(=OC)$ measures *retentivity or remanence* of the material.

To demagnetise the iron bar we have to apply the magnetising force H in the reverse direction. When H is reversed, by reversing current through the solenoid, then B is reduced to zero at point D where $H=OD$. This value of H required to wipe off residual magnetism is known as *coercive force* and is a measure of the *coercivity* of the material *i.e.* its 'tenacity' with which it holds on to its magnetism.

If after the magnetisation has been reduced to zero, value of H is further increased in the 'negative' *i.e.* reversed direction, the iron bar again reaches a state of magnetic saturation represented by point E. By taking H back from its value corresponding to negative saturation $(=OL)$ to its value for positive saturation $(=OM)$, a similar curre *EFGA* is obtained. If we again start from G, the same curve *GACDEFG* is obtained once again.

It is seen that B always lags behind H. The two never attain zero value simultaneously. This lagging of B behind H is given the name 'hysteresis' which literally means *'to lag behind'*. The closed loop *ACDEFGA* which is obtained when iron bar is taken through one complete cycle of reversal of magnetisation is known as 'hysteresis loop'.

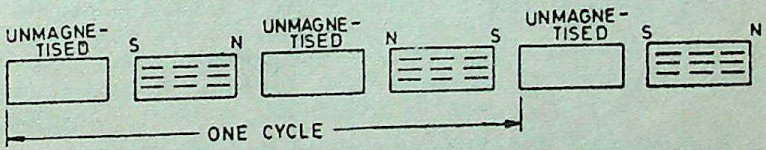

Fig. 9-2

By one cycle of reversal of magnetisation of a magnetic material is meant its being first magnetised in one direction, then demagnetised and being magnetised again but in opposite direction as shown in Fig. 9-2.

(A.M.I.E. Winter 1985)

9-2. Area of Hysteresis Loop

Just as the area of an indicator diagram measures the energy made available in a machine when taken through a cyclic operation, so also the area of the hysteresis loop represents the net energy spent in taking the iron bar through one cycle of magnetisation.

According to *Weber's Molecular Theory* of magnetism, when a magnetic material is magnetised, its molecules are forced along a straight line. So energy is spent in this process. Now, if iron had no retentivity, then energy spent in straightening the molecules could be recovered by reducing H to zero in the same way as the energy stored up in a spring can be recovered by allowing the spring to release its energy by driving some kind of load. Hence, in the case of magnetisation of a material of *high retentivity*, all the energy put into it originally for straightening the molecules is not recovered when H is reduced to zero. We will now proceed to find this loss of energy per cycle of magnetisation.

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Let l = mean length of the iron bar
A = its area of cross-section
N = No. of turns of wire in the solenoid

If B is the flux density at any instant, then $\Phi = BA$

When current through the solenoid changes, flux also changes and so produces an induced e.m.f. whose value is

$$e = N\frac{d\Phi}{dt} \text{ volts} \qquad \text{(neglecting —ve sign)}$$

$$= N\frac{d}{dt}(BA) = NA\frac{dB}{dt} \text{ volts}$$

Now $$H = \frac{NI}{l} \quad \text{or} \quad I = \frac{Hl}{N}$$

The power or rate of expenditure of energy in maintaining the current 'i' against induced e.m.f. 'e' is

$$= ei \text{ watt}$$

$$= NA\frac{dB}{dt} \times \frac{Hl}{N} \text{ watt}$$

$$= Al\ H\frac{dB}{dt} \text{ watt}$$

Energy spent in time 'dt' during which flux has changed, is

$$= Al.H\ \frac{dB}{dt} \times dt \text{ joules}$$

$$= Al\ H\ .\ dB \text{ joules}$$

Total *net* work done for one cycle of magnetisation is

$$W = Al \oint H\ dB \text{ joules}$$

where $\oint$ stands for integration over the whole cycle. Now '$H\ dB$' stands for the area of horizontal shaded strip in Fig. 9-1. Hence, $\oint H\ dB$ = area of the loop, *i.e.* the area between the B/H curve and the B-axis.

$\therefore$ work done/cycle = $Al \times$ (area of the loop) joules

Now Al = volume of the material

$\therefore$ net work done/cycle/m^3 = (loop area) joules

$\therefore$ W = (Area of B/H loop) joules/m^3/cycle

Precaution

Scales of B and H should be taken into consideration while calculating the actual loop area.

For example, if the scales are

1 cm = x AT/m —for H

and 1 cm = y Wb/m^2 —for B

then $W = xy$ (Area of B/H loop) joules/m^3/cycle

As seen from above, hysteresis loop measures the energy dissipated due to hysteresis which appears in the form of heat and so

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raises the temperature of that portion of the magnetic circuit which is subjected to magnetic reversals. The shape of the hysteresis loop depends on the nature of the magnetic substance (Fig. 9-3).

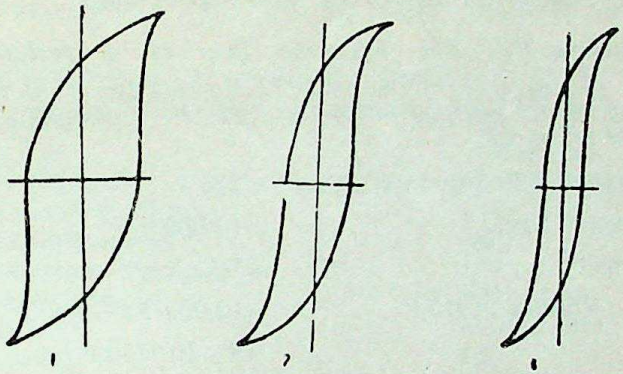

Fig. 9-3

Loop 1 is for hard steel. Due to its high retentivity and coercivity, it is well suited for making permanent magnets. But due to large hysteresis loss (as shown by large loop area) it is not suitable for rapid reversals of magnetisation. Certain alloys of aluminium, nickel and steel called alnico alloys have been found extremely suitable for making permanent magnets because they have high coercivity and high magnetic stability against the effects of heat, vibration and demagnetising fields but mechanically they are very hard, impossible to forge and difficult to machine except by grinding.

Loop 2 is for wrought iron and cast steel. It shows that these materials have high permeability and fairly good coercivity, hence making them suitable for cores of electromagnets.

Loop 3 is for alloyed sheet steel and it shows high permeability and low hysteresis loss. Hence, such materials are most suited for making armatures and transformer cores which are subjected to rapid reversals of magnetisation.

Example 9-1. *The area of hysteresis loop for a certain magnetic material is 60 cm² and the scales of the graph on which it is drawn are*

H-axis, 1 cm = 200 AT/m

B-axis, 1 cm = 0·2 Wb/m²

Calculate the loss in watt/kg. The frequency is 50 Hz and the density of the material is 8·0 g/cm³.

Solution. loss $= xy$ (area of B/H loop) J/m³/cycle

Here $x = 200,\ y = 0{\cdot}2$

$$\text{Loss} = 200 \times 0{\cdot}2 \times 60 = 2400 \text{ J/m}^3\text{/cycle}$$

$$\text{Volume of 1 kg} = \frac{1000}{8} \times 10^{-6} \text{ m}^3 = 10^{-3}/8 \text{ m}^3$$

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$\therefore$ loss per kg $= 2400 \times 10^{-3}/8 = 0{\cdot}3$ J/cycle

No. of reversals/second $= 50$

$\therefore$ hysteresis loss $= 0{\cdot}3 \times 50 =$ **15 J/s or watts**

Example 9-2. *The hysteresis loop of a specimen of iron weighing 10 kg is equivalent in area to 250 joules/m³ of iron. Find the loss of energy per hour at the rate of 50 Hz. Assume the density of iron as 7·5 g/cm³.*

Solution. Energy loss/m³/cycle $= 250$ J

Mass of iron $= 10{,}000$ g

Density $= 7{\cdot}5$ g/cm³

$\therefore$ Volume of iron $= 10{,}000/7{\cdot}5 = 4{,}000/3$ cm³

$= 4 \times 10^{-3}/3$ m³

No. of cycles/second $= 50$

Total number of cycles/hour $= 3600 \times 50$

$\therefore$ Total loss $= 250 \times \dfrac{4 \times 10^{-3}}{3} \times 3600 \times 50$ J

$=$ **60,000 J**

9-3. Steinmetz Law

It was experimentally found by Steinmetz that hysteresis loss per m³ per cycle of magnetisation of a magnetic material depends on (*i*) the maximum flux density established in it, *i.e.* B_{max} and (*ii*) the magnetic quality of the material.

$\therefore$ Hysteresis loss $W_h \propto B_{max}^{1{\cdot}6}$ joules/m³/cycle

$= \eta B_{max}^{1{\cdot}6}$ joules/m³/cycle

where η is a constant depending on the nature of the magnetic material and is known as **Steinmetz hysteresis coefficient.** The index 1·6 is empirical and holds good if the value of B_{max} lies between 0·1 and 1·2 Wb/m². If B_{max} is either lesser than 0·1 Wb/m² or greater than 1·2 Wb/m², then index is greater than 1·6.

If V is the volume of the magnetic material in m³ and is subjected to f reversals/second, then loss is given by

$$W_h = \eta\, B_{max}^{1{\cdot}6}\, f\, V \text{ watt}$$

The armatures of electric motors and generators and transformer cores etc. which are subjected to rapid reversals of magnetisation, should obviously, be made of substances having low hysteresis coefficients in order to reduce hysteresis loss. Values of hysteresis coefficients for some of the most commonly used materials are given as follows :

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TABLE No. 9-1

Hysteresis Coefficients

Material		*Hysteresis Coefficient* η (*joules*/m^3) $\times 10^2$
Cast iron	...	27·63 to 40·2
Sheet iron	...	10·05
Cast steel	...	7·54 to 30·14
Hard cast steel	...	63 to 70·34
Silicon steel (4·8% Si)	...	1·91
Hard tungsten steel	...	145·1
Good dynamo sheet steel	...	5·02
Mild steel castings	...	7·54 to 22·61
Nickel	...	32·66 to 100·5
Permalloy	...	0·25

Example 9-3. *A cylinder of iron of volume* 8×10^{-3} m^3 *revolves for 20 minutes at a speed of 3000 r.p.m. in a two pole field of strength of flux density 0·8* Wb/m^2. *If the hysteresis coefficient of iron is 753·6 joules/*m^3, *specific heat of iron is 0·11, the loss due to eddy current is equal to that due to hysteresis and 25% of the heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as 7·8* g/cm^3.

Solution. An armature revolving in a multipolar field undergoes one magnetic reversal after passing under one pair of poles. In other words, number of magnetic reversals is the same as the number of *pair* of poles. If P is the number of poles, then magnetic reversals in one revolution are $P/2$. If speed of armature rotation is N r.p.m., then number of revolutions/second $= N/60$.

$\therefore$ No. of reversals/second = reversals in one revolution $\times$ revolutions/second

$$= \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ reversals/second}$$

Here $N = 3000$ r.p.m. $\quad P = 2$

$$\therefore \quad f = \frac{3000 \times 2}{120} = 50 \text{ reversals/second}$$

According to Steinmetz, hysteresis loss

$$W_h = \eta B_{max}^{1.6} f V \text{ watt}$$

It may be noted that f here stands for magnetic reversals/second and not for frequency of armature rotation.

$\therefore \quad W_h = 753{\cdot}6 \times (0{\cdot}8)^{1.6} \times 50 \times 8 \times 10^{-3}$ J/s

Loss in 20 minutes $= 753{\cdot}6 \times (0{\cdot}8)^{1.6} \times 50 \times 8 \times 10^{-3} \times 1200$ J

$= 253{\cdot}2 \times 10^3$ J

Eddy current loss $= 253{\cdot}2 \times 10^3$ J

Total loss $= 506{\cdot}4 \times 10^3$ J

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Heat produced $= 506{\cdot}4 \times 10^3/4{\cdot}2 = 120{,}570$ cal

Heat utilized $= 120{,}570 \times 0{\cdot}75 = 90{,}430$ cal

Heat absorbed by iron $= (8000 \times 7{\cdot}8) \times 0{\cdot}11t$ cal

$\therefore \quad (8000 \times 7{\cdot}8) \times 0{\cdot}11 \times t = 90{,}430 \qquad \therefore \quad t = \mathbf{13{\cdot}17°C}$

Example 9-4. *Calculate the energy lost per hour by hysteresis in an iron armature weighing 50·86 kg in a 4-pole dynamo running at 1500 r.p.m., the limits of induction being $\pm 0{\cdot}3$ Wb/m², the hysteresis coefficient being 376·8 joules/m³ and specific gravity of the armature iron being 7·75.*

Solution. Assuming that for this B_{max}, the index of 1·6 is applicable, we have

$$W_h = \eta B_{max}^{1.6} f V \text{ watt}$$

Here $\quad f = 1500 \times 4/120 = 50$ Hz

$$V = \frac{50{\cdot}86 \times 1000}{7{\cdot}75} \times 10^{-6} = 6{\cdot}561 \times 10^{-3} \text{ m}^3$$

$\therefore \quad W_h = 376{\cdot}8 \times (0{\cdot}3)^{1.6} \times 50 \times 6{\cdot}561 \times 10^{-3}$ J/s

$= 18$ J/s

Loss/hour $= 18 \times 3600 = 64{,}800$ J

$= 64{,}800/36 \times 10^5 = \mathbf{0{\cdot}018}$ **kWh**

Example 9-5. *In a certain transformer, the hysteresis loss is 300 W when the maximum flux density is 0·9 Wb/m² and the frequency 50 Hz. What would be the hysteresis loss if the maximum flux density were increased to 1·1 Wb/m² and the frequency reduced to 40 Hz. Assume the hysteresis loss over this range to be proportional to $B_{max}^{1.7}$.*

Solution. Hysteresis loss, $W_h = \eta B_{max}^{1.7} f V$ joules/second

$\therefore \quad W_h \propto B_{max}^{1.7} f$

In the first case,

$$W_h = 300 \text{ W} ; B_{max} = 0{\cdot}9 \text{ Wb/m}^2, f = 50 \text{ Hz}$$

In the second case

$$B_{max} = 1{\cdot}1 \text{ Wb/m}^2 ; f = 40 \text{ Hz} ; W_h = ?$$

$$\frac{300}{W_h} = \frac{(0{\cdot}9)^{1.7} \times 50}{(1{\cdot}1)^{1.7} \times 40}$$

$$W_h = 300 \times (1{\cdot}1/0{\cdot}9)^{1.7} \times 4/5 = \mathbf{337{\cdot}7 \text{ W}}$$

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9-4. Energy Stored in a Magnetic Field

(A.M.I.E. Summer 1986)

For establishing a magnetic field, energy must be spent, though no energy is required to *maintain* it. Take the example of the exciting coils of an electromagnet. The energy supplied to it is spent in two ways (*i*) part of it goes to meet I^2R loss and is lost once for all (*ii*) part of it goes to create flux and is stored in the magnetic field as potential energy and is similar to the potential energy of a raised weight. When a mass m is raised through a height of h, then potential energy stored in it is mgh. Work is done in *raising* this mass, but once raised to a certain height, no further expenditure of energy is required to *maintain* it at that position. This mechanical potential energy can be recovered, so can the electrical energy stored in a magnetic field.

When current through an inductive coil is gradually changed from zero to a maximum value I then every change of it is opposed by the self-induced e.m.f. produced due to this change. Energy is needed to overcome this opposition. This energy is stored in the magnetic field of the coil and is, later on, recovered when that field collapses. The value of this stored energy can be found in the following two ways.

(*i*) **First Method**

Let i = instantaneous value of current

e = induced e.m.f. at the instant $= L\, di/dt$

Then work done in time dt in overcoming this opposition is

$$dW = e\, i\, dt$$

$$= L \frac{di}{dt} \times i \times dt \qquad (\because \quad e = L\, di/dt)$$

$$= L\, i\, di$$

Total work done in establishing the maximum steady current of I is

$$\int_0^W dW = \int_0^I L\, i\, di = \frac{1}{2} LI^2$$

or

$$W = \frac{1}{2} LI^2$$

If L is in henrys and I in amperes, then this value is in joules.

$\therefore$

$$W = \frac{1}{2} LI^2 \text{ joules.}$$

(*ii*) **Second Method**

If current grows uniformly from zero value to its maximum value I, then average current is $I/2$. If L is the inductance of the circuit, then self-induced e.m.f. is $e = L \cdot I/t$ where 't' is the time for the change from zero to I.

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$\therefore$ Average power absorbed = induced e.m.f. $\times$ average current

$$= L.\frac{I}{t} \times \frac{1}{2} I = \frac{1}{2} \frac{LI^2}{t}.$$

Total energy absorbed = Power $\times$ time

$$= \frac{1}{2} \frac{LI^2}{t} \times t = \frac{1}{2} LI^2$$

$\therefore$ Energy stored $= \frac{1}{2} LI^2$ joules

Example 9-6 *A solenoid 1 m in length and 10 cm in diameter has 5000 turns Calculate (i) approximate inductance and (ii) energy stored in the magnetic field when a current of 2A flows in the solenoid.* (*A.M.I.E. Summer 1986*)

Solution. $L = \mu_0 \mu_r AN^2/l$ henry

Here, $A = \pi \times 10^2/4 = 25\pi$ cm^2 $= 25\pi \times 10^{-4}$ m^2

$N = 5 \times 10^3$, $l = 1$m, $\mu_r = 1$ (assumed)

$\therefore$ $L = 4\pi \times 10^{-7} \times 1 \times 25\pi \times 10^{-4} \times (5 \times 10^3)^2/1 = \mathbf{0{\cdot}25}$ **H**

Energy stored $= \frac{1}{2} \times 0{\cdot}25 \times 2^2 = \mathbf{0{\cdot}5}$ **J.**

9-5. Energy Stored Per Unit Volume of Magnetic Field

It has already been shown that the energy stored in a magnetic field of length l metre and of cross-section A m^2 is

$$E = \frac{1}{2} LI^2 \text{ joules}$$

$$= \frac{1}{2} \times \frac{\mu_0 \mu_r A N^2}{l} I^2 \text{ joules}$$

Now $$H = \frac{NI}{l} \text{ AT/m}$$

$\therefore$ $$E = \left(\frac{NI}{l}\right)^2 \times \frac{1}{2} \mu_0 \mu_r Al \text{ joules}$$

$$= \tfrac{1}{2} \mu_0 \mu_r H^2 \times Al \text{ joules}$$

Now Al = volume of the magnetic field in m^3

$\therefore$ Energy stored/m^3 $= \frac{1}{2} \mu_0 \mu_r H^2$

$= \frac{1}{2} BH$ joules ($\because$ $\mu_0 \mu_r H = B$)

$= \frac{B^2}{2 \mu_0 \mu_r}$ joules —in a medium

$= \frac{B^2}{2\mu_0}$ joules —in air

$= \frac{\mu_0 H^2}{2}$ joules —in air

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9-6. Lifting Power of a Magnet

In Fig. 9-4 let

P = force in newtons between two poles

A = pole area in m²

If one of the poles (say, upper one) is pulled apart against this attractive force through a distance of dx metres (dx being so small that neither P nor B are altered appreciably) then work done

$$= P \times dx \text{ joules} \qquad ...(i)$$

This work goes to provide energy for the additional volume of the magnetic field created.

Additional volume of magnetic field created is

$$= A \times dx \text{ m}^3$$

Energy required per m³

$$= \frac{B^2}{2\mu_0} \text{ joules} \quad \text{...Art. 9-5}$$

$\therefore$ Energy required for the new volume

$$= \frac{B^2}{2\mu_0} \times A\,dx \qquad ...(ii)$$

Equating (i) and (ii), we get

$$P.dx = \frac{B^2 \times A\,.\,dx}{2\mu_0}$$

$$\therefore \quad P = \frac{B^2 A}{2\mu_0} \text{ N}$$

or $$P = \frac{B^2 A}{9{\cdot}81 \times 2\mu_0} = \frac{B^2 A}{19{\cdot}62\ \mu_0} \text{ kg wt}$$

Also $$P = \frac{B^2}{2\mu_0} \text{ N/m}^2$$

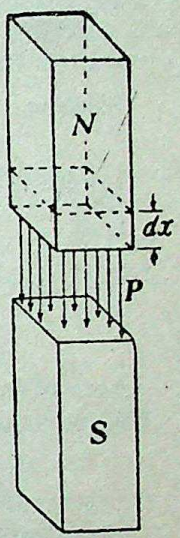

Fig. 9-4

Example 9-7. *A horse-shoe magnet is formed out of a bar of wrought iron 45·72 cm long, having a cross-section of 6·45 cm². Exciting coils of 500 turns are placed on each limb and connected in series. Find the exciting current necessary for the magnet to lift a load of 64 kg assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of iron = 700.*

Solution. Horse-shoe magnet is shown in Fig. 9-5.

Force of attraction at each pole = 64/2 = 32 kg wt

$$= 32 \times 9{\cdot}81 \text{ N}$$

$$= 314 \text{ N}$$

$$A = 6{\cdot}45 \times 10^{-4} \text{ m}^2$$

Since $F = \frac{B^2 A}{2\mu_0}$

$$\therefore \quad 314 = \frac{B^2 \times (6{\cdot}45 \times 10^{-4})}{2 \times 4\pi \times 10^{-7}}$$

$$\therefore \quad B = 1{\cdot}1 \text{ Wb/m}^2$$

Now, $H = B/\mu_0\mu_r$

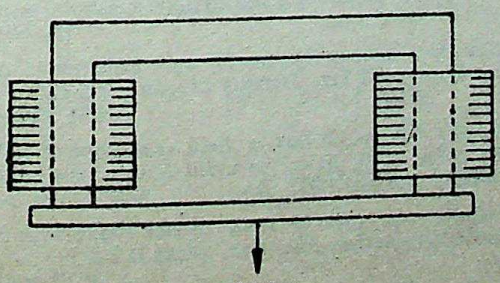

Fig. 9-5

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$= 1 \cdot 1/4\pi \times 10^{-7} \times 700 = 1296$ AT/m

Length of the path $= 0 \cdot 4572$ m

$\therefore$ A.T. required $= 1296 \times 0 \cdot 4572 = 592 \cdot 6$

No. of turns $= 500 \times 2 = 1000$

$\therefore$ current required $= 592 \cdot 6/1000 =$ **0·5926 A**

Example 9-8. *An iron ring having a mean diameter of 20 cm and a cross-sectional area of 10 cm² is cut across a diameter into two halves. The faces of the two halves are fitted to make a close contact and each half is wound with a coil of 100 turns carrying a current of 4 A. Calculate the force in kg required to separate the two halves. Take relative permeability of iron as 800.*

Solution. Let us first find the value of B.

Total AT $= 200 \times 4 = 800$

Length of iron path $= 20\pi$ cm $= 0 \cdot 2\ \pi$ metre

Now $$AT = H \times l = \frac{B}{\mu_0\ \mu_r} \times l$$

$\therefore$ $$800 = \frac{B \times 0 \cdot 2\ \pi}{4\pi \times 10^{-7} \times 800} = 625\ B$$

$\therefore$ $B = 800/625 = 1 \cdot 28$ Wb/m²

Area of each face $= 10$ cm² $= 10^{-3}$ m²

Force required at one face of each half ring

$$= \frac{B^2 A}{2\ \mu_0} \text{ N}$$

Force required for two faces of each half ring

$$= 2 \times \frac{B^2 A}{2\mu_0} = \frac{B^2 A}{\mu_0} \text{ N}$$

$$= \frac{1 \cdot 28^2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1303 \text{ N}$$

$= 1303/9 \cdot 81 =$ **132·8 kg**

Tutorial Problems—12

1. The armature of a 4-pole d.c. motor has a volume af 0·012 m³. In a test on the sheet iron used in the armature, carried out to the same value of maximum flux density as exists in the armature, the area of the hysteresis loop obtained represented a loss of 200 J/m³. Determine the hysteresis loss in watt when the armature rotates at a speed of 900 r.p.m. **[72 W]**

2. The hysteresis loop for a specimen weighing 12 kg is equivalent to 300 J/m³. Find the loss of energy per hour at 50 Hz. Density of iron is 7·5 g/cm.³ **[0·024 kWh]**

3. A 4-pole, 500-V shunt-wound d.c. motor has a field resistance of 500 ohm and a flux of 0·025 Wb per pole. Calculate the inductance of the field circuit and the energy stored if each field coil has 1200 turns. **[120 H ; 60 J]**

4. A lifting magnet is required to raise a load of one tonne with a factor of safety of 1·5. If the flux density across the pole faces is 0·8 Wb/m², calculate the area of each pole face. **[577 cm²]**

5. A ring of iron has a mean diameter of 30 cm and a circular cross-sectional area of 6 cm². It is wound with 1000 turns of wire carrying a current

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of 3 A. The ring is split across a diameter and an air-gap of 0·25 mm is formed each side. Assuming $\mu_r = 500$ for iron, determine the approximate pull in kg wt required to separate the two parts. [122 kg]

6. The two halves of a cast-iron ring of mean circumference 50 cm and mean cross-sectional area 10 cm² are separated by brass plates 2·5 mm thick. Calculate the flux density and current required to hold the two halves of the ring together with an attractive force of 25 kg. The ring is uniformly wound with 1000 turns and a relative permeability of 2500 may be assumed.

[0·556 Wb/m² ; 2·3 A]

7. The two halves of a cast-iron ring of mean circumference 50 cm and cross-sectional area 10 cm² are placed together with negligible air-gap. If the ring is uniformly wound with 1000 turns each carrying 2·5 A and the two halves of the ring are held together with an attractive force of 90 kg, find the relative permeability of iron. [168]

9-7. Rise of Current in an Inductive Circuit

In Fig. 9-6 is shown a resistance of $R\ \Omega$ in series with a coil of self-inductance L henrys ; the two being put across a battery of V volts. The R-L combination becomes connected to battery when switch S is in position 1 and is short-circuited when S is in position 2. The inductive coil is assumed to be resistanceless, its actual small resistance being included in R.

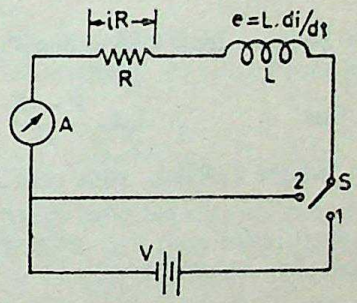

Fig. 9-6

When S is connected to 1, the R-L combination is suddenly put across the voltage of V volts. Let us take the instant of closing S as the starting zero time. It is found that current does not reach its maximum value instantaneously but takes some finite time. It is easily explained by recalling that the coil possesses electrical inertia *i.e.* self-inductance and hence, due to the production of the counter e.m.f. of self-inductance, delays the instantaneous full establishment of current through it.

We will now investigate the growth of current i through such an inductive circuit.

The applied voltage V must, at any instant, supply not only the ohmic drop iR over the resistance R but must also overcome the e.m.f. of self-inductance *i.e.* $L\ di/dt$.

$$\therefore \quad V = iR + L\ di/dt \qquad \ldots(i)$$

or $$(V - iR) = L\ di/dt$$

$$\therefore \quad \frac{di}{V - iR} = \frac{dt}{L}$$

Multiplying both sides by $(-R)$, we get

$$(-R)\frac{di}{(V - iR)} = -\frac{R}{L}\ dt$$

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Integrating both sides, we get

$$\int \frac{(-R)\,di}{(V-iR)} = -\frac{R}{L}\int dt$$

$$\therefore \quad \log_e (V-iR) = -\frac{R}{L}t + K \qquad \ldots(ii)$$

where e is the Napierian logarithmic base $= 2{\cdot}718$ and K is constant of integration whose value can be found from the initial known conditions.

To begin with, when $t=0$, $i=0$, hence putting these values in Eq. (ii) above, we get

$$\log_e V = K$$

Substituting this value of K in the above given equation, we have

$$\log_e (V-iR) = -\frac{R}{L}t + \log_e V$$

or $$\log_e (V-iR) - \log_e V = -\frac{R}{L}t$$

or $$\log_e \frac{V-iR}{V} = -\frac{R}{L}t = -\frac{t}{\lambda}$$

where $L/R = \lambda$...time constant.

$$\therefore \quad \frac{V-iR}{V} = e^{-t/\lambda}$$

or $$i = \frac{V}{R}(1-e^{-t/\lambda}) \qquad \ldots(iii)$$

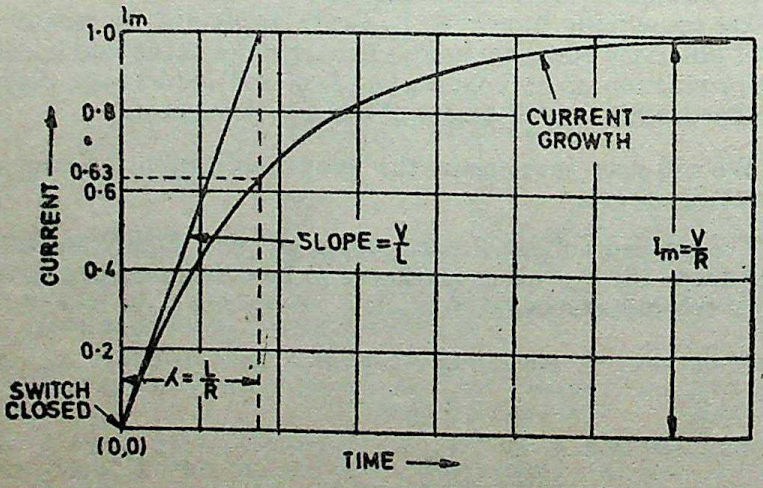

Fig. 9-7

Now V/R represents the maximum steady value of current I_m that would eventually be established through the R-L circuit.

$$\therefore \quad i = I_m\,(1-e^{-t/\lambda}) \qquad \ldots(iv)$$

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This is an exponential equation whose graph is shown in Fig. 9-7. It is seen from it that current rise is rapid at first and then decreases until at $t=\infty$, it becomes zero. Theoretically, current does not reach its maximum steady value I_m until infinite time. However, in practice, it reaches this value in a relatively short time.

The rate of rise of current *i.e.* di/dt at any stage can be found by differentiating Eq. (*ii*) above w.r.t. time. However, the *initial* rate of rise of current can be obtained by putting $t=0$ and $i=0$ in Eq. (*i*) above*

$$\therefore \quad V=0\times R+L\frac{di}{dt}$$

$$\therefore \quad \frac{di}{dt}=\frac{V}{L} \quad \text{...initial value}$$

The constant $\lambda=L/R$ is known as the *time-constant* of the circuit. It can be variously defined as :

(*i*) *It is the time during which current would have reached its maximum value of* I_m $(=V/R)$ *had it maintained its initial rate of rise.*

$$\text{Time taken}=\frac{I_m}{\text{initial rate of rise}}=\frac{V/R}{V/L}=\frac{L}{R}$$

(*ii*) But actually the current takes more time because its rate of rise decreases gradually. In actual practice, in a time equal to the time constant, it merely reaches 0·632 of its maximum value as shown below :—

Putting $t=L/R=\lambda$ in Eq. (*iv*) above, we get

$$i=I_m\left(1-e^{-\lambda/\lambda}\right)=I_m\left(1-\frac{1}{e}\right)$$

$$=I_m\left(1-\frac{1}{2\cdot718}\right)=0\cdot632\ I_m$$

Hence, the time-constant λ of an *R-L* circuit may also be defined *as the time during which the current actually rises to 0·632 of its maximum steady value* (*Fig. 9-7*).

This delayed rise of current in an inductive circuit is utilised in providing time-lag in the operation of electric relays and trip coils etc.

9-8. Decay of Current in an Inductive Circuit

When the switch *S* (Fig. 9-6) is put in position 2, then the *R-L* circuit is short-circuited. It is found that the current does not cease immediately, as it would do in a non-inductive circuit, but continues to flow and is reduced to zero only after an appreciable time has elapsed since the instant of short-circuit

* Initial value of di/dt can also be found by differentiating Eq. (*iv*) and putting $t=0$ in it

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The equation for decay of current with time is found by putting $V=0$ in Eq. (*i*) of Art. 9-7.

$$0=iR+L\frac{di}{dt}$$

or $$\frac{di}{i}=-\frac{R}{L}\,dt$$

Integrating both sides, we have

$$\int\frac{di}{i}=-\frac{R}{L}\int dt$$

$$\therefore \quad \log_e i=-\frac{R}{L}\,t+K \qquad \ldots(i)$$

Now, at the instant of switching off the current, $i=I_m$ and if time is counted from this instant, then $t=0$.

$$\therefore \quad \log_e I_m=0+K$$

Putting this value of K in Eq. (*i*) above, we get

$$\log_e i=-\frac{t}{\lambda}+\log_e I_m$$

$$\therefore \quad \log_e i/I_m=-\frac{t}{\lambda} \qquad \text{or} \qquad \frac{i}{I_m}=e^{-t/\lambda}$$

or $$i=I_m\,e^{-t/\lambda} \qquad \ldots(ii)$$

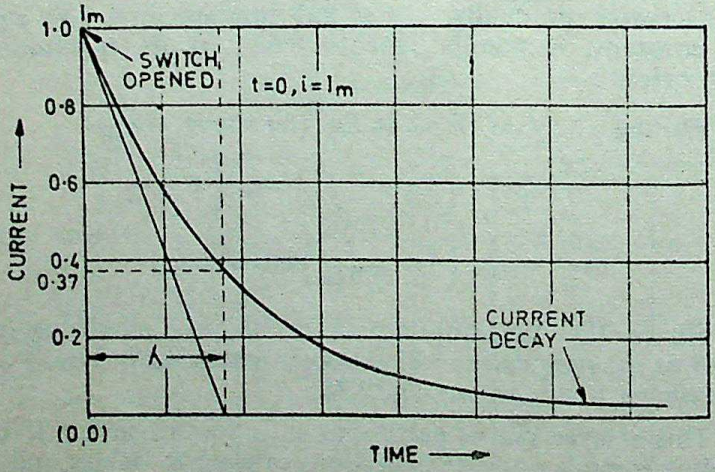

Fig. 9-8

It is a decaying exponential function and is plotted in Fig. 9-8. It can be shown again that, theoretically, current should take infinite time to reach zero value although in actual practice it does so in a relatively short time.

Again putting $t=\lambda$ in Eq. (*ii*) above, we get

$$i=\frac{I_m}{e}=\frac{I_m}{2{\cdot}178}=0{\cdot}37\ I_m$$

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Hence, time constant (λ) of an R-L circuit may be defined as *the time during which current falls to 0·37 of its maximum steady value while decaying (Fig. 9-8).*

Example 9-9. *A coil of resistance 20 ohms and inductance 0·8 henry is connected to a 200 volts d.c. supply. Find (i) rate of change of current at instant of closing switch (ii) final steady value of current (iii) time constant of circuit (iv) time taken for current to rise to half its final value and (v) energy stored in the field when steady state is reached.* (*A.M.I.E. Summer 1977*)

Solution. (*i*) Initial rate of change of current at the instant of closing the switch is $(di/dt)_{t=0} = V/L = 200/0·8 = 250$ A/s (*ii*) Final steady value of the current is $I_m = V/R = 200/20 = 10$ A (*iii*) Time constant, $\lambda = L/R = 0·8/20 = 0·04$ second (*iv*) Here, $i = I_m/2$. Also, for growth, $i = I_m(1 - e^{-t/\lambda})$ $\therefore$ $I_m/2 = I_m(1 - e^{-t/0·04})$; $t = 0·0277$ second (*v*) $E = \frac{1}{2} LI^2 = \frac{1}{2} \times 0·8 \times 10^2 = 40$ J

Example 9-10. *The field winding of a d. c. machine takes a steady current of 10 A when connected to a d.c. source of 230 volts. On switching the voltage to the field, it is observed that it takes 0·3 second to reach 5 A. Find the value of the inductance of the field. Find also the time for the current to fall to 30% of the final steady value if the field terminals are short-circuited.*

(*A.M.I.E. Winter 1977*)

Solution.

Here, $i = 5$A, $I_m = 10$ A, $t = 0·3$ second ; $R = 230/10 = 23\ \Omega$

Now, for growth of current, the relation is

$$i = I_m(1 - e^{-t/\lambda}) \quad \text{or} \quad 5 = 10\,(1 - e^{-0·3/\lambda})$$

$$\therefore \quad e^{-0·3/\lambda} = \frac{1}{2} \text{ or } e^{0·3/\lambda} = 2 \quad \text{or} \quad \frac{0·3}{\lambda} \log_e e = \log_e 2$$

$$\therefore \quad 0·3/\lambda = 2·3 \log_{10} 2 = 2·3 \times 0·3 \ ; \ \lambda = 0·4348 \text{ second}$$

Now, $\lambda = L/R$; $L = \lambda R = 0·4348 \times 23 = 10$ H

For decay of current, $i = I_m\, e^{-t/\lambda}$

Here, $i = 30\%$ of 10 A $= 3$ A, $\lambda = 0·4348$ second

$$\therefore \quad 3 = 10\, e^{-t/\lambda} \text{ or } e^{-t/\lambda} = 0·3$$

$$\therefore \quad \frac{-t}{\lambda} \log_e e = \log_e 0·3 \text{ or } \frac{-t}{\lambda} = 2·3 \log_{10} 0·3$$

$$\therefore \quad -t/\lambda = 2·3 \times \bar{1}·4771 \text{ or } t/\lambda = 2·3 \times 0·5229$$

$$t = 0·4348 \times 2·3 \times 0·5229 = 0·523 \text{ second}$$

Example 9-11. *A d.c. voltage of 120 V is applied to a coil having a resistance of 8 Ω and inductance of 12 H. Calculate the value of the current 0·3 second after switching on the supply. With the current having reached the final steady value, how much time would it take for the current to reach a value of 6A after switching off the supply?*

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Solution. **For Growth of Current**

$$i = I_m\,(1 - e^{-t/\lambda})$$

$$I_m = V/R = 120/8 = 15 \text{ A}$$

$$\lambda = L/R = 12/8 = 1{\cdot}5 \text{ second}$$

$$t = 0{\cdot}3 \text{ second}$$

$$\therefore \quad i = 15(1 - e^{-0.3/1.5}) = 15(1 - e^{-0.2})$$

$$= 15(1 - 0{\cdot}8485) = \mathbf{2{\cdot}72 \text{ A}}$$

For Decay of Current

$$i = I_m\, e^{-t/\lambda}$$

$$6 = 15\, e^{-t/1.5} \text{ or } e^{-t/1.5} = 0{\cdot}4$$

or $$e^{t/1.5} = 2{\cdot}5$$

$$\therefore \quad t = \mathbf{1{\cdot}374 \text{ second}}$$

Example 9-12. *What is meant by the time-constant of a circuit ?*

A 50-V battery, a 100-Ω resistance and a coil having an inductance of 4 H and a resistance of 100 Ω are connected in series. Find (a) the current after a period equivalent to the time constant of the coil and (b) the rate at which the current will then be increasing.

Solution. $$i = I_m\,(I - e^{-R/L \,.\, t})$$

Total resistance $$R = 100 + 100 = 200\ \Omega\ ;\ R/L = 200/4 = 50$$

$$I_m = 50/200 = 0{\cdot}25 \text{ A}$$

Hence, the above equation becomes

$$i = 0{\cdot}25\,(1 - e^{-R/L \,.\, t})$$

Time constant of the *coil*

$$= 4/100 = 0{\cdot}04 \text{ second}$$

Hence putting $t = 0{\cdot}04$ second in the above equation, we get

(*a*) $$i = 0{\cdot}25\,(1 - e^{-50 \times 0.04}) = 0{\cdot}25\,(1 - e^{-2})$$

$$= \mathbf{0{\cdot}216 \text{ A}}$$

(*b*) $$L\, di/dt = V - iR$$

$$\therefore \quad 4.di/dt = 50 - 0{\cdot}216 \times 200 = 6{\cdot}8$$

$\therefore$ di/dt at that time $= 6{\cdot}8/4 = \mathbf{1{\cdot}7 \text{ A/s}}$

Example 9-13. *A relay coil has a time constant of 2 milliseconds, an operating current of 0·25 A and a hold-on current of 0·1 A. When a 50-V d.c. supply is suddenly switched on to the coil, there is a delay of 2 milliseconds before the relay operates. Calculate*

(*a*) *the resistance of the coil*

(*b*) *the inductance of the coil*

(*c*) *the delay before the relay releases when the supply is suddenly disconnected from the coil.*

Solution. After switching on, it takes 2 milliseconds to operate the coil *i.e.* for the current to reach a value of 0·25 A. Using the equation

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$\therefore$ $i=I_m(1-e^{-t/\lambda})$, we get

$$0{\cdot}25=I_m\,(1-e^{-2/2})$$

Now $I_m=V/R=50/R$

$\therefore$ $$0{\cdot}25=\frac{50}{R}\,(1-e^{-1})=\frac{50}{R}\left(1-\frac{1}{2{\cdot}718}\right)$$

(*a*) $R=126{\cdot}1\ \Omega$

(*b*) Now $\lambda=\dfrac{L}{R}=2\times10^{-3}$ $\therefore$ $L=252{\cdot}2$ **mH**

The moment current decreases to 0·1 A or less the hold-on coil would no longer work. Using the relation

$$i=I_m\,e^{-t/\lambda}\ \text{we get}$$

$$0{\cdot}1=\frac{50}{126{\cdot}1}\,e^{-t/2}\ \text{or}\ e^{-t/2}=0{\cdot}252$$

$\therefore$ $$-\frac{t}{2}=2{\cdot}3\log_{10}{}^{0{\cdot}252}$$

$$=2{\cdot}3\times\bar{1}{\cdot}4014=2{\cdot}3\times-0{\cdot}5986$$

$\therefore$ $t=2{\cdot}75$ **millisecond.**

Tutorial Problems—13

1. A coil with a self-inductance of 2·4 H and resistance of 12 Ω is suddenly switched across a 120-V d.c. supply of negligible internal resistance. Determine the time-constant of the coil, the instantaneous value of the current after 0·1 second, the final steady value of the current and the time taken for the current to reach 5A. **[0·2 second : 3·94A ; 10A ; 0·139 second]**

2. A relay has a resistance of 300 Ω and is switched on to a 110-V d.c. supply. If the current reaches 63·2 per cent of its final steady value in 0·002 second, determine

(*a*) the time constant of the circuit

(*b*) the inductance of the circuit

(*c*) the final steady value of the current

(*d*) the initial rate of rise of current.

[(*a*) 0·002 second (*b*) 0·6 H (*c*) 0·366 A (*d*) 183 A/second]

3. A coil of 200 turns has a resistance of 50Ω and produces a magnetic flux of 5 mWb when a steady current of 4 A is passed through it. The coil is connected across a d.c. supply of 200 V. Calculate

(*a*) time constant of the coil

(*b*) the induced e.m.f. in the coil at the instant when the current has risen to 3A

(*c*) the rate of growth of the current at this instant

(*d*) the energy stored in the magnetic field when the current has reached its steady value.

[(*a*) 50 ms (*b*) 50 V (*c*) 20 A/s (*d*) 20 J]

4. A coil has a self-inductance of 0·75 H and a resistance of 25 Ω. It is connected to a d.c. supply of 50 V. If the supply is suddenly removed and replaced by a short-circuit, calculate the time taken for the current to decay from 1·5A to 0·5A. **[33 ms]**

5. A solenoid of inductance 30 H and of resistance 30 Ω are connected in series with a battery of 100 V. In how much time, after connecting the battery, will the current reach half of its final value. ***(A.M.I.E. May 1975)***

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HIGHLIGHTS

1. By magnetic hysteresis is meant the lagging of induction flux density (B) behind magnetising force (H).

It results in some net loss (known as hysteresis loss) when a magnetic material is carried through one complete cycle of reversal of magnetisation.

2 Hysteresis loss is equal to

(*a*) xy (area of hysteresis loop) J/m³/cycle

(*b*) $\eta B_{max}^{1.6} fV$ watt —Steinmetz Law

3. Energy stored in an inductive coil is

$$E = \tfrac{1}{2} LI^2 \text{ joules}$$

4. Energy stored per unit volume of a magnetic field is

$$= 2\frac{B^2}{\mu_0} \text{ J/m}^3$$

5. Lifting power of a magnet is

$$= \frac{B^2 A}{2\mu_0} \text{ N}$$

6. In an inductive circuit, the current rises according to the relation

$$i = I_m (1 - e^{-t/\lambda})$$

Similarly, the current decays according to the relation

$$i = I_m e^{-t/\lambda}$$

The time constant of the circuit is

$$\lambda = L/R \text{ second}$$

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D.C. GENERATORS 10

1. Generator Principle 2. Simple Loop Generator 3. Practical Generator 4. Yoke 5. Pole Cores and Pole Shoes 6. Pole Coils 7. Armature Core 8. Armature Windings 9. Commutator 10. Brushes and Bearings 11. Armature Winding 12. Armature Resistance 13. Types of Generators 14. E.M.F. Equation of a Generator 15. Iron Losses in Armature 16. Total Losses in a D.C. Generator 17. Stray Losses 18. Constant and Standing Losses 19. Power Stages 20. Condition for Maximum Efficiency 21. Armature Reaction 22. Commutation 23. Highlights

10-1. Generator Principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power).

This energy conversion is based on the principle of the production of dynamically (or motionally) induced e.m.f. As seen from Fig. 8-2 whenever a conductor cuts magnetic flux, dynamically induced e.m.f. is produced in it according to Faraday's Laws of Electromagnetic Induction. This e.m.f. will cause a current to flow if the conductor circuit is closed.

Hence, the basic essential parts of an electrical generator are (*i*) a magnetic field and (*ii*) a conductor or conductors which can move so as to cut the flux.

10-2. Simple Loop Generator

Construction

In Fig. 10-1 is shown a single-turn rectangular copper coil *ABCD* moving about its own axis in a magnetic field provided by either permanent magnets or electromagnets. The two ends of the coil are joined to two sliprings or discs '*a*' and '*b*' which are insulated from each other and from the central shaft. Two collecting brushes (of carbon) press against the sliprings. Their function is to collect the current induced in the coil and to convey it to the external load resistance *R*.

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The rotating coil may be called armature and the magnets as field magnets.

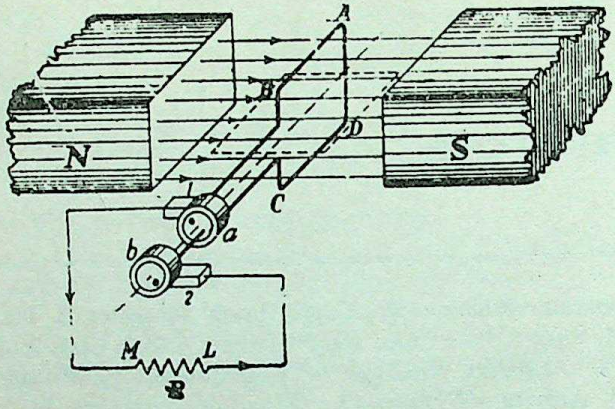

Fig. 10-1

Working

Imagine the coil to be rotating in clockwise direction (Fig. 10-2) As the coil assumes successive positions in the field, the flux linked with it changes. Hence, an e.m.f. is induced in it which is proportional to the rate of change of flux linkages ($e = -Nd\Phi/dt$). When the plane of the coil is at right angles to the lines of flux *i.e.* when it is in position 1, the flux linked with the coil is maximum but *rate of change of flux linkages is minimum.* This is so because in this position, the coil sides *AB* and *CD* do not cut or shear the lines of flux, rather they slide along them *i.e.* they move parallel to them. Hence, there is no induced e.m.f. in the coil. Let us take this no e.m.f. or vertical position of the coil as the starting position. The angle of rotation or time will be measured from this position.

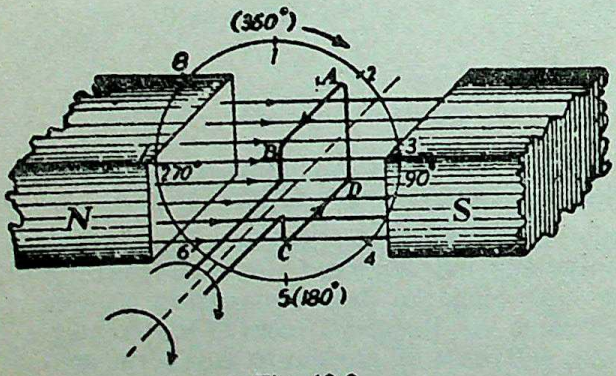

Fig. 10-2

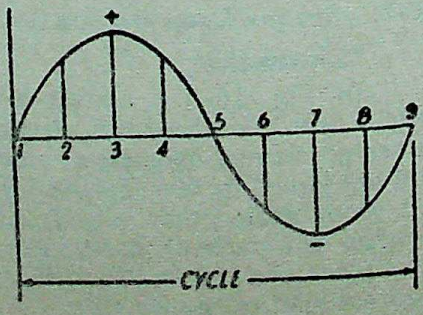

Fig. 10-3

As the coil continues rotating further, the rate of change of flux linkages (and hence induced e.m.f. in it) increases, till position 3 is reached where

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$\theta = 90°$. Here the coil plane is horizontal *i.e.* parallel to the lines of flux. As seen, the flux linked with the coil is minimum but *rate of change of flux linkages is maximum*. Hence, maximum e.m.f. is induced in the coil when in this position (Fig. 10-3).

In the next quarter revolution *i.e.* from 90° to 180°, the flux linked with the coil gradually *increases* ; but rate of change of flux *decreases*. Hence, the induced e.m.f. decreases gradually till in position 5 of the coil, it is reduced to zero value.

So, we find that in the first half revolution of the coil, no (or minimum) e.m.f. is induced in it when in position 1, maximum e.m.f. is induced when in position 3 and no e.m.f. is induced when in position 5. The direction of this induced e.m.f. can be found by applying Fleming's Right-hand rule which gives its direction from *A* to *B* and *C* to *D*. Hence, the direction of current flow is *ABMLCD* (Fig. 10-1). The current through the load resistance *R* flows from *M* to *L* during the first half revolution of the coil.

In the next half revolution *i.e.* from 180° to 360°, the variations in the magnitude of e.m.f. are similar to those in the first half revolution. Its value is maximum when coil is in position 7 and minimum when it is in position 1. But it will be found that the direction of the induced current is from *D* to *C* and *B* to *A*. Hence, the path of current flow is along *DCLMBA* which is just the reverse of the previous direction of flow.

Therefore, we find that the current which we obtain from such a simple generator reverses its direction after every half revolution. Such a current undergoing periodic reversals is known as alternating current (A.C.). It is, obviously, different from a direct current (D.C.) which continuously flows in one and the same direction. It should be noted that A.C. not only reverses its direction, it does not even keep its magnitude constant while flowing in any one direction. The two half-cycles may be called positive and negative half-cycles respectively (Fig. 10-3).

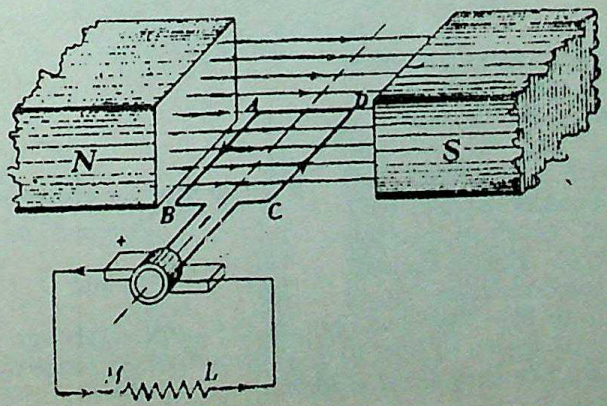

Fig. 10-4

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For making the flow of current unidirectional in the external

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circuit, the slip-rings are replaced by split-rings (Fig. 10-4). The split-rings are made out of a conducting cylinder which is cut into two halves or segments insulated from each other by a thin sheet of mica or some other insulating material (Fig. 10-5).

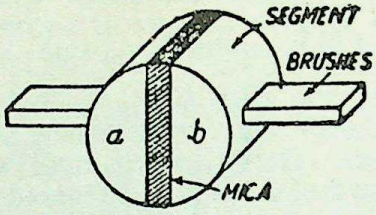

Fig. 10-5

As before, the coil ends are joined to these segments on which rest the carbon brushes.

It is seen [Fig. 10-6 (*a*)] that in the first half revolution, current flows along *ABLMCD i.e.* brush No. 1 which is in contact with segment '*a*', acts as the positive end of the supply and brush No. 2 and '*b*' as the negative end. In the next half revolution Fig. 10-6 (*b*), the

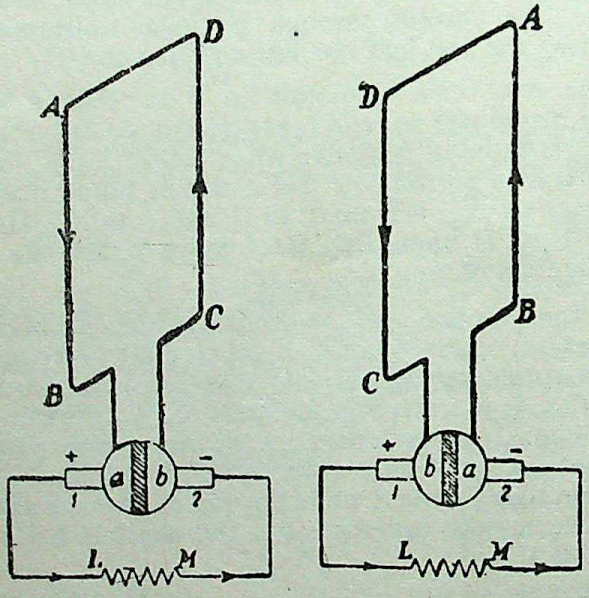

Fig. 10-6

direction of the induced current in the coil is reversed. But at the same time, the positions of segments '*a*' and '*b*' are also reversed with the result that brush No. 1 comes in touch with that segment which is positive *i.e.* segment '*b*'. Hence, the current in the load resistance again flows from *L* to *M*. The wave-form of the current through the external circuit is as shown in Fig. 10-7. *This current is unidirectional but not continuous like pure direct current.*

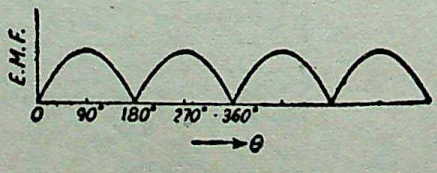

Fig. 10-7

It should be noted that the position of brushes is so arranged that the changeover of segments '*a*' and '*b*' from one brush to the

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other takes place when the plane of the rotating coil is at right angles to the plane of the lines of flux because in that position the induced e.m.f. in the coil is zero.

Another important point to remember is that even now the current induced in the coil sides is alternating as before. It is only due to the rectifying action of the split rings (also called commutator) that it becomes unidirectional in the external circuit. Hence, it should be clearly understood that even in the armature of a d.c. generator, the induced current is alternating.

10-3. Practical Generator *(A.M.I.E. May 1973)*

The simple loop generator has been considered in detail merely to bring out the basic principle, underlying the construction and

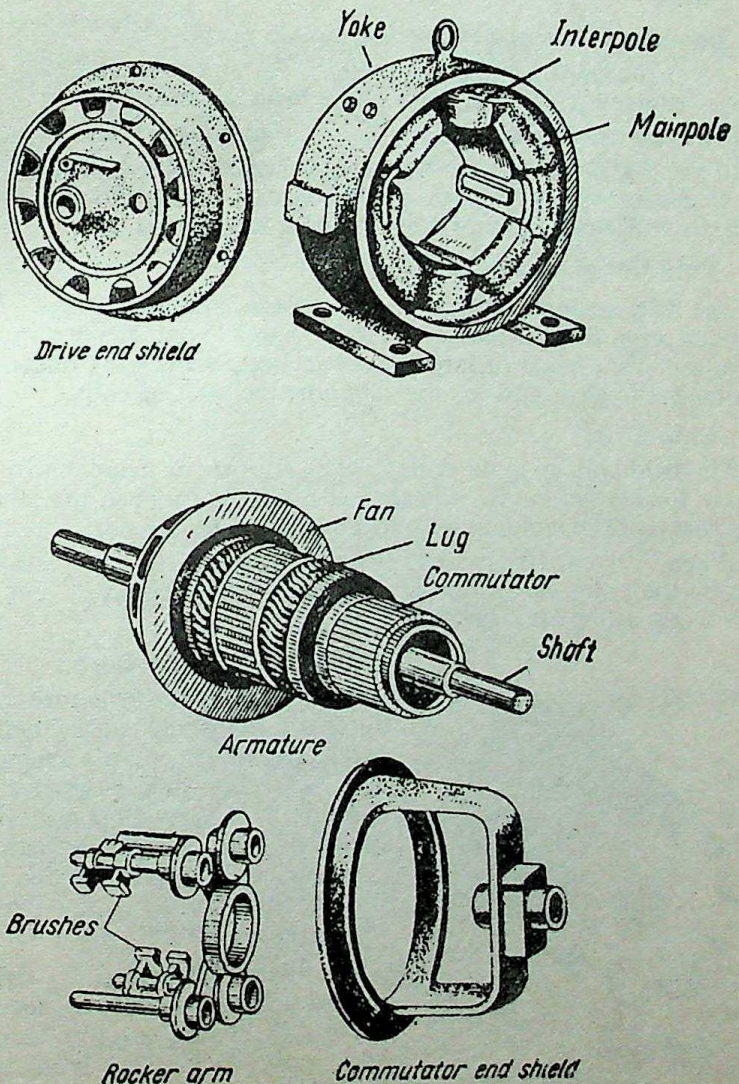

Fig. 10-8

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working of an actual generator illustrated in Fig. 10-8 which consists of the following essential parts :

(*i*) Magnetic Frame of Yoke
(*ii*) Pole Cores and Pole Shoes
(*iii*) Pole Coils or Field Coils
(*iv*) Armature Core
(*v*) Armature Windings or Conductors
(*vi*) Commutator
(*vii*) Brushes and Bearings

Of these, the yoke, the pole cores, the armature core and air gaps between the poles and the armature core form the magnetic circuit whereas the rest form the electrical circuit.

Fig. 10-8 gives a view of a d.c. machine taken apart. Full details of each component are given below.

10-4. Yoke

The outer frame or yoke serves double purpose :

(*i*) it provides mechanical support for the poles and acts as a protecting cover for the whole machine and

(*ii*) it carries the magnetic flux produced by the poles.

In small generators where cheapness rather than weight is the main consideration, yokes are made of cast iron. But for large machines usually cast steel or rolled steel is employed.

10-5. Pole Cores and Pole Shoes

The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes : (*i*) they spread out the flux in the air-gaps and also, being of larger cross-section, reduce the reluctance of the magnetic path and (*ii*) they support the exciting coils.

10-6. Pole Coils

The field coils or pole coils, which consist of copper wire or strip, are former-wound for the correct dimension. Then the former is removed and the wound coil put into place over the core.

When current is passed through these coils, they electromagnetise the poles which produce the necessary flux that is cut by the revolving armature conductors.

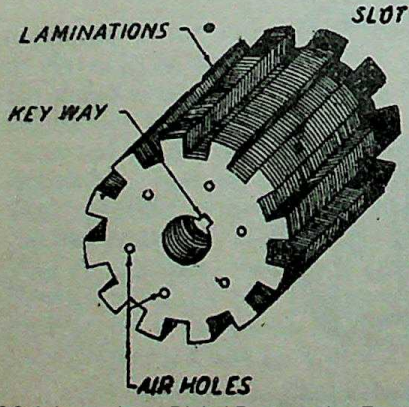

Fig. 10-9

10-7. Armature Core

It houses the armature conductors or coils and causes them to rotate and hence cut the magnetic flux of the field magnets. In addition to this, its most important function is to provide a path of very low reluctance to the flux passing through the armature from a *N*-pole to a *S*-pole.

It is cylindrical or drum-shaped and is built up of usually circular sheet steel discs or laminations approximately 0·064 mm thick (Fig. 10-9).

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The purpose of using laminations is to reduce the loss due to eddy currents. Thinner the laminations, greater is the resistance offered to the induced e.m.f., smaller the current and hence less the I^2R loss in the core.

10-8. Armature Windings

The armature windings are usually former-wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in the slot and is secured in place by special hard wooden or fibre wedges.

10-9 Commutator

The function of the commutator is to facilitate the collection of current from the armature conductors. As shown in Art 10-2, it rectifies *i.e.* converts the alternating current induced in the armature into unidirectional current. It is of cylindrical structure and is built up of wedge-shaped segments of high-conductivity hard-drawn or drop-forged copper. These segments are insulated from each other by thin layers of mica. Each commutator segment is connected to the armature conductors by means of a copper lug or strip (or riser).

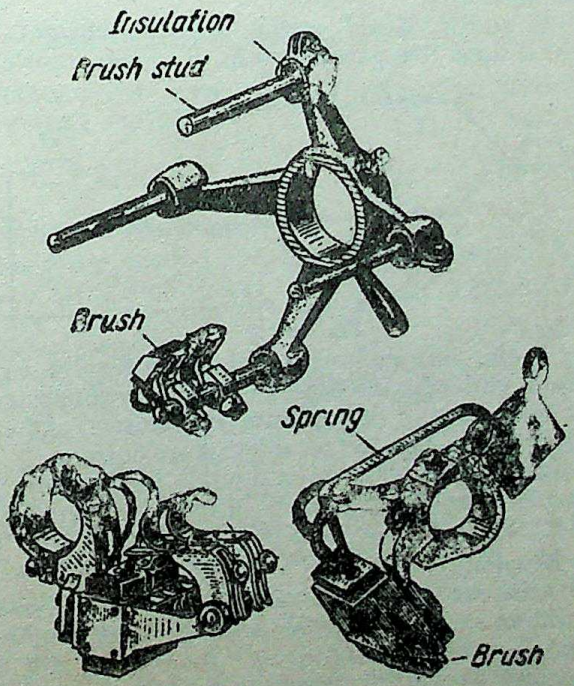

Fig. 10-10

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10-10. Brushes and Bearings

The brushes whose function is to collect current from commutators, are usually made of carbon and are in the shape of a rectangular block. These brushes are housed in brush-holders (usually of the box-type variety which are mounted an brush-holder studs or brackets.

In turn, the brush-holder studs are mounted on a brush yoke or rocker arm. The brush-holder studs are insulated from the brush yoke, by means of insulation sleeves and discs. The brush yoke, brush-holders and brushes make up the brush gear which is shown in Fig. 10-10.

Because of their reliability, ball bearings are frequently employed ; though for heavy duties, roller bearings are preferable. The ball and rollers are generally packed in hard oil for quieter operation and for reduced bearing wear, sleeve bearings are used, which are lubricated by ring oilers fed from oil reservoir in the bearing bracket.

10-11. Armature Winding

Two basic types of winding mostly employed for drum-type armature are known as (*i*) Wave winding and (*ii*) Lap winding.

For the purposes of this book, it is sufficient to know the following facts about these windings :

(*a*) Wave Winding

The most distinguishing feature of this winding is that electrically it divides the armature conductors into two parallel paths between the positive and negative brushes irrespective of the number of poles of the machine.

As shown in Fig. 10-11 as the armature current enters the negative brush, it finds two parallel paths of equal resistance available for going to the positive brush. Hence, it divides equally into two parts. Each path consists of $Z/2$ conductors connected in series (Z—being the total number of armature conductors) and each carries a current of $I_a/2$ where I_a is the total armature current.

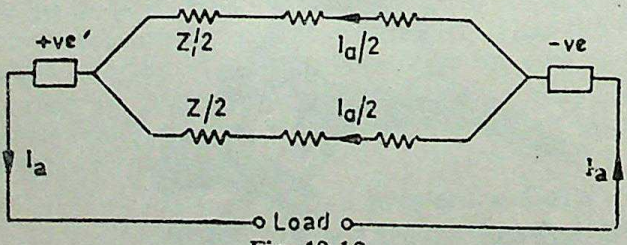

Fig. 10-12

For example, in the case of a 4-pole, wave-wound generator having 30 armature conductors, each of the two parallel paths will have 15 conductors.

(*b*) Lap Winding

In this case the armature conductors are divided into as *many parallel paths as the number of poles of the generator.* If there are P poles and Z armature conductors, then there are P parallel paths, each consisting of Z/P conductors connected in series between the positive and negative set of brushes.

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Fig. 10-12 shows the case of a 4-pole machine. As armature current enters the negative brush, it has four parallel paths available for going to the positive brush. Each path has $Z/4$ conductors and carries a current of $I_a/4$.

10-12. Armature Resistance

Let, l = length of each armature conductor
S = its cross-section
A = no. of parallel paths
= 2—for wave-winding
= P—for lap-winding
R = resistance of the whole winding

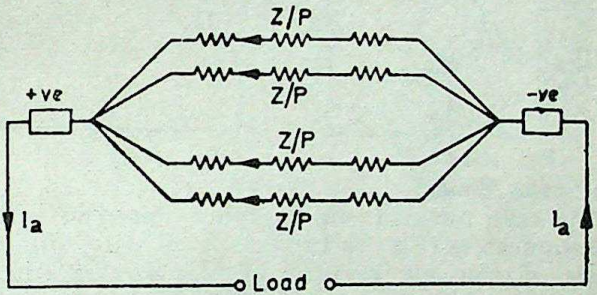

Fig. 10-12

Then, $$R = \frac{\rho l}{S} \times Z$$

Resistance of each parallel path $= \frac{\rho l Z}{SA}$

There are such A paths in parallel, hence equivalent resistance called armature resistance is

$$R_a = \frac{1}{A} \quad \frac{\rho LZ}{SA} \qquad \text{or} \qquad R_a = \frac{\rho LZ}{SA^2}$$

10-13. Types of Generators

Generators are usually classified according to the way in which their fields are excited. Generators may be divided into (*a*) separately-excited generators and (*b*) self-excited generators.

(*a*) **Separately-excited** generators are those whose field magnets are energised from an independent external source of direct current. It is shown diagrammatically in Fig. 10-13.

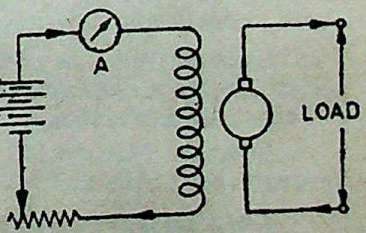

Fig- 10-13

(*b*) **Self-excited** generators are those whose field magnets are energised by the current produced by the generators themselves. Due to residual magnetism, there is always present some flux in the poles. When the armature is rotated, some e.m.f. and hence some induced current is produced which is partly or fully passed through the field coils thereby strengthening the residual pole flux further.

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There are three types of self-excited generators named according to the manner in which their field coils (or windings) are connected to the armature.

(*i*) Shunt Wound

The field windings are connected across or in parallel with the armature conductors, and have the full voltage of the generator applied across them (Fig. 10-14).

The field coil consists of many turns of fine gauge copper wire. Such generators are in much common use.

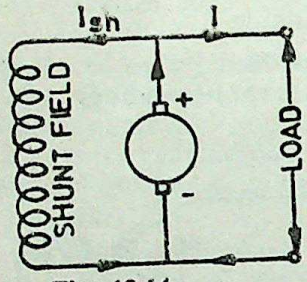

Fig. 10-14

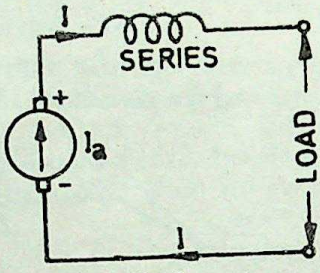

Fig. 10-15

(*ii*) Series Wound

In this case, the field windings are joined in series with the armature conductors (Fig. 10-15). As they carry full-load current, they consist of relatively few turns of thick wire or strip. Such generators are rarely used except for special purposes *i.e.* as boosters etc.

(*iii*) Compound Wound

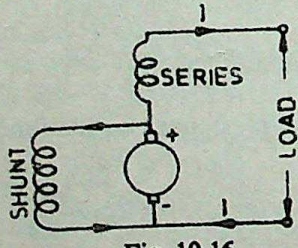

Fig. 10-16

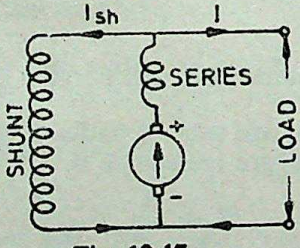

Fig. 10-17

It is a combination of a few series and a few shunt windings and can be either short-shunt or long-shunt as shown in Fig. 10-16 and Fig. 10-17 respectively. Various types of d.c. generators have been shown separately in Fig. 10-18.

Example 10-1. *A d.c. shunt generator supplies a load of 7·5 kW at 200 V. Calculate the induced e.m.f. if armature resistance is 0·6 ohm and field resistance is 80 ohms.* (*A.M.I.E. Summer 1979*)

Solution. Generator circuit is shown in Fig. 10-19. Current through shunt field winding is $I_{sh} = 200/80 = 2{\cdot}5$A

Load current

$$= 7{\cdot}5 \times 1000/200$$

$$= 37{\cdot}5 \text{ A}$$

Armature current is

$$I_a = I + I_{sh} = 37{\cdot}5 + 2{\cdot}5$$

$$= 40 \text{ A}$$

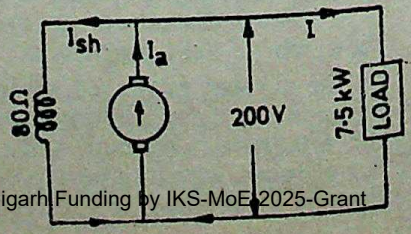

Fig. 10-19

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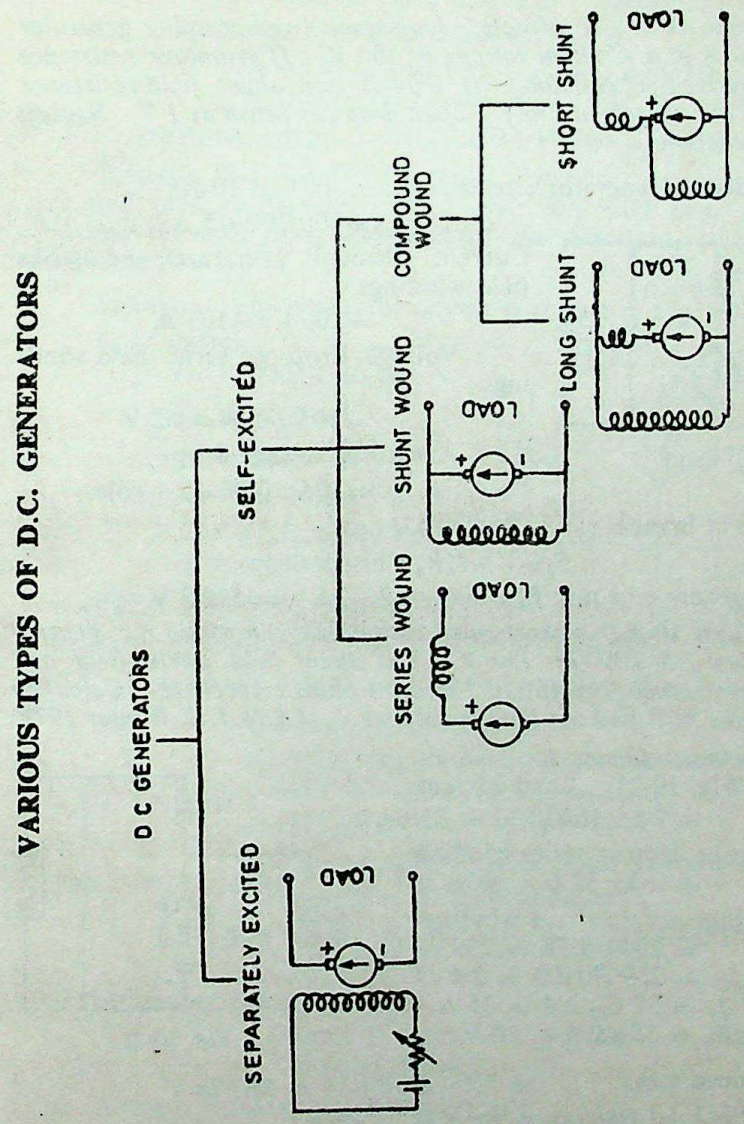

Fig. 10-18

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Armature voltage drop is

$$I_a R_a = 40 \times 0{\cdot}6 = 24 \text{ V}$$

Now, E_g = terminal voltage + armature drop

$\therefore$ e.m.f. generated in armature $E_g = 200 + 24 = \mathbf{224\ V}$.

Example 10-2. *A 4-pole, long-shunt compound generator supplies 100 A at a terminal voltage of 500 V. If armature resistance is 0·02 Ω, series field resistance is 0·04 Ω and shunt field resistance 100 Ω, find the generated e.m.f. Take drop per brush as 1 V. Neglect armature reaction.*

Solution. Generator circuit is shown in Fig. 10-20.

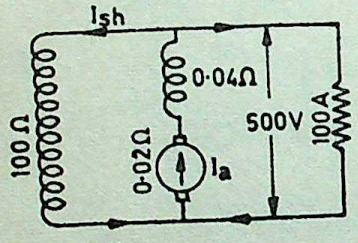

Fig. 10-20

$$I_{sh} = 500/100 = 5 \text{ A}$$

Current through armature and series field windings

$$= 100 + 5 = 105 \text{ A}$$

Voltage drop on series field windings

$$= 105 \times 0{\cdot}04 = 4{\cdot}2 \text{ V}$$

Armature voltage drop

$$= 105 \times 0{\cdot}02 = 2{\cdot}1 \text{ volt}$$

Drop at brushes $= 2 \times 1 = 2$ V

Now $E_g = V + I_a R_a$ + brush drop

$\therefore$ generated e.m.f. $E_g = 500 + 4{\cdot}2 + 2{\cdot}1 + 2 = \mathbf{508{\cdot}3\ V}$

Example 10-3. *A short-shunt cumulative compound d.c. generator supplies 7·5 kW at 230 V. The shunt field, series field and armature resistances are 100, 0·3 and 0·4 ohms respectively. Calculate the induced e.m.f. and the load resistance.* **(A.M.I.E. Winter 1979)**

Solution. Generator circuit is shown in Fig. 10-21. Load current

$$= 7{\cdot}5 \times 1000/230 = 32{\cdot}6 \text{A}$$

Voltage drop in series winding

$$= 0{\cdot}3 \times 32{\cdot}6 = 9{\cdot}78 \text{ V}$$

Voltage across shunt winding

$$= 230 + 9{\cdot}78 = 239{\cdot}78 \text{ V}$$

$$I_{sh} = 239{\cdot}78/100 = 2{\cdot}4 \text{ A}$$

$$I_a = 32{\cdot}6 + 2{\cdot}4 = 35 \text{ A}$$

$$I_a R_a = 35 \times 0{\cdot}4 = 14 \text{ V}$$

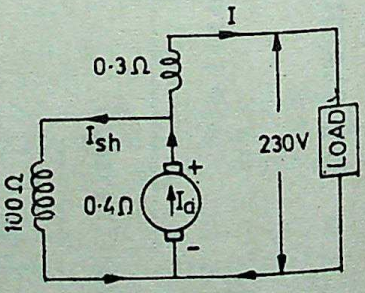

Fig. 10-21

Induced e.m.f. $= 230 + 9{\cdot}78 + 14 = \mathbf{253{\cdot}78\ V}$

If R is load resistance, then $W = V^2/R$

$\therefore$ $R = V^2/W = 230^2/7{\cdot}5 \times 1000 = \mathbf{7{\cdot}05\ \Omega}$.

Example 10-4. *The resistance of the field circuit of a shunt-excited d.c. generator is 200 Ω. When the output of the generator is 100 kW, the terminal voltage is 500 V and the generated e.m.f. 525 V. Calculate (a) the armature resistance and (b) the value of the generated e.m.f. when the output is 60 kW, if the terminal voltage then is 520 V.*

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Solution. (*a*) Output current is

$$I = 100{,}000/500 = 200 \text{ A}$$

$$I_{sh} = 500/200 = 2{\cdot}5 \text{ A} ; \quad I_a = 202{\cdot}5 \text{ A}$$

Armature drop $= 525 - 500 = 25$ V

$\therefore \quad I_a R_a = 25$ or $202{\cdot}5 \times R_a = 25 ; \quad R_a = \mathbf{0{\cdot}123\ \Omega}$

(*b*) Output current $= 60{,}000/520 = 115{\cdot}4$ A

$$I_{sh} = 520/200 = 2{\cdot}6 \text{ A} ; \quad I_a = 115{\cdot}4 + 2{\cdot}6 = 118 \text{ A}$$

$\therefore \quad I_a R_a = 118 \times 0{\cdot}123 = 14{\cdot}56$ V

$\therefore$ generated e.m.f. $E_g = 520 + 14{\cdot}56 = \mathbf{534{\cdot}56\ V}$

Example 10-5. *A 30-kW, 300-V, d.c. shunt generator has armature and field resistances of 0·05 ohm and 100 ohm respectively. Calculate the total power developed by the armature when it delivers full output power.* (*A.M.I.E. Summer 1975*)

Solution. The generator circuit is shown in Fig. 10-22.

Load current,

$$I = 30{,}000/300 = 100 \text{ A}$$

$$I_{sh} = 300/100 = 3 \text{ A}$$

$$I_a = 100 + 3 = 103 \text{ A}$$

$$I_a R_a = 103 \times 0{\cdot}05 = 5{\cdot}15 \text{ A}$$

$$E_g = V + I_a R_a = 300 + 5{\cdot}15 \text{ V}$$

$$= 305{\cdot}15 \text{ V}$$

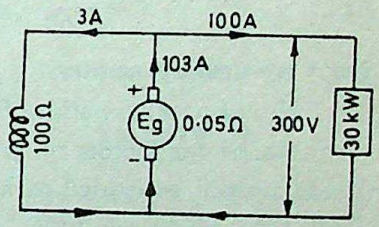

Fig. 10-22

Power developed in armature

$$= E_g I_a = 305{\cdot}15 \times 103/1000 = \mathbf{31{\cdot}43\ kW}$$

10-14. Generated E.M.F. or E.M.F. Equation of a Generator

Let Φ = flux/pole in webers (*A.M.I.E. Winter 1980*)

Z = total number of armature conductors

= No. of slots × conductors/slots

P = No. of poles

A = No. of parallel paths in armature

N = armature rotation in r.p.m.

E_g = e.m.f. induced in any parallel path in the armature

Generated e.m.f. E_g

= e.m.f. generated in any one of the armature parallel paths.

Average e.m.f. generated/conductor

$$= n \frac{d\Phi}{dt} \text{ volt} \qquad \text{(Ref. Art. 8-3)}$$

$$= \frac{d\Phi}{dt} \text{ volt} \qquad (\because\ n = 1)$$

Now, flux cut/conductor in one revolution $= \Phi P$ weber

$\therefore \quad d\Phi = \Phi \times P$

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No. of revolutions/second $= N/60$

$\therefore$ Time for one revolution $= 60/N$ second

$\therefore$ $dt = 60/N$ second

$\therefore$ Flux cut per conductor/second

$$= \frac{d\Phi}{dt} = \frac{\Phi P}{60/N} = \frac{\Phi PN}{60} \text{ Wb/s}$$

$\therefore$ e.m.f. generated/conductor $= \dfrac{\Phi PN}{60}$ volt

For a wave-wound generator

No. of parallel paths $= 2$

No. of conductors (in series) in one path $= Z/2$

$\therefore$ E.M.F. generated/path

$$= \frac{\Phi PN}{60} \times \frac{Z}{2} = \frac{\Phi ZPN}{120} \text{ volt}$$

For a lap-wound generator

No. of parallel paths $= P$

No. of conductors (in series) in one path $= Z/P$

$\therefore$ e.m.f. generated/path

$$= \frac{\Phi PN}{60} \times \frac{Z}{P} = \frac{\Phi ZN}{60} \text{ volt}$$

In general, generated e.m.f. is

$$E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

where $A = 2$ —for wave-winding

$= P$ —for lap-winding

Example 10-6. *Calculate the e.m.f. generated by a 4-pole, wave-wound armature having 45 slots with 18 conductors per slot when driven at 1200 r.p.m. The flux per pole is 0·016 Wb.*

Solution.

$$E = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

Here $\Phi = 0{\cdot}016$ Wb ; $N = 1200$ r.p.m.

$Z = 45 \times 18 = 810$; $P = 4$

$A = 2$ **(wave-winding)**

$$E_g = \frac{0{\cdot}016 \times 810 \times 1200}{60}\left(\frac{4}{2}\right) = 518{\cdot}4 \text{ volt}$$

Example 10-7. *A 6-pole, lap-wound d. c. generator has 600 conductors on its armature. The flux per pole is 0·02 Wb. Calculate (a) the speed at which the generator must be run to generate 300 V, (b) what would be the speed if the generator were wave-wound?*

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Solution.

$$(a)\quad 300=\frac{0{\cdot}02\times 600\times N}{60}\left(\frac{6}{6}\right) \qquad \therefore\ N=\mathbf{1500\ r.p.m.}$$

$$(b)\quad 200=\frac{0{\cdot}02\times 600\times N}{60}\left(\frac{6}{2}\right) \qquad \therefore\ N=\mathbf{500\ r.p.m.}$$

Example 10-8. *A d.c. generator develops an e.m.f. of 200 V when driven at 1000 r.p.m. with a flux per pole of 0·02 Wb. It is desired that this e.m.f. be increased to 210 V at 1100 r.p.m. What should be the value of the flux per pole under the new circumstances ?*

Solution.

$$E_g=\frac{\Phi\, ZN}{60}\left(\frac{P}{A}\right)\text{ volt}$$

$$\therefore \qquad E\propto \Phi N$$

In the first case,

$$200\propto 0{\cdot}02\times 1000 \qquad \ldots(i)$$

In the second case,

$$210\propto \Phi\times 1100 \qquad \ldots(ii)$$

$$\therefore \qquad \frac{210}{200}=\frac{1100\,\Phi}{1000\times 0{\cdot}02}\ ;\quad \mathbf{\Phi=0{\cdot}019\ Wb}$$

Example 10-9. *The armature of a 4-pole d.c. generator is required to generate an e.m.f. of 520 V on open circuit when revolving at a speed of 660 r.p.m. Calculate the magnetic flux per pole required if the armature has 144 slots with 2 coil sides per slot, each coil consisting of three turns. The armature is wave-wound.*

Solution. Formula used :

$$E_g=\frac{\Phi ZN}{60}\times\left(\frac{P}{A}\right)\text{ volt}$$

Here $E_g=520$ V ; $N=660$ r.p.m.

$$Z=144\times 2\times 3=864\ ;\ P=4\ ;\ A=2$$

$$\therefore \qquad 520=\frac{\Phi\times 864\times 660}{60}\times\left(\frac{4}{2}\right)$$

$$\therefore \qquad \Phi=\frac{60\times 520}{2\times 864\times 660}=\mathbf{0{\cdot}0274\ Wb}$$

Example 10-10. *A 4-pole, lap-wound, d.c. shunt generator has a useful flux per pole of 0·07 Wb. The armature winding consists of 220 turns each of 0·004 Ω resistance. Calculate the terminal voltage when running at 900 r.p.m. if the armature current is 50 A.*

Solution. Since each turn has two sides

$$Z=220\times 2=440\ ;\ N=900\text{ r.p.m.}$$

$$\Phi=0{\cdot}07\text{ Wb}\ ; \qquad P=A=4$$

$$E_g=\frac{\Phi ZN}{60}\times\left(\frac{P}{A}\right)\text{ volt}$$

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$$= \frac{0{\cdot}07 \times 440 \times 900}{60} \times \left(\begin{matrix} 4 \\ 4 \end{matrix} \right) = 462 \text{ volt}$$

Total resistance of 220 turns or 440 conductors

$$= 220 \times 0{\cdot}004 = 0{\cdot}88\ \Omega$$

Since there are 4 parallel paths in armature,

$\therefore$ resistance of each path $= 0{\cdot}88/4 = 0{\cdot}22\ \Omega$

Now, there are four such resistances in parallel each of value $0{\cdot}22\ \Omega$.

$\therefore$ armature resistance $R_a = 0{\cdot}22/4 = 0{\cdot}055\ \Omega$

Armature drop $= I_a R_a = 50 \times 0{\cdot}055 = 2{\cdot}75$ volt

Now, terminal voltage $V = E_g - I_a R_a = 462 - 2{\cdot}75 = \mathbf{459{\cdot}25}$ V

Example 10-11. *A 4-pole d.c. shunt generator with a wave-wound armature having 390 conductors has to supply a load of 500 lamps each of 100 W at 250 V. Allowing 10 V for the voltage drop in the connecting leads between the generator and the load and a contact drop of 1 volt per brush, calculate the speed at which the generator should be driven. The flux per pole is 30 mWb and the armature and shunt field resistances are respectively 0·05 Ω and 65 Ω.*

(A.M.I.E. Winter 1980)

Solution.

Load current $= 500 \times 100/250 = 200$ A

Voltage across shunt $= 250 + 10 = 260$ V

$$I_{sh} = 260/65 = 4 \text{ A}$$

$$I_a = 200 + 4 = 204 \text{ A}$$

$$I_a R_a = 204 \times 0{\cdot}05 = 10{\cdot}2 \text{ V}$$

Brush drop $= 2 \times 1 = 2$ V

Drop in connecting leads $= 10$ V

Generated e.m.f. $= 250 + 10 + 2 + 10{\cdot}2 = 272{\cdot}2$ V

Now $$E_g = \frac{\Phi Z N}{60} \times \left(\frac{P}{A} \right) \text{ volt}$$

$\therefore$ $$272{\cdot}2 = \frac{30 \times 10^{-3} \times 390 \times N}{60} \times \left(\frac{4}{2} \right)$$

$\therefore$ $$N = \mathbf{698 \text{ r.p.m.}}$$

Example 10-12. *A 4-pole, d.c. shunt generator with wave-wound armature has 41 slots each having 12 conductors. The armature resistance is 0·5 Ω and the shunt field resistance is 200 Ω. The flux per pole is 25 mWb. If a load of 10 Ω resistance is connected across the armature terminals, calculate the voltage across the load when the generator is driven at 1000 r.p.m.*

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Solution.

$$E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

Here $P=4$; $A=2$

$Z=41\times 12 = 492$

$N=1000$ r.p.m.

$\Phi = 25\times 10^{-3}$ Wb

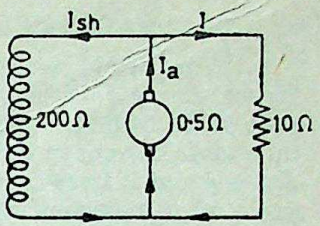

Fig. 10-23

$$\therefore \quad E_g = \frac{25\times 10^{-3}\times 492\times 1000}{60} \times \frac{4}{2} = 410 \text{ V}$$

As seen from Fig. 10-23, the shunt and load resistances are in parallel with each other.

$$\therefore \text{ their combined resistance} = \frac{10\times 200}{210} = \frac{200}{21} \ \Omega$$

Total circuit resistance $= 0{\cdot}5 + 200/21 = 421/42 \ \Omega$

$$\therefore \text{ armature current} = \frac{410}{421/42} = 40{\cdot}9 \text{ A}$$

Armature drop $= 40{\cdot}9\times 0{\cdot}5 = 20{\cdot}45$ volt

$\therefore$ terminal p.d. $= 410 - 20{\cdot}45 = \mathbf{389{\cdot}55\ V}$

10-15. Iron Loss in Armature

Due to the rotation of the iron core of the armature in the magnetic flux of the field poles, there are some losses taking place continuously in the core and are known as Iron losses or Core losses. Iron loss consists of (*i*) **Hysteresis loss** and (*ii*) **Eddy Current loss.**

(*i*) **Hysteresis Loss** (W_h).

This loss is due to the reversal of magnetism of the armature core. Every portion of the rotating core passes under *N*- and *S*-pole alternately, thereby attaining *S*- and *N*-polarity respectively. The core undergoes one complete cycle of magnetic reversal after passing under one *pair* of poles. If P is the number of poles and N the armature speed in r.p.m., then frequency of magnetic reversals is

$$f = PN/120 \text{ reversals/second}$$

The loss depends upon the volume and grade of iron, maximum value of flux density B_{max} and frequency of magnetic reversals. For normal flux densities (*i.e.* up to 1·5 Wb/m^2), hysteresis loss is given by **Steinmetz** formula. According to this formula

$$W_h = \eta B_{max}^{1.6} f V \text{ watt}$$

where V—volume of the core in m^3

η = Steinmetz hysteresis coefficient

Value of η for

Good dynamo sheet steel = 502·4, silicon steel = 191.

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(*ii*) Eddy Current Loss *(A.M.I.E. Winter 1983)*

When the armature core rotates, then it also cuts the flux. Hence, an e.m.f. is induced in the body of the core according to the laws of electromagnetic induction. This e.m.f., though small, sets up a large current in the body of the core due to its small resistance. This current is known as eddy current The power loss due to the flow of this current is known as 'eddy current' loss. This loss would be considerable if solid iron core were used. In order to reduce this loss and the consequent heating of the core to a small value the core is built up of thin laminations, which are stacked and then riveted at right angles to the path of the eddy currents. These core laminations are insulated from each other by a thin coating of varnish. The effect of laminations is shown in Fig. 10-24. Due to the core body being one continuous solid iron piece [Fig. 10-24 (*a*)] the magnitude of eddy current is large. As armature cross-sectional area is large, its resistance is very small, hence eddy current as well as the loss is large. In Fig. 10-24 (*b*) the same core has been split

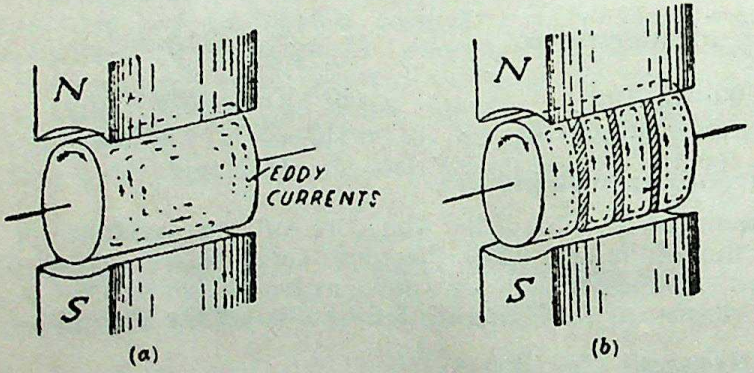

Fig. 10-24

up into thin circular discs insulated from each other. It is seen that now each current path, being of much less cross-section, has a very high resistance. Hence, magnitude of eddy currents is reduced very much, thereby drastically reducing eddy current loss.

It is found that eddy current loss W_e is given by the following relation :

$$W_e = k B_{max}^2 f^2 t^2 V \text{ watt}$$

where B_{max} = maximum flux density
f = frequency of magnetic reversals
t = thickness of each lamination
V = volume of armature core

It is seen from above that this loss varies directly as the square of the thickness of laminations, hence it should be kept as small as possible. Another point to note is that $W_h \propto f$ but $W_e \propto f^2$. This fact makes it possible to separate the two losses experimentally if so desired.

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As said earlier, these iron losses if allowed to take place unchecked, not only reduce the efficiency of the generator but also raise the temperature of the core. As the output of the machines is limited, in most cases, by the temperature rise, these losses have to be kept as small as is economically possible.

Eddy current loss is reduced by using laminated core, but hysteresis loss cannot be reduced this way. For reducing the hysteresis loss, those metals are chosen for the armature core which have a low hysteresis coefficient. Generally, special silicon steels, such as stalloys are used which not only have a low hysteresis coefficient, but which possess high electrical resistivity.

10-16. Total Losses in a D.C. Generator

The various losses occurring in a generator can be subdivided as follows :

(*a*) **Copper Losses** (or I^2R loss)

(*i*) Armature copper loss $= I_a^2 R_a$ (not $E_g I_a$)

where R_a = resistance of armature and interpoles and series field winding etc.

This loss is about 30 to 40% of full load losses.

(*ii*) Field copper loss : In the case of shunt generators, it is practically constant and $= I_{sh}^2 R_{sh}$ (or VI_{sh}). In the case of series generator, it is $= I_{se}^2 R_{se}$ where R_{se} is resistance of the series field winding.

This loss is about 20 to 30% of F.L. losses.

(*iii*) The loss due to brush contact resistance. It is usually included in the armature copper loss.

(*b*) **Magnetic Losses** (also known as iron or core losses).

(*i*) Hysteresis loss, $W_h \propto B_{max}^{1.6}$

(*ii*) Eddy current loss, $W_e \propto B_{max}^2 f^2$

These losses are practically constant for shunt and compound-wound generators, because field current, in their case, is approximately constant.

Both these losses total up to about 20 to 30% of F.L. losses.

(*c*) **Mechanical Losses**

These consist of

(*i*) friction loss at bearings and commutator.

(*ii*) air-friction or windage loss of rotating armature.

These are about 10 to 20% of F.L. losses.

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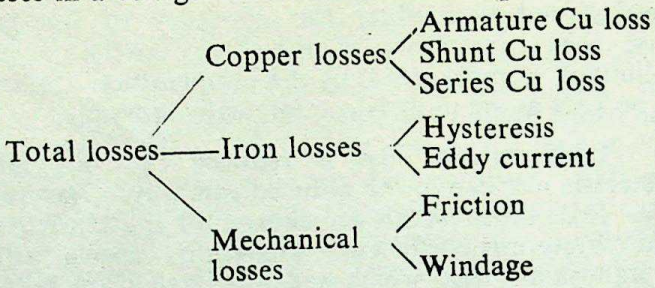
The total losses in a d.c. generator are summed up below.



10-17. Stray Losses

Usually magnetic and mechanical losses are collectively known as **stray losses.**

10-18. Constant and Standing Losses

As said above, field Cu loss is constant for shunt and compound generators. Hence, stray losses and shunt Cu losses are constant in their case. These losses are known as *standing* or *constant* losses W_c.

Hence, for shunt and compound generators,

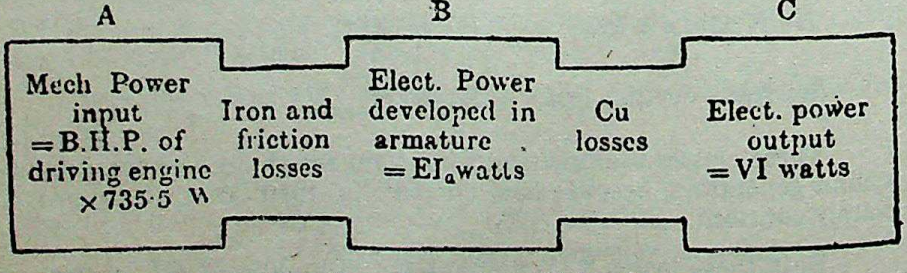
$$\text{Total losses} = \text{armature copper loss} + W_c$$
$$= I_a^2 R_a + W_c = (I + I_{sh})^2 R_a + W_c$$

Armature Cu loss, $I_a^2 R_a$ is known as *variable* loss because it varies with the load current.

$\therefore$ Total losses = variable losses + constant losses W_c

10-19. Power Stages

Various power stages in the case of a d.c. generator are shown below :



Following are the three generator efficiencies :

(I) Mechanical Efficiency

$$\eta_m = \frac{B}{A} = \frac{\text{total watts generated in armature}}{\text{mechanical power supplied}}$$

$$= \frac{E_g I_a}{B.H.P. \times 735{\cdot}5}$$

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(2) **Electrical Efficiency**

$$\eta_e = \frac{C}{B} = \frac{\text{watts available in load circuit}}{\text{total watts generated}} = \frac{VI}{E_g I_a}$$

(3) **Overall or Commercial Efficiency**

$$\eta_c = \frac{C}{A} = \frac{\text{watts available in load circuit}}{\text{mechanical power supplied}}$$

$$= \frac{VI}{B.H.P. \times 735{\cdot}5}$$

It is obvious that overall efficiency $\eta_c = \eta_m \times \eta_e$. For good generators, its value may be as high as 95%.

Note. Unless specified otherwise, commercial efficiency is always to be understood.

10-20. Condition for Maximum Efficiency

Generator output $= VI$ watts

Generator input $=$ output $+$ losses

$$= VI + I_a^2 R_a + W_c$$

$$= VI + (I + I_{sh})^2 R_a + W_c \quad (\because \; I_a = I + I_{sh})$$

However, if I_{sh} is negligible as compared to load current I, then

$$I_a = I \text{ (approx)}$$

$$\therefore \quad \eta = \frac{\text{output}}{\text{input}} = \frac{VI}{VI + I_a^2 R_a + W_c}$$

$$= \frac{VI}{VI + I^2 R_a + W_c} \quad (\because \; I_a = I)$$

$$= \frac{1}{1 + \left(\frac{IR_a}{V} + \frac{W_c}{VI} \right)}$$

Now, efficiency is maximum when denominator is minimum

i.e. when $\frac{d}{dI}\left(\frac{IR_a}{V} + \frac{W_c}{VI} \right) = 0$

or $\frac{R_a}{V} - \frac{W_c}{VI^2} = 0$ or $I^2 R_a = W_c$

Hence, efficiency is maximum when

variable loss = constant loss

The load current corresponding to maximum efficiency is given by the relation

$$I^2 R_a = W_c$$

or $$I = \sqrt{\frac{W_c}{R_a}}$$

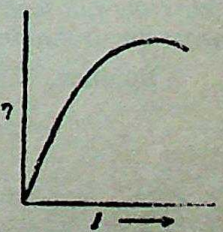

Fig. 10-25

Variation of η with load current is shown in Fig. 10-25.

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Example 10-13. *A shunt generator delivers 195 A at a terminal p.d. of 250 V. The armature resistance and shunt field resistance are 0·02 Ω and 50 Ω respectively. The iron and friction losses equal 950 W. Find*

(a) e.m. f. generated *(b) Cu losses*
(c) Output of the prime mover in kW
(d) Commercial, mechanical and electrical effciencies.

Solution. (*a*) $I_{sh}=250/50=5$ A, $I_a=195+5=200$ A

Armature voltage drop $=I_aR_a=200\times0{\cdot}02=4$ V
$\therefore$ Generated e.m.f. $=250+4=\mathbf{254}$ V
(*b*) Armature Cu loss $=I_a^2R_a=200^2\times0{\cdot}02=800$ W
Shunt Cu loss $=V.I_{sh}=250\times5=1250$ W
$\therefore$ Total Cu loss $=1250+800=\mathbf{2050}$ W
(*c*) Stray losses $=950$ W
Total losses $=2050+950=3{,}000$ W
Output $=250\times195=48{,}750$ W
Input $=48{,}750+3{,}000=51{,}750$ W
$\therefore$ Output of the prime mover $=51{,}750$ W $=51{\cdot}75$ kW

(*d*) Input $=51{,}750$ W ; Stray loss $=950$ W

Electrical power produced in armature
$=51{,}750-950=50{,}800$ W

$$\therefore \quad \eta_m=\frac{50{,}800}{51{,}750}\times100=\mathbf{98{\cdot}2\%}$$

Electrical or Cu losses
$=2050$ W

$$\therefore \quad \eta_e=\frac{48{,}750}{48{,}750+2{,}050}\times100=\mathbf{95{\cdot}9\%}$$

$$\eta_c=\frac{48{,}750}{51{,}750}\times100=\mathbf{94{\cdot}2\%}$$

Example 10-14. *A 400-V shunt generator has a full-load current of 200 A, its armature resistance is 0·06 Ω and field resistance 100 Ω ; the stray losses are 2,000 W. Find the h.p. of prime mover when it is delivering full load, and find the load for which the efficiency of the generator is maximum.*

Solution. Full-load output current $=200$ A
$\therefore$ F.L. output $=400\times200=80{,}000$ W
$I_{sh}=400/100=4$ A
$\therefore$ $I_a=200+4=204$ A
Armature Cu loss $=204^2\times0{\cdot}06=2{,}497$ W
Shunt Cu loss $=400\times4=1{,}600$ W
Stray losses $=2{,}000$ W

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$\therefore$ Total losses in the generator

$$= 2{,}000 + 2{,}497 + 1{,}600 = 6{,}097 \text{ W}$$

$\therefore$ Total input $= 80{,}000 + 6{,}097 = 86{,}097$ W

$\therefore$ H.P. of prime mover $= 86{,}097/735{\cdot}5 = \mathbf{117}$

The load current for which efficiency is maximum is given by

$$I = \sqrt{W_c/R_a}$$

Now, constant losses = Stray losses + Shunt Cu loss

$$= 2{,}000 + 1{,}600 = 3{,}600 \text{ W}$$

$$I = \sqrt{\frac{3600}{0{\cdot}06}} = 245 \text{ A}$$

Example 10-15. *A shunt generator has a F.L. current of 196 A at 220 V. The stray losses are 720 W and the shunt field coil resistance is 55 Ω. It has a F.L. efficiency of 88%, find the armature resistance. Also find the load current corresponding to maximum efficiency.*

Solution. Output $= 220 \times 196 = 43{,}120$ W

$\eta = 88\%$ (overall efficiency)

$\therefore$ Electrical input $= 43{,}120/0{\cdot}88 = 49{,}000$ W

$\therefore$ Total losses $= 49{,}000 - 43{,}120 = 5{,}880$ W

Shunt field current $= 220/55 = 4$ A

$\therefore$ $I_a = 196 + 4 = 200$ A

$\therefore$ Shunt Cu loss $= 220 \times 4 = 880$ W

Stray losses $= 720$ W

Constant losses $= 880 + 720 = 1{,}600$ W

$\therefore$ Armature Cu loss $= 5{,}880 - 1{,}600 = 4{,}280$ W

$\therefore$ $I_a^2 R_a = 4{,}280$ W

$$200^2 R_a = 4{,}280 \quad \text{or} \quad R_a = \frac{4{,}280}{200 \times 200} = 0{\cdot}107 \ \Omega$$

For maximum efficiency,

$$I^2 R_a = \text{constant losses} = 1{,}600 \text{ W}$$

$$I = \sqrt{\frac{1600}{0{\cdot}107}} = 122{\cdot}3 \text{ A}$$

Example 10-16. *A shunt generator gives a full-load output of 6 kW at a terminal potential of 200 V. The armature and field resistances are 0·5 Ω and 50 Ω respectively. If the mechanical and iron losses combined amount to 500 W, calculate the horse-power required at the driving shaft on full load and the full-load efficiency.*

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Solution. $I_{sh} = 200/50 = 4$ A

Output current $= 6{,}000/200 = 30$ A

$\therefore \quad I_a = 30 + 4 = 34$ A

$I_a^2 R_a = 34^2 \times 0{\cdot}5 = 578$ W

Shunt field Cu loss $= 200 \times 4 = 800$ W

Total losses $= 578 + 800 + 500 = 1{,}878$ W

$\therefore$ Input $= 6{,}000 + 1{,}878 = 7{,}878$ W

H.P. (metric) $= 7{,}878/735{\cdot}5 = \mathbf{10{\cdot}71}$

Full load $\eta = \dfrac{6{,}000 \times 100}{7{,}878} = \mathbf{76{\cdot}2\%}$

Tutorial Problems—14

1. Calculate the flux per pole required on full-load for a 50-kW, 400 V, 8-pole, 600 r.p.m. d.c. shunt generator with 256 conductors arranged in a lap-connected winding. The armature winding resistance is 0·1 Ω, the shunt field resistance is 200 Ω and there is a brush contact voltage drop of 1 volt at each brush on full-load. **[162 mWb]**

2. A 4-pole, short-shunt compound-generator has armature, shunt field and series field resistances of 0·4 Ω, 160 Ω and 0·2 Ω respectively.

The armature is lap-wound with 440 conductors and is driven at 600 r.p.m. Calculate the flux per pole when the machine is delivering 120 A at 400 V. **[108 mWb]**

3. A 6-pole, wave-wound shunt generator delivers 75 kW at 120 V. The armature has 270 conductors and a total resistance of 0·02 Ω. The flux per pole is 0·01 Wb. Calculate the speed of the machine in r.p.m. if the field resistance is 60 Ω and the total voltage drop across the brushes is 2 V. **[1000 r.p.m.]**

4. A d.c., six-pole shunt generator has a wave-wound armature with 412 conductors. The flux per pole is 20 mWb and it is driven at a speed of 1250 r.p.m. The armature resistance is 0·15 Ω, the field resistance is 125 Ω and the total brush contact drop is 2·4 V. If the terminal p.d. is maintained at 500 V, calculate the current supplied to the load. **[80 A]**

5. A 4-pole machine has a lap-wound armature with 90 slots each containing six conductors. If the machine runs at 1500 r.p.m. and the flux per pole is 0·03 Wb, calculate the e.m.f. generated.

If the machine is run as a shunt generator with the same field flux, the armature and field resistances being 1·0 Ω and 200 Ω respectively, calculate the output current when the armature current is 25 A. If due to a fall in speed, the e.m.f. becomes 380 V, calculate the load current in a 40 Ω load. **[405 V ; 23·1 A ; 9·2 A]**

6. In a d.c. motor-generator set, the generator supplies a load of 198 A at 60 V. The resistance of the generator armature is 0·05 Ω and the shunt field resistance is 30 Ω. The iron, friction and windage losses in the generator amount to 1·2 kW. The 500 V d.c. motor which drives the generator at 600 r.p.m. has an efficiency of 80%. Calculate (*a*) the output of the motor in kW (*b*) current taken by the motor. **[(*a*) 15·12 kW (*b*) 38 A]**

7. A 20-kW, 440-V, short-shunt compound d.c. generator has a full-load efficiency of 87%. If the resistance of the armature is 0·4 Ω and that of the series and shunt field is 0·25 Ω and 240 Ω respectively, calculate the combined bearing, windage and core loss of the machine. **[725 W]**

8. A short-shunt compound d.c. generator supplies a current of 100 A at a voltage of 220 V. If the resistance of the shunt field is 50 Ω, of the series field 0·025 Ω, of the armature 0·05 Ω, the total brush drop is 2 V and the iron and friction loss amount to 1 kW, find

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(*a*) the generated e.m.f.

(*b*) the copper losses

(*c*) the output of the prime mover driving the generator

(*d*) generator efficiency.

[(*a*) 229·7 V (*b*) 1·995 kW (*c*) 24·99 kW (*d*) 0·88]

10-21. Armature Reaction

By armature reaction is meant the effect of magnetic field set up by armature current on the distribution of flux under main poles of a generator (or motor). This armature field has two effects :

(*i*) it demagnetises or weakens the main pole flux and

(*ii*) it distorts the main flux.

Accordingly, there are two components of armature reaction, one is called the 'demagnetising component' and the other 'distorting component'. Both these components increase with increase in the armature current, that is, with increase in the load on the generator. Under severe overloads or short circuit, the demagnetising component of armature reaction may become so strong as to reverse the polarity of the main poles. In general, when the main flux Φ is decreased due to armature reaction, the e.m.f. induced in the armature of a d.c. generator is also decreased because $E \propto \Phi$.

The demagnetising effect of armature reaction is neutralized by adding a few extra ampere-turns to the main field winding. The distorting effect is neutralized by using 'compensating windings'. These windings are embedded in slots in the pole-shoes and are connected in series with armature in such a way that current in them flows in a direction opposite to that of the armature current flowing in the armature conductors directly below the pole shoes.

10-22. Commutation

As briefly explained in Art. 10-2, the induced currents in the armature conductors of a d.c. generator are alternating currents *i.e.* these currents flow in one direction when conductors are under *N*-pole and in exactly opposite direction when they are under *S*-pole. As conductors pass out of the influence of a *N*-pole and enter that of a *S*-pole, the current in them is reversed. This reversal of current takes place along magnetic neutral axis (*M.N.A.*) or brush axis *i.e.* when the brush spans and hence short-circuits the particular coil undergoing reversal of current through it.

This process by which current in the short-circuited armature coil is reversed while it crosses the *M.N.A.* is called 'commutation'.

The brief period during which coil remains short-circuited is known as commutation period T_c.

If current reversal is completed within the time T_c, then commutation is ideal. If not, then sparking is produced between the brush and the commutator which damages both.

The main factor which does not allow the armature current to completely reverse its direction within the specified period of T_c

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is the production of the self-induced e.m.f. called *reactance voltage* in the conductors through which the current is reversing. If current changes from $+I$ to $-I$ in time T_c, then reactance voltage is $L\ 2I/T_c$ where L is the inductance of the armature conductors. Commutation can be improved *i.e.* current reversal can be made sparkless by using interpoles. These are small poles fixed to the yoke and spaced in between the main poles as shown in Fig. 10-26. They are wound with a few thick copper-wire turns and are connected in series with the armature so that they carry full armature current. Their polarity is the same as that of the main pole *ahead* in the direction of rotation.

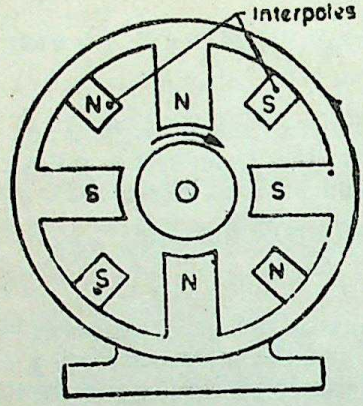

Fig. 10-26

HIGHLIGHTS

1. The e.m.f. generated within the armature of d.c. generator given by

$$E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volts} \quad \text{... when } N \text{ is in r.p.m.}$$

$$= \Phi ZN\,(P/A) \text{ volts} \quad \text{... when } N \text{ is in r.p.s.}$$

2. The losses in a generator are (*i*) Cu or electrical losses (*ii*) iron or magnetic losses and (*iii*) mechanical losses.

3. Magnetic and mechanical losses are collectively known as stray losses.

4. In the case of a shunt generator, the shunt Cu loss is practically constant. Hence, stray losses plus Cu loss are collectively referred to as constant losses (W_c).

5. The condition for maximum efficiency of a d.c. shunt generator is

Armature Cu loss = Constant loss

6. The load current corresponding to maximum efficiency is given by

$$I = \sqrt{\frac{W_c}{R_a}}$$

7. By 'armature reaction' is meant the effect of the magnetic field set up by armature current on the main-pole field. There are two components of armature reaction :

(*i*) demagnetising component which weakens the main pole flux and

(*ii*) distorting component which distorts it

8. The process by which current in the short-circuited armature coil is reversed while it crosses the magnetic neutral axis is called 'commutation'.

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GENERATOR CHARACTERISTICS

11

1. Characteristics of D.C Generators. 2. Open Circuit Characteristic 3. Critical Resistance for a Series Generator. 4. Critical Resistance for Shunt Generator. 5. How to Find Critical Resistance R_c 6. How to Draw O.C.C. at Different Speeds 7. Critical Speed N_c 8. Internal and External Characteristics of a Series Generator 9. Voltage Build up of a Shunt Generator 10. Conditions for Build up of a Shunt Generator 11. Internal and External Characteristics of a Shunt Generator 12. Voltage Regulation 13. Highlights.

11·1. Characteristics of D.C. Generators

Following are the three *most* important characteristics or curves of a d.c. generator :

1. **Open-circuit Characteristic (O.C.C.)**

It is also known as Magnetic Characteristic or No-load Saturation Characteristic. It shows the relation between the no-load generated e.m.f. in the armature E_0 and the field or exciting current I_f at a given fixed speed. In fact, it is just the magnetisation curve for the material of the electromagnets. Its shape is practically the same for all generators whether separately excited or self-excited.

2. **Internal or Total Characteristic (E/I_a)**

It gives the relation between e.m.f. E *actually* induced in the armature (after allowing for the demagnetising effect of armature reaction) and the armature current $_a$. This characteristic is of interest mainly to the designer.

3. **External Characteristic (V/I)**

It is also referred to as performance characteristic or sometimes voltage regulation curve.

It gives relation between the terminal voltage V and the load current I. This curve lies below the internal characteristic because it takes into account the voltage drop over the armature circuit resistance. The values of V are obtained by subtracting I_aR_a from

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corresponding values of E. This characteristic is of great importance in judging the suitability of a generator for a particular purpose. It may be obtained in two ways : (*i*) by making simultaneous measurements with a suitable voltmeter and ammeter on a loaded generator or (*ii*) graphically from the *O.C.C* provided the armature and field resistances are known and also if the demagnetising effect, under rated load conditions, of the armature reaction (from the short-circuit test) is known.

11-2. Open Circuit Characteristic

The *O.C C.* for d.c. generators is found as described below :

The field winding of the generator (whether shunt or series wound) is disconnected from the machine and, instead, connected to an external separate d.c. source like a battery as shown in Fig. 11-1. The exciting current is changed rheostatically and its value read on the ammeter. The armature is driven at a constant speed, say, N and the generated e.m.f. on no-load E_0 is measured by the voltmeter connected across the armature. The field (or exciting) current I_f is increased by suitable steps (starting from zero) and the corresponding values of E_0 are measured. On plotting the relation between E_0 and I_f, a curve of the form shown in Fig. 11-1 is obtained.

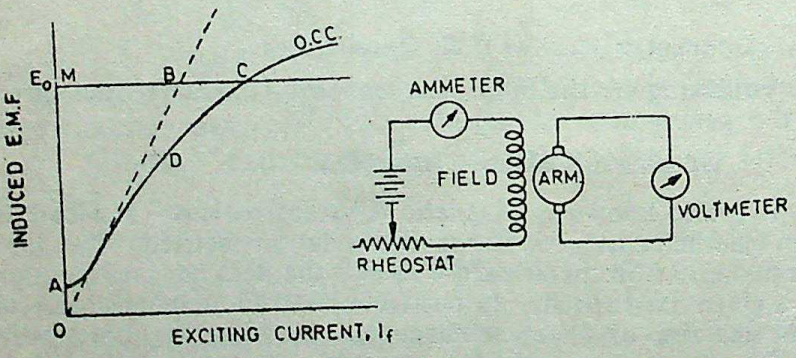

Fig. 11-1

Due to residual magnetism in the poles, some e.m.f. is generated even when exciting current is zero. Hence, the curve starts a little way up instead of starting from the origin. The length *OA* represents the e.m.f. induced due to residual flux only. It is seen that the initial portion of the curve is practically straight. This is due to the fact that at low flux densities, the reluctance of the magnetic path is almost constant. The magnetic path (in the generator) lies partly through air (clearance between the pole-shoes and the armature) and partly through iron (*i.e.* poles). At low flux densities, the reluctance of iron path is negligible because of its high permeability, hence the total reluctance is very nearly represented by the reluctance of the air-gap alone. Therefore, the flux (and hence E_0) increases linearly with exciting current*. But at

*The initial slope of the O.C.C. is determined by the reluctance of the air-gap *i.e.* by the air-gap width.

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higher flux densities, where permeability of iron is less, the reluctance of iron path is no longer negligible, rather it goes on increasing as flux density is increased*. The result is that the total reluctance is no longer constant which means that the straight line relationship between E_0 and I_f no longer holds good.

At point *D*, the magnetic saturation of the poles sets in and hence the curve bends over. Over the saturated portion of the curve, a greater increase in exciting current is required for producing a given increase in the generated e.m.f.

It may be pointed out here that in all subsequent figures, the modification of *O.C.C.* due to the effects of residual magnetism would be considered negligibly small and hence the *O.C.C.* would be drawn starting from the origin.

11-3. Critical Resistance for a Series Generator

Now, connect the field windings back to the armature and in series with it and run the machine as a series generator. Suppose the load on the generator consists of a variable resistance. This resistance can be represented by a straight line passing through the origin in Fig. 11-2 because the axes of the graph are volts and amperes. Suppose the load resistance is 20 Ω. Then corresponding to a voltage of 100 volts, current is 100/20 = 5A. Hence, point *B* (5 A, 100 V) is the point which lies on the 20-Ω resistance line. Join point *O* to *B* and produce further. This line is known as the resistance line. The slope of this line, obviously, represents 20 Ω. Resistance line *OA* represents a resistance of value less than 20 Ω. But lines *OT* and *OD* represent resistances of higher values. Obviously, steeper the line is, greater is the resistance.

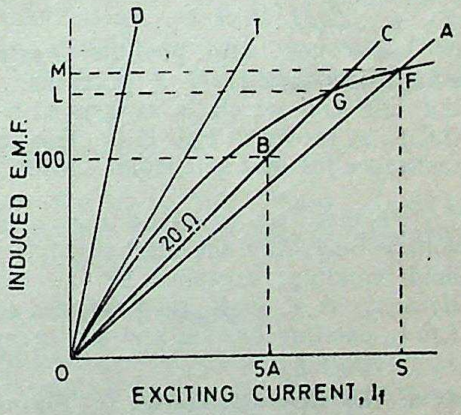

Fig. 11-2

Suppose the load resistance is 20 Ω as represented by resistance line *OBC*. Let the armature of this series generator be set into rotation at *N* r.p.m. Due to residual flux, a small e.m.f. would be produced initially. If the load resistance is not large, this e.m.f. will be able to set up a small current which will increase the field ampere-turns, which in turn, will increase the pole flux. Increased pole flux will lead to more induced e.m.f. which will result in more field

*For example for point *C* in Fig. 11-1.

MB=ampere-turns required by the air-gap.

BC=ampere-turns required by the iron path.

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ampere-turns and so on. In this way, the cumulative building up action of the generator will start till equilibrium is reached at some point G where the resistance line cuts the $O.C.C.$ The voltage to which the machine will build up $= OL$.

If the load resistance is decreased (say, equal to that represented by resistance line OA) then the machine will build up to a maximum voltage of OM which is somewhat greater than the previous value of OL.

On the other hand, if the load resistance is increased, then the maximum voltage to which the machine builds up is dereased.

If the resistance is increased too much, in fact, so much that its resistance line does not intersect $O.C.C.$ at all (as line OD), then with this load resistance, the machine will never start building up *i.e.* the generator will fail to excite with this high resistance· It is so because under these conditions, the electrical power induced in the armature is not sufficient to increase the pole flux after supplying the I^2R loss. Hence, the cumulative building-up does not commence.

The maximum possible resistance with which the generator would *just* excite, at a particular speed, is given by the slope of the line OT which is tangent to the initial straight portion of the $O.C.C.$ as shown in Fig. 11-2. This resistance is known as the *critical resistance* for that particular speed.

Hence, we conclude that a series generator will build up the voltage only when the total circuit resistance (*i.e.* load resistance and field winding resistance etc.) is less than the critical resistance. Because, it is only then that the mutual reinforcement of induced e.m.f., exciting current and flux takes place.

11.4. Critical Resistance for Shunt Generator

Now, connect the field windings back to armature and run the machine as a shunt generator. Due to residual magnetism, some initial e.m.f. and hence current, would be generated. This current while passing through the field coils, will strengthen the magnetism of the poles (provided field coils are properly connected). This will increase the pole flux which will further increase the generated e.m.f. Increased e.m.f. means more current which further increases the flux and so on. This mutual reinforcement of e.m.f. and flux proceeds on till equilibrium is reached at some point like P (Fig. 11-3). This point lies on the resistance line OA of the field winding. Let R be the resistance of the field winding. Line OA is drawn such that its slope equals the field winding resistance R *i e.* every point on this curve is such that

$$\frac{\text{volt}}{\text{ampere}} = R$$

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The voltage OL, corresponding to point P, represents the maximum voltage to which the machine will build up with R as field resistance. OB represents smaller resistance and the corresponding voltage OM is slightly greater than OL. If field resistance is increased, then slope of the resistance line increases and hence the maximum voltage to which the generator will build up at a given speed, decreases. If R is increased so much that the resistance line does not cut the $O.C.C.$ (like OT), then obviously, the machine will fail to excite *i.e.* there will be no 'build up' of the voltage. If the resistance line just lies along the slope, then with that value of field resistance the machine will just excite. The value of resistance represented by the tangent to the curve, is known as *critical resistance* R_c for *a given speed.*

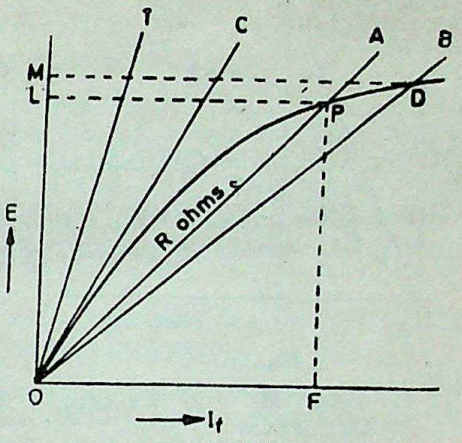

Fig. 11-3

11-5. How to Find Critical Resistance R_c ?

First, $O.C.C.$ is plotted from the given data. Then tangent is drawn to it. The slope of this curve gives the critical resistance for the speed at which the data were obtained.

11-6. How to Draw O.C.C. at Different Speeds ?

Suppose we are given the data for $O.C.C.$ of a generator run at a fixed speed, say, N_1. It will be shown that $O.C.C.$ at any other constant speed N_2 can be deduced from the $O.C.C.$ for N_1. In Fig 11-4, curve OA represents the $O.C.C.$ at N_1.

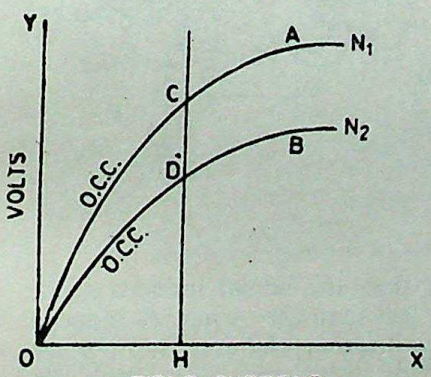

Fig. 11-4

Since $E \propto N$ for any fixed excitation, hence

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

or $$E_2 = E_1 \times \frac{N_2}{N_1}$$

As seen, for $I_f = OH$, $E_1 = HC$. The value of new voltage for the same I_f but at N_2 is

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

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In this way, point *D* is located. In a similar way, other such points can be found and the new *O.C.C.* drawn (see Ex. 11-1).

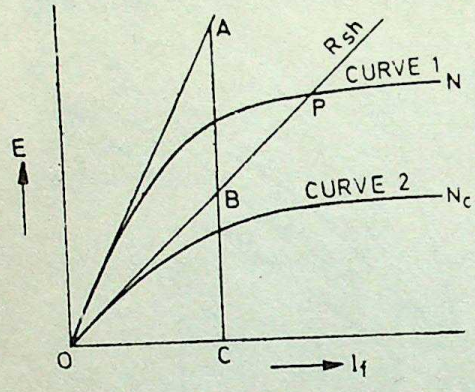

Fig. 11-5

11-7. Critical speed N_c

Critical speed of a shunt generator is that speed for which the *given* shunt field resistance represents critical resistance. In Fig. 11-5, curve 2 corresponds to critical speed because R_{sh} line is tangential to it. Obviously

$$\frac{BC}{AC}=\frac{N_c}{\text{full speed}}=\frac{N_c}{N}$$

$$\therefore\ N_c=\frac{BC}{AC}\times\text{full speed } N$$

11-8. Internal and External Characteristics of a Series Generator

(A.M.I.E. Nov. 1969)

Since, in this generator, the field windings are in series with the armature, hence they carry the full armature current I_a (which is also the load current). If there were nothing to reduce the flux of the generator when it delivers load, then as I_a is increased, the flux and hence the generated e.m f would also have increased as given by the *O.C.* characteristic *OA*. However, due to armature reaction, the flux on load, for a given current, would be less than

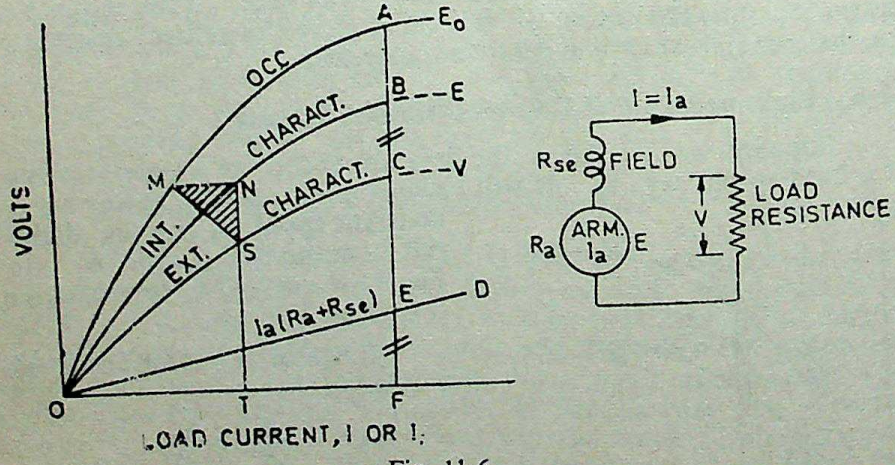

Fig. 11-6

the flux on no-load, with the result that the actual induced e.m.f. *E* (under load conditions) is a little less than the e.m.f. E_0 induced under no-load conditions. Hence, the internal characteristic *OB* lies somewhat below the *O.C.C.* But for this armature reaction the two would have coincided.

The external characteristic *i.e.* *V/I* curve lies still lower because the terminal p.d. *V* is less than *E* by an amount equal to the ohmic drop in the generator.

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If R_a = armature resistance

and R_{se} = resistance of series field winding

Then ohmic drop is $= I_a (R_a + R_{se})$

$\therefore \quad V = E - I_a(R_a + R_{se})$

The ohmic voltage drop is represented by the straight line *OD* in Fig. 11-6. If ordinates of curve *OD* are subtracted from those of internal characteristic *OB*, then we get the external characteristic *OC*. For example, for a load current $= OF$, the ohmic drop is *EF*. If we mark off $BC = EF$, then, we get point *C* on the external characteristic. By obtaining a number of similar points, we can draw the external characteristic.

It will be noticed that a series generator is a variable voltage generator *i.e.* its voltage increases with load current.

It would also be seen that with increase in load current, the demagnetising effect of armature reaction is also increased. In the shaded right-angled triangle '*MNS*', for a load current of *OT*,

MN = effect of armature reaction in terms of current

NS = internal voltage drop.

If '*n*' is the number of turns per pole, then the product '$n \times MN$' represents the total demagnetising effect of armature reaction for a load current $= OT$.

Beyond point *C*, when load current is increased, then due to excessive demagnetising effect of armature reaction, terminal p.d. *V* is decreased instead of increasing because the induced e.m.f. itself is reduced. The portion beyond point *C* represents unstable conditions.

It is seen that the initial portion of the external characteristic is straight *i.e.* over a restricted range, the terminal p.d. *V* is pro-

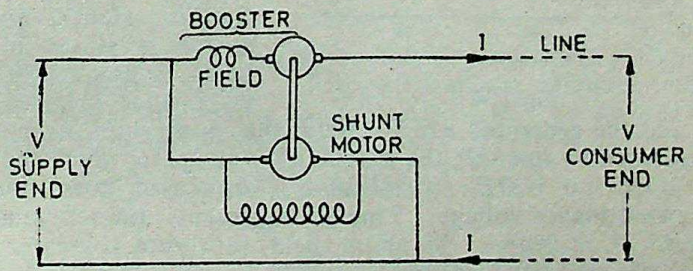

Fig. 11-7

portional to the load current. This proportionality makes a series generator suitable for being used as a booster *i.e.* for 'boosting up' the supply voltage. We know that in a supply system the voltage at the consumer end is always less than that at the supply end by an amount equal to the drop in the cable. This voltage drop due to line resistance is directly proportional to the current. Hence, if a series generator is connected in the line (as shown in Fig. 11-7) and

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is so adjusted that at all load currents, it supplies an additional voltage which is just equal to the line drop, then the voltage at the consumer's end can be maintained constant. The booster of Fig. 11-7 is driven by a shunt motor.

11-9. Voltage Build up of a Shunt Generator

Before loading a shunt generator, it is allowed to build up its voltage. As usually there is always present some residual magnetism in the poles, a small e.m.f. is produced initially. This e.m.f. circulates a small current in the field circuit which increases the pole flux (provided field circuit is properly connected to armature, otherwise this current may wipe off the residual magnetism). When flux is increased, generated e.m.f. is increased which further increases the flux and so on.

Now, the generated e.m f. in the armature has to

(*a*) supply the ohmic drop $I_f R_{sh}$ in the winding and (*b*) to overcome the opposing self-induced e.m.f. in the field coils *i.e.* $L\ dI/dt$ because field coils have appreciable self-inductance.

If (and so long as) the generated e.m.f. is in excess of the ohmic drop $I_f\ R_{sh}$, energy would continue being stored in the pole fields. For example, as shown in Fig. 11-8 corresponding to field current *OA*, the generated e.m.f. is *AC*. Of this, *AB* goes to supply ohmic drop $I_f R_{sh}$ and *BC* goes to overcome self-induced e.m.f. in the coil. Corresponding to $I_f = OF$, whole of the generated e.m f. is used to overcome ohmic drop. None is left to overcome $L.dI/dt$. Hence, no further energy is stored in the pole fields. Subsequently, there is no further increase in pole flux and the generated e.m.f. With the given shunt field resistance represented by line *OP*, the maximum voltage to which the machine will build up is *OE*. If resistance is decreased, it will build up to somewhat higher voltage. Line *OR* represents the resistance known as *critical resistance*. If shunt field resistance is greater than this value, the generator will fail to excite.

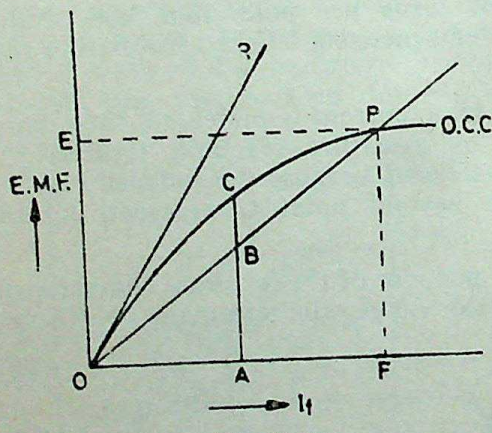

Fig. 11-8

11-10. Conditions for the Build up of a Shunt Generator

We may summarise the conditions necessary for the build up of a (self-excited) shunt generator as follows :

(*i*) There must be some residual magnetism in the poles.

(*ii*) For the given direction of rotation, the shunt field coils should be correctly connected to the armature *i.e.* they should be

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so connected that the induced current reinforces the e.m.f. produced initially due to residual magnetism.

(*iii*) If excited on open circuit, its shunt field resistance should be less than the critical resistance (which can be found from its *O.C.C.*)

(*iv*) If excited on load, its shunt field resistance should be more than a certain minimum value of resistance which is given by internal characteristic (Fig. 11-10).

11-11. Internal and External Characteristics of a Shunt Generator

(A.M.I.E. Summer 1986)

A shunt generator is usually first excited on no-load, so that it gives its full open circuit voltage $=OM$. The value of this *O.C.* voltage is adjusted with the help of field rheostat *Rh* as shown in Fig. 11-9. Then the load is switched on. If there were no demagnetising effect of armature reaction and no ohmic drop in armature circuit, then the voltage would have remained constant as shown by the horizontal line *MA*. But, in actual practice, due to armature reaction, especially at higher loads, the pole flux and hence the *actual*

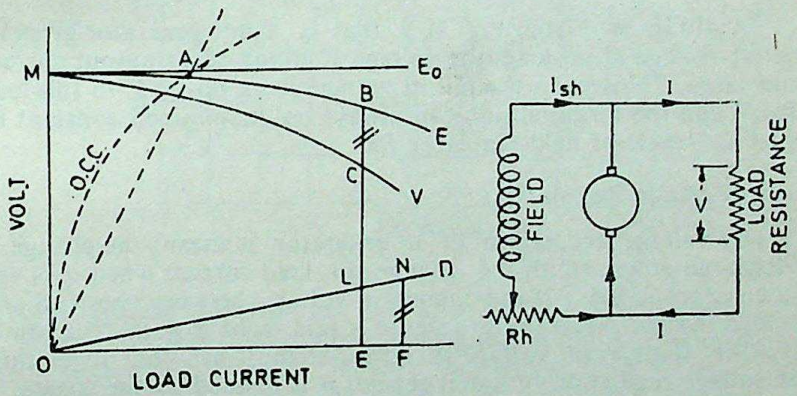

Fig. 11-9

induced e.m.f. is reduced. Therefore, the internal (or E/I_a) characteristic droops down slightly. The external characteristic can be obtained by further subtracting ohmic voltage drop (for various load currents) from the internal characteristic.

We know that for a shunt generator

$$I_a = I + I_{sh}$$

The resistance line *OD* represents the shunt field resistance (and the interpole winding resistance). For a load current *OE*, the armature current is $OE + EF$ where *EF* is the shunt field current I_{sh}. The voltage drop for an armature current *OE* is *FN*. Now, for a load current of *OE*, the actual induced e.m.f. is *BE*. Out of this if we subtract $BC = FN$, then we get the terminal p.d. of *EC*. The point *C* will, therefore, lie on the external characteristic. Similar other points can be found and the *V/I* graph can be drawn which is found to be still more drooping.

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If load resistance is decreased, then armature current increases up to a certain value. After that any decrease in load resistance is not accompanied by any increase in load current as might be expected. Rather, the load current is decreased and the curve turns back as shown in Fig. 11-10. If the load resistance is too small, then the generator is short-circuited and hence there is no generated e.m.f. due to heavy demagnetisation of the main poles.

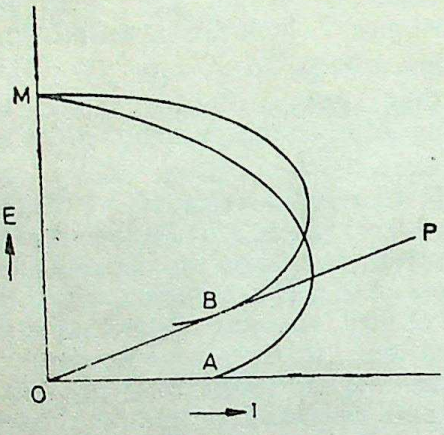

Fig. 11-10

Line *OP* is tangential to the internal characteristic *MB* and its slope gives the value of *minimum* resistance with which the shunt generator will excite if excited on load.

It will be seen from Fig. 11-9 that a shunt generator gives its greatest voltage at no-load, the voltage *V* falling off as output current is increased. However, the fall in voltage from no-load to full-load is small and the terminal p.d. can always be maintained constant by adjusting the shunt field regulator *Rh*.

11-12. Voltage Regulation

By voltage regulation of a generator is meant the change in its terminal voltage with the change in load current when it is run at a constant speed. If the change in voltage between no-load and full load is small, then the generator is said to have good regulation, but if the change in voltage is large, then it has poor regulation. The voltage regulation of a d.c. generator is defined as the change in voltage when the load is reduced from rate dvalue to zero, expressed as a percentage of the rated load voltage.

If no-load voltage is 240 V and rated-load voltage is 220 V,

then $$\text{regulation} = \frac{240-220}{220} = 0{\cdot}091 \quad \text{or} \quad 9{\cdot}1\%.$$

Example 11-1. *The O.C.C. of a d.c. shunt generator running at 1000 r.p.m. is as follows :*

O.C. volts :	*52·5*	*107·5*	*155*	*196·5*	*231*	*256·5*	*275*	*287·5*
I_f in amps :	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*

Estimate the voltage to which machine will build up if the speed is 800 r.p.m. and field circuit resistance is 30 Ω.

Solution. The *O.C C* for 1000 r.p.m. has been plotted in Fig. 11-11.

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Resistance line OA for 30 Ω is drawn as follows :

Take 5 ampere current. Multiplying it by 30 Ω, we get 150 V.

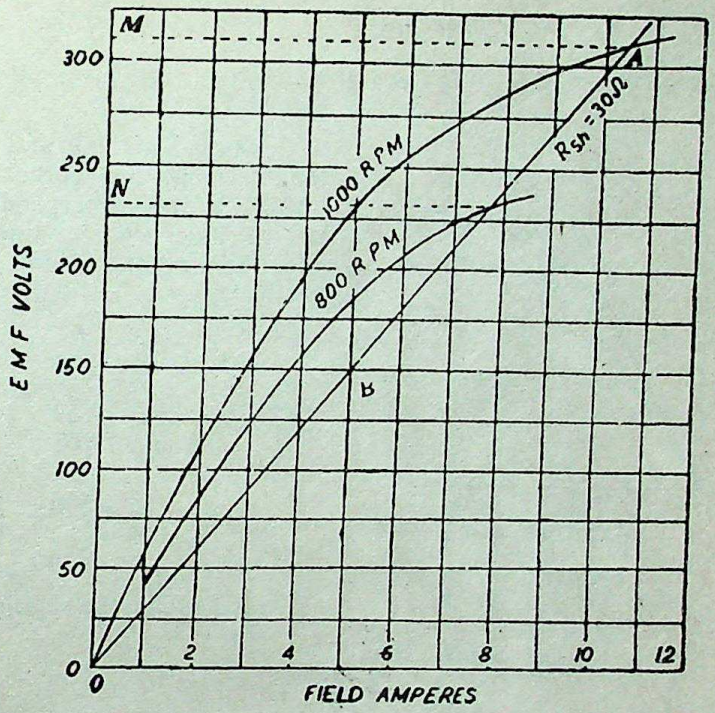

Fig. 11-11

Hence, join the ordinates of 5 A and 150 V. We get the point B (5 A, 150 V). Now, draw a line joining the origin (0, 0) and B. This gives 30 Ω resistance line. It cuts the $O C C.$ at A. Draw a horizontal line from A cutting the Y-axis at M. OM gives the maximum voltage to which the machine will build up with 30 Ω resistance in the shunt field and at 1000 r.p.m. It will be found that OM = **310 V.**

For finding the e.m f. induced at 800 r p.m , a new curve is drawn. Since e.m.f. is proportional to the speed, hence voltage values at 1000 r.p.m. are multiplied by 800/1000 = 0 8 and $O.C.C.$ is plotted as before. With R_{sh} = 30 Ω, e.m.f. is = ON = **230 V.**

Example 11-2. *The magnetisation curve at 800 r.p.m. of a 4-pole, 220-V, shunt generator with 576 conductors lap-wound armature is as follows :*

Field current (Amp.)	0	0 5	1	2	3	4	5
E.M.F. (Volts)	10	50	100	175	220	245	262

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(*a*) *If field circuit resistance is 75 Ω, calculate* (*i*) *the speed at which the machine just fails to excite* (*ii*) *the flux per pole if the open-circuit p.d. is 225 V.*

(*b*) *What is the residual flux ?*

Solution. *O.C.C.* has been drawn from the given data in Fig. 11-12.

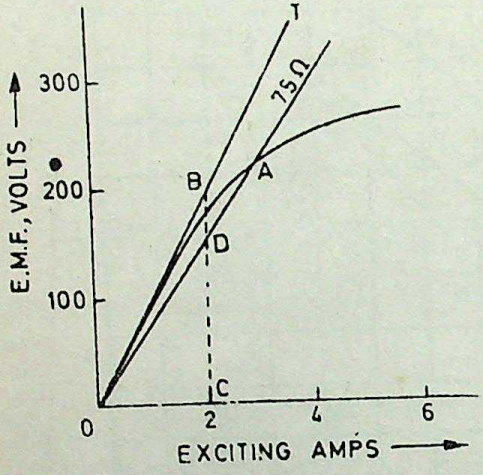

Fig. 11-12

OA is 75-Ω line and *OT* is tangential to the curve. Any line *BC* is drawn perpendicular to the base, cutting lines *OA* and *OT* at points *D* and *B* respectively.

(*a*) (*i*) $\frac{N_c}{N} = \frac{DC}{BC}$

or $\frac{N_c}{800} = \frac{150}{194}$

$\therefore \quad N_c = 800 \times \frac{150}{194}$

$= \mathbf{620\ r.p.m.}$

(*ii*) Using the relation

$$E = \frac{\Phi Z N}{60}\left(\frac{P}{A}\right) \text{ volts} \qquad \ldots(i)$$

we get $$225 = \frac{\Phi \times 576 \times 800}{60} \times \left(\frac{4}{4}\right) \qquad \therefore\ \Phi = \mathbf{29{\cdot}3\ mWb}$$

(*b*) With no exciting current, the e.m.f. induced due to residual flux is 10 V (as given in the table).

Hence, using again equation (*i*) above, we get

$$10 = \frac{\Phi \times 576 \times 800}{60} \times \left(\frac{4}{4}\right) \qquad \therefore\ \Phi = \mathbf{1{\cdot}3\ mWb}$$

Example 11-3. *The open-circuit characteristic of a d c. shunt generator running at 1000 r.p.m. is as follows :*

Field Current Amps :	*2*	*4*	*6*
Open Circuit Voltage :	*180*	*225*	*235*

Determine the voltage to which the machine builds up if the field circuit resistance is 50 Ω.

If the armature resistance is 0·1 Ω, calculate load current for a terminal voltage of 200 volts. Neglect the armature reaction.

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Solution. As shown in Fig. 11-13, the *O.C.C.* has been plotted from the given data. The shunt resistance line is drawn as usual. It cuts the *O.C.C.* at point *A*.

Hence, the voltage to which the machine will build up is **230 V.**

When terminal voltage is 200 V, then shunt current is

$$I_{sh}=200/50=4 \text{ A}$$

As seen from *O.C.C.*, with shunt field current of 4 A, the total e.m.f. induced in the armature (neglecting armature reaction) is 225 V. As armature reaction is negligible, the difference of $225-200=25$ V is the drop over the armature resistance alone.

$$\therefore \quad I_a R_a=25$$

or $$I_a=25/0{\cdot}1=250 \text{ A}$$

As load current $I=I_a-I_{sh}$

$$\therefore \quad I=250-4=\mathbf{246 \text{ A}}$$

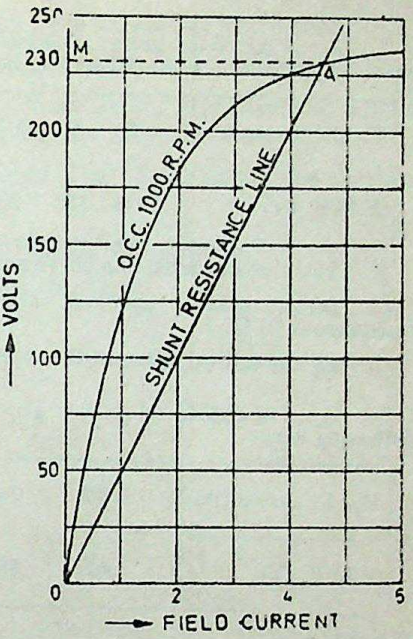

Fig. 11-13

Tutorial Problems-15

1. A d.c. shunt-wound generator has the following open-circuit magnetisation curve at its rated speed :

Field Current (A)	0·5	1·0	1·5	2·0	3·0	4·0
E.M.F. (V)	180	340	450	500	550	570

The resistance of the field circuit is 200 Ω and that of the armature circuit is 0·5 Ω. If the generator is driven at its rated speed, find the terminal voltage (*i*) on open-circuit (*ii*) when the armature current is 60 A.

[540 V ; 505 V]

2. A d.c. generator has the following magnetisation characteristic at 1200 r.p.m.

Field Current (A)	1	2	4	6	8	10
Generated E.M.F. (V)	192	312	468	566	626	660

If the generator is shunt-excited and driven at 1,000 r.p.m., determine

(*i*) The voltage to which it will excite on open-circuit

(*ii*) The approximate value of the critical resistance of the shunt field circuit

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(*iii*) The terminal p.d. and load current for a load resistance of 3 Ω.

Field resistance = 60 Ω ; armature resistance = 0·5 Ω

[(*i*) **540 V** (*ii*) **160 Ω** (*iii*) **426 V ; 142 A**]

3. A d.c. shunt generator which runs at a constant speed of 1200 r.p.m. has a magnetisation curve which can be plotted from the following data :

Field Current (A)	5	4	3·25	2·75	2·25	2·0	1·5	1·0
E.M.F. (V)	375	338	300	268	225	202	150	95

Plot the magnetisation curve and find :

(*a*) the no-load terminal voltage of the machine when the shunt field resistance is 90 Ω.

(*b*) the critical value of the shunt field resistance.

[(*a*) **315 V** (*b*) **100 Ω**]

4. The *O.C.C.* of a d.c. generator driven at 1000 r.p.m. is given by the following data :

Field Current (A)	0·2	0·4	0·6	0·8	1·0	1·2
E.M.F. (V)	46	88	126	152	165	173

Estimate :

(*a*) the critical field resistance when the generator is run as a shunt machine at 1200 r.p.m.

(*b*) the *O.C.* voltage at 1200 r.p.m. with a field resistance of 190 Ω.

(*c*) the resistance to be inserted in series with the shunt field to reduce the *O.C.* voltage to 180 V at 1200 r.p.m. [(*a*) **266 Ω** (*b*) **202 Ω** (*c*) **44 Ω**]

HIGHLIGHTS

1. The three main important characteristics of a d.c. generator are as follows :—

(*a*) No-load Saturation Characteristic (E_0/I_f). It is also known as Magnetic Characteristic or Open Circuit Characteristic. It is, in fact, magnetisation curve of the material of the electromagnets and is the same for all the generators.

(*b*) Internal or Total Characteristic (E/I_a).

It gives relation between the e.m.f. E *actually* induced in the armature and the armature current I_a.

(*c*) External Characteristic (V/I)

It gives relation between terminal voltage V and the load current I. It is also known as performance characteristic or voltage regulation curve.

2. The maximum possible resistance with which the generator would just excite, at a particular speed, is called the critical resistance of the generator at that particular speed.

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3. A series generator has got risi ng voltage characteristic *i.e.* its voltage rises with current.

4. Conditions for the build-up of a shunt generator are :

(*i*) there must be some residual magnetism in the poles

(*ii*) for the given direction of rotation, the shunt field coils should be correctly connected to the armature

(*iii*) its shunt resistance must be less than the critical resistance.

5. For a shunt generator, the voltage equation is

$$E_g = V + I_a R_a + \text{brush drop}$$

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D. C. MOTOR 12

1. Motor Principle 2. Comparison of Generator and Motor Action 3. Significance of Back E.M.F. 4. Voltage Equation of the Motor 5. Condition for Maximum Power 6. Torque 7. Armature Torque of a Motor. 8. Shaft Torque 9. Speed of D.C. Motor 10. Speed Regulation 11. Motor Characteristics 12. Characteristics of Series Motor 13. Characteristics of Shunt Motor 14. Characteristics of Compound Motor 15. Characteristics of Cummulative Compound Motors 16. Characteristics of Differential Compound Motors. 17. Comparison of Shunt and Series Motors 18. Losses and Efficiency 19. Power Stages 20. Highlights.

12-1. Motor Principle *(A.M.I.E. May 1972)*

An electric motor is a machine which converts electrical energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's left hand rule (Art. 7-16) and whose magnitude is given by

$$F = BIl \text{ newton} \qquad \text{(Art. 7-16)}$$

Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 12-1 is shown a part ot a multipolar d.c. motor. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under *N*-pole are assumed to carry current downwards (crosses) and those under *S*-poles to carry current upwards (dots). By applying Fleming's left-hand rule, the direction of the force on each conductor can be found. It is shown

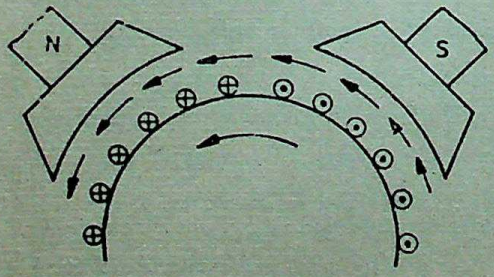

Fig. 12-1

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by small arrows placed above each conductor. It will be seen that each conductor experiences a force F which tends to rotate the armature in anti-clockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

It should be noted that the function of the commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

12-2. Comparison of Generator and Motor Action

As said above, the same d.c. machine can be used, at least theoretically, interchangeably as a generator and as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which, in turn, produces mechanical rotation.

Let us first consider its operation as a generator and see how exactly and through which agency, mechanical power is converted into electric power.

In Fig. 12-2 is shown part of a generator whose armature is being driven clockwise by its prime mover.

Fig. 12-2 (*a*) represents the fields set up independently by the main poles and the armature conductors like A in the figure. The

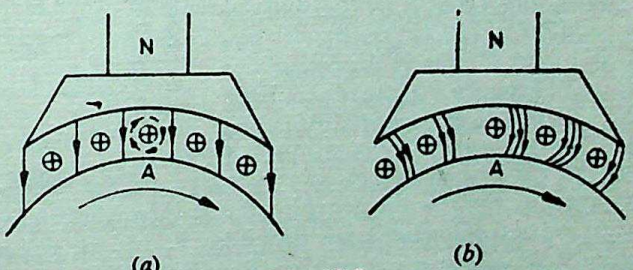

Fig. 12-2

resultant field or magnetic lines of force are shown in Fig. 12-2(*b*). It is seen that there is a crowding of lines of force on the right-hand side of A. These magnetic lines of force may be likened to the rubber bands under tension. Hence, the bent lines of force set up a mechanical force on A much in the same way as the bent elastic rubber band of a catapult produces a mechanical force on the stone piece. It will be seen that this force is in a direction opposite to that of armature rotation. Hence, it is known as backward force or magnetic drag on the conductors. It is against this drag acting on all armature conductors that the prime mover has to work. The work done in overcoming this opposition is converted into electric energy. Therefore, it should be clearly understood that it is only through the instrumentality of this magnetic drag that energy conversion is possible in a d.c. generator.

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Next, suppose that the above d.c. machine is uncoupled from its prime mover and that current is sent through the armature conductors under a *N*-pole in the downward direction as shown in Fig. 12-3. The conductors will again experience a force in the anti-clockwise direction (Fleming's Left-hand Rule). Hence, the machine will start rotating anti-clockwise, thereby developing a torque which can produce mechanical rotation. The machine is then said to be motoring.

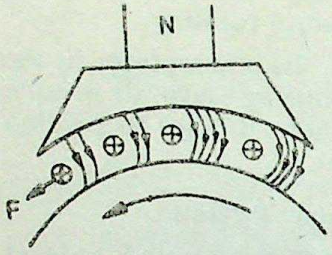


Fig. 12-3

As said above, energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In the case of a generator, it was the magnetic drag which provided the necessary opposition. But what is the equivalent of that in the case of a motor ? Well, it is the back e.m.f. It is explained in this manner :

As soon as the armature starts rotating, dynamically (or motionally) induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f. as found by Fleming's Right-hand Rule, is outward *i.e.* in direct opposition to the applied voltage (Fig. 12-4). That is why it is known as back e.m.f. E_b or counter e.m.f. Its value is the same as for the motionally induced e.m.f. in the generator ; $E_b = (\Phi ZN) \times (P/A)$ volts. The applied voltage V has to force current through the armature against this back e.m.f. E. The electric work done in overcoming this opposition is converted into mechanical energy developed in the armature. Therefore, it is obvious that but for the production of this opposing e.m.f., energy conversion would not have been possible.

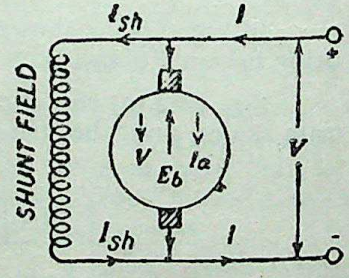
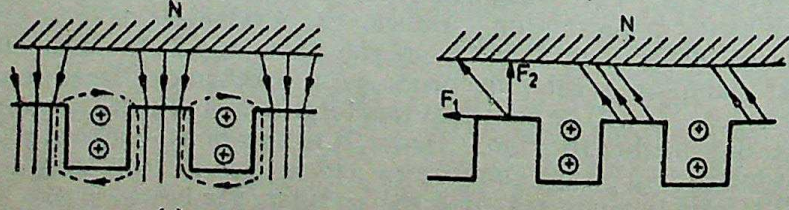


Fig. 12-4

Now, before leaving this topic, let it be pointed out that in an actual motor with slotted armature, the torque is not due to mechanical force on the conductors themselves, but due to tangential pull on the armature teeth as shown in Fig. 12-5.



(*a*) (*b*)

Fig. 12-5

It is seen in Fig. 12-5 (*a*) that the main flux is concentrated in the form of tufts at the armature teeth while the armature flux

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is shown by the dotted lines embracing the armature slots. The effect of armature flux on the main flux, as shown in Fig. 12-5 (*b*), is two-fold :

(*i*) It increases the flux in the left-hand side of the teeth and decreases it in the right-hand side, thus making the distribution of the flux density across the tooth section unequal.

(*ii*) It inclines the direction of lines of force in the air-gap so that they are not radial but are disposed in a manner shown in Fig. 12-5 (*b*). The pull exerted by the poles on the teeth can now be resolved into two components. One is the tangential component F_1 and the other vertical component F_2. The vertical component F_2, when considered for all the teeth round the armature, adds up to zero. But the component F_1 is not cancelled, and it is this tangential component which, acting on all the teeth, gives rise to the torque.

12-3. Significance of the Back E.M.F.

As explained in Art 12-2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming's Right-hand Rule, is in opposition to the applied voltage (Fig. 12-6). Because of its opposing direction, it is referred to as back e.m.f. E_b. The equivalent circuit of a motor is shown in Fig. 12-6 (*b*). The rotating armature generating the back e.m.f. E_b is like a battery of e.m.f. E_b put across a supply mains of V volts. Obviously, V has to drive I_a against the opposition of E_b. The power required to overcome this opposition is $E_b I_a$ watts.

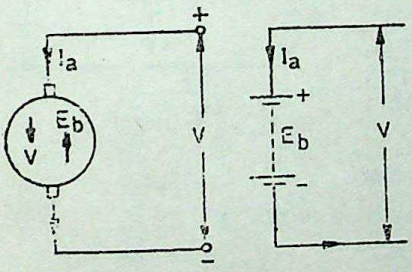

Fig. 12-6

In the case of a battery, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

It will be seen that $I_a = \dfrac{\text{net voltage}}{\text{resistance}} = \dfrac{V - E_b}{R_a}$ where R_a is the resistance of armature circuit.

As pointed out above

$$E_b = \Phi Z N \times (P/A) \text{ volts} \qquad \ldots N \text{ in r.p.s.}$$

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, E_b is large, hence armature current I_a, as seen from the above equation, is small. If the speed is less, then E_b is less, hence more current flows which develops more torque

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(Art. 12-7). So, we find that E_b acts like a governor *i.e.* it makes a motor self-regulating so that it draws as much current as is just necessary.

12-4. Voltage Equation of the Motor

The voltage V applied across the motor armature (Fig. 12-7) has to

(*i*) overcome the back e.m.f. E_b and

(*ii*) supply the armature ohmic drop I_aR_a

$\therefore \quad V=E_b+I_aR_a$

This is known as voltage equation of a motor.

Now, multiplying both sides by I_a, we get

$$VI_a=E_bI_a+I_a^2R_a$$

As shown in Fig. 12-7,

VI_a = electrical input to the armature.

E_bI_a = electrical equivalent of mechanical power P_m developed in the armature.

$I_a^2R_a$ = Cu loss in the armature.

Hence, out of the armature input, some is wasted as I^2R loss and the rest is converted into mechanical work within the armature.

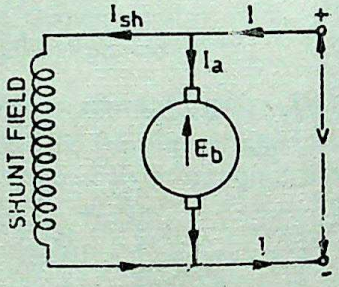

Fig. 12-7

12-5. Condition for Maximum Power

The mechanical power developed by a motor is

$$P_m=VI_a-I_a^2R_a$$

Differentiating both sides with respect to I_a, we get

$$dP_m/dI_a=V-2\,I_aR_a=0 \qquad \therefore \quad I_aR_a=V/2$$

As $V=E_b+I_aR_a$ and $I_aR_a=V/2$ $\therefore$ $E_b=V/2$

The mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 per cent.

Example 12-1. *A 230-V motor has an armature circuit resistance of 0·6 Ω. If the full load armature current is 30 A and the no-load armature current is 4 A, find the change in back e.m.f. from no-load to full-load.*

Solution. $E_b=V-I_aR_a$

At F.L. ; $E_b=230-(30\times0{\cdot}6)=212$ V

$E_{bo}=230-(4\times0{\cdot}6)=227{\cdot}6$ V

$\therefore$ **Change in back e.m.f.** $=227{\cdot}6-212=15{\cdot}6$ V

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Example 12-2. *A 440-V shunt motor has an armature resistance of 0·8 Ω and a field resistance of 200 Ω. Determine the back e.m.f. when giving an output of 7·46 kW at 85 per cent efficiency.*

Solution. Motor input power $= 7{\cdot}46 \times 1000/0{\cdot}85$ W

Motor input current $= 7460/0{\cdot}85 \times 440 = 19{\cdot}95$ A

$$I_{sh} = 440/200 = 2{\cdot}2 \text{ A}$$

$$\therefore \quad I_a = 19{\cdot}95 - 2{\cdot}2 = 17{\cdot}75 \text{ A}$$

Now $$E_b = V - I_a R_a = 440 - (17{\cdot}75 \times 0{\cdot}8) = \mathbf{425{\cdot}8\ V}$$

Example 12-3. *A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0·06 Ω and 100 Ω respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.*

Solution. As Generator (Fig. 12-8)

Output current $= 25{,}000/250 = 100$ A

$$I_{sh} = 250/100 = 2{\cdot}5 \text{ A} \; ; \; I_a = 102{\cdot}5 \text{ A}$$

Generated e.m.f. $= 250 + I_a R_a = 250 + 6{\cdot}15 = 256{\cdot}15$ V

Power developed in armature

$$= E_g I_a = \frac{256{\cdot}15 \times 102{\cdot}5}{1000} = \mathbf{26{\cdot}25\ kW}$$

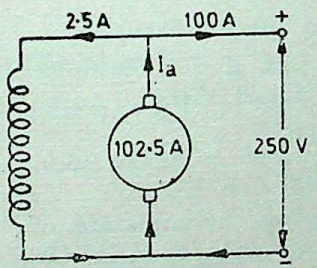

Fig. 12-8

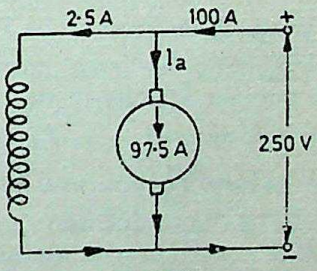

Fig. 12-9

As Motor (Fig. 12-9)

Motor input current $= 100$ A

$$I_{sh} = 2{\cdot}5 \text{ A} \; ; \; I_a = 97{\cdot}5 \text{ A}$$

$$E_b = 250 - (97{\cdot}5 \times 0{\cdot}06) = 250 - 5{\cdot}85 = 244{\cdot}15 \text{ V}$$

Power developed in armature $= E_b I_a$ watt

$$= \frac{244{\cdot}15 \times 97{\cdot}5}{1000} = \mathbf{23{\cdot}8\ kW}$$

Tutorial Problems—16

1. What do you understand by the term back e.m.f. ? A d.c. motor connected to a 460-V supply has an armature resistance of 0·15 Ω. Calculate

(*a*) the value of back e.m.f. when the armature current is 120 A.

(*b*) the value of armature current when the back e.m.f. is 447·4 V.

[(*a*) **442 V** (*b*) **84 A**]

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2. A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of 0·25 Ω, calculate the value of the back e.m.f. at this load. [430 V]

12-6. Torque

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

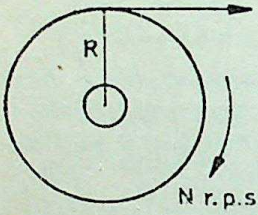

Fig. 12-10

Consider a pulley of radius R metres acted upon by a circumferential force F newton which causes it to rotate at N r.p.m. (Fig. 12-10).

Then torque $T = F \times R$ N-m

Work done by this force in one revolution

$= \text{Force} \times \text{distance} = F \times 2\pi R$ joules

Work done per second

$$W = F \times 2\pi R \times N \text{ joules/second}$$

$$= (F \times R) \times 2\pi N \text{ joules/second}$$

Now $2\pi N = \text{angular velocity } \omega \text{ in radian/second}$

and $F \times R = \text{torque } T$

$\therefore$ Work done/second $= T \times \omega$ joules/second

It is also obvious from above that if T is the torque in N-m and ω the angular velocity in radian/second, then

Power developed $= T \times \omega$ watt

12-7. Armature Torque of a Motor

Let T_a be the torque developed by the armature of a motor running at N r.p.s. *i.e.* with an angular velocity of $\omega = 2\pi N$ rad/s. If T_a is in N-m then,

Power developed $= T_a \times \omega = T_a \times 2\pi N$ watt ...(*i*)

We also know that electrical power converted into mechanical power in the armature is (Art. 12-4)

$$= E_b I_a \text{ watt} \quad \text{...(ii)}$$

Equating (*i*) and (*ii*), we get

$$T_a \times 2\pi N = E_b I_a \quad \text{...(iii)}$$

Since $E_b = \Phi Z N \times (P/A)$ volt

$\therefore$ $T_a \times 2\pi N = \Phi Z N\,(P/A).\, I_a$

or $$T_a = \frac{1}{2\pi}\, \Phi Z I_a\, (P/A) \text{ N-m}$$

$$= 0{\cdot}159\, \Phi Z I_a\, (P/A) \text{ N-m}$$

$$= (0{\cdot}159/9{\cdot}81)\, \Phi Z I_a\, (P/A) \text{ kg-m}$$

$$= 0{\cdot}0162\, \Phi Z I_a\, (P/A) \text{ kg-m}$$

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From the equation for the torque, we find that $T_a \propto \Phi I_a$

(*a*) In the case of series motor, Φ is directly proportional to I_a (before saturation) because field windings carry full armature current.

$$\therefore \quad T_a \propto I_a^2$$

(*b*) For shunt motors, Φ is practically constant, hence

$$T_a \propto I_a$$

It is also seen from Eq. (*iii*) above that

$$T_a = \frac{1}{2\pi} \cdot \frac{E_b I_a}{N} \text{ N-m}$$

$$= 0{\cdot}159 \frac{E_b I_a}{N} \text{ N-m}$$

$$= 0{\cdot}0162 \frac{E_b I_a}{N} \text{ kg-m}$$

12-8. Shaft Torque (T_{sh})

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of this is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque T_{sh}. It is so called because it is available at the shaft.

$$\text{Output in watts} = T_{sh} \times 2\pi N \quad \therefore \; T_{sh} = \frac{\text{Output in watts}}{2\pi N}$$

The difference $T_a - T_{sh}$ is known as lost torque.

$$\therefore \text{ Lost torque} = 0{\cdot}159 \times \frac{\text{iron and friction losses}}{N} \text{ N-m}$$

$$= 0{\cdot}0162 \times \frac{\text{iron and friction losses}}{N} \text{ kg-m}$$

Note. The value of back e.m.f. E_b can be found from

(*i*) the equation $E_b = V - I_a R_a$

(*ii*) the formula $E_b = \Phi ZN \times (P/A)$ volt

Example 12-4. *Determine the value of torque in N-m units established by the armature of a 4-pole motor having 774 conductors, two paths in parallel, 24 mWb per pole when the total armature current is 50 amperes.*

Solution. Formula used :

$$T_a = 0{\cdot}159 \, \Phi Z I_a \, (P/A) \text{ N-m}$$

Here $Z = 774$; $I_a = 50$ A

$\Phi = 24 \times 10^{-3}$ Wb, $P = 4$, $A = 2$

$$T_a = 0{\cdot}159 \times 24 \times 10^{-3} \times 774 \times 50 \times \frac{4}{2} \text{ N-m}$$

$$= 295{\cdot}3 \text{ N-m}$$

Example 12-5. *A cutting tool exerts a tangential force of 400 newtons on a steel bar of diameter 10 cm, which is being turned in a*

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simple lathe. The lathe is driven by chain at 840 r.p.m. from a 220 V, d.c. motor which runs at 1800 r.p.m.

Calculate the current taken by the motor if its efficiency is 80%. What size is the motor pulley if the lathe pulley has a diameter of 24 cm ?

Solution.

Torque $=$ Tangential force $\times$ radius

$=400 \times 0{\cdot}05 = 20$ N-m

Output power $= T_a \times 2\pi N$ watt

$= 20 \times 2\pi \times (840/60) = 1760$ W

Motor $\eta = 0{\cdot}8$

$\therefore$ Motor input $= 1760/0{\cdot}8 = 2{,}200$ W

Current drawn by motor $= 2{,}200/220 =$ **10 A**

Let N_1 and D_1 be the speed and diameter of the driver pulley respectively and N_2 and D_2 the respective speed and diameter of the lathe pulley.

Then $N_1 \times D_1 = N_2 \times D_2$

$\therefore$ $1800 \times D_1 = 840 \times 0{\cdot}24$

$\therefore$ $D_1 = 840 \times 0{\cdot}24/1800 = 0{\cdot}112$ m $=$ **11·2 cm**

12-9. Speed of a D.C. Motor

From the voltage equation of a motor (Art. 12-4), we get

$$E_b = V - I_a R_a$$

or $$\Phi Z N (P/A) = V - I_a R_a$$

$\therefore$ $$N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{A}{ZP}\right) \text{ r.p.s.}$$

Now $$V - I_a R_a = E_b$$

$\therefore$ $$N = \frac{E_b}{\Phi} \times \left(\frac{A}{ZP}\right) \text{ r.p.s. or } N = \frac{kE_b}{\Phi}$$

It shows that speed is directly proportional to back e.m.f. E_b and inversely to the flux Φ.

or $$N \propto \frac{E_b}{\Phi}$$

For Series Motor

Let N_1 = speed in the 1st case

I_{a1} = armature current in the 1st case

Φ_1 = flux/pole in the 1st case

N_2, I_{a2}, Φ_2 = corresponding values in the second case.

Then, using the above relation, we get

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$$N_1 \propto \frac{E_{b1}}{\Phi_1} \qquad \text{where} \quad E_{b1}=V-I_{a1}R_a$$

$$N_2 \propto \frac{E_{b2}}{\Phi_2} \qquad \text{where} \quad E_{b2}=V-I_{a2}R_a$$

$$\therefore \quad \frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}\times\frac{\Phi_1}{\Phi_2}$$

Prior to saturation of magnetic poles, $\Phi \propto I_a$

$$\therefore \quad \frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}\times\frac{I_{a1}}{I_{a2}}$$

For Shunt Motor

In this case, the same equation applies,

$$\frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}\times\frac{\Phi_1}{\Phi_2}$$

If $\Phi_2=\Phi_1$, then $\frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}$

12-10. Speed Regulation

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed controlling devices.

The speed regulation is defined as *the change in speed when the load on the motor is reduced from rated value to zero, expressed as per cent of the rated load speed.*

$$\therefore \quad \%\ \text{speed regulation} = \frac{N.L.\ \text{speed}-F.L.\ \text{speed}}{F.L.\ \text{speed}}\times 100$$

$$=\frac{N_0-N}{N}\times 100=\frac{dN}{N}\times 100$$

Example 12-6. *A 6-pole, lap-wound shunt motor has 500 conductors in the armature. The resistance of the armature path is 0·05 Ω. The resistance of the shunt field is 25 Ω. Find the speed of the motor when it takes 120A from a d.c. main of 100V supply. Flux per pole =20 mWb.* (A.M.I.E. Winter, 1978)

Solution. $N=\frac{E_b}{\Phi}\left(\frac{A}{ZP}\right)$ r.p.s.

$A=6, P=6, Z=500$; $E_b=V-I_aR_a$

Now, $I=120\ A$ (given) ; $I_{sh}=100/25=4A$

$I_a=120-4=116A$

$\therefore \quad E_b=100-(116\times 0{\cdot}05)=94{\cdot}2V$

$\therefore \quad N=\frac{94{\cdot}2}{20\times 10^{-3}}\left(\frac{6}{500\times 6}\right)=9{\cdot}42$ r.p.s. $=565$ r.p.m.

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Example 12-7. *A d.c. shunt generator delivers 50 kW at 250 V when running at 400 r.p.m. The armature and field resistances are 0·02 Ω and 50 Ω respectively. Calculate the speed of the same machine when running as a shunt motor and taking 50 kW input at 250 V. Allow 1 volt per brush for contact drop.*

Solution. **As Generator** (Fig. 12-11)

$$I_{sh}=250/50=5 \text{ A}$$

$$\therefore \quad I_a=I+I_{sh}=200+5=205 \text{ A}$$

$$I_aR_a=205\times 0{\cdot}02=4{\cdot}1 \text{ V}$$

$$\text{Brush drop} \quad =2\times 1=2 \text{ V}$$

$$\therefore \quad E_{b1}=250+4{\cdot}1+2=256{\cdot}1 \text{ V}$$

$$N_1=400 \text{ r.p.m.}$$

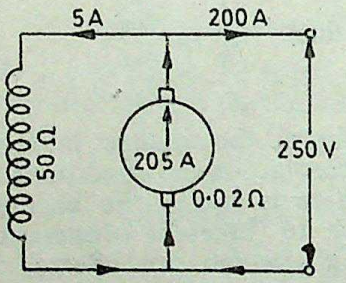

Fig. 12-11

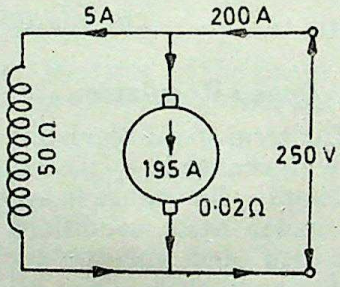

Fig. 12-12

As Motor (Fig. 12-12)

$$E_{b2}=V-I_aR_a\text{—brush drop}$$

$$=250-(195\times 0{\cdot}02)-2=244{\cdot}1 \text{ V}$$

$$\frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}\times\frac{\Phi_1}{\Phi_2}$$

$$\therefore \quad \frac{N_2}{400}=\frac{244{\cdot}1}{256{\cdot}1} \qquad (\because \quad \Phi_1=\Phi_2)$$

$$\therefore \quad N_2=400\times 244{\cdot}1/256{\cdot}1=\mathbf{381{\cdot}4 \text{ r.p.m.}}$$

Example 12-8. *A 4-pole, 250-V, 7·46 kW d.c. shunt motor with a full-load efficiency of 85 per cent has 382 wave-connected armature conductors. The shunt field resistance is 125 Ω and the armature resistance is 1 Ω. If the motor takes a current of 4 A from mains on no load and the flux per pole is 20 mWb, calculate its no load speed.*

At what speed will the motor run on full load?

Solution. As seen from Art. 12-9

$$N=\frac{E_b}{\Phi}\left(\frac{A}{ZP}\right) \text{ r.p.s.}$$

Now $\quad I_{sh}=250/125=2 \text{ A}; \quad I_a=4-2=2 \text{ A}$

$$\therefore \quad E_{b0}=V-I_{a0}R_a=250-(2\times 1)=248 \text{ V}$$

$$\Phi=20 \text{ mWb}=20\times 10^{-3} \text{ Wb}$$

$$Z=382; \quad A=2; \quad P=4$$

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$\therefore$ No-load speed $N_0 = \frac{248}{20\times10^{-3}}\left(\frac{2}{382\times4}\right) = 16{\cdot}23$ r.p.s.

$= 16{\cdot}23\times60 = \mathbf{973{\cdot}8}$ **r.p.m.**

Full-load motor input current

$I = 7{\cdot}46\times1000/0{\cdot}85\times250 = 35{\cdot}1$ A

$I_{sh} = 2$ A (same as before)

Full-load $I_a = 35{\cdot}1 - 2 = 33{\cdot}1$ A

$E_b = 250 - (33{\cdot}1\times1) = 216{\cdot}9$ V

Now $\frac{N}{N_0} = \frac{E_b}{E_{b0}}$

$N = 973{\cdot}8\times216{\cdot}9/248 = \mathbf{851{\cdot}7}$ **r.p.m.**

Example 12-9. *A 250-volt series motor has an armature resistance of 0·08 Ω and field resistance of 0·02 Ω and produces full-load torque when running at 500 r.p.m. taking a current of 40 A.*

Calculate (a) armatu.e current (b) the speed when producing ½ full-load torque. Neglect effect of armature reaction and saturation.

Solution. (*a*) Since saturation is neglected

$T \propto \Phi I_a \propto I_a^2$

$T_{a1} \propto I_{a1}^2 \propto 40^2$ and $T_{a2} \propto I_{a2}^2$

$T_{a2}/T_{a1} = I_{a2}^2/40^2$

Now $T_{a2}/T_{a1} = 1/2$ or $I_{a2}^2/40^2 = 1/2$

or $I_{a2} = 40/\sqrt{2} = \mathbf{28{\cdot}28}$ **A**

(*b*) $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}\times\frac{I_{a1}}{I_{a2}}$

$E_{b1} = 250 - (40\times0{\cdot}1) = 246$ V

$E_{b2} = 250 - (28{\cdot}28\times0{\cdot}1) = 247{\cdot}2$ V

$\therefore \quad \frac{N_2}{500} = \frac{247{\cdot}2}{246}\times\frac{40}{28{\cdot}28} \qquad \therefore \quad N_2 = \mathbf{711}$ **r.p.m.**

Tutorial Problems—16

1. A d.c. machine has the following details :

Poles = 4 ; flux per pole = 58·7 mWb :

Armature winding = wave ; No. of slots = 48

Conductors per slot = 4 ; Armature resistance = 0·2 Ω

When the machine was connected to a 400-V supply, it ran as a motor at a speed of 1050 r.p.m.

Calculate the value of back e.m.f. and hence find the value of the gross torque developed by the motor. **[395 V ; 90 N-m]**

2. The armature of a 4-pole, d.c. shunt motor has a lap-connected armature winding with 740 conductors. The no-load flux per pole is 30 mWb. If the armature current is 40 A and the effect of armature reaction at this current is to reduce the air-gap flux per pole by 4%, determine the torque developed. **[135·7 N-m]**

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3. A shunt machine has an armature resistance of 0·05 Ω and a field resistance of 50 Ω. When connected across a 200-V supply, it runs as a motor at 500 r. p. m. and takes a total input of 100 kW. At what speed must the machine be driven as a generator to supply an output of 100 kW at a terminal voltage of 200 V? **[642 r.p.m.]**

4. A shunt motor supplied at 460 V runs light at 750 r.p.m. If it is then connected to a 230-V supply, at what speed will it run; assuming that flux is proportional to field current. **[750 r.p.m.]**

5. A 37·3 kW, 500-V shunt motor has a full-load efficiency of 84%; it has six poles, the flux per pole is 40 mWb; the armarure is wave-wound and has 492 conductors; the resistance of the armature circuit is 0·12 Ω and the resistance of the shunt field is 250 Ω.

Calculate (*i*) full-load speed (*ii*) the useful-load torque.

[(*i*) 497 r.p.m. (*ii*) 719·2 N-m]

6. A d.c. shunt-excited machine runs at 750 r.p.m. and supplies a load of 45 kW to 225 V bus-bars.

Determine the speed at which the machine will run as a shunt motor when taking 36 kW from the 225-V bus-bars.

Field resistance = 45 Ω; Armature resistance = 0·1 Ω. Neglect armature reaction effect. **[640 r.p.m.]**

7. The armature of a 6-pole, d.c. motor is (*i*) wave wound (*ii*) lap wound. If the armature has 61 slots with 8 conductors per slot, calculate the torque developed in the armature for both the above types of winding when the input current to the armature is 840 A and the flux per pole is 35 mWb.

[(*i*) 1140 N-m (*ii*) 380 N-m]

12-11. Motor Characteristics

The characteristic curves of a motor are those curves which show relation between the following quantities :

(1) **Torque and armature current** *i.e.* T_a/I_a characteristic. This is also known as *electrical characteristic.*

(2) **Speed and armature current** *i.e.* N/I_a characteristic.

(3) **Speed and torque** *i.e.* N/T_a characteristic. This is also known as *mechanical characteristic.* This can be found from (1) and (2) above.

While discussing motor characteristics, the following relations should always be kept in mind.

$$(i)\ T_a \propto \Phi I_a \quad (ii)\ I_a = \frac{V - E_b}{R_a} \text{ and } \quad (iii)\ N \propto \frac{E_b}{\Phi}$$

12-12. Characteristics of Series Motors (*A.M.I.E. Summer 1984*)

(1) **T_a/I_a characteristic.** We have seen that $T_a \propto \Phi I_a$. In this case, as field windings also carry the armature current, hence, $\Phi \propto I_a$ upto the point of magnetic saturation. Hence, before saturation,

$$T_a \propto \Phi I_a \quad \text{or} \quad T_a \propto I_a^2$$

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At light loads, I_a and hence Φ is small. But as I_a increases, T_a increases as the square of the current. Hence, T_a/I_a curve is a parabola as shown in Fig. 12-13. After saturation, Φ is almost indipendent of I_a, hence $T_a \propto I_a$ only. So the characteristic becomes a straight line. The shaft torque T_{sh} is less than armature torque due to stray losses. It is shown dotted in the figure. So we conclude that (prior to magnetic saturation) on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in case where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc, series motors are used.

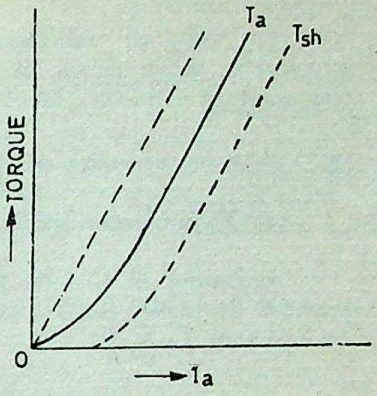

Fig. 12-13

(2) N/I_a Characteristic : Variations of speed can be deduced from the formula :

$$N \propto \frac{E_b}{\Phi}$$

Change in E_b for various load currents is small and hence may be neglected for the time being. With increased I_a, Φ also increases. Hence, speed varies inversely as armature current as shown in Fig. 12-14.

When load is heavy, I_a is large, Hence, speed is low (this decreases E_b and allows more armature current to flow). But when

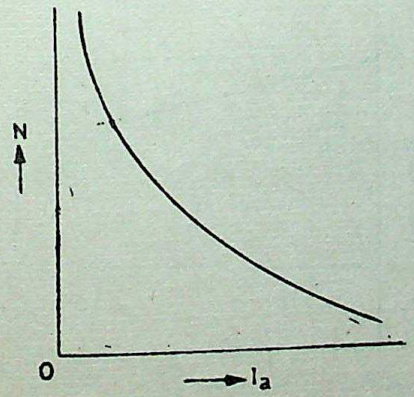

Fig. 12-14

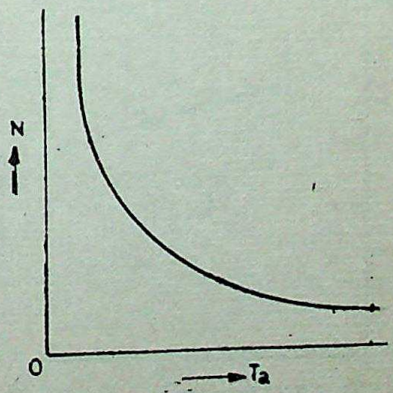

Fig. 12-15

load and hence I_a falls to a small value, speed becomes dangerously high. Hence, a series motor should never be started without some mechanical (not belt-driven) load on it, otherwise it may develop

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excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

(3) N/T_a or **machanical characteristic** : It is found from above that when speed is high, torque is small and *vice-versa.* The relation between the two is as shown in Fig. 12-15.

12-13. Characteristics of Shunt Motors

(1) T_a/I_a Characteristic.

Assuming Φ to be practically constant (though at heavy loads, Φ decreases somewhat due to increased armature reaction) we find that $T_a \propto \Phi$

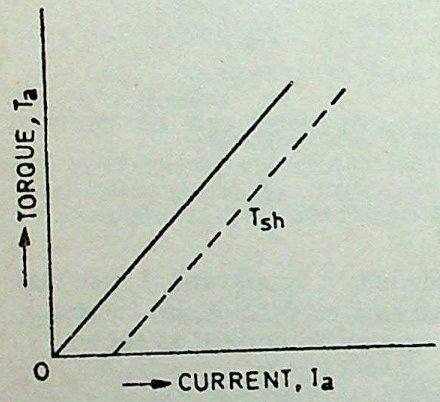

Fig. 12-16

Hence, the electrical characteristic as shown in Fig. 12-16 is practically a straight line through the origin. Shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on (heavy) load.

(2) N/I_a Characteristic

If Φ is assumed constant, then $N \propto E_b$. As E_b is also practically constant, speed, for most purposes, is constant (Fig. 12-17).

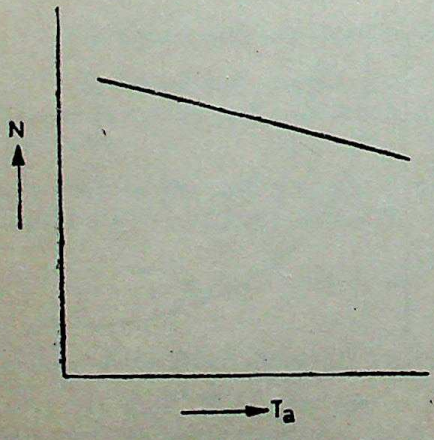

Fig. 12-17

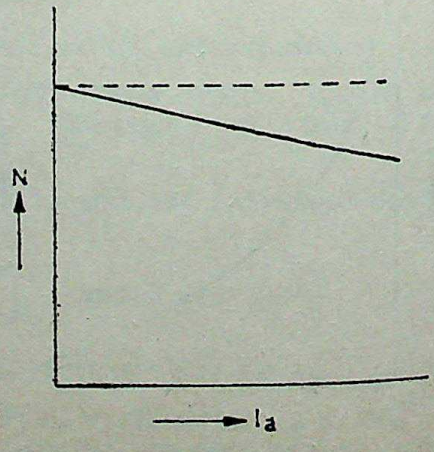

Fig. 12-18

But strictly speaking, both E_b and Φ decrease with increasing load. However, E_b decreases slightly more than Φ hence, on the

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whole, there is some decrease in speed. Hence, the actual speed curve is slightly drooping as shown by the dotted line in Fig. 12-17. But for all practical purposes, shunt motor is taken as a constant-speed motor.

Because there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

(3) **N/T_a characteristic** can be deduced from (1) and (2) above and is shown in Fig. 12-18.

12-14. Compound Motors

In such motors, a part of the field winding is in parallel with the armature and the other part is in series with it. Compound motors are of two types :

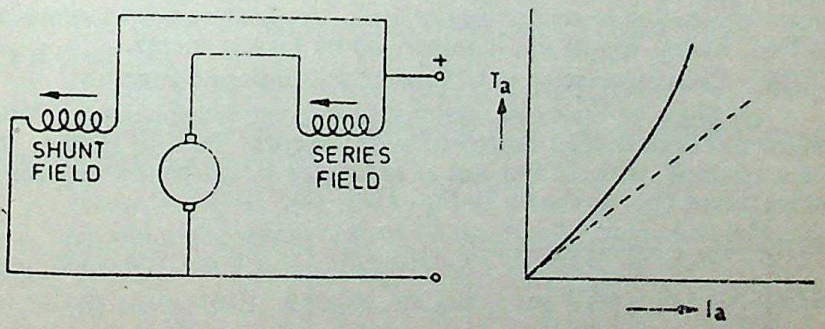
(*i*) *Cumulative compound motors* : In their case, series field winding helps or aids the shunt winding as shown in Fig. 12-19 (*a*).

(*ii*) *Differential compound motors*. In their case, the fluxes of the two windings oppose each other as shown in Fig. 12-21 (*a*).

12-15. Characteristics of Cumulative Compound Motors

(*A.M.I.E. Summer 1975*)

(*i*) It is seen from Fig. 12-19(*b*) that as armature current increases, series flux increases thereby increasing the total flux of the



(*a*) (*b*)

Fig. 12-19

motor. Consequently, torque is increased. However, one point worth noting is that this increase in T_a with I_a is greater than it would be for shunt motor alone. Obviously, such motors develop a high torque with sudden increase in load.

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(*ii*) As mentioned above, as I_a increases, the series flux and hence total motor flux increases. This leads to a decrease in motor speed which has a definite value N_0 at no-load Again, it should be noted that decrease in speed is more rapid than would be the case for a shunt motor only (or the decrease is less rapid, than would be the case for series motor only). Variation of N with I_a is shown in Fig. 12-20 (*a*).

(*iii*) Since in such motors, series excitation helps shunt excitation, their mechanical characteristics lie in between those of shunt and series motors as shown in Fig. 12-20 (*b*).

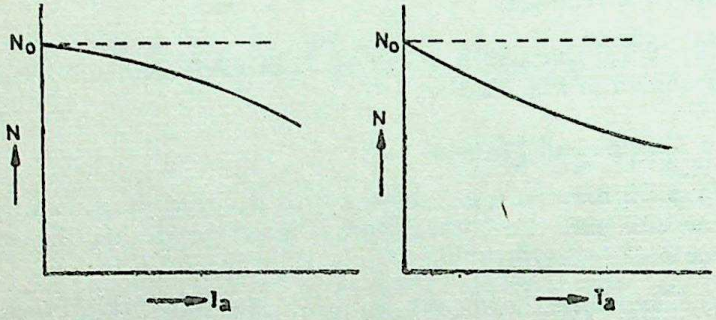

Fig. 12-20

Such machines are used where series characteristics are required and where, in addition, the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to take sudden deep cuts quite often. Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy load. In conjunction with fly-wheel (functioning as load equalizer), it is employed where there are sudden temporary loads as in rolling mills. The fly-wheel supplies its stored kinetic energy when motor slows down due to sudden heavy load. And when due to removal of load, motor speeds up, it gathers up its kinetic energy.

12-16. Characteristics of Differential Compound Motors

(*i*) Since in their case, series field opposes the shunt field, total motor flux is somewhat decreased with increase in I_a. Hence, T_a does increase with I_a but not as rapidly as would be the case for a series motor only as shown in Fig. 12-21 (*b*).

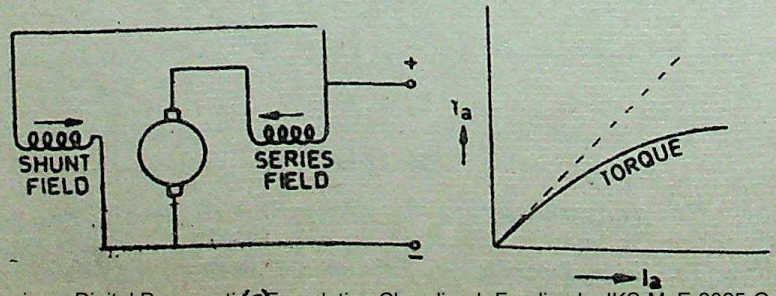

Fig. 12-21

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(*ii*) Because of overall decrease in motor flux with increase in I_a, the motor speed increases slightly as shown in Fig. 12-22 (*a*).

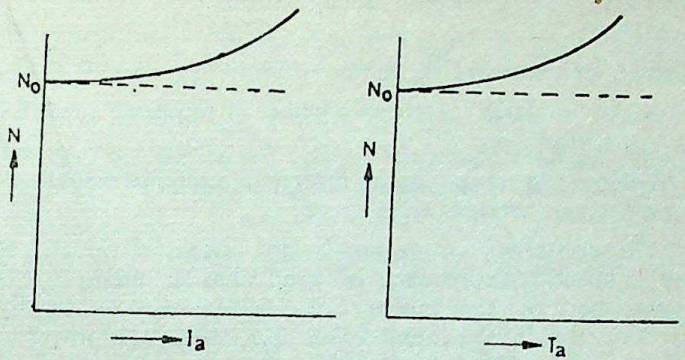

Fig. 12-22

(*iii*) From the mechanical characteristic shown in Fig. 12-22(*b*), it is seen that motor torque increases very slightly with increase in speed. However, in practice, since speed of such a motor remains practically constant, its torque is also almost constant.

Such motors are rarely used in practice because

(*i*) due to weakening of the field with increase in I_a, there is a tendency towards speed instability with a consequent possibility of motor running away.

(*ii*) unless the series field is short-circuited during starting, there is a strong possibility of the motor starting in the wrong direction. This happens when the starting current in the series field is so large that series ampere-turns overbalance the shunt field turns.

12-17. Comparison of Shunt and Series Motors

1, Shunt Motors

The different characteristics have been discussed in Art. 12-13. It is clear that

(*a*) speed of a shunt motor is sufficiently constant.

(*b*) for the same current input, its starting torque is not as high as that of a series motor.

Hence it is used

(*i*) when the speed has to be maintained approximately constant from *N.L.* to *F.L.* *i.e.* for driving a line of shafting etc.

(*ii*) when it is required to drive the load at various speeds, any one speed being kept constant for a relatively long period *i.e.* for individual driving of such machines as lathes. The shunt regulator enables the required speed control to be obtained easily and economically.

2. Series Motors

The operating characteristics have been discussed in Art. 12-12. These motors

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(*a*) have a relatively huge starting torque.

(*b*) have low speed at high loads and dangerously high speed at low loads.

Hence, such motors are used

(*i*) when a large starting torque is required *i.e.* for driving hoists, cranes, trams etc.

(*ii*) when the motor can be directly coupled to a load such as a fan whose torque increases with speed.

(*iii*) if constancy of speed is not essential, then, in fact, the decrease of speed with increase of load, has the advantage that the power absorbed by the motor does not increase as rapidly as the torque. For instance, when torque is doubled, the power approximately increases by about 50 to 60% only. $(\because I_a \propto \sqrt{T_a})$

(*iv*) A series motor should not be used where there is a possibility of the load decreasing to a very small value. Thus it should not be used for driving centrifugal pumps or for a belt drive of any kind.

Summary of Applications

Type of Motor	*Characteristics*	*Applications*
Shunt	Approximately constant speed Adjustable speed Medium starting torque (up to 1·5 F.L. torque)	For driving constant-speed line shafting Lathes Centrifugal pumps Machine tools Blowers and fans Reciprocating pumps
Series	Variable speed Adjustable varying speed High starting torque	For traction work *i.e.* Electric Locomotives Rapid Transit systems Trolley Cars etc. Cranes and hoists Conveyors
Cumulative Compound	Variable speed Adjustable varying speed High starting torque	For intermittent high torque loads For shears and punches Elevators Conveyors Heavy planers Rolling mills

12-18. Losses and Efficiency

The losses taking place in the motor are the same as in generators (Art. 10-16). These are (*i*) Copper losses (*ii*) Magnetic losses and (*iii*) Mechanical losses.

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The condition for maximum *power* developed by the motor is

$$I_a R_a = V/2 = E_b \qquad \text{(Art. 12-3)}$$

The condition for maximum *efficiency* is that armature Cu losses should be equal to constant losses (Art. 10-20).

12-19. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown diagrammatically in Fig. 12-23.

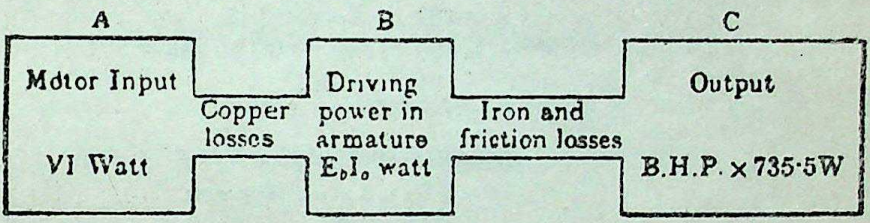

Fig. 12-23

Overall or commercial efficiency $\eta_c = \frac{C}{A}$

Electrical efficiency $\eta_e = \frac{B}{A}$

Mechanical efficiency $\eta_m = \frac{C}{B}$

The efficiency curve for a motor is similar in shape to that for a generator (Art. 10-25).

Example 12-10. *A 250-volt, d.c. shunt motor takes 4A at rated voltage on no-load. Armature and field resistances are 0·5 Ω and 250 Ω respectively. Calculate the efficiency of the motor for a load current of 40 A.* (*A.M.I.E. Nov. 1969*)

Solution. No-load motor input

$$= 250 \times 4 = 1000 \text{ W}$$

This input goes to meet no-load losses which consist of (*i*) armature Cu loss and (*ii*) constant losses. This no-load data can be used to find the constant losses of the motor.

$$I_{sh} = 250/250 = 1 \text{ A} \; ; \; I_a = 4 - 1 = 3 \text{ A}$$

Armature Cu loss $= I_a^2 R_a = 3^2 \times 0{\cdot}5 = 4{\cdot}5$ W

Constant losses of motor

$$= 1000 - 4{\cdot}5 = 995{\cdot}5 \text{ W}$$

With load current of 40 A

$$I_a = 40 - 1 = 39 \text{ A}$$

Armature Cu loss $= 39^2 \times 0{\cdot}5 = 760{\cdot}5$ W

Motor input $= 250 \times 40 = 10{,}000$ W

Total losses $= 760{\cdot}5 + 995{\cdot}5 = 1{,}756$ W

Motor output $= 10{,}000 - 1{,}756 = 8{,}244$ W

Motor $\eta = 8{,}244/10{,}000 = 0{\cdot}8244$ or 82·44%

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Example 12-11. *A 4-pole, 500-V, 74·6-kW wave-connected motor has 492 conductors. The flux per pole is 50 mWb and full-load efficiency is 92%. Its shunt field resistance is 250 Ω and armature circuit resistance is 0·1 Ω. Calculate for full-load (a) motor and (b) the useful torque developed.*

Solution. Motor input $=74{\cdot}6\times1000/0{\cdot}92$

$$\text{Motor input current} = \frac{74{,}600}{0{\cdot}92\times500} = 162{\cdot}1 \text{ A}$$

$$I_{sh} = 500/250 = 2 \text{ A}$$

$$I_a = 162{\cdot}1 - 2 = 160{\cdot}1 \text{ A}$$

$$E_b = 500 - (160{\cdot}1\times0{\cdot}1) = 484 \text{ V}$$

(a) $$E_b = \Phi ZN\times\left(\frac{P}{A}\right) \text{ volt}$$

$\therefore$ $$484 = 50\times10^{-3}\times492\times N\times(4/2)$$

$\therefore$ $$N = 9{\cdot}83 \text{ r.p.s.} = \mathbf{590 \text{ r.p.m.}}$$

(b) $$I_{sh} = \frac{746\times100}{2\pi\times9{\cdot}83} = \mathbf{1207 \text{ N-m}}$$

Tutorial Problems—17

1. A 7·46-kW, 230-V, shunt motor has a full-load speed of 1200 r.p.m. The resistances of the armature and the field circuit are 0·3 Ω and 115 Ω respectively. The full-load efficiency of the motor is 85%. If the total input to the motor is 620 W when it runs light, determine the no-load speed of the motor.

Neglect brush contact drop and the effect of the armature reaction. **[1260 r.p.m.]**

2. A d.c. shunt motor has the following particulars :

8·952 kW output ; 440 V, armature resistance = 1·5 Ω, brush contact drop = 2 V ; shunt resistance = 700 Ω. Iron and friction losses on full-load = 450 W. Calculate the efficiency when taking the full-load current of 24 A. **[85%]**

3. A d.c. series motor on full-load takes 50 A from 230-V d.c. mains. The total resistance of the motor is 0·22 Ω. If the iron and friction losses together amount to 5% of the input, calculate the output furnished by the motor. Total voltage drop due to brush contact is 2 V. **[10·26 kW]**

4. A 250-V shunt motor takes a current of 80 A and runs at 850 r.p.m. The resistance of the shunt field is 125 Ω and the armature circuit resistance including the brushes is 0·1 Ω. If the iron and friction losses are 1·5 kW, calculate (*i*) the output (*ii*) shaft torque (*iii*) the efficiency.
[(*i*) 17·35 kW (*ii*) 195·4 N-m (*iii*) 86·95%]

5. A 250-V, d.c. compound motor, connected long-shunt. with its series field directly in series with the armature, takes an armature current of 80 A at full-load. Estimate the efficiency at full-load, given

Armature resistance = 0·09 Ω
Shunt field resistance = 125 Ω
Series field resistance = 0·04 Ω
Friction and iron loss at full-load = 750 W **[89·8%]**

6. A 2-pole d.c. shunt motor operating from a 200-V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500 Ω and the armature has a resistance of 0·6 Ω. Calculate the full-load efficiency of the motor. Take the brush drop as being equal to 1·5 V per brush arm. Neglect temperature rise. **[82·6%]**

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HIGHLIGHTS

1. Armature current of a d.c. motor is given by

$$I_a = \frac{V - E_b}{R_a}$$

2. Voltage equation of the motor is

$$V = E_b + I_a R_a + \text{brush drop}$$

3. Condition for maximum power developed in armature is

$$E_b = V/2$$

4. Armature torque of a motor is given by

$$T_a = 0{\cdot}159\ \Phi Z I_a\ (P/A)\ \text{N-m}$$
$$= 0{\cdot}0162\ \Phi Z I_a\ (P/A)\ \text{kg-m}$$

Also $$T_a = 0{\cdot}159 \frac{E_b I_a}{N}\ \text{N-m}$$

$$= 0{\cdot}0162 \frac{E_b I_a}{N}\ \text{kg-m}$$

5. Shaft or useful torque is given by

$$T_{sh} = \frac{\text{output in watts}}{2\pi N}\ \text{N-m}$$

6. Lost torque which is the difference of the armature and shaft torque is given by

$$\text{Lost torque} = 0{\cdot}159 \frac{\text{Iron and friction losses}}{N}\ \text{N-m}$$

$$= 0{\cdot}0162 \frac{\text{Iron and friction losses}}{N}\ \text{kg-m}$$

7. Speed of a d.c. motor is given by

$$N = \frac{E_b}{\Phi}\left(\frac{A}{ZP}\right)\ \text{r.p.s.}$$

8. Speed regulation of a motor is given by

$$\%\ \text{speed regn.} = \frac{N.L.\ \text{speed} - F.L.\ \text{speed}}{F.L.\ \text{speed}} \times 100$$

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SPEED CONTROL OF D.C. MOTORS 13

1. Factors Controlling the Speed 2. Speed Control of Shunt Motors 3. Speed Control of Series Motors 4. Motor Starters—their Necessity 5. Shunt Motor Starter with Protective Devices 6. Merits and Demerits of Rheostatic Control Method 7. Advantages of Field Control Method 8. Highlights.

13-1. Factors Controlling the Speed

It has been shown earlier that the speed of a motor is given by the relation

$$N = \frac{V - I_a R_a}{\Phi} \times \frac{60\,A}{ZP} = K\,\frac{V - I_a R_a}{\Phi} \text{ r.p.m}$$

where R_a = armature circuit resistance.

It is obvious that the speed can be controlled by varying (*i*) flux per pole, Φ (flux control) (*ii*) resistance R_a of armature circuit (rheostatic control). These methods as applied to shunt and series motors will be discussed below.

13-2. Speed Control of Shunt Motors

(i) Variation of Flux or Flux Control Method

It is seen from above that $N \propto 1/\Phi$ By decreasing the flux, the speed can be increased and *vice versa* Hence, the name *flux* or *field control* method. The flux of a d.c. motor can be changed by changing I_{sh} with the help of a shunt field rheostat (Fig. 13-1). Since I_{sh} is relatively small, shunt field rheostat has to carry only a small current, which means I^2R loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpolar machines, the speed can be increased by this method in the ratio 2 : 1. Any further weakening of flux Φ adversely affects the commutation and

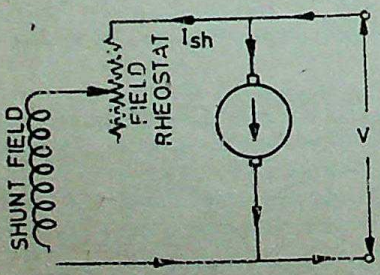

Fig. 13-1

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hence puts a limit to the maximum speed obtainable with this method. In machines fitted with interpoles, a ratio of maximum to minimum speeds of 6 : 1 is fairly common.

Example 13-1. *Explain, in detail, what happens when the flux is reduced by 10% in a 200-V, d.c. shunt motor, having an armature resistance of 0·2 Ω, carrying a current of 50 A and running at 960 r.p.m. prior to weakening the field. The total torque may be assumed to remain constant, iron and friction losses may be neglected.*

Solution. By decreasing the flux, speed can be increased as shown by the following relation

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

Now $T_a \propto \Phi I_a$

Since torque is the same in both cases,

$$\Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \text{or} \quad \Phi_1 \times 50 = 0{\cdot}9\ \Phi_1 \times I_{a2}$$

$$\therefore \quad I_{a2} = 50/0{\cdot}9 = 55{\cdot}55 \text{ A} ; \quad E_{b1} = 200 - (0{\cdot}2 \times 50) = 190 \text{ V}$$

$$E_{a2} = 200 - (0{\cdot}2 \times 55{\cdot}55) = 188{\cdot}9 \text{ V} ; \quad \Phi_2 = 0{\cdot}9\ \Phi_1 ; \quad N_1 = 960 \text{ r.p.m}$$

$$\frac{N_2}{960} = \frac{188{\cdot}9}{190} \times \frac{\Phi_1}{0{\cdot}9\ \Phi_1}, \quad \therefore \quad N_2 = 1058 \text{ r.p.m.}$$

Tutorial Problems—18

1. A 250-V d.c. shunt motor has an armature circuit resistance of 0·5 Ω and a field circuit resistance of 125 Ω. It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to 150 Ω. If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current.
[1186 r.p.m. ; 33·6 A]

2. A 250-V, shunt motor with an armature resistance of 0·5 Ω and a shunt field resistance of 250 Ω drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be effected in the shunt field resistance ? Assume that the magnetisation curve of the motor is a straight line. [88 Ω]

(*ii*) Armature or Rheostatic Control Method

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 13-2. As controller resistance is increased, p.d. across the armature is decreased. thereby decreasing the armature speed. For a load of constant torque, speed is approximately proportional to the p.d. across the armature. From the speed *Vs* armature current characteristic

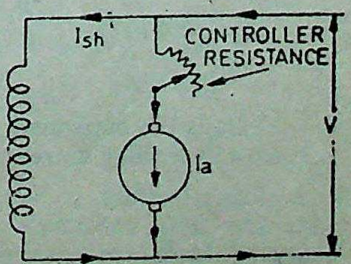

Fig. 13-2

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[Fig. 13-3] it is seen that greater the resistance in the armature circuit, greater is the fall in speed.

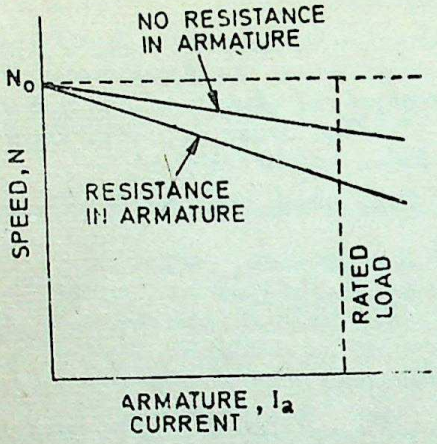

Fig. 13-3

Let I_{a1}=armature current in the first case

I_{a2}=armature current in the second case

(If $I_{a1}=I_{a2}$ then the load is of constant torque)

N_1, N_2=corresponding speeds

V=supply voltage

Then $N_1 \propto V-I_{a1}R_a$

Now $V-I_{a1}R_a=E_{b1}$

$\therefore \quad N_1 \propto E_{b1}$

Let some controller resistance of value R be added to the armature circuit so that total armature circuit resistance becomes $(R+R_a)=R_t$

Then $\quad N_2 \propto V-I_{a2}R_t \propto E_{b2} \quad \therefore \quad \dfrac{N_2}{N_1}=\dfrac{E_{b2}}{E_{b1}}$

(In fact, it is a simplified form of relation given in Art. 12-9 because here $\Phi_1=\Phi_2$)

Considering no-load speed, we have

$$\frac{N}{N_0}=\frac{V-I_aR_t}{V-I_{a0}R_a}$$

Neglecting $I_{a0}R_a$ with respect to V, we get

$$N=N_0\left(1-\frac{I_aR_t}{V}\right)$$

It is seen that for a given resistance R_t, the speed is a linear function of armature current I_a as shown in Fig. 13-4.

The load current for which the speed would be zero is found by putting $N=0$ in the above relation.

$$\therefore \quad 0=N_0\left(1-\frac{I_aR_t}{V}\right)$$

or $\quad I_a=V/R_t$

This is the maximum current and is known as stalling current.

This method is very wasteful, expensive and unsuitable for rapidly changing loads, because for a given value of R_t, speed will change with load.

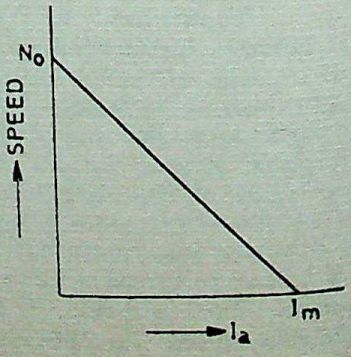

Fig. 13-4

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Example 13-2. *A shunt motor runs at 500 r.p.m. on a 200-V circuit. Its armature resistance is 0·5 Ω and the current taken is 30 A in addition to field current. What resistance must be placed in series in order that the speed may be reduced to 300 r.p.m., the current in armature remaining the same ?*

Solution. Since field current remains the same, $\Phi_2 = \Phi_1$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$E_{b1} = 200 \quad (0{\cdot}5 \times 30) = 185 \text{ V}, \qquad E_{b2} = (200 - 30\ R_t) \text{ volt}$$

$$\therefore \quad \frac{300}{500} = \frac{200 - 30R_t}{185} \qquad \therefore \quad R_t = 2{\cdot}97\ \Omega$$

Additional resistance required $= 2{\cdot}97 - 0{\cdot}5 = \mathbf{2{\cdot}47\ \Omega}$

Example 13-3. *A shunt motor, fed from a 400-V direct current supply, takes an armature current of 100 A when running at 800 r.p.m. If the total torque developed remains unchanged, find the speed at which the motor will run if the flux is increased to 120% of its original value and a resistance of 0·8 Ω is connected in series with the armature. The armature resistance is 0·2 Ω.*

Solution. Formula used : $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\Phi_1}{\Phi_2}$

The main thing required is I_{a2}. It can be found from the fact that motor torque remains constant.

Now, $T_a \propto \Phi\, I_a$

$\therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}$

or $I_{a2} = I_{a1} \times \Phi_1/\Phi_2 = 100/1{\cdot}2 = 250/3$ A

Hence $E_{b1} = V - I_{a1}R_a = 400 - (100 \times 0{\cdot}2) = 380$ V

$$E_{b2} = 400 - \frac{250}{3}(0{\cdot}8 + 0{\cdot}2) = \frac{950}{3} \text{ V}$$

$\Phi_1/\Phi_2 = 100/120 = 1/1{\cdot}2$; $N_1 = 800$ r.p.m.

$$\therefore \quad \frac{N_2}{800} = \frac{950/3}{380} \times \frac{1}{1{\cdot}2} \qquad \therefore \quad \mathbf{N_2 = 555\ r.p.m.}$$

13-3. Speed Control of Series Motors

(*i*) Flux Control Method

Variations in the flux of a series motor can be brought about in any one of the following ways :

(a) Field Divertors

The series windings are shunted by a variable resistance known as field divertor (Fig. 13-5). Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence, the flux can be decreased and consequently the speed of the motor increased.

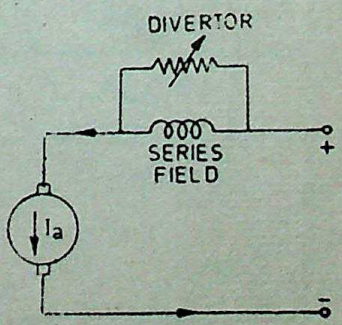

Fig. 13-5

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(*b*) *Armature Divertor*

A divertor across the armature can be used for giving speeds lower than the normal speeds (Fig. 13-6). For a *given constant load*

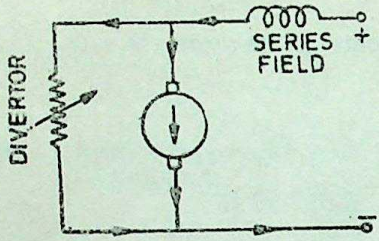

Fig. 13-6

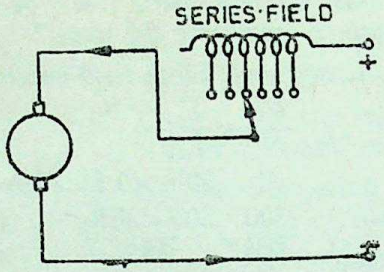

Fig. 13-7

torque, if I_a is reduced due to armature divertor, then Φ must increase ($\because T_a \propto \Phi I_a$). This results in an increase in current taken from the supply (which increases the flux) and a fall in speed ($\because N \propto 1/\Phi$). The variations in speed can be controlled by varying the divertor resistance.

(*c*) *Tapped Field Control*

This method is often used in electric traction and is shown in Fig. 13-7.

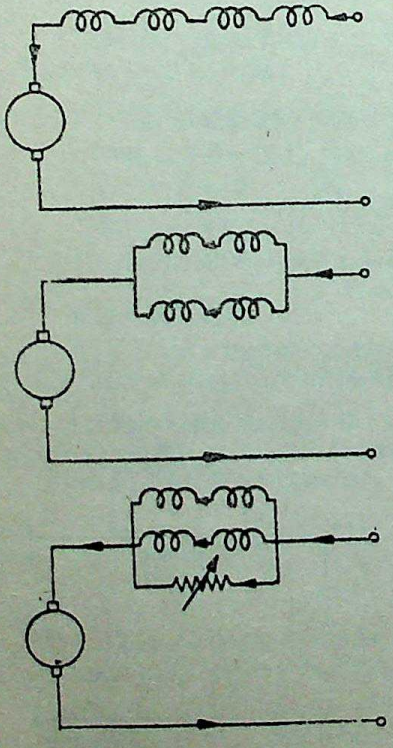

Fig. 13-8

The number of series field turns in the circuit can be changed at will as shown. With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

(*d*) *Paralleling Field Coils*

In this method, used for fan motors, several speeds can be obtained by regrouping the field coils as shown in Fig. 13-8. It is seen that for a 4-pole motor, three fixed speeds can be obtained easily.

(*ii*) *Variable Resistance in Series with Motor Armature*

By increasing the resistance in series with the armature (Fig. 13-9) the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.

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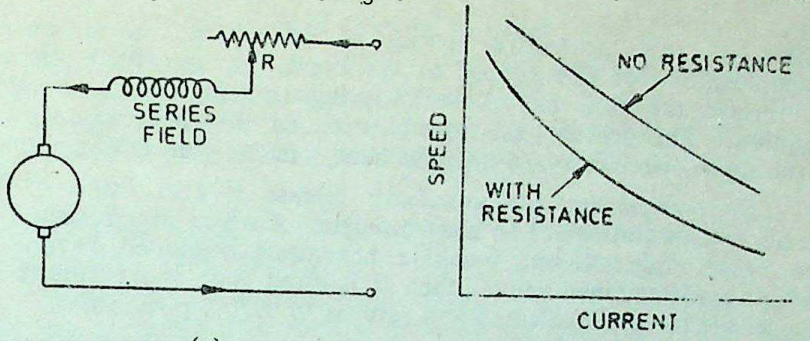

Fig. 13-9

13-4. Motor Starters—their Necessity (*A.M.I.E. Summer 1984*)

When a motor is at rest, there is, as yet, no back e.m.f. and if full supply voltage is applied, then the starting current is very high because armature resistance is very small. Suppose, a 200-kW motor has a cold armature resistance of 0·28 Ω and a *F.L.* current of 50 A. If this motor is directly switched on to a supply of 440-V, an armature current of 440/0·28 = 1572 A would flow which is 1572/50 = 31·4 times its *F.L.* current. This excessive current will blow out fuses and may damage the brushes etc. To avoid this excessive starting current, a resistance is inserted in series with the armature and is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

For series motors, the starter resistance is in series with both the armature and the field, but with shunt and compound motors, full shunt field is established on the first contact and maintained during the starting period while the starter resistance is progressively decreased.

13-5. Shunt Motor Starter With Protective Devices

It is shown in Fig. 13-10. It consists of an arm or handle *A* which moves over the studs. When the arm touches the first stud, field circuit is completed through brass arc *B* and full resistance is placed in the armature which is gradually cut out as the handle is moved over. The handle moves against a strong spring as shown. It has a piece of soft iron *C* attached to it which in the '*Full ON*' position is attracted and held by the electromagnet *E* which is energised by shunt field current. This is known as '*hold on*' coil or '*low voltage*' (formerly *No-voltage*) release. The

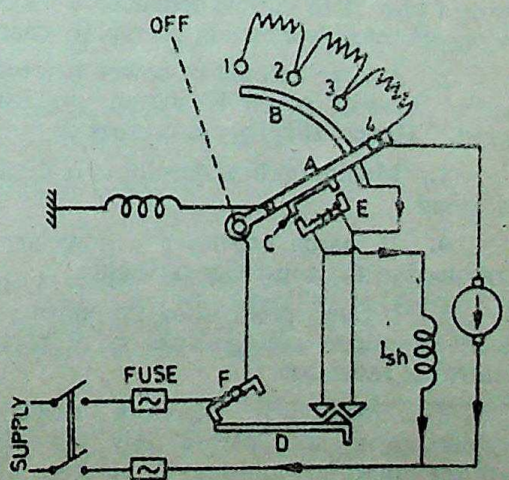

Fig. 13-10

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action of this protective device is, in case of a failure or disconnection of the supply or a break in the field circuit, to release the arm and allow the spring to bring it back to '*OFF*' position. This prevents the fuses blowing, as they otherwise would, if the supply were restored with the handle in the '*Full ON*' position.

An *over-current* (or *over-load*) release is also fitted in the starter. This consists of an electromagnet *F* which is connected in the supply line. If the machine becomes over-loaded beyond a certain predetermined value, then *D* is lifted and short-circuits *E*. Hence, the handle is released and returns to '*OFF*' position.

Example 13-4. *A 200-V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0·3 Ω and that of the field 0·5 Ω. If the current taken remains constant, calculate the resistance necessary to reduce the speed to 350 r.p.m.*

Solution. Total armature circuit resistance is

$$=0{\cdot}3+0{\cdot}5=0{\cdot}8\ \Omega$$

$$I_{a1}=I_{a2}=25\ \text{A}$$

$$\therefore \quad E_{b1}=200-(25\times 0{\cdot}8)=180\ \text{V}$$

$$E_{b2}=(200-25\,R)\ \text{V}$$

where R = controller resistance $+ R_a + R_f$

$$\because \quad \frac{N_2}{N_1}=\frac{E_{b2}}{E_{b1}}$$

$$\therefore \quad \frac{350}{500}=\frac{200-25\,R}{180} \qquad \therefore \quad R=2{\cdot}96\ \Omega$$

Controller resistance $=2{\cdot}96-0{\cdot}8=\mathbf{2{\cdot}16\ \Omega}$

13-6. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.

2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.

3. Maximum h.p. developed is diminished in the same ratio as speed.

4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.

5. It gives speeds *below* the normal value, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method is, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where the motor is continually started and stopped.

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13-7 Advantages of Field Control Method

This method is economical, more efficient and convenient though it can give speeds *above* (not below) the normal. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

Example 13-5. *A series motor, with an unsaturated field and negligible resistance when running at a certain speed on a given load takes 50 A at 460-V. If the load torque varies as the cube of speed, calculate the resistance required to reduce the speed by 25%.*

Solution. As series field is unsaturated, $\therefore \quad \Phi \propto I_a$

Now $T_a \propto \Phi I_a$

or $T_a \propto I_a^2$ ...(*i*)

Also $T_a \propto N^3$ ... given ...(*ii*)

From (*i*) and (*ii*) $I_a^2 \propto N^3$

In the first case, $I_{a1}^2 \propto N_1^3$

In the second case, $I_{a2}^2 \propto N_2^3$

$$\therefore \quad \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \left(\frac{N_2}{N_1}\right)^3 = \left(\frac{3}{4}\right)^3 \qquad \left(\because N_2 = \frac{3}{4} N_1\right)$$

$$\left(\frac{I_{a2}}{I_{a1}}\right)^2 = \frac{27}{64}$$

$$I_{a2} = I_{a1}\sqrt{\frac{27}{64}} = 50 \times \sqrt{\frac{27}{64}} = 32{\cdot}45 \text{ A}$$

Hence $E_{b1} = 460 - I_{a1} \times 0 = 460$ V

$E_{b2} = (460 - 32{\cdot}45\ R_t)$ V

where R_t is controller resistance in series with armature resistance.

Now $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \qquad (\because \quad \Phi \propto I_a)$$

$$\frac{3}{4} = \frac{460 - 32{\cdot}45\ R_t}{460} \times \frac{50}{32{\cdot}45}$$

$$R_t = 0{\cdot}523\ \Omega \text{ (approx)}$$

Tutorial Problems—19

1. The full-load efficiency of a 250-V, 7·46 kW, 1000 r.p.m., shunt motor is 80%. The field and armature resistances are 130 Ω and 0·4 Ω respectively. Calculate the value of resistance to be inserted in series with the armature circuit to reduce the speed to 800 r.p.m. with full load torque being developed. [1·34 Ω]

2. A d.c. 4-pole shunt motor has a wave-wound armature with 738 conductors, flux per pole 0·02 Wb, armature circuit resistance 0·2 Ω and total brush contact drop of 2·5 V. When connected to a 500-V supply, the armature current is 20 A.

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Calculate the value of the resistance which must be connected in series with the armature so that the motor speed is reduced to 700 r.p.m. when the armature current is 20 A. **[7·53 Ω]**

3. A 220-V. d.c. shunt motor has an armature resistance of 0·5 Ω and runs at 850 r.p.m. when taking a full-load current of 32 A. The shunt field resistance is 110 Ω. Calculate the speed at which the motor will run

(*a*) if a 1·5 Ω resistor were connected in series with the armature

(*b*) if a 30 Ω resistor were connected in series with the field winding, the load torque remaining the same throughout. Assume that the field flux is proportional to the field current. **[(*a*) 664 r.p.m. (*b*) 1062 r.p.m.]**

4. A 500-V d.c. series motor has a total resistance of 0·2 Ω and when taking a current of 60 A its speed is 500 r.p.m. When taking 30 A, the flux is 65% of that at 60 A. Calculate under the new conditions (*i*) the speed (*ii*) the new torque expressed as a percentage of the original torque.
[(*i*) 778 r.p.m. (*ii*) 32·5%]

5. A 200-V shunt motor running at 1000 r.p.m. takes an armature current of 17·5 A. It is required to reduce the speed to 500 r.p.m. by the addition of a resistance in the armature circuit, the armature current being unaltered. What must be the magnitude of this resistance if the armature resistance is 0·4 Ω ? **[5·51 Ω]**

6. A 220-V d.c. shunt motor has a speed of 1200 r.p.m., an armature resistance of 0·2 Ω and negligible voltage drop at the brushes. The motor draws an armature current of 20 A when connected to a rated supply voltage for a given load. As the mechanical load is increased, the field flux is also increased by 15 % and the armature current is found to rise to 45 A. Find (*i*) the back e.m.f. at 45 A load (*ii*) the speed at 45 A load (*iii*) the internal power developed at 20 A and 45 loads respectively.
[(*i*) 211 V (*ii*) 1019 rpm (*iii*) 4·32 kW ; 9·495 kW] (*A.M.I.E. Summer 1984*)

HIGHLIGHTS

1. Two factors controlling the speed of a given motor are
   (*i*) $N \propto E_b$ and
   (*ii*) $N \propto 1/\Phi$

2. Based on these two factors, there are two main methods of speed control
   (*i*) Flux or field control method
      (*a*) For shunt motors, this method consists of putting a variable field rheostat in the shunt winding for changing the shunt current and hence the flux of d.c. shunt motor.
      (*b*) For series motors, flux is changed by using a diverter across the series field windings.

3. In the armature or rheostatic control method, a variable controlling resistance is used in series with the armature circuit.

4. Flux control method is more economical and convenient whereas rheostatic control method is expensive and less efficient.

5. Flux control method gives speeds above the normal whereas rheostatic control method gives speeds below the normal.

6. Motor starters are essential for starting the motors in order to avoid excessive starting current.

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CHEMICAL EFFECTS OF CURRENT 14

1. Types of Electric Conductors 2. Ionization or Dissociation 3. Electrolysis 4. Electrode Reactions 5. Some Definitions 6. Faraday's Laws of Electrolysis 7. Polarisation or Back e.m.f. 8. Value of Back e.m.f. 9. Storage Cells—Definition 10. Materials of Lead-acid, Cells 11. Chemical Changes 12. Formation of Plates 13. Plante Process 14. Internal Resistance and Capacity of the Cell 15. Two Efficiencies of the Cell 16. Electrical Characteristics of Lead-acid Cell 17. Indications of Fully-charged Cell 18. Applications of Lead-acid Batteries 19. Charging Systems 20. Constant Current System 21. Constant Voltage System 22. Sulphation : Causes and Cure 23. Maintenance of Lead-acid Cell 24. Alkaline Accumulators 25. Edison Cell 26. Construction 27. Chemical Changes 28. Electrical Characteristics 29. Nickel-Cadmium Cell 30. Comparison Between Lead-acid and Edison Cell. 31. Highlights.

14-1. Types of Electric Conductors

In a very broad sense, conductors of electricity may be divided into two general classes as under :

(*i*) *Those conductors whose composition is not affected due to the passage of current through them.*

This class includes metals, alloys and liquids like mercury. Consider the case of copper wire. When current passes through it, there is no change in its physical and chemical states except for a rise in temperature. This is so because current flow in such metallic conductors is due to the movement of free electrons present in them and because the number of electrons entering at one end of the conductor is the same as that leaving it from the other end.

(*ii*) The second type of conductors are those *which undergo decomposition when electric current is passed through them.*

Such substances which conduct an electric current and are, at the same time, decomposed by it are known as *electrolytes.*

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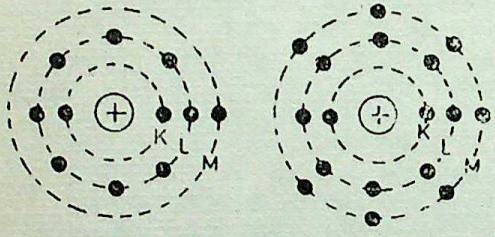
To this class belong acids (like H_2SO_4, HNO_3 and HCl etc.), bases (like NaOH, KOH etc.), salt solutions (like NaCl, $CuSO_4$ and $AgNO_3$ etc.) and fused salts. The phenomenon of conduction through fused salt is the basis of some important electrochemical industries like the production of aluminium and caustic soda etc.

It has been found that the conductivity of electrolytes is due to the presence in them of small electrified particles, to which, Faraday gave the name of '*ions*'. The process of ionization is discussed below.

14-2. Ionization or Dissociation

It is found that when a substance is dissolved in water, some of its molecules break up spontaneously into two groups (of atoms) *carrying equal but opposite charges*. These two groups of charged atoms are known as *ions*.



(a) Na ATOM (b) Cl ATOM

Fig. 14-1

Ions are composed of atoms or groups of atoms but they differ from them in that they carry electric charge, positive or negative. This process of splitting up of molecules into negative and positive ions is known as *dissociation* or *ionization* although it is also referred to as *ionic* dissociation.

To understand the process of ionization, let us first consider the formation of a NaCl (sodium chloride) molecule. The atomic number of Na is 11 and that of Cl is 17. Hence, the number of electrons is 11 in Sodium and 17 in Chlorine and these electrons are arranged in their orbits as shown in Fig. 14-1. The various orbits are numbered as *K*, *L*, *M* etc. starting from the orbit nearest to the nucleus. The maximum number of electrons that can occupy an orbit is given by $2n^2$ where n is the serial number of the orbit. Thus the *K* orbit ($n=1$) can hold a maximum number of two electrons, the *L* orbit ($n=2$) 8 electrons and *M* orbit ($n=3$) 18 electrons and so on.

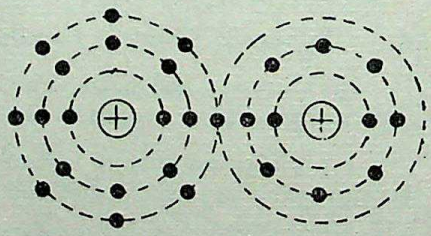


Fig. 14-2

It is seen that the *valency* orbit of Na atom contains only *one* electron and that of Cl contains *seven* electrons. The orbits of both the atoms are incomplete. hence the atoms are chemically active. When these atoms are brought together, Na atom loses its outermost electron, thereby acquiring a positive charge whereas Cl atom gains one electron thereby acquiring a negative charge. This exchange of electrons results in the completion of the outermost orbits of both the atoms, hence they no longer show any chemical activity. But

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the electrostatic force existing between a positive sodium atom and a negative chlorine atom brings the two atoms together to form a sodium chloride (NaCl) molecule as shown in Fig. 14-2. The formation of HCl may likewise be explained by the association of a hydrogen atom (which becomes positive due to loss of one electron) and a chlorine atom (which becomes negative due to the gain of one electron) because of electrostatic force existing between them.

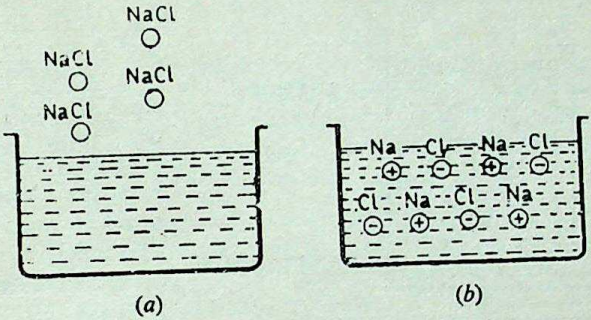

Fig. 14-3

When a NaCl molecule is now dissolved in water, then the interatomic electrostatic force $\left[F = \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{Q_1Q_2}{d^2}\right]$ is reduced to 1/80 of its value in air (or vacuum) because the relative permittivity of water $\epsilon_r = 80$. This reduced force is too small to hold the two atoms together, hence the molecule breaks up under the action of normal thermal agitation (which is ever present) and now exists in solution as positively-charged sodium ion (Na^+) and negatively-charged chlorine ion (Cl^-) as in Fig. 14-3(*b*). Similarly shown, HCl is decomposed or dissociated into H^+ ions and Cl^- ions. *The electric charge associated with an ion is decided by its valency. Monovalent* ions are those which have gained or lost one electron each *i.e.* sodium or chlorine or hydrogen ions. In the case of dissociation of a $CuSO_4$ (copper sulphate) molecule, Cu^{++} ions and SO_4^{--} ions are produced. Both these ions are *divalent* because Cu atom loses two electrons and SO_4 gains two electrons.

Acids always dissociate into positive hydrogen ions H^+ and negative ions of acid radical.

Bases always dissociate into positive metal ions and negative hydroxyl ions (OH).

In the case of salts, the metallic elements like Na, Cu, Ag develop into positive ions, whereas non-metallic elements like Cl, SO_4, NO_3, etc. give negative ions.

14-3. Electrolysis

Electrolysis (electric analysis) was the name given by Faraday to the chemical decomposition which occurs in electrolytes when

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traversed by an electric current. Consider a solution of $CuSO_4$ in water. As discussed above, the moment $CuSO_4$ is dissolved in water its molecules split up into Cu^{++} ions and SO_4^{--} ions. If now two Cu conductors or Cu electrodes are dipped in $CuSO_4$ solution and a p.d. is applied, then the positive copper ions (Cu^{++}) are attracted towards the negative electrode or *cathode* and the negative SO_4^{--} ions are attracted towards the positive electrode *i.e.* *anode*. Since SO_4^{--} ions move towards the anode, they are sometimes called *anions* whereas Cu^{++} ions which move towards the cathode are called *cations*. This process is shown in a simple way in Fig. 14-4. The movement of ions constitutes a flow of current through the electrolyte because each ion is a carrier of electricity. But it should be noted that there are two continuous streams of ions, positive ions moving in one direction and the negative ions in the opposite direction. This oppositely-directed movement of ions gives us the total current in the cell. However, in the external circuit the current is due to the axial motion of electrons only and is equal to the total current flowing in the electrolyte.

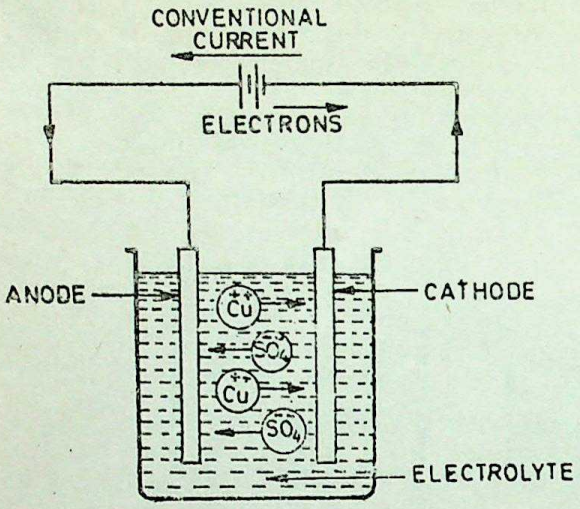

Fig. 14-4

When the SO_4^{--} ion reaches the anode, it '*gives up its charge*' and ceases to be an ion. The two electrons surrendered by SO_4^{--} ion enter the anode and become part of the electron stream in the external circuit. Similarly, when Cu^{++} ion reaches the cathode, it makes up its deficiency of two electrons from the cathode.

14-4. Electrode Reactions

Now, let us see what happens to the ions when they give up their charges at their respective electrodes. It is found that if the material of the electrode is suitable, they enter into combination with it and if, it is not, then they are removed by some reaction with the electrolyte or pass off as a gas or else in the case of metal ions, they are deposited in the metallic state on the electrode.

Take the case of two copper electrodes dipped in an electrolytic solution of $CuSO_4$ as shown in Fig. 14-4 above. The Cu^{++} ions move to the cathode, give up their charge, become Cu atoms and are deposited as metallic copper on the cathode. Similarly SO_4^{--} ions move to the anode, give up their negative charge, and then react with it to re-form $CuSO_4$ and enter the electrolyte. Hence, the final result of such an electrolysis (Cu electrodes with $CuSO_4$ as electro-

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lyte) is the transference of metallic copper from anode to cathode. If the electrodes were of such a material that SO_4 could not react with it *i.e.* they were made of platinum, then SO_4 would have combined with the hydrogen of the water thereby liberating oxygen according to the following reaction.

$$2SO_4 + 2H_2O \rightarrow 2H_2SO_4 + O_2$$

Similarly, in a cell with silver electrodes and silver nitrate ($AgNO_3$) as electrolyte, the ions are Ag^+ and NO_3^-, the Ag^+ ions moving to the cathode and NO_3^- ions to the anode. At cathode, Ag^+ ions leave the ionic state and are deposited as metallic silver on the surface of the cathode. At the anode, NO_3^- ions leave their ionic state and combine with the anode to form $AgNO_3$. So the cathode gains in weight exactly by the same amount what the anode loses, the electrolyte thereby remaining unchanged.

Next, consider a cell with platinum electrodes and a weak solution of H_2SO_4 as an electrolyte. The products of dissociation are H^+ ions and SO_4^{--} ions. These ions can form no compounds with platinum, so at cathode hydrogen is evolved as a gas and collects in the form of bubbles on the face of cathode. At the anode, the SO_4^{--} gives up its charge, and since it cannot react with platinum, it combines with water of the electrolytic solution and liberates oxygen thus :

$$2SO_4 + 2H_2O = 2H_2SO_4 + O_2$$

Now, oxygen is bivalent, hence it carries a double negative charge whereas hydrogen, being monovalent, carries a single positive charge. Therefore, for a given quantity of electricity passing through the cell, the volume of hydrogen liberated is twice that of oxygen.

In the end, it is thought worthwhile to emphasize that *ions are not produced by the action of the current in any way.* The molecules of a substance in solution are already dissociated or ionized by the mere act of going into solution. The ionization is due to the action of water and not that of the electric current. All that the current or applied potential difference does is to make use of the ions as carriers of electric charge.

14-5. Some Definitions

(*i*) **Atomic Weight (A)** : It is the weight of one atom of the given element as compared to the weight of one hydrogen atom. For example, when we say that atomic weight of sodium is 23, it simply means that one sodium atom is as heavy as 23 hydrogen atoms put together.

(*ii*) **Valency** (*v*). It measures the combining capacity of an atom. The valency of an atom is equal to the member of hyrogen atoms which the given atom can combine with or replace in any chemical reaction.

For example in HCl, one atom of chlorine combines with one hydrogen atom. Hence, valency of chlorine is one.

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Similarly, when aluminium (Al) reacts with HCl, it forms $AlCl_3$. In other words, one Al atom replaces *three* hydrogen atoms in HCl to form one molecule of $AlCl_3$. Hence, valency of Al is three.

(*iii*) **Chemical Equivalent Weight** (E). It is given by the ratio of the above two quantities.

$$\therefore \quad E = \frac{A}{v}$$

(*iv*) **Electrochemical Equivalent Weight** (Z). It is equal to the mass of ions of a substance liberated by the passage of one coloumb charge through its electrolytic solution. Its unit is kg/C.

$$\text{Also} \quad Z = \frac{E}{F} = \left(\frac{1}{F} \cdot \frac{A}{v}\right)$$

(*v*) **Faraday's Constant** (F). It is defined as the charge required to liberate electrolytically one gram-equivalent of any substance.

It is also given by the ratio of chemical equivalent weight (E) and the electro-chemical equivalent weight (Z)

$$\therefore \quad F = \frac{E}{Z}$$

It is a universal constant and its value is 96,500 C/gram-equivalent (or $96{\cdot}5 \times 10^6$C kg-equivalent).

(*vi*) **Gram-equivalent Weight.** It is the unit of mass. It is equal to the equivalent weight of a substance expressed in grams. Silver has a chemical equivalent weight of 118. Hence, for silver one gram-equivalent means an amount of 118 gram. Also, one kg-equivalent would mean an amount of 118 kg.

14-6. Faraday's Laws of Electrolysis *(A.M.I.E. Summer 1986)*

From his experiments, Faraday deduced two fundamental laws which govern the phenomenon of electrloysis. These are :

(*i*) **First Law.** The mass of an ion liberated at an electrode is directly proportional to the quantity of electricity which passes through the electrolyte.

(*ii*) **Second Law.** The masses of ions of different substances liberated by the same quantity of electricity are proportional to their chemical equivalent weights.

Explanation of the First Law

If m = mass of the ions liberated

Q = quantity for electricity passed

$= I \times t$ where I is the current and t is the time,

then according to the first law

$$m \propto Q \qquad \text{or} \quad m = ZQ \qquad \text{or} \quad m = ZIt$$

where Z is a constant and is known as the electro-chemical equivalent (E.C.E.) of the substance.

If $Q = 1$ coulomb *i.e.* $I = 1$ amp and $t = 1$ second, then $m = Z$.

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Hence E.C.E. of a *substance is equal to the mass of its ions liberated by the passage of one ampere current for one second through its electrolytic solution or by the passage of a charge of one coulomb.*

Faraday's First Law can also be expressed as given below :

$$m=\left(\frac{1}{F}\cdot\frac{A}{v}\right)It=\left(\frac{1}{F}\cdot\frac{A}{v}\right)Q \quad =\frac{E}{F}\cdot Q$$

where E is the chemical equivalent weight $(=A/v)$.

Explanation of the Second Law

Suppose an electric current is passed for the same time through acidulated water, solution of $CuSO_4$ and $AgNO_3$, then for every 1·0078 (or 1·008) gram of hydrogen evolved, 107·88 gram of silver and 31·54 gram of Cu are liberated. The values 107·88 and 31·54 represent the equivalent weights* of silver and copper respectively *i.e.* their atomic weights (as referred to hydrogen) divided by their respective valencies.

Example 14-1. *Calculate the quantity of electricity and the steady current required to deposit 5 gram of copper from copper sulphate solution in one hour. Electrochemical equivalent of copper is 0·3294 mg/C.* *(A.M.I.E. May 1970)*

Solution. $m=ZQ$

$$5=0{\cdot}3294\times10^{-3}\times Q$$

$\therefore$ $Q=5\times10^3/0{\cdot}3294=$ **15,180 coulomb**

Now $I=Q/t=15{,}180/3600=$ **42·2 A**

Example 14-2. *A steady current of 20 A is passed through a solution of copper sulphate for 30 minutes. Find the weight of copper deposited. The chemical equivalent for copper is 31·8 and E.C.E. of hydrogen is 0·01044 milligram per coulomb.*

Solution. Let us first find out *E.C.E.* of copper.

$$\frac{E.C.E.\text{ of Cu}}{E.C.E.\text{ of }H_2}=\frac{\text{Chemical Equivalent of Cu}}{\text{Chemical Equivalent of }H_2}$$

$\therefore$ $E.C.E.$ of Cu $=E.C.E.$ of hydrogen $\times\dfrac{C.E.\text{ of copper}}{C.E.\text{ of hydrogen}}$

$$=0{\cdot}01044\times10^{-3}\times\frac{31{\cdot}8}{1}=0{\cdot}000332\text{ g/C}$$

Now $m=ZIt=0{\cdot}000332\times20\times(30\times60)=$ **11·95 g**

---

*The electro-chemical equivalents and chemical equivalents of different substances are inter-related thus

$$\frac{\text{E.C.E. of }A}{\text{E.C.E. of }B}=\frac{\text{chemical equivalent of }A}{\text{chemical equivalent of }B}$$

Further if m_1 and m_2 are masses of ions deposited at or liberated from an electrode, E_1 and E_2 their chemical equivalents and Z_1 and Z_2 their electro-chemical equivalent weights, then

$$m_1/m_2=E_1/E_2=Z_1/Z_2$$

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Example 14-3. *In a copper refinery, copper is deposited on the cathodes with a current density of 0·02 A/cm². Find approximately the time needed to deposit a layer of copper on the cathode 1 cm thick. The density of copper may be taken as 8·9 g/cm³ and the E.C.E. as 0·000328 g/C. If 0·3 V is needed to send the current through the cell, find the number of kWh needed to deposit one kg of copper.*

Solution. Let A cm² be the cross-section of the cathode.

Then $I = 0{\cdot}02\ A$ amperes

Vol. of Cu $= A \times 1 = A$ cm³

Mass of Cu $= 8{\cdot}9\ A$ gram

Using $m = ZIt$, we get

$$t = m/ZI = 8{\cdot}9\ A/0{\cdot}000328 \times 0{\cdot}02\ A$$
$$= 1{\cdot}357 \times 10^6 \text{ seconds} = 377 \text{ hr}$$

We know that a current of $0{\cdot}02\ A$ amperes is needed to deposit $8{\cdot}9\ A \times 10^{-3}$ kg of Cu, therefore, current required for depositing 1 kg is given by

$$0{\cdot}02\ A/8{\cdot}9\ A \times 10^{-3} = 2{\cdot}247 \text{ A}$$

$\therefore$ Electric energy used $= 2{\cdot}247 \times 0{\cdot}3 \times 377/1000 = \mathbf{0{\cdot}254\ kWh}$

Example 14-4. *A coating of nickel 1 mm thick is to be built on a cylinder 20 cm in diameter and 30 cm in length in 2 hours. Calculate the electrical energy used in the process if the voltage is 10V. Electrochemical equivalent of nickel is 0·000304 g/C. Specific gravity of nickel is 8·9.*

Solution. Area of the curved surface to be coated

$$= \pi D \times l = \pi \times 20 \times 30 = 1885 \text{ cm}^2$$

Volume of nickel deposited

$$= 1885 \times 0{\cdot}1 = 188{\cdot}5 \text{ cm}^3$$

Weight of nickel deposited

$$= 188{\cdot}5 \times 8{\cdot}9 = 1677{\cdot}7 \text{ gram}$$

Now $m = ZIt$

$\therefore$ $1677{\cdot}7 = 0{\cdot}000304 \times I \times 2 \times 3600$

$\therefore$ $I = 1677{\cdot}7/2 \times 3600 \times 0{\cdot}000304 = 766{\cdot}5$ A

Watt-hours reqd. $= VIt$ ...t in hours

$$= 10 \times 766{\cdot}5 \times 2 = 15{,}330$$

$\therefore$ Energy reqd. $= 15{,}330/1000 = \mathbf{15{\cdot}33\ kWh}$

Example 14-5. *A current of 0·5 A is passed through a dilute solution of sulphuric acid for 15 minutes. Given that E.C.E. of hydrogen is $10{\cdot}44 \times 10^{-6}$ g/C and that 1 gram of hydrogen occupies 11·2 litres at N.T.P., calculate the volume of hydrogen liberated at an atmospheric pressure of 750 mm of Hg at 20°C. (A.M.I.E. Nov. 1972)*

Solution. $m = ZIt = 10{\cdot}44 \times 10^{-6} \times 0{\cdot}5 \times (15 \times 60) = 0{\cdot}0047$ g

Volume at N.T.P. $= 0{\cdot}0047 \times 11{\cdot}2 = 0{\cdot}0526$ litre

$= 52{\cdot}6$ cm³

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This volume can be converted to the value for the given condition by using general gas equation

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\therefore \quad \frac{76\times 52\cdot 6}{273}=\frac{75\times V_2}{293} \quad ; \quad V_2=57\cdot 23 \text{ cm}^3$$

Tutorial Problems—20

1. A metal plate having a surface of 100 cm^2 is plated with silver 0·0019 mm thick. To deposit this silver a current of 2·5 amperes is used for 12 minutes. Calculate the electrochemical equivalent of silver. Density of silver is 10·6 g/cm^3. **[0·001119 g/C]**

2. The atomic weights of silver and copper are 108 and 63 respectively and *E.C.E.* of silver is 0·001118, calculate the *E.C.E.* of copper in cupric condition (valency=2). Also calculate the quantity of silver deposited in an hour by a current of 3 amperes. **[$31\cdot 5\times 10^{-5}$ g/C ; 11·99 g]**

3. A metal plate having a surface of 120 cm^2 is to be silver-plated. What thickness of silver will be deposited if a current of 1·5 A is used for 2 hours. Density of silver is 10·6 and its *E.C.E.*=0·00118 g/C. **[0·095 mm]**

4. An ammeter connected in series with a silver electrolyte cell reads 0·5 ampere and it is found that 0·322 gram of the metal deposits in a run lasting 10 minutes. What is the error of the ammeter at this point of its range (*E.C.E.* of silver=1·118 mg/C). **[0·02 A]**

5. Electric energy is supplied to an electroplating company at 200 V. It electroplates 1013 gram of nickel per day by working 5 hours daily. If the E.C.E. of nickel be $20\cdot 26\times 10^{-5}$ g/C, calculate the power that the company is using. **[55·5 kW]**

6. A steady current was passed for 10 minutes through an ammeter in series with a silver voltameter and 3·489 gram of silver were deposited. The reading of the ammeter was 5 A. Calculate the percentage error. Electrochemical equivalent of silver is 1·1183 milligram per coulomb. **[3·85%]**

14-7. Polarisation or Back E.M.F.

Let us consider the case of two platinum electrodes dipped in dilute sulphuric acid solution. When a small potential difference is applied across the electrodes, no current is found to flow. When, however, the applied voltage is increased, a time comes when a temporary flow of current takes place. The H^+ ions move towards the cathode and O^{--} ions move towards the anode and are absorbed there. These absorbed ions have a tendency to go back into the electrolytic solution, thereby leaving them as oppositely charged electrodes. This tendency produces an e.m.f. which is in opposition to the applied voltage which is consequently reduced.

This opposing e.m.f. which is produced in an electrolyte due to the absorption of gaseous ions by the electrolyte from the two electrodes is known as the back e.m.f. of electrolysis or polarisation.

The value of this back e.m.f. is different for different electrolytes. The minimum voltage required to decompose an electrolyte is called the *decomposition* voltage for that electrolyte.

14-8. Value of Back E.M.F.

For producing electrolysis, it is necessary that the applied voltage must be greater than the back e.m.f. of electrolysis for that electrolyte. The value of this back e.m.f. of electrolysis can be found thus :

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Let us, for example, find the decomposition voltage of water. We will assume that the energy required to separate water into its constituents (*i.e.* oxygen and hydrogen) is equal to the energy liberated when hydrogen and oxygen combine to form water. Let H be the amount of heat energy absorbed when 9 gram of water are decomposed into 1 gram of hydrogen and 8 gram of oxygen. If the electrochemical equivalent of hydrogen is Z gram/coulomb, then a passage of Q coulombs liberates ZQ gram of hydrogen. Now, H is the heat energy required to release 1 gram of hydrogen, hence for releasing ZQ gram of hydrogen, heat energy required is HZQ calories or $JHZQ$ joules. If E is the decomposition voltage, then energy spent in circulating Q coulombs of charge is EQ joules. Equating the two amounts of energies, we have

$$EQ = JHZQ \quad \text{or} \quad E = JHZ$$

where J is 4·2 joules/cal.

The e.m.f. of a cell can be calculated by determining the two electrode potentials. The electrode potential is calculated on the assumption that the electrical energy comes entirely from the heat of the reactions of constituents. Let us take a zinc electrode. Suppose it is given that 1 gram of zinc when dissolved liberates 540 calories of heat and that the electro-chemical equivalent of zinc is 0·000338 gram/coulomb. As calculated above,

$$E = JHZ = 4{\cdot}2 \times 540 \times 0{\cdot}000338 = 0{\cdot}76 \text{ volt}$$

The electrode potentials are usually referred to in terms of the potential of a standard hydrogen electrode *i.e.* an electrode of hydrogen gas at normal atmospheric pressure and in contact with a normal acid solution. In Table 14-1 are given the electrode potentials of various elements as referred to the standard hydrogen electrode. The elements are assumed to be in normal solution and at atmospheric pressure.

In the case of Daniel cell having copper and zinc electrodes, copper electrode potential with respect to hydrogen ion is +0·345 V and that of the zinc electrode is −0·758 V. Hence, the cell e.m.f. is $= 0{\cdot}345 - (-0{\cdot}758) = 1{\cdot}103$ volt. The e.m.f. of other primary cells can be found in a similar way.

Table No. 14-1

Electrodes	*Potential (volts)*
Cadmium	−0·398
Copper	+0·345
Hydrogen	0
Iron	−0·441
Lead	−0·122
Mercury	+0·799
Nickel	−0·231
Potassium	−2·922
Silver	+0·80
Zinc	−0·758

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Example 14-6. *One gram of hydrogen on burning to form water yields 34,000 calories and E.C.E. of hydrogen is $1{\cdot}044 \times 10^{-5}$ gram per coulomb. What is the minimum e.m.f. necessary to decompose water ?*

Solution. The decomposition voltage is given by

$$E = JHZ = 4{\cdot}2 \times 34{,}000 \times 1{\cdot}044 \times 10^{-5} = \mathbf{1{\cdot}49\ V}$$

Example 14-7. *Calculate the weight of zinc and MnO_2 required to produce 1 ampere-hour in a Leclanche cell.*

Atomic weight :—Mn, 55 ; O, 16 ; Zn, 65.

E.C.E. of hydrogen = 0·0000104 g/C

Solution. 1 ampere-hour = 3600 A-C = 3600 coulombs

Wt. of hydrogen liberated $= ZQ = 0{\cdot}0000104 \times 3600 = 0{\cdot}03744$ g

Now, the chemical reactions in the cell are,

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + H_2$$

It is seen that 1 *atom* of zinc is used up in liberating *two atoms* of hydrogen. In other words, to produce 2 gram of hydrogen, 65 gram of zinc will have to go into chemical combination.

$\therefore$ zinc required to produce 0·03744 gram of hydrogen

$$= 0{\cdot}03744 \times 65/2 = 1{\cdot}217 \text{ gram}$$

The hydrogen liberated combines with manganese dioxide as under :

$$2MnO_2 + H_2 = H_2O + Mn_2O_3$$

Atomic weight of $MnO_2 = 2(55 + 16 \times 2) = 174$

It is seen that 174 gram of MnO_2 combine with 2 gram of hydrogen, hence weight of MnO_2 needed to combine with 0·03744 gram of hydrogen

$$= 0{\cdot}03744 \times 174/2 = 3{\cdot}258 \text{ gram}$$

Hence, for 1 ampere-hour, **1·217** gram of zinc and **3·258** gram of MnO_2 are needed.

14-9. Storage Cells—Definition

The function of 'storage' cells is to convert electrical energy into chemical energy during the process known as 'charging' and the reverse of it when 'discharging'.

During charging the cell, when current is passed through it, certain chemical changes take place in the active materials of the cell. Such chemical changes absorb energy during their formation. When these chemical reactions are completed and the electric current produces no further chemical changes, the cell is said to be fully charged.

When the cell is next connected to an external circuit, the active materials of the cell revert to their original condition, thereby reversing the changes which occurred during charging. In this process of undoing the chemical changes, absorbed energy is released in the form of electric current, the process being known as discharging.

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It should be noted that the cell does not 'store' electricity as such but absorbs electric energy in the form of chemical energy, the whole process being reversible.

We will discuss two types of storage cells or accumulators or secondary cells *i.e.* Lead-acid cell and Edison alkali cell.

14-10. Materials of Lead-acid Cell (*A.M.I.E. Summer 1983*)

Those substances of the cell which take active part in chemical combination and hence absorb or produce electricity during charging or discharging, are known as the **active materials** of the cell.

The active materials of a lead acid cell are :

(1) *Lead Peroxide* (PbO_2) for +ve plate

(2) *Sponge Lead* (Pb) for —ve plate

(3) *Dilute Sulpuric Acid* (H_2SO_4) as electrolyte

(1) Lead Peroxide

It is a combination of lead and oxygen, is dark chocolate brown in colour and is quite hard but brittle substance. It is made up of one atom of lead (Pb) and two atoms of oxygen (O_2) and its chemical formula is PbO_2. As said earlier, it forms the positive active material.

(2) Sponge Lead

It is pure lead in soft spongy or porous condition. Its chemical formula is Pb and forms the negative active material.

(3) Dilute Sulphuric Acid

It is approximately 3 parts water and one part sulphuric acid. The chemical formula of the acid is H_2SO_4. The positive and negative plates are immersed in this solution which is known as 'electrolyte'. It is this medium through which the current produces chemical changes.

Hence, the lead-acid cell depends for its action on the presence of two plates of PbO_2 and Pb in a solution of dilute H_2SO_4 of specific gravity 1·21 or so.

Lead in the form of PbO_2 or sponge Pb has very little mechanical strength, hence it is supported by plates of pure lead. Those plates covered with or otherwise supporting PbO_2, are known as +ve plates and those supporting spongy lead are called —ve plates. The +ve and —ve plates are arranged alternately and are connected to two common +ve and —ve terminals. These plates are assembled in a suitable jar or container to make a complete cell.

14-11. Chemical Changes (*A.M.I.E. Summer 1986*)

Following chemical changes take place during the charging and discharging of a lead-acid cell.

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DISCHARGING (Fig. 14-5).

When the cell is fully charged, its positive plate or anode is PbO_2 (dark chocolate brown) and the negative plate or cathode is Pb (slate grey). When the cell discharges *i.e.* it sends current through the external load, then H_2SO_4 is dissociated into positive H_2 ions and negative SO_4 ions. As the current within the cell is flowing from cathode to anode, H_2 ions move to anode and SO_4 ions move to the cathode.

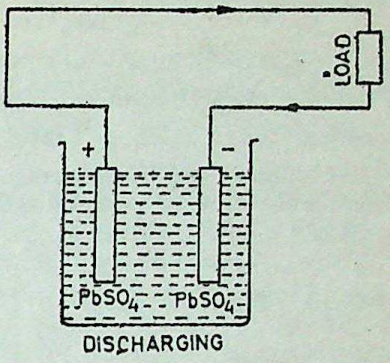

Fig. 14-5

At anode (PbO_2), H_2 combines with the oxygen of PbO_2 and H_2SO_4 attacks lead to form $PbSO_4$.

$$PbO_2 + H_2 + H_2SO_4 \rightarrow PbSO_4 + 2H_2O$$

At the cathode (Pb), SO_4 combines with it to form $PbSO_4$

$$Pb + SO_4 \rightarrow PbSO_4$$

It will be noted that during discharging,

(*i*) *Both anode and cathode become $PbSO_4$ which is somewhat whitish in colour*

(*ii*) *Due to formation of water, specific gravity of the acid decreases*

(*iii*) *Voltage of the cell decreases*

(*iv*) *The cell gives out energy.*

CHARGING (Fig. 14-6)

When the cell is re-charged, H_2 ions move to cathode and SO_4 ions go to anode and the following changes take place.

At cathode, $PbSO_4 + H_2 \rightarrow Pb + H_2SO_4$

At anode, $PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4$

Hence, the anode and cathode again become PbO_2 and Pb respectively.

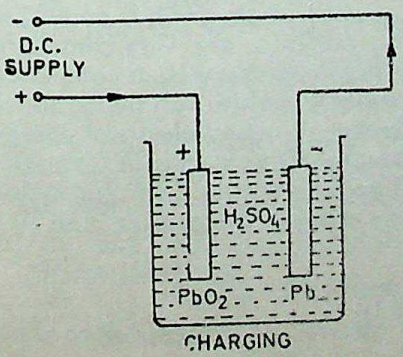

Fig. 14-6

It will be noticed that during charging :—

(*i*) *The anode becomes dark chocolate brown in colour (PbO_2) and cathode becomes grey metallic lead (Pb).*

(*ii*) *Due to consumption of water, specific gravity of H_2SO_4 is increased.*

(*iii*) *There is a rise in voltage.*

(*iv*) *Energy is absorbed by the cell.*

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The charging and discharging of the cell can be represented by a single reversible equation given below :

$$\underset{\text{Pos. Plate}}{PbO_2} + 2H_2SO_4 + \underset{\text{Neg. Plate}}{Pb} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{Pos. Plate}}{PbSO_4} + 2H_2O + \underset{\text{Neg. Plate}}{PbSO}$$

For discharge, the equation should be read from left to right and for charge from right to left.

14-12. Formation of Plates

There are, in general, two methods of producing the active materials of the cell and attaching them to lead plates. These are known after the names of their inventors (*i*) **Plante** plates or formed plates (*ii*) **Faure** plates or pasted plates.

14-13. Plante Process

In this process, two sheets of lead are taken and immersed in dilute H_2SO_4. When a current is passed into this lead-acid cell from a dynamo or some other external source of supply, then due to electrolysis, hydrogen and oxygen are evolved. At anode, oxygen attacks lead converting it into PbO_2 whereas cathode is unaffected because hydrogen can form no compound with Pb.

If the cell is now discharged (or current is reversed through it) then peroxide coated plate becomes cathode, so hydrogen forms on it and combines with the oxygen of PbO_2 to form water thus

$$PbO_2 + 2H_2 \rightarrow Pb + 2H_2O$$

At the same time, oxygen goes to anode (the plate previously unattacked) which is lead and reacts to form PbO_2. Hence, the anode becomes covered with a thin film of PbO_2.

By continuous reversal of the current or by charging and discharging the above electrolytic cell, the thin film of PbO_2 will become thicker and thicker and the polarity of the cell will take increasingly longer time to reverse. Two lead plates after being subjected to hundreds of reversals will acquire a skin of PbO_2 thick enough to possess sufficiently high capacity. This process of making positive plates is known as *formation*. The negative plates are also made by the same process. They are turned from positive to negative plates by reversing the current through them until whole PbO_2 is converted into spongy lead. Although Plante positives are very commonly used for stationary work. Plante negatives have been completely replaced by the Faure or pasted type plates. However, owing to the length of time required and enormous expenditure of electrical energy, this process is commercially impracticable. The process of formation can be accelerated by forming agents such as acetic, nitric or hydrochloric acid or their salts, but still this method is expensive and slow and plates are heavy.

14-14. Internal Resistance and Capacity of a Cell

The secondary cell possesses internal resistance due to which some voltage is lost in the form of potential drop across it when current is flowing. Hence, the internal resistance of the cell has to be kept to the minimum.

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One obvious way to lessen internal resistance is to increase the size of the plates. However, there is a limit to this because the cell will become too big to handle. Hence, in practice it is usual to multiply the number of plates inside the cell and to join all the negatives together and all the positives together.

The effect is equivalent to joining many cells in parallel. At the same time, the length of the electrolyte between the electrodes is decreased with a consequent reduction in the internal resistance.

The 'capacity' of a cell is given by the product of current in amperes and the time in hours during which the cell can supply current until its e.m.f. falls to 1·8 volts. It is expressed in ampere-hours (Ah).

The interlacing of plates not only decreases the internal resistance but additionally increases the capacity of the cell also. There is always one more negative plate than the positive plates *i.e.* there is a negative plate at both ends. This gives not only more mechanical strength but assures that both sides of a positive plate are used.

Since in this arrangement, the plates are quite close to each other, something must be done to make sure that a positive plate does not touch the negative plate otherwise an internal short-circuit will take place. This separation between the two plates is achieved by using separators which, in the case of small cells, are made of wood, ebonite or hard rubber and in the case of large stationary cells, are in the form of glass rods.

14-15. Two Efficiencies of the Cell

The efficiency of a cell can be considered in two ways :

(1) **The quantity or ampere hour (Ah) efficiency**

(2) **The energy or watt-hour (Wh) efficiency**

The Ah efficiency does not take into account the varying voltages of charge and discharge. The Wh efficiency does so and is always less than Ah efficiency because average p.d. during discharging is less than that during charging. Usually, during discharge the e.m.f. falls from about 2·1 V to 1·8 V whereas during charge, it rises from 1·8 volts to about 2·6 V.

$$\therefore \quad \text{Ah eff.} = \frac{\text{ampere-hour on discharge}}{\text{ampere-hour on charge}}$$

The Ah efficiency of a lead-acid cell is normally between 90 to 95%, meaning that about 100 Ah must be put back into the cell for every 90—95 Ah taken out of it.

If Ah efficiency is given, Wh efficiency can be found from the following relation :

$$\text{Wh efficiency} = \text{Ah efficiency} \times \frac{\text{average volts on discharge}}{\text{average volts on charge}}.$$

The Wh efficiency varies between 72 – 80%.

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From the above, it is clear that anything that increases the charge volts or reduces the discharge volts will decrease Wh efficiency. Because high charge and discharge rates will do this. it is advisable to avoid these.

14-16. Electrical Characteristics of the Lead-acid Cell

The three important features of an accumulator, of interest to an engineer are (*i*) voltage (*ii*) capacity and (*iii*) efficiency.

(1) Voltage

The open-circuit voltage of a fully-charged cell is approximately 2·1 volts. This value is not fixed but depends on (*a*) length of time since it was last charged (*b*) specific gravity—voltage increasing with increase in specific gravity and *vice versa*. If specific gravity comes near to density of water *i.e* 1·00 then voltage of the cell will disappear altogether. (*c*) temperature—voltage increases, though not much, with increase in temperature.

CHARGE AND DISCHARGE VOLTAGE CURVES

The variations in the terminal p.d. of a cell on charge and discharge are shown in Fig. 14-7. The voltage fall depends on the

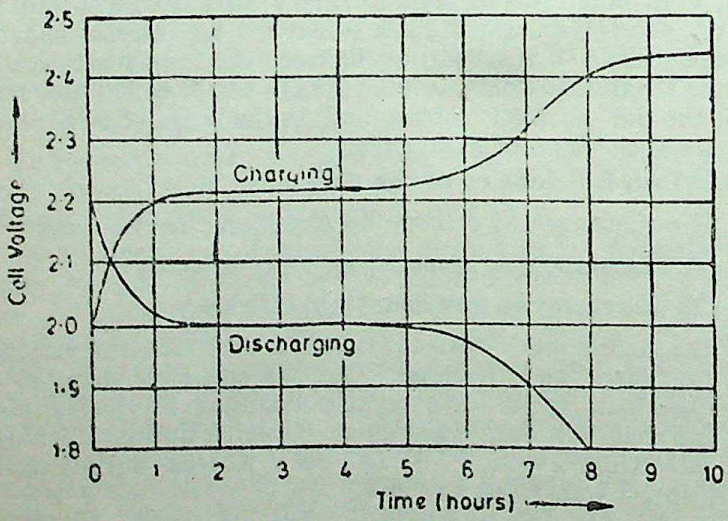

Fig. 14-7

rate of discharge. Rates of discharge are generally specified by the number of hours during which the cell will sustain the rate in question before falling to 1·8 V. The voltage falls rapidly in the beginning (rate of fall depending on the rate of discharge), then very slowly up to 1·85 and again suddenly to 1·8 V. The voltage should not be allowed to fall to lower than 1·8 V, otherwise hard insoluble lead sulphate is formed on the plates which increases the internal resistance of the cell.

The general form of the voltage-time curves corresponding to 1-, 3-, 5- and 10- hours rate of discharge, are shown in Fig. 14-8

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i.e. corresponding to the steady currents which would discharge the cell in the above mentioned times (in hours). It will be seen that both the terminal voltage and the rate at which the voltage falls depend on the rate of discharge. The more rapid fall in voltage at higher rates of discharge is due to rapid increase in the internal resistance of the cell.

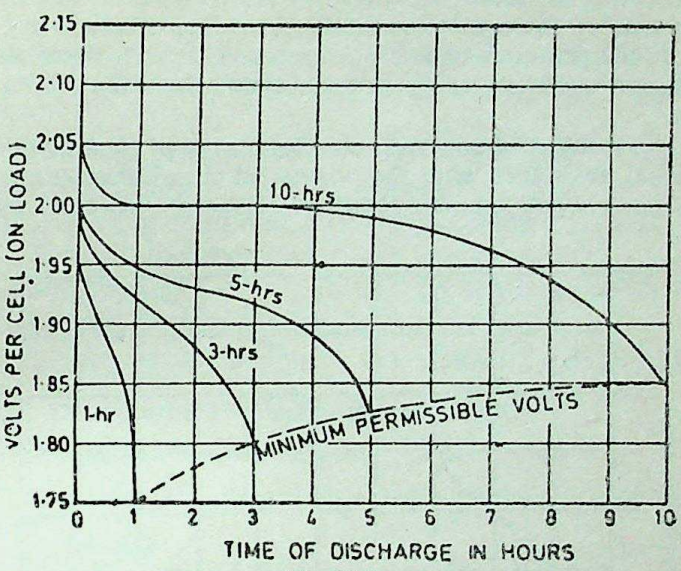

Fig. 14-8

During charging, the p.d. increases (Fig. 14-7). The curve is similar to the discharge curve reversed but is everywhere higher due to the increased density of H_2SO_4 in the pores of the positive plates.

(2) **Capacity**

It is measured in ampere-hours (Ah). One ampere-hour (Ah) is the amount of electricity conveyed by one ampere in one hour. The capacity is always given at a specified rate of discharge (10-hour discharge in U.K., 8-hour discharge in U.S.A.). The capacity of a cell depends on the amount of the active material on its plates. In other words, it depends on the size and thickness of the plates. However, for a given battery, the capacity is affected by the following factors :

(*i*) *Rate of Discharge.* The capacity of a cell, as measured in Ah, depends on the discharge rate. It decreases with increased rate of discharge. Rapid rate of discharge means greater fall in p.d. of the cell due to internal resistance of the cell. Moreover, with rapid discharge, the weakening of the acid in the pores of the plates is also greater. Hence, the chemical change produced at the plates by 1 ampere for 10 hours is not the same as produced by 2 amperes for 5 hours or 4 amperes for 2·5 hours. It is found that a cell having a 100 Ah capacity at 10 hour discharge rate, has its capacity reduced

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to 82·5 Ah at 5 hour rate and 50 Ah at 1-hour rate. The variation of capacity with discharge rate is shown in Fig. 14-9.

(*ii*) *Temperature.* Capacity increases with increase in temperature, the increase in capacity being more marked at higher rates of discharge. This is due to the fact that at higher temperatures (*a*) chemical action is more vigorous (*b*) the resistance of the acid decreases and (*c*) there is better diffusion of the electrolyte. With decrease in temperature, available voltage and capacity decrease until at freezing point, the capacity is zero even when the cell is fully charged.

(*iii*) *Density of electrolyte.* As the density of electrolyte affects the internal resistance and the vigour of chemical reaction, it has an important effect on the capacity. Capacity increases with the density.

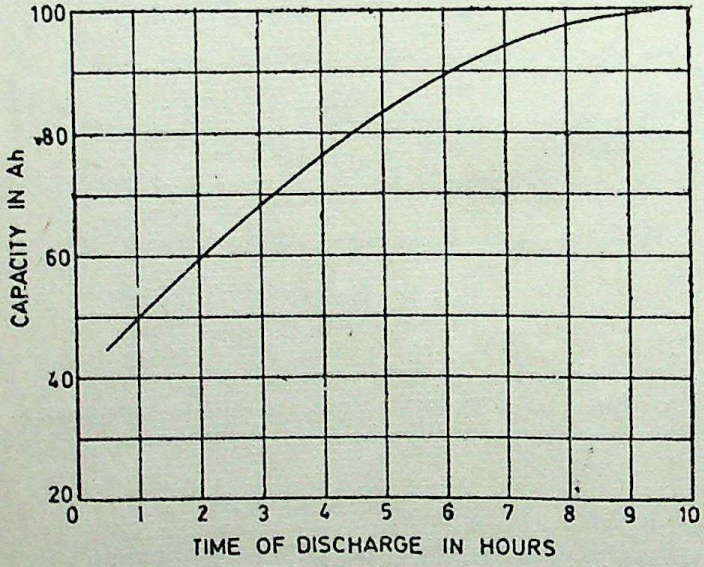

Fig. 14-9

(3) Efficiency

It has already been discussed in Art. 14-14.

Example 14.8. *A discharged battery is put on charge at 5 A for 3½ hours at a mean charging voltage of 13·5 V. It is then discharged in 6 hours at a constant terminal voltage of 12 V through a resistance of R ohm. Determine :*

(i) value of R for an Ah efficiency of 85%

(ii) Wh efficiency of the battery. (*A.M.I.E. Nov. 1969*)

Solution. Ah on charge $= 5 \times 3 \cdot 5 = 17 \cdot 5$

Discharging current $= 12/R$ ampere

Ah on discharge $= 6 \times 12/R = 72/R$

(*i*) Ah efficiency $= \dfrac{\text{Ah of discharge}}{\text{Ah of charge}} = \dfrac{72}{17 \cdot 5R}$

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$$\therefore \quad \frac{72}{17.5\,R} = 0.85 \quad \text{or} \quad R = 4.84\ \Omega$$

(*ii*) Wh efficiency $= \frac{12}{13.5} \times 0.85 = 0.756$ or **75.6%**

Example 14-9. *An emergency battery has 175 alkaline cells connected in series and the input capacity per cell is 1000 Ah. If the quantity efficiency is 80 per cent, calculate the average kW output during an 8-hour discharge if the average voltage during discharge is 1·2 V per cell.*

Solution. Input of all cells $= 175 \times 1000$ Ah

Ah efficiency $= 0.8$

$\therefore$ Output of all cells $= 175 \times 1000 \times 0.8 = 175 \times 800$ Ah

Time of discharge $= 8$ hours

Output current $= 175 \times 800/8 = 17{,}500$ A

Average discharge voltage $= 1.2$ V

$\therefore$ Power output $= 17{,}500 \times 1.2$ watt $=$ **21 kW**

14-17. Indications of a Fully-charged Cell

The indications of a fully-charged cell are :—

(*i*) *gassing* (*ii*) *voltage* (*iii*) *specific gravity* (*iv*) *colour of plates.*

(*i*) Gassing

When the cell is fully charged, it freely gives off hydrogen at cathode and oxygen at the anode, the process being known as "gassing". Gassing at *both* plates indicates that the current is no longer doing any useful work and hence should be stopped.

(*ii*) Voltage

The voltage ceases to rise when the cell becomes fully charged. The value of the voltage of a fully-charged cell is a variable quantity being affected by the rate of charging, the temperature and specific gravity of the electrolyte etc. The approximate value of the e.m.f. is 2·1 V or so.

(*iii*) Specific Gravity of the Electrolyte

A third indication of the state of charge of a battery is given by the specific gravity of the electrolyte. We have seen from the chemical equations of Art. 14-11, that during discharging, the density of electrolyte decreases due to the production of water, whereas it increases during charging due to the absorption of water. The value of density when the cell is fully charged is 1·21 and 1·18 when discharged up to 1·3 V. Specific gravity can be measured with a suitable hydrometer.

(*iv*) Colour

The colour of plates, on full charge, is deep chocolate brown for positive plates and slate grey for negative plates and the cell looks quite brisk and alive.

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14-18. Applications of Lead-acid Batteries

(A.M.I.E. Summer 1986)

Storage batteries are, these days, used for a great variety and range of purposes, some of which are summarised below :

(*i*) In central stations for supplying the whole load during light load periods, also to assist the generating plant during peak load periods, for providing reserve emergency supply during periods of plant breakdown and finally to store energy at times when load is light for use at times when load is at its peak value.

(*ii*) In private generating plants both for industrial and domestic use, for much the same purpose as in Central Stations.

(*iii*) In sub-stations, they assist in maintaining the declared voltage by meeting a part of the demand and so reducing the load on and the voltage drop in feeder during peak-load periods.

(*iv*) As a power source for industrial and mining battery locomotives and for road vehicles, like cars and trucks.

(*v*) As a power source for submarines when submerged.

(*vi*) For petrol motor-car starting and ignition etc.

(*vii*) As a low-voltage supply for operating purposes in many different ways such as high-tension switchgear, automatic telephone exchange and repeater stations, broadcasting stations and for wireless receiving sets.

14-19. Charging Systems

In various installations, batteries are kept 'floating' on the line and are so connected that they are being charged when load demands are light and automatically discharged during peak periods when load demands are heavy or when the usual power supply fails or is disconnected. In some other installations, the battery is connected to the feeder circuit as and when desired, allowed to discharge to a certain point, then removed and re-charged for further requirements.

For batteries other than the 'floating' and 'system-governed' type, the following two general methods (though there are some variations of these) are employed :

(*i*) *The constant-current system*

(*ii*) *The constant-voltage system*

14-20. Constant Current System

In this method (Fig. 14-10), the charging current is kept constant by varying the supply voltage to overcome the increased back e.m.f. of cells. If a charging booster (which is just a shunt dynamo directly driven by a motor) is used, then current supplied by it can be kept constant by adjusting its excitation. If charged on a d.c. supply, then current is controlled by varying the Rheostat connected in the circuit. The value of charging current should be so chosen

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that there would be no excessive gassing during final stages of charging and also the cell temperature does not exceed 45°C. This method takes a comparatively longer time.

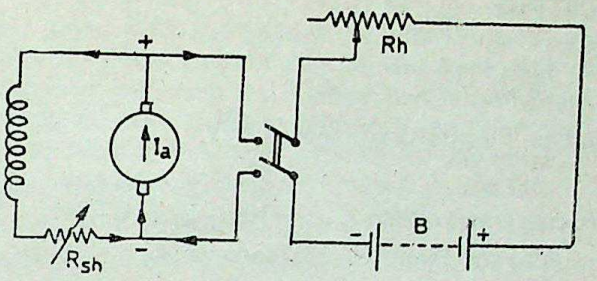

Fig. 14-10

14-21. Constant Voltage System

In this method (Fig. 14-11), the voltage is kept constant, but it results in very large charging current in the beginning when the back e.m.f. of the cells is low and a small current when their back e.m.f. increases on being charged. This automatic decrease in charging current as cells near their full-charge condition, is the most satisfactory feature of this system.

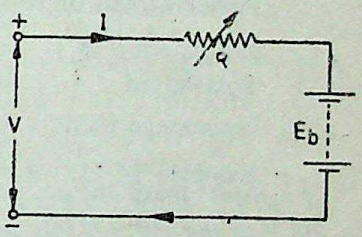

Fig. 14-11

With this method, time of charging is almost reduced to half. It increases the capacity by approximately 20% but reduces the efficiency by 10% or so.

Calculations

When a secondary cell or a battery of such cells is being charged, the e.m.f. of the cells acts in opposition to the applied voltage. If V is the supply voltage which sends a charging current of I against the back e.m.f. E_b, then input is VI, but the work done in overcoming the opposition (Fig. 14-11) is E_bI. This work E_bI minus that lost as heat due to circuit resistance is converted into the chemical energy which is stored in the cell. The charging current can be found from the following equation

$$I = \frac{V - E_b}{R}$$

where V = supply voltage (assumed constant)
E_b = back e.m.f. of the battery
R = total circuit resistance including internal resistance of the battery
I = charging current

By varying R, the charging current can be kept constant throughout.

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Example 14-10. *A battery of accumulators of e.m.f. 50 volts and internal resistance 2 Ω is charged on 100-volt direct mains. What series resistance will be required to give a charging current of 2 A ?*

If the price of energy is 10 paise per kWh, what will it cost to charge the battery for 8 hours and what percentage of energy supplied will be used in the form of heat ?

Solution. Applied voltage $= 100$ V

$E_b = 50$ V

Net charging voltage $= 100 - 50 = 50$ V

Let R be the required resistance, then

$$2 = \frac{50}{R+2} \qquad \therefore \quad R = 46/2 = \mathbf{23\ \Omega}$$

Input for eight hours $= 100 \times 2 \times 8 = 1600$ Wh $= 1{\cdot}6$ kWh

Cost $= 1{\cdot}6 \times 10 = \mathbf{16}$ **paise**

Power wasted on total resistance

$= 25 \times 2^2 = 100$ W

Total input $= 100 \times 2 = 200$ W

Percentage waste $= 100 \times 100/200 = \mathbf{50\%}$

Example 14-11. *A battery of 60 cells is charged from a supply of 250 V. Each cell has an e.m.f. of 2 volts at the start of charge and 2·5 V at the end. If internal resistance of each cell is 0·1 Ω and if there is an external resistance of 19 Ω in the circuit, calculate (a) the initial charging current (b) the final charging current and (c) the additional resistance which must be added to give a finishing charge at 2 A rate.*

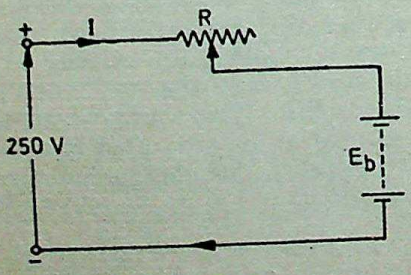

Fig. 14-12

Solution. Supply voltage $V = 250$ V

Back e.m.f. of the battery E_b at start $= 60 \times 2 = 120$ V and at the end $= 60 \times 2{\cdot}5 = 150$ V

Internal resistance of the battery $= 60 \times 0{\cdot}1 = 6\ \Omega$

Total circuit resistance $= 19 + 6 = 25\ \Omega$

(*a*) Net charging voltage at start $= 250 - 120 = 130$ V

$\therefore$ Initial charging current $= 130/25 = \mathbf{5{\cdot}2\ A}$

(*b*) Final charging current $= 100/25 = \mathbf{4\ A}$

(*c*) Let R be the external resistance, then

$$2 = \frac{100}{R+6} \qquad \therefore$$

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$\therefore \quad R = 88/2 = 44\ \Omega$

$\therefore$ Additional resistance required $= 44 - 19 = 25\ \Omega$

Example 14-12. *Twenty-five lead-acid cells each of discharge capacity of 200 Ah at 10-hour rate are to be fully charged at constant current for 12 hours. The Ah efficiency is 80 per cent and the d.c. supply is 110 V. The e.m.f. of each cell at the beginning and at the end of the charge is 1·8 V and 2·6 V respectively. Calculate the maximum and minimum values of the external resistance necessary. Internal resistance of the cells should be neglected.*

Solution. Output/cell $= 200$ Ah , Ah $\eta = 0{\cdot}8$

Input/cell $= 200/0{\cdot}8 = 250$ Ah

Constant charging current $= 250/12 = 125/6$ A

New charging current is given by

$$I = \frac{V - E_b}{R} \qquad \text{—Art 14-20}$$

At the beginning of charge

$$E_b = 25 \times 1{\cdot}8 = 45 \text{ V}$$

$$\therefore \quad \frac{125}{6} = \frac{110 - 45}{R} \qquad \text{or} \quad R = \mathbf{3{\cdot}12\ \Omega}$$

At the end of the charge

$$E_b = 25 \times 2{\cdot}6 = 65 \text{ V}$$

$$\therefore \quad \frac{125}{6} = \frac{110 - 65}{R} \qquad \therefore \quad R = \mathbf{2{\cdot}16\ \Omega}$$

Example 14-13. *A charging booster (shunt generator) is to charge a storage battery of 100 cells each of internal resistance 0·001 Ω. Terminal p.d. of each cell at completion of charge is 2·55 V. Calculate the e.m.f. which the booster must generate to give a charging current of 20 A at the end of charge. The armature and shunt field resistances of the generator are 0·2 and 258 Ω respectively and the resistance of the cable connectors is 0·05 Ω.*

Solution.

Terminal p.d./cell $= 2{\cdot}55$ volt

The charging voltage across the battery must be capable of overcoming the back e.m.f. and also to supply the voltage drop across the internal resistance of the battery.

Back e.m.f. $= 100 \times 2{\cdot}55 = 255$ V

Voltage drop on internal resistance

$= 100 \times 0{\cdot}001 \times 20 = 2$ V

$\therefore$ P.D. across points A and B (Fig. 14-13)

$= 255 + 2 = 257$

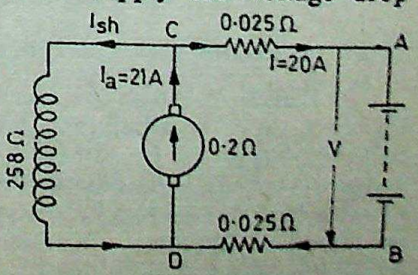

Fig. 14-13

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P.D. across terminals C and D of the generator

$$= 257 + (20 \times 0{\cdot}05) = 258 \text{ V}$$

$$\therefore \quad I_{sh} = 258/258 = 1 \text{ A}$$

$$I_a = 20 + 1 = 21 \text{ A}$$

$$I_a R_a = 21 \times 0{\cdot}2 = 4{\cdot}2 \text{ V}$$

$$\therefore \quad \text{Generated e.m.f.} = 258 + 4{\cdot}2 = 262{\cdot}2 \text{ V}$$

14-22. Sulphation—Causes and Cure

If the cell is left incompletely charged or is not fully charged periodically, then the lead sulphate formed during discharge, is not converted back into PbO_2 and Pb. Some of the unreduced $PbSO_4$ which is left, gets deposited on the plates which are then said to be sulphated. $PbSO_4$ is in the form of minute crystals which gradually increase in size if not reduced by thoroughly charging the cell. It increases the internal resistance of the cell thereby reducing its efficiency and capacity. Sulphation also sets in if the battery is over-charged or left discharged for a long time.

Sulphated cells can be cured by giving them successive over-charges, for which purpose they are cut out of the battery during discharge, so that they can get two charges with no intervening discharge. The other method, in which sulphated cells need not be cut out of the battery is to continue them charging with a **'milking booster'** even after the battery as a whole has been charged. A milking booster is a motor-driven low-voltage dynamo which can be connected directly across the terminals of the sulphated cells.

14-23. Maintenance of Lead-acid Cells

The following important points should be kept in mind for keeping the battery in good condition :—

(*i*) Discharging should not be prolonged after the minimum value of the voltage for the particular rate of discharge is reached.

(*ii*) It should not be left in discharged condition for long.

(*iii*) The level of the electrolyte should always be above the top of the plates which must not be left exposed to air. Evaporation of electrolyte should be made up by adding distilled water occasionally.

14-24. Alkaiine Accumulators

Alkaline batteries are ideally suited for portable work. There are two types of alkaline batteries which are in general use (*i*) the nickel-iron type (or Edison type) and (*ii*) Nickel-cadmium type (or Junger type) also commercially known as Nife battery. Another alkaline battery which differs from the above only in the mechanical details of its plates, is known as Alkum accumulator which uses nickel hydroxide and graphite in the positive plates and a powdered alloy of iron and chromium in the negative plates.

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14-25. Edison Alkali Cell *(A.M.I.E. Winter 1986)*

The active materials in the cell are nickel oxide for the positive plate and iron oxide for the negative plate. The electrolyte is 21 per cent solution of caustic potash KOH (potassium hydrate) to which is added a small quantity of lithium hydrate LiOH for increasing the capacity of the cell.

14-26. Construction

The positive plates are built up from a number of tubes made of nickel-plated perforated steel ribbon wound spirally. The tubes are usually 10 cm long and 6 mm in diameter and are held in shape by steel rings. The active material, consisting of apple-green oxide of nickel NiO_2 or $Ni(OH)_4$ and flakes of metallic nickel (added to increase conductivity of the electrolyte) is packed into the steel tubes in alternate layers. After being filled up, the tubes are then clamped in a nickel-plated steel frame.

The negative plate consists of a number of oblong or rectangular pockets stamped from a finely-perforated nickelled-steel ribbon or strip, the pockets being filled with powdered iron oxide. A little bit of mercury is added to improve its conductivity.

The plates, in the form of tubes, are held in a nickel-plated steel container with welded seams and are kept insulated from each other by hard rubber strip separators.

14-27. Chemical Changes *(A.M.I.E. Nov. 1969)*

The exact nature of the chemical changes taking place in such a cell is not clearly understood because the exact formula for the nickel oxide is not yet well established, but the action of the cell can be understood by assuming the peroxide NiO_2 or its hydrated form $Ni(OH)_4$.

First, let us assume that at positive plate, nickel oxide is in its hydrated form $Ni(OH)_4$. During discharge, electrolyte KOH splits up into K ions and OH ions. The K ions go to anode and reduce $Ni(OH)_4$ to $Ni(OH)_2$. The OH ions travel towards the cathode and oxidise iron. During charging, just the opposite reactions take place *i.e.* K ions go to cathode and OH ions go to anode. The chemical reactions can be written thus

$$KOH \rightarrow K + OH$$

During Discharge

Positive Plate : $Ni(OH)_4 + 2K \rightarrow Ni(OH)_2 + 2KOH$

Negative Plate : $Fe + 2OH \rightarrow Fe(OH)_2$

During Charging

Positive Plate : $Ni(OH)_2 + 2OH \rightarrow Ni(OH)_4$

Negative Plate : $Fe(OH)_2 + 2K \rightarrow Fe + 2KOH$

The charge and discharge can be represented by a single reversible equation thus

$$\underset{\text{+ Plate}}{Ni(OH)_4} + KOH + \underset{\text{−ve Plate}}{Fe} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{+ Plate}}{Ni(OH)_2} + KOH + \underset{\text{−ve Plate}}{Fe(OH)_2}$$

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It will be observed from the above equation that as no water is formed, there is no overall change in the strength of electrolyte. Its function is merely to serve as a conductor or as a vehicle for the transfer of OH ions from one plate to another. Hence, the specific gravity of the electrolyte remains practically constant, both during charging and discharging. That is why only a small amount of electrolyte is required which fact enables the cells to be small in bulk.

Note. If, however, we assume the nickel oxide to be in the form NiO_2, then the above reactions can be represented by the following reversible equation.

$$\underset{\text{+Plate}}{6NiO_2} + 8KOH + \underset{\text{−ve Plate}}{3Fe} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \underset{\text{+Plate}}{2Ni_3O_4} + 8KOH + \underset{\text{−ve Plate}}{Fe_3O_4}$$

14-28. Electrical Characteristics

The e.m.f. of an Edison cell, when fully charged, is nearly 1·4 V which decreases rapidly to 1·3 V and then very slowly to 1·1 or 1·0 V on discharge. The average discharge voltage for a 5-hour discharge rate is 1·2 V. Hence, for the same average value of the voltage, an alkali accumulator will consist of 1·6 to 1·7 times as many cells as in a lead-acid battery. The internal resistance of an alkali cell is nearly five times that of the lead-acid cell, hence there is a relatively greater difference between its terminal voltages when charging and discharging.

The average charging voltage for an alkali cell is about 1·7 V. The general shapes of the charge and discharge curves for such cells are, however, similar to those for lead-acid cells. The rated capacity of nickel accumulators usually refers to 5-hour discharge rate unless stated otherwise.

The plates of such cells have greater mechanical strength because of all-steel construction. They are comparatively lighter because (*i*) their plates are lighter and (*ii*) they require less quantity of electrolyte. They can withstand heavy charge and discharge currents and do not deteriorate even if left discharged for long periods.

Due to its relatively higher internal resistance, the efficiencies of an Edison cell are lower than those of the lead acid cell. On the average, its Ah efficiency is about 80% and Wh efficiency 60 or 50%.

With increase in temperature, e.m.f. is increased slightly but capacity increases by an appreciable amount. With decrease in temperature, the capacity decreases becoming practically zero at 4°C even though the cell is fully charged. This is a serious drawback in the case of electrically-driven vehicles in cold weather and precautions have to be taken to heat up the battery before starting, though in practice the I^2R loss in the internal resistance of the battery is sufficient to keep the battery cells warm when running.

The principal disadvantage of the Edison battery or nickel-iron battery is its high initial cost (which will probably be sufficiently reduced when patents expire). At present, an Edison battery costs approximately twice as much as a lead-acid battery designed for similar service. But since the alkaline battery outlasts an

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indeterminate number of lead-acid batteries, it is cheaper in the end.

Because of their lightness, compact construction, increased mechanical strength, ability to withstand rapid charging and discharging without injury and freedom from corrosive liquids and fumes, alkali batteries are idealy suited for traction work such as propulsion of electric factory trucks, mine locomotives, miner's lamps, lighting and starting of public service vehicles and other services involving rough usage etc.

14-29. Nickel-cadmium Cell

The positive plate of such a cell is similar to that of an Edison cell with the same active materials, and the same electrolyte but its negative plate is of cadmium. The use of cadmium results in reduced internal resistance of the cell. Such batteries are more suitable than Edison batteries for floating duties in conjunction with a charging dynamo, because in their case, the difference between charging and discharging e.m.fs. is not as great as in nickel-iron cell batteries.

14-30. Comparison : Lead acid Cell and Edison Cell

The relative strong and weak points of the cells are summarized as under :

Particulars	*Lead-acid cell*	*Edison cell*
1. Positive plate	PbO_2, lead-peroxide	Nickel hydroxide $Ni(OH)_4$ or NiO_2
2. Negative plate	Spongy lead	Iron oxide
3. Electrolyte	diluted H_2SO_4	KOH
4. Average e.m.f.	2·0 V/cell	1·2 V/cell
5. Internal resistance	Comparatively low	Comparatively higher
6. Efficiency : amp-hour watt-hour	90—95% 72—80%	nearly 80% about 60%
7. Cost	comparatively less than alkali cell	almost twice that of Pb-acid cell. Easy maintenance.
8. Life	gives nearly 1250 charges and discharges	Five years at least
9. Strength	needs much care and maintenance. Sulphation occurs often due to incomplete charge or discharge.	due to all-steel construction, they are robust, mechanically strong, can withstand vibration, are light, unlimited rates of charge and discharge. Can be left discharged, free from corrosive liquids and fumes.

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Tutorial Problems—21

1. An alkaline cell is discharged at a steady current of 4 A for 12 hours, the average terminal voltage being 1·2 V. To restore it to its original state of charge, a steady current of 3 A for 20 hours is required, the average terminal voltage being 1·44 V. Calculate the ampere-hour and watt-hour efficiencies. [80% ; 66·7%]

2. Twenty alkaline secondary cells, each of discharge capacity 60 Ah at the 10-hr rate, are to be charged with constant current for 9 hours.

Calculate the maximum and minimum values of the charging resistance required if the direct current supply is 120 V and the Ah efficiency of the cells is 80%. The e.m.f. of each cell at commencement and end of charge respectively is 1·3 V and 1·7 V. [11·28 Ω ; 10·32 Ω]

3. A battery of 10 cells is to be charged at a constant current of 8 A from a generator of e.m.f. 35 V and internal resistance 0·2 Ω. The internal resistance of each cell is 0·08 Ω and its e.m.f. ranges from 1·85 V discharged to 2·15 V charged. Determine the maximum and minimum values of series resistance which must be included in the circuit. [1·0625 Ω ; 0·6875 Ω]

4. A separately-excited d.c. generator has an armature resistance of 0·2 Ω. It is used to charge a battery having an open-circuit e.m.f. of 105 V and an internal resistance of 0·1 Ω. The copper conductors between generator and battery have a total length of 100 m and are 2·5 mm in diameter. What will be the value of the generated e.m.f. of the generator when the battery is being charged at 10 A ?

How much power will be wasted in the conductors between generator and battery ?

The resistance of a copper wire, 1 metre long and 1 mm² cross-section is 1/58 Ω. [111·5 V ; 35·1 W]

HIGHLIGHTS

1. Faraday's Laws of electrolysis are :

(*i*) First Law : The mass of an ion liberated at an electrode is directly proportional to the quantity of electricity that passes through the electrolyte. Mathematically,

$$m = ZIt$$

$$= \left(\frac{1}{F} \times \frac{A}{v}\right) It = \frac{E}{F} It = \frac{E}{F} \cdot Q$$

where Z = *E.C.E.* of the element

E = chemical equivalent weight

= atomic wt./valency.

(*ii*) Second Law : The masses of ions of different substances liberated by the same quantity of electricity are proportional to their electro-chemical equivalent weights. In other words,

$$m_1 : m_2 : m_3 :: Z_1 : Z_2 : Z_3$$

2. Electrochemical equivalent weight of a substance is defined as the mass of its ions liberated by the passage of a charge of one coulomb through the electrolytic solution.

3. Faraday's constant is defined as the charge required to liberate one gram-equivalent of a substance.

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For all substances,

$$\frac{\text{chemical equivalent}}{\text{electrochemical equivalent}} = \text{Faraday's constant}$$

$$= 96{,}500 \text{ C}$$

4. The *E.C.E.*s and chemical equivalents of different substances are inter-related thus

$$\frac{E.C.E. \text{ of } A}{E.C.E. \text{ of } B} = \frac{\text{chemical equivalent of } A}{\text{chemical equivalent of } B}$$

or

$$\frac{E_1}{E_2} = \frac{Z_1}{Z_2}$$

5. The value of back e.m.f. of electrolysis is

$$E = JHZ$$

6. The two efficiencies of a cell are :

(*i*) The quantity or ampere-hour efficiency

(*ii*) The energy or watt-hour efficiency

$$\text{Ah } \eta = \frac{\text{ampere-hours on discharge}}{\text{ampere-hours on charge}}$$

$$\text{Wh } \eta = \frac{\text{watt-hours on discharge}}{\text{watt-hours on charge}}$$

$$= \text{Ah } \eta \times \frac{\text{average volts on discharge}}{\text{average volts on charge}}$$

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ELECTRICAL INSTRUMENTS AND MEASUREMENTS

15

1. Absolute and Secondary Instruments 2. Electrical Principles of Operation 3. Essentials of Indicating Instruments 4. Deflecting Torque 5. Controlling Torque 6. Damping Torque 7. Moving-iron Ammeters and Voltmeters 8. Attraction Type M.I. Instruments 9. Repulsion Type M.I. Instruments 10. Sources of Error 11. Advantages and Disadvantages 12. Extension of Range by Shunts and Multipliers 13. Moving-coil Instruments 14. Permanent-magnet Moving-coil Type Instruments 15. Extension of Range 16. Electrodynamic or Dynamometer Type Instruments 17. Hot-wire Instruments 18. Range Extension 19. Induction Type Instruments 20. Induction Ammeters 21. Disc Ammeter with Split-phase Winding 22. Shaded-pole Type Ammeters 23. Induction Voltmeters 24. Errors in Induction Ammeters and Voltmeters 25. Advantages and Disadvantages 26. Electrostatic Voltmeters 27. Attracted Disc Type Voltmeter 28. Quadrant Type Voltmeters 29. Kelvin's Multicellular Voltmeter 30. Advantages and Limitations of Electrostatic Voltmeters 31. Range Extension of Electrostatic Voltmeters 32. Wattmeters 33. Dynamometer Wattmeters 34. Induction Wattmeters 35. Advantages and Limitations of Induction Wattmeters 36. Energy Meters 37. Electrolytic Meter 38. Motor Meters 39. Errors in Motor Meters 40. Quantity or Ampere-hour Meter 41. Ampere-hour Mercury Motor Meter 42. Friction Compensation 43. Mercury Meter Modified as Watthour Meter 44. Induction Type Single-phase Watthour Meter 45. Errors in Induction Watthour Meter 46. Megger 47. Wheatstone Bridge 48. D.C. Potentiometer 49. Direct-reading Potentiometer 50. Standardizing the Potentiometer 51. Calibration of Ammeters 52. Calibration of Voltmeters 53. Highlights.

15-1. Absolute and Secondary Instruments

The various electrical instruments may, in a very broad sense, be divided into (*i*) *absolute* instruments and (*ii*) *secondary* instruments. Absolute instruments *are those which give the value of the quantity to be measured in terms of the constant of the instrument and their deflection only.* No previous calibration or comparison

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is necessary in their case. The example of such an instrument is tangent galvanometer which gives the value of current in terms of the tangent of deflection produced by the current and of the radius and number of turns of wire used and the horizontal component of earth's field.

Secondary instruments are those in which the value of electrical quantity to be measured can be determined from the deflection of the instrument only when they have been pre-calibrated by comparison with an absolute instrument. Without calibration, the deflection of such instruments is meaningless.

It is the secondary instruments which are most generally used in everyday work, the use of the absolute instruments being merely confined within laboratories as standardizing instruments.

15-2. Electrical Principles of Operation

All electrical measuring instruments depend for their action on one of the many physical effects of an electric current or potential and are generally classified according to which of these effects is utilized in their operation. The effects generally utilized are :—

(*i*) Magnetic Effect...for ammeters and voltmeters usually.

(*ii*) Electrodynamic Effect...for ammeters, voltmeters but particularly for wattmeters.

(*iii*) Electromagnetic effect...for ammeters, voltmeters, wattmeters and watthour meters.

(*iv*) Thermal Effect...for ammeters and voltmeters.

(*v*) Chemical Effect...for d.c. ampere-hour meters.

(*vi*) Electrostatic Effect...for voltmeters only.

Another way to classify secondary instruments is to divide them into (*i*) *indicating instruments* (*ii*) *recording instruments* and (*iii*) *integrating instruments.*

Indicating instruments are those which indicate the instantaneous value of the electrical quantity being measured *at the time* at which it is being measured. Their indications are given by pointers moving over calibrated dials. Ordinary ammeters, voltmeters and wattmeters belong to this class.

Recording instruments are those which, instead of indicating by means of a pointer and a scale the instantaneous value of an electrical quantity, give a *continuous record* of the variations of such a quantity over a selected period of time. The moving system of the instrument carries an inked-pen which rests lightly on a chart or graph that is moved, at a uniform and low speed, in a direction perpendicular to that of the deflection of the pen. The path traced out by the pen presents a continuous record of the variations in the deflection of the instruments.

Integrating instruments are those which measure and register by a set of dials and pointers either the *total* quantity of electricity

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(in ampere-hours) or the *total* amount of electrical energy (in watt-hours or kWh) supplied to a circuit in a given time. Their summation gives the product of time and the electrical quantity but gives no direct indication as to the *rate* at which the quantity or energy is being supplied because their registrations are independent of this rate provided the current flowing through the instrument is sufficient to operate it.

Ampere-hour and watt-hour meters fall in this class.

15-3. Essentials of Indicating Instruments

As defined above, indicating instruments are those which indicate the value of the quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale and which is attached to a moving system pivoted in jewelled bearings. The moving system is subjected to the following three torques :

(1) A deflecting (or operating) torque

(2) A controlling (or restoring) torque

(3) A damping torque

15-4. Deflecting Torque

The deflecting or operating torque (T_d) is produced by utilizing one or other effects mentioned in Art. 15-2 *i.e.* magnetic, electrostatic, electrodynamic, thermal or chemical etc. The actual method of torque production depends on the type of instrument and will be discussed in the succeeding paragraphs. This deflecting torque causes the moving system (and hence the pointer attached to it) to move from its '*zero*' position *i.e.* its position when the instrument is disconnected from the supply

15-5. Controlling Torque

The deflection of the moving system would be indefinite if there were no controlling or restoring torque. This torque opposes the deflecting torque and increases with the deflection of the moving system. The pointer is brought to rest at a position where the two opposing torques are numerically equal. The deflecting torque ensures that currents of different magnitudes shall produce deflections of the moving system in proportion to their size. Without such a torque, the pointer would swing over to the maximum deflected position irrespective of the size of the current to be measured. Moreover, in the absence of a restoring torque, the pointer once deflected, would not return to its zero position on removing the current. The controlling or restoring or balancing torque in indicating instruments is either obtained by a spring or by gravity as described below.

(*a*) Spring Control

A hair-spring, usually of phosphor-bronze, is attached to the moving system of the instrument as shown in Fig. 15-1 (*a*).

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With the deflection of the pointer, the spring is twisted in the opposite direction. This twist in the spring produces restoring torque which is directly proportional to the angle of deflection of the moving system. The pointer comes to a position of rest (or equilibrium) when the deflecting torque (T_d) and controlling torque (T_c) are equal. For example, in permanent magnet moving-coil type of instruments, the deflecting torque is proportional to the current passing through them.

$\therefore \quad T_d \propto I$ and for spring control $T_c \propto \theta$

As $\quad T_c = T_d \quad \therefore \quad \theta \propto I$

Since deflection θ is directly proportional to current I, the spring-controlled instruments have a uniform or equally-spaced scales over the whole of their range as shown in Fig. 15-1 (*b*).

To ensure that controlling torque is proportional to the angle of deflection, the spring should have a fairly large number of turns

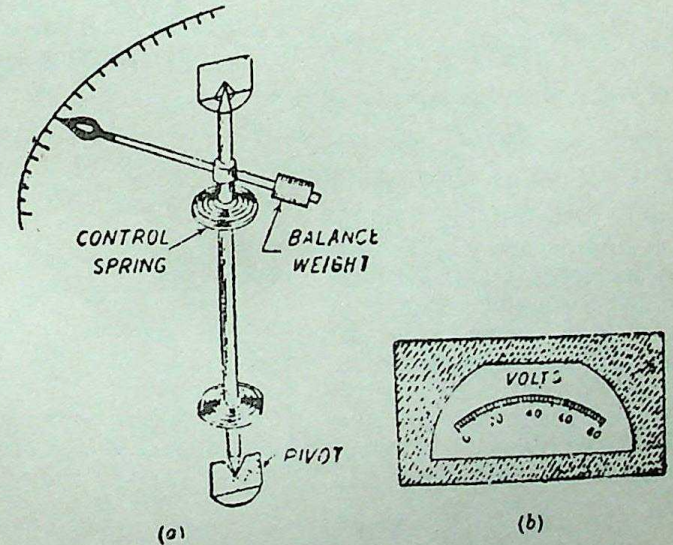

Fig. 15-1

so that angular deformation per unit length, on full-scale deflection, is small. Moreover, the stress in the spring should be restricted to such a value that it does not produce a permanent set in it.

Springs are made of such materials which

(*i*) are non-magnetic.

(*ii*) are not subject to much fatigue.

(*iii*) have low specific resistance—especially in cases where they are used for leading the current in or out of the instrument.

(*iv*) have low temperature-resistance coefficient.

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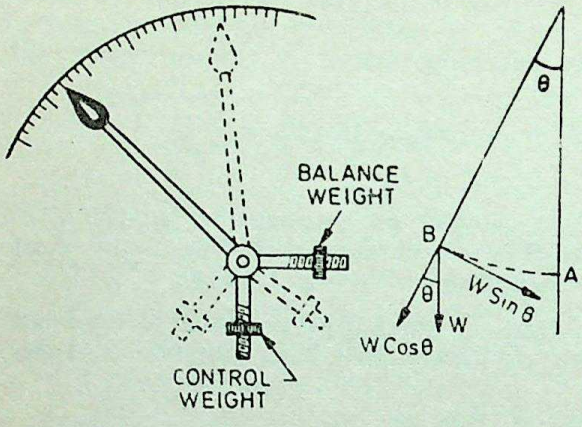
(*b*) Gravity Control

Gravity control is obtained by attaching a small adjustable weight to some part of the moving system such that the two exert torques in the opposite directions. The usual arrangement is shown in Fig. 15-2 (*a*).



(*a*) (*b*)

Fig. 15-2

It is seen from Fig. 15-2 (*b*) that the controlling or restoring torque is proportional to the sine of the angle of deflection *i.e.* $T_c \propto \sin\theta$. The degree of control is adjusted by screwing the weight up or down the carrying system.

If $T_d \propto I$, then for position of rest

$$T_d = T_c$$

or $$I \propto \sin\theta \quad (not\ \theta)$$

It will be seen from Fig. 15-2 (*b*) that as θ approaches 90°, the distance AB increases by a relatively small amount for a given change in the angle than when θ is just increasing from its zero value. Hence, gravity-controlled instruments have scales which are not uniform but are cramped or 'crowded' at their lower ends as shown in Fig. 15-3.

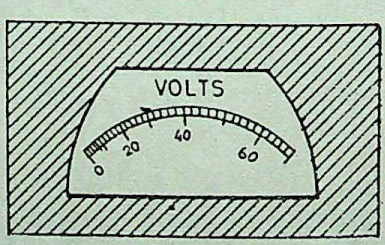


Fig. 15-3

As compared to spring control, the disadvantages of gravity control are :

(*i*) it gives cramped scale

(*ii*) the instrument has to be kept vertical.

However, gravity control has the following advantages :

(*i*) it is cheap

(*ii*) it is unaffected by temperature

(*iii*) it is not subject to fatigue or deterioration with time.

15-6. Damping Torque

(*A.M.I.E. Nov. 1969*)

A damping force is one which acts on the moving system of the instrument *only when it is moving* and always opposes its motion.

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Such a stabilizing or damping force is necessary to bring the pointer to rest *quickly*, otherwise due to inertia of the moving system, the pointer will oscillate about its final deflected position quite for some time before coming to rest in the steady position. The degree of damping should be adjusted to a value which is sufficient to enable the pointer to rise quickly to its deflected position without overshooting. In that case, the instrument is said to be *dead-beat*. Any increase of damping above this limit *i.e.* over-damping will make the instrument slow and lethargic. In Fig. 15-4 is shown the effect of damping on the variation of position, with time, of the moving system of an instrument.

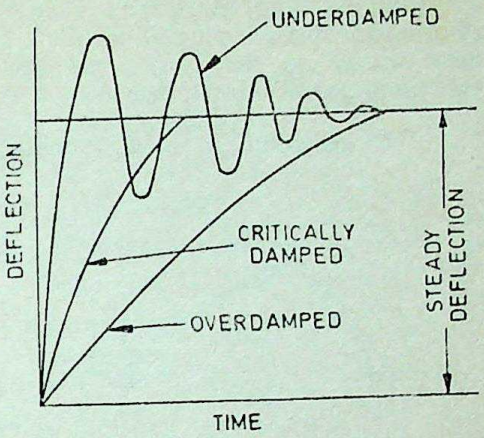

Fig. 15-4

The damping force can be provided by (*i*) *air friction* (*ii*) *eddy current* and (*iii*) *fluid-friction* (used occasionally).

Two methods of air-friction damping are shown in Fig. 15-5 (*a*) and (*b*). In Fig. 15-5 (*a*) the light aluminium piston attached to the moving system of the instrument is arranged to travel with a very small clearance, in a fixed air chamber closed at one end. The cross-section of the chamber is either circular or rectangular. Damping of the oscillations is affected by the compression and suction actions of the piston on the air enclosed in the chamber. Such a system of damping is not much favoured these days, those shown in Fig. 15-5 (*b*) and (*c*) being preferred. In the latter method, one or two light aluminium vanes are mounted on the spindle of the moving system which moves in a closed sector-shaped box as shown.

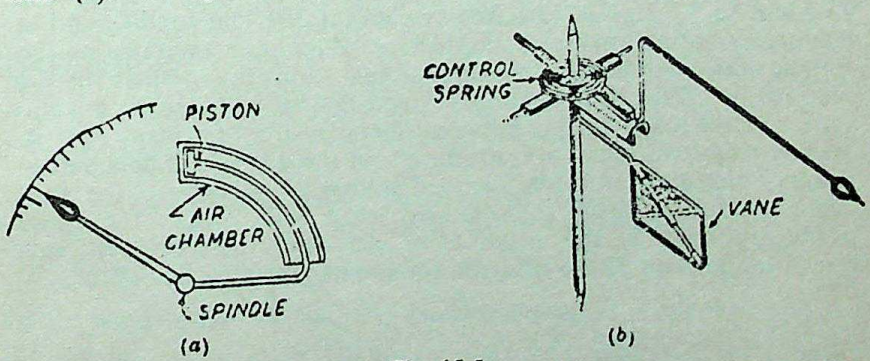

Fig. 15-5

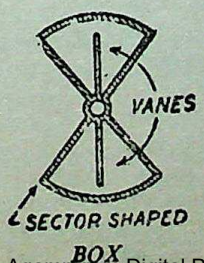

Fig. 15-5 (*c*)

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Fluid friction is similar in action to the air friction. Due to greater viscosity of oil, the damping is more effective. However, oil damping is not much used because of several disadvantages such as objectionable creeping of oil, the necessity of using the instrument always in the vertical position and its obvious unsuitability for use in portable instruments.

The eddy-current form of damping is the most efficient of the

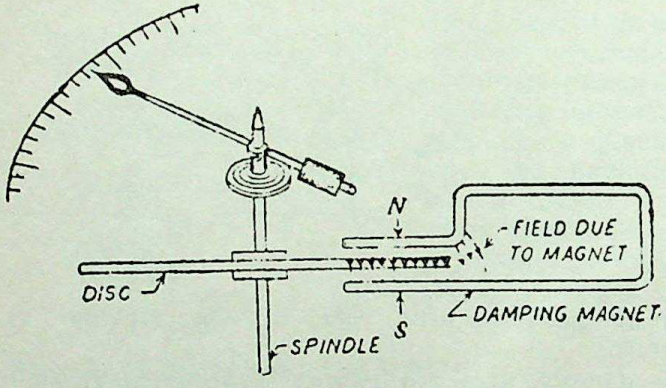

Fig. 15-6

three. The two forms of such a damping are shown in 15-6 and 15-7. In Fig. 15-6 is shown a thin disc of a conducting but *non-magnetic* material like copper or aluminium mounted on the spindle which carries the moving system and the pointer of the instrument. The disc is so positioned that its edge, when in rotation, cuts the magnetic flux between the poles of a permanent magnet. Hence, eddy currents are produced in the disc which flow and so produce a damping force in such a direction as to oppose the very cause producing them (Lenz's Law, Art. 8-6). Since the cause producing them is the rotation of the disc, these eddy currents retard the motion of the disc and the moving system as a whole.

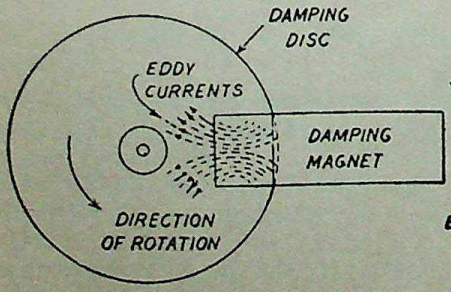

Fig. 15-7

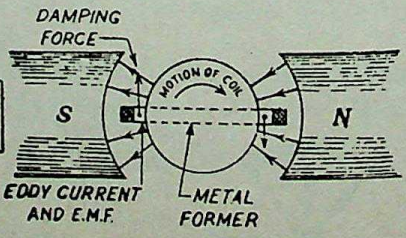

Fig. 15-8

In Fig. 15-8 is shown the second type of eddy-current damping generally employed in permanent-magnet moving-coil instruments. The coil is wound on a thin light aluminium former in which eddy

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currents are produced when the coil moves in the field of the permanent magnet. The directions of the induced currents and of the damping force produced by them are shown in the figure given above.

The various types of instruments and the order in which they would be discussed in this chapter are given below :—

Ammeters and voltmeters

(1) Moving-iron type (both for D.C./A.C.)

(*a*) *the attraction type* (*b*) *the repulsion type*

(2) Moving-coil type

(*a*) *permanent-magnet type* (for D.C. only)

(*b*) *electrodynamic or dynamometer type* (for D.C./A.C.)

Hot-wire type (for D.C./A.C.)

(4) Induction type (for A.C. only)

(*a*) *Split-phase winding type* (*b*) *Shaded-pole type*

(5) Electrostatic type...for voltmeters only (for D.C./A.C.)

Wattmeters

(6) Dynamometer type (for D.C./A.C.)

(7) Induction type (for A.C. only)

Energy Meters

(8) Electrolytic type (for D.C. only)

(9) Motor Meters

(*i*) *Mercury Motor Meter.* For D.C. work only. Can be used as ampere-hour or watt-hour meter.

(*ii*) *Induction type.* For A.C. only.

15-7. Moving-iron Ammeters and Voltmeters (1978)

There are two basic forms of these instruments *i.e.* the *attraction* type and the *repulsion* type. The operation of the attraction type depends on the attraction of a single piece of soft iron into a magnetic field and that of repulsion type depends on the repulsion of two adjacent pieces of iron magnetised by the same magnetic field. For both types of these instruments, the necessary magnetic field is produced by the ampere-turns of a current-carrying coil. In case the instrument is to be used as an ammeter, the coil has comparatively few turns of thick wire so that the ammeter has low resistance because it is connected in series with the circuit. In case it is to be used as a voltmeter, the coil has high impedance so as to draw as small a current as possible since it is connected in parallel with the circuit. As the current through the coil is small, it has large number of turns in order to produce sufficient ampere-turns.

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15-8. Attraction Type M.I. Instruments (*A.M.I.E. Summer 1986*)

The basic working principle of an attraction type moving-coil instrument is illustrated in Fig. 15-9. It is well-known that if a piece of unmagnetised soft iron is brought up near either of the two ends of a current-carrying coil, it would be attracted into the coil in the same way as it would be attracted by the pole of a bar magnet. Hence, if we pivot an oval-shaped disc of soft iron on a spindle between bearings and near the coil, the iron disc will swing into the coil when the latter has an electric current passing through it. As the field strength would be strongest at the centre of the coil, the oval-shaped iron disc is pivoted in such a way that the greatest bulk of iron moves into the centre of the coil. If a pointer is fixed to the spindle carrying the disc, then the passage of current through the coil will cause the pointer to deflect. The amount of deflection produced would be greater when the current producing the magnetic field is greater. Another point worth noting is that *whatever the direction of current through the coil, the iron disc would always be magnetised in such a way that it is pulled inwards.* Hence, such instruments can be used both for direct as well as for alternating currents.

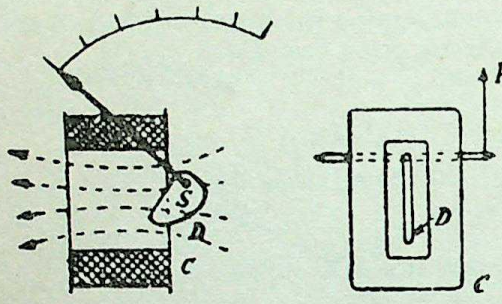

Fig. 15-9

When the current to be measured is passed through the coil *C*, the magnetic field is produced which attracts the eccentrically mounted disc inwards thereby deflecting the pointer which moves over a calibrated scale which is made as uniform as possible by shaping the disc suitably.

Deflecting Torque

The force (F) pulling the iron disc inwards depends on (i) the strength of the field H produced by the coil and (ii) the pole strength (m) developed by the disc which again depends on H. In other words,

$$F \propto mH \propto H^2$$

Hence, the deflecting torque,

$$T_d \propto F \propto H^2$$

If the relative permeability of iron is assumed constant, then $H \propto$ current I.

$$\therefore \quad T_d \propto H^2 \propto I^2 \qquad (i)$$

If spring control is used, then the controlling torque is given by,

$$T_c \propto \text{deflection of the disc, } \theta$$

or

$$T_c \propto \theta \qquad ...(ii)$$

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In the steady position of deflection,

$$T_d = T_c$$

Hence, from Eq. (*i*) and (*ii*), we get

$$\theta \propto I^2$$

If alternating current is used, then

$$\theta \propto I_{r.m.s.}^2$$

Obviously in both cases, the scale is uneven, although, as stated earlier, the disc is specially shaped to give a scale as nearly uniform as possible.

Alternate Treatment

The value of the deflecting torque can also be found in terms of the inductance L of the instrument. The inductance of the coil of the instrument changes with the deflection θ in both the attraction type and repulsion type moving-iron instruments. In the attraction type, inductance of the instrument increases with increase in the deflection of the iron disc. In the case of repulsion type the increase in instrument inductance is due to the increase of flux produced by the decrease in demagnetising effect of one iron rod on the other as the two move farther apart. Hence, there is relationship between the deflecting torque and inductance of the instrument as shown below.

Suppose that when a direct current of I passes through the instrument, its deflection is θ and the inductance is L. Also suppose that when current changes from I to $(I+dI)$, deflection changes from θ to $(\theta+d\theta)$ and L changes to $(L+dL)$. Then the increase in the energy stored in the magnetic field is given by

$$dE = d(\tfrac{1}{2}LI^2) = \tfrac{1}{2}L.2I.dI + \tfrac{1}{2}I^2.dL$$
$$= LI.dI + \tfrac{1}{2}I^2dL \text{ joules}$$

If T newton-metres is the controlling torque for deflection θ, then extra energy stored in control is $T \times d\theta$ joules. Hence, the total increase in energy of the system is

$$= LI.dI + \tfrac{1}{2}I^2dL + T \times d\theta \qquad \ldots(i)$$

The e.m.f. induced in the coil is

$$e = \frac{d}{dt}(N\Phi)$$

Now $L = N\Phi/I$ or $N\Phi = LI$

$$\therefore \quad e = \frac{d}{dt}(LI)$$

The energy drawn from the supply to overcome this e.m.f.

$$= eI.dt = \frac{d}{dt}(LI).I.dt = I.d(LI)$$
$$= I(L.dI + I\,dL) = LI.dI + I^2.dL \text{ joules} \qquad \ldots(ii)$$

Equating Eq. (*i*) and (*ii*), we get

$$LI.dI + \tfrac{1}{2}I^2dL + T\,d\theta = LI.dI + I^2.dL$$

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$$\therefore \qquad T = \tfrac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m}$$

Damping

As shown, air-friction damping is provided, the actual arrangement being a light piston moving in an air-chamber.

15-9. Repulsion Type M.I. Instruments (*A.M.I.E. Summer 1986*)

A simple sketch of such an instrument is shown in Fig. 15-10. The arrangement consists of a fixed coil *C* inside which are placed two soft-iron rods (or bars) *A* and *B* parallel to one another and along the axis of the coil *C*. One of them, *i.e.* *A* is fixed and the other, *B*, which is movable, carries a pointer that moves over a calibrated scale. When the current to be measured is passed through the fixed coil, it sets up its own magnetic field which magnetises the two rods similarly *i.e.* the adjacent points on the lengths of the rods will have the same magnetic polarity. Hence, they repel each other with the result that the pointer is deflected against the controlling torque of a spring or gravity. The force of repulsion is approximately proportional to the square of the current passing through the coil. Moreover, whatever may be the direction of the current through the coil, the two rods will be magnetised similarly and hence will repel each other.

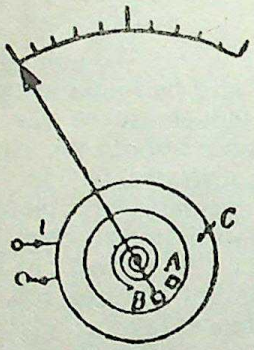

Fig. 15-10

In order to achieve uniformity of scale, two tongue-shaped strips of iron are used instead of two rods. As shown in Fig. 15-11(*a*) the fixed iron consists of a tongue-shaped sheet iron bent into a cylindrical form, the moving iron also consists of another sheet of iron and is so mounted as to move parallel to the fixed iron and towards its narrower end.

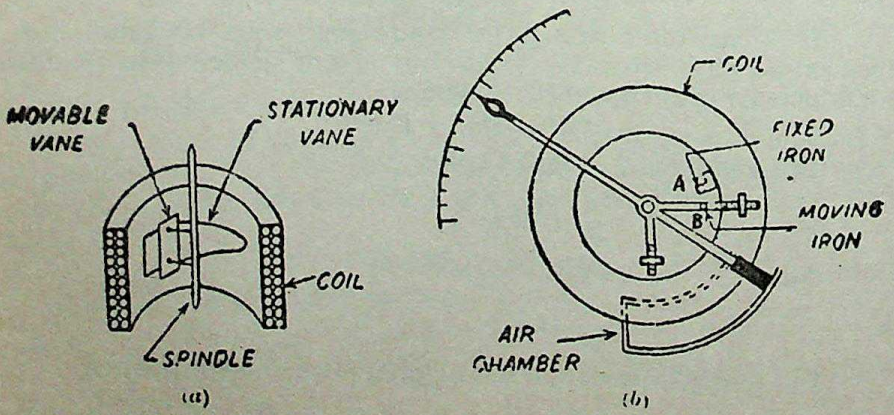

Fig. 15-11

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Deflecting torque

The deflecting torque is due to the repulsive force between the two similarly-magnetised iron rods or sheets.

Instantaneous torque $\propto$ repulsive force

$\propto m_1 m_2$ ...product of pole strengths

Since pole strengths are proportional to the magnetising field H of the coil.

$\therefore$ Instantaneous torque $\propto H^2$

Since H itself is proportional to current (assuming *constant* permeability) passing through the coil,

$\therefore$ Instantaneous torque $\propto i^2$

Hence, the deflecting torque, which is proportional to the mean torque is, in effect, proportional to the mean value of i^2. Therefore when used on alternating current, the instrument reads the r.m.s. value of the current.

Scales of such instruments are uneven if rods are used and uniform if suitably-shaped pieces of iron sheet are used.

The instrument is spring-controlled.

Damping is pneumatic, eddy current damping cannot be employed because the presence of a permanent magnet required for such purpose would affect the deflection and hence the reading of the instrument.

Since the polarity of both iron rods reverses simultaneously, the instrument can be used both for alternating and direct currents *i.e.* instrument belongs to the unpolarised class.

15-10. Sources of Error *(A.M.I.E. Winter 1978)*

There are two types of possible errors in such instruments, firstly, those which occur both in alternating current and direct current work and secondly, those which occur in alternating current work alone.

(*a*) Errors with both D.C. and A.C.

(*i*) *Error due to hysteresis.* Because of hysteresis in the iron parts of the moving system, the readings are higher for descending values but lower for ascending values.

The hysteresis error is almost completely eliminated by using Mumetal or Permalloy which have negligible hysteresis loss.

(*ii*) *Error due to stray fields.* Unless shielded effectively from the effect of stray external fields, it will give wrong readings. Magnetic shielding of the working parts is obtained by using a covering case of cast-iron.

(*b*) Errors with A.C. only

Changes of frequency produce (*i*) change in the impedance of the coil and (*ii*) change in the magnitude of the eddy currents. The

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increase in impedance of the coil with increase in the frequency of the alternating current is of importance in voltmeters* (Ex. 15-1). For frequencies higher than the one used for calibration, the instrument gives lower values. However, this error can be removed by connecting a capacitor of suitable value in parallel with the swamp resistance R of the instrument. It can be shown that the impedance of the whole circuit of the instrument becomes independent of frequency if $C=L/R^2$ where C is the capacitance of the capacitor**

15-11. Advantages and Disadvantages

Such instruments are cheap and robust, give a reliable service and can be used both on alternating and direct current circuits, although they cannot be calibrated with a high degree of precision with D.C. on account of the effect of hysteresis in the iron rods or vanes. Hence, they are usually calibrated by comparison with an alternating current standard.

15-12. Extension of Range by Shunts and Multipliers

(*i*) As Ammeter

The range of the moving-iron instrument as ammeter can be extended by using a suitable shunt across its terminals. So far as the operation with direct currents is concerned, there is no trouble but with alternating currents, the division of current between the instrument and the shunt changes with the change in the applied frequency. For a.c. work, both the inductance and the resistance of the instrument coil and shunt have to be considered. Obviously,

$$\frac{\text{Current through instrument, } i}{\text{Current through shunt, } I_s}=\frac{\sqrt{R_s^2+\omega^2L_s^2}}{\sqrt{R^2+\omega^2L^2}}=\frac{Z_s}{Z}$$

where R and L are the resistance and inductance of the instrument coil and R_s and L_s are the corresponding values for the shunt. It can be shown that the above ratio *i.e.* division of current between the instrument and the shunt would be independent of frequency if the time constant of the instrument coil and shunt are the same *i.e.* if L/R equals L_s/R_s. The multiplying power (N) of the shunt is given by

$$N=\frac{I}{i}=1+\frac{R}{R_s}$$

where I=line current

i=full-scale deflection current of the instrument (without shunt)

---

*At higher frequencies, an error is introduced due to the capacitance existing between different turns of the coil of the instrument when used as an ammeter. This capacitance error can be compensated by an additional circuit consisting of resistance and capacitance in series, connected across the instrument terminals.

**This is the value of capacitance required when resistance of the instrument coil r is not negligible as compared to high series resistance R. However, if r is negligible as compared to R, then $C=0{\cdot}41\ L/R^2$.

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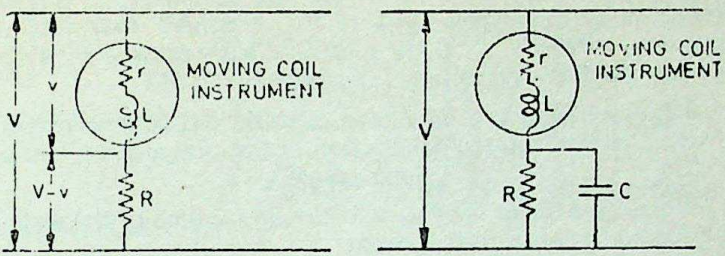
(*ii*) As Voltmeter

The range of this instrument, when used as a voltmeter can be extended or multiplied by using a high non-inductive series resistance R connected in series with it as shown in Fig. 15-12 (*a*). That



(*a*) (*b*)

Fig. 15-12

is why these series resistances are known as 'multipliers'. When used for d.c. work, suppose, its range is to be extended from v to V. Then, obviously, the excess voltage of $(V-v)$ is to be dropped across R. If i is the full-scale deflection current of the instrument, then

$$iR = V - v$$

$$\therefore \quad R = \frac{V-v}{i} = \frac{V-ir}{i} = \frac{V}{i} - r$$

Now, voltage multiplication $= V/v$

Since $iR = V - v \quad \therefore \quad \frac{iR}{v} = \frac{V}{v} - 1$

or $\frac{iR}{ir} = \frac{V}{v} - 1 \quad \therefore \quad \frac{V}{v} = \left(1 + \frac{R}{r}\right)$

Hence, greater the value of R, greater is the extension in the voltage range of the instrument. For d.c. work, the principal requirement of R is that its value should remain constant *i.e.* it should have a low temperature coefficient. For a.c. work, it is essential that the total impedance of the voltmeter and the series resistance R should remain as nearly constant as possible at different frequencies. That is why R is made as non-inductive as possible in order to keep the inductance of the whole circuit to the minimum. The frequency error introduced by the inductance of the instrument coil can be compensated by shunting R by a capacitor C as shown in Fig. 15-12 (*b*).

Example 15-1. *The coil of a 250-V moving iron voltmeter has a resistance of 500 Ω and an inductance of 1 henry. The current taken by the instrument when placed on 250-V, d.c. supply is 0·05 A. Determine the percentage error when the instrument is placed on 250-V, a.c. supply at 100 Hz.* (*A.M.I.E. Summer 1975*)

Solution. Here, original calibration of the instrument is with direct current.

Total ohmic resistance $= 250/0{\cdot}05 = 5000\ \Omega$

Reactance of coil $= 2\pi \times 100 \times 1 = 628\ \Omega$

Coil impedance $= \sqrt{5000^2 + 628^2} = 5040\ \Omega$

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$\therefore$ Voltage reading on a.c. $= 250 \times 5000/5040 = 244{\cdot}5$ V

$\therefore$ error $= -5{\cdot}5$ V

percentage error $= 5{\cdot}5 \times 100/250 = 2{\cdot}2$

Example 15-2. *A 15-V moving-iron voltmeter has a resistance of 500 Ω and an inductance of 0·12 H. Assuming that this instrument reads correctly on D.C., what will be its reading on A.C. at 15 volts when the frequency is (i) 25 Hz and (ii) 100 Hz.*

Solution. When used for d.c. work, only ohmic resistance is involved. Hence, full-scale deflection of the instrument on d.c. is

$$= 15/500 = 0{\cdot}03 \text{ A}$$

When used for a.c. work, it is the impedance of the instrument which has to be taken into account.

(*i*) Impedance at 25 Hz

$$Z = \sqrt{[500^2 + (2\pi \times 25 \times 0{\cdot}12)^2]} = 500{\cdot}3 \ \Omega$$

$\therefore$ Current at 15 V, 25 Hz is $= 15/500{\cdot}3$ A

$\therefore$ Instrument reading $= 15 \times \dfrac{15/500{\cdot}3}{0{\cdot}03} = \mathbf{14{\cdot}99\ V}$*

(*ii*) Impedance at 100 Hz

$$Z = \sqrt{[500^2 + (2\pi \times 100 \times 0{\cdot}12)^2]} = 505{\cdot}8 \ \Omega$$

$\therefore$ Current at 15 V, 100 Hz is $= 15/505{\cdot}8$ A

$\therefore$ Instrument reading $= 15 \times \dfrac{15/505{\cdot}8}{0{\cdot}03} = \mathbf{14{\cdot}83\ V}$**

Incidentally, it may be noted that as the frequency is increased, the impedance of the instrument is also increased. Hence, the current is decreased and, therefore, the instrument readings are lower.

Example 15-3. *A moving-iron instrument gives full-scale deflection with 100 V. It has a coil of 10,000 turns and a resistance of 2,000 Ω. If the instrument is to be used as an ammeter to give full-scale deflection at 20 A, calculate the necessary number of turns in the coil.* (*A.M.I.E. May 1968*)

Solution. It should be kept in mind that in such instruments, the strength of the magnetic field (and hence the deflecting torque) depends on the number of ampere-turns.

Full-scale deflection current $= 100/2{,}000 = 1/20$ A

Full-scale deflection ampere-turns $= 10{,}000 \times 1/20 = 500$ AT

When current is 20 A, the number of turns required is

$$= 500/20 = \mathbf{25}$$

Example 15-4. *The inductance of a certain moving-iron ammeter is $(8 + 4\theta - \frac{1}{2}\theta^2)$ μH where θ is the deflection in radians from the zero position. The control spring torque is 12×10^{-6} N-m/rad. Calculate the scale position in radians for a current of 2A.*

---

*Or reading $= 15 \times 500/500{\cdot}3 = 14{\cdot}95$ V

**Similarly, reading $= 15 \times 500/505{\cdot}8 = 14{\cdot}83$ V

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Solution. The torque is given by

$$T=\frac{1}{2}\ I^2\ \frac{dL}{d\theta}$$

Now $L=(8+4\theta-\frac{1}{2}\theta^2)\times 10^{-6}$ H

$\therefore$ $\frac{dL}{d\theta}=(0+4-2.\ \frac{1}{2}\theta)\times 10^{-6}$ H/rad

$=(4-\theta)\times 10^{-6}$ H/rad

Let the deflection be θ radians for a current of 2A, then

$$12\times 10^{-6}\times\theta=\tfrac{1}{2}\times 2^2\times(4-\theta)\times 10^{-6}$$

$$12\theta=8-2\theta \quad \therefore \quad \theta=8/14=0{\cdot}571 \text{ radian}$$

Tutorial Problems—22

1. The total resistance of a moving-iron voltmeter is 1000 Ω and the coil has an inductance of 0·765 H. The instrument is calibrated with a full-scale deflection on 50 V d.c. Calculate the percentage error when the instrument is used on (*a*) 25 Hz supply, (*b*) 250 Hz supply, the applied voltage being 50 V in each case. [(*a*) 0·72% (*b*) 36%]

2. A moving-iron voltmeter has full-scale deflection of 75 V and was calibrated to read correctly at 50 Hz when the current taken for full-scale deflection was 50 mA. When a capacitor of 6·36 μF was connected in series with the meter, the current taken was again 50 mA with 75 V applied. Calculate the percentage error in the reading if the meter were used on a 75 V d.c. supply. [−1·4%]

15-13. Moving-coil Instruments

There are two types of such instruments *i.e.* (*i*) *permanent magnet type* which can be used for d.c. only and (*ii*) the *dynamometer type* which can be used both for A.C. and D.C.

15-14. Permanent-magnet Moving-coil Type Instruments

(A.M.I.E. Winter 1986)

The operation of moving-coil permanent-magnet type instrument is based upon the principle that when a current-carrying conductor is placed in a magnetic field, it is acted upon by a force which tends to move it to one side and out of the field.

Construction

As the name indicates, the instrument consists of a permanent magnet and a rectangular coil of many turns wound on light aluminium or copper former inside which is an iron core as shown in Fig. 15-13. The powerful U-shaped permanent magnet is made of Alnico and has soft-iron pole pieces which are bored out cylindrically. Between the magnetic poles is fixed a soft-iron

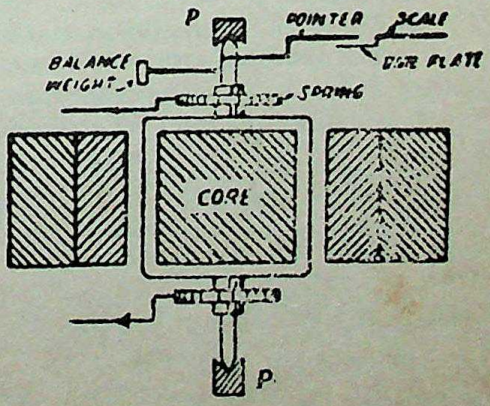

Fig. 15-13

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cylinder whose function is (*i*) to make the field radial and uniform and (*ii*) to decrease the reluctance of the air path between the poles and hence increase the magnetic flux. Surrounding the core is a rectangular coil of many turns wound on a light aluminium frame which is supported by delicate bearings and to which is attached a light pointer. The aluminium frame not only provides support for the coil but also provides damping by eddy currents induced in it. The sides of the coil are free to move in the two air-gaps between the poles and core as shown in Fig. 15-13 and 15-14. Control of the coil movement is affected by two phosphor-bronze hair springs, one above and one below, which additionally serve the purpose of leading the current in and out of the coil. The two springs are spiralled in opposite directions for neutralizing the effects of temperature changes.

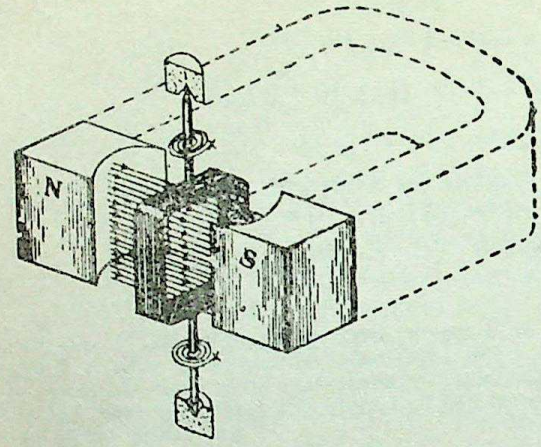

Fig. 15-14

Deflecting Torque

When current is passed through the coil, forces are set up on its both sides which produce a deflection torque as shown in Fig. 15-15. Let

B = flux density in Wb/m^2

l = length or depth of the coil in metres

b = breadth of coil in metres

N = number of turns in the coil

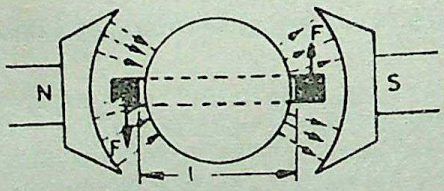

Fig. 15-15

Then, if I amperes is the current passing through the coil, the magnitude of the force experienced by each of its sides is

$$= BIl \text{ newton}$$

For N turns, the force on each side of the coil is

$$= NBIl \text{ newton}$$

$\therefore$ Deflecting torque T_d = Force × perpendicular distance

$$= NBIl \times b = NBI(l \times b) = NBIA \text{ N-m}$$

where A is the face area of the coil.*

*Now $T_d = NBIlA$ N-m. Suppose the deflection of the coil is θ and it changes by $d\theta$, then the change in the flux linked with the coil is $d\phi = BA.d\theta$

$$\frac{d\phi}{d\theta} = BA$$

Hence $$T_d = NIBA = NI \frac{d\phi}{d\theta}$$

This is another useful expression for the deflecting torque.

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It is seen that if B is constant, then T_c is proportional to the current passing through the coil.

Since such instruments are invariably spring-controlled, $T_c \propto$ deflection θ.

Since in the final deflected position $T_d = T_c$

$$\therefore \qquad \theta \propto I$$

Hence, such instruments have uniform scales. Damping is electromagnetic *i.e.* by eddy currents induced in the metal frame over which the coil is wound. Since the frame moves in an intense magnetic field, the induced eddy currents are large and damping is very effective.

Advantages and Disadvantages

The moving-coil permanent-magnet type instruments have the following advantages and disadvantages :

Advantages

(*i*) they have low power consumption.

(*ii*) their scales are uniform and can be designed to extend over an arc of 270° or so.

(*iii*) they possess high (torque/weight) ratio.

(*iv*) they can be modified with the help of shunts and resistances to cover a wide range of currents and voltages.

(*v*) they have no hysteresis loss.

(*vi*) they have very effective and efficient eddy-current damping.

(*vii*) since the operating fields of such instruments are very strong, they are not much affected by stray magnetic fields.

Disadvantages

(*i*) due to delicate construction and the necessary accurate machining and assembly of various parts, such instruments are somewhat costlier as compared to moving-iron instruments.

(*ii*) some errors are set in due to the ageing of control springs and the permanent magnets.

Such instruments are mainly used for d.c. work alone, but they have been sometimes used in conjunction with rectifiers or thermo-junctions for a.c. measurements over a wide range of frequency.

Moving-coil permanent-magnet instruments can be used as (*i*) ammeters (with the help of a low resistance shunt) or (*ii*) voltmeters (with the help of a high series resistance) (*iii*) flux-meters (by eliminating the control springs) and (*iv*) as ballistic galvanometers (by making control springs of large moment of inertia).

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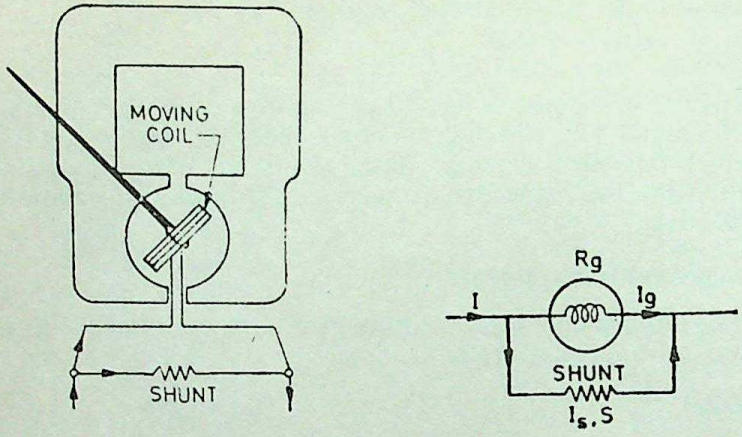
15-15. Extension of Range (*A.M.I.E. Summer 1984*)

(*i*) *As ammeter*

When such an instrument is used as an ammeter, its range can be extended with the help of a low-resistance shunt. This has been



(*a*) Fig. 15-16 (*b*)

fully discussed in Art. 2-16. Fig. 15-16 (*a*) shows the actual arrangement whereas Fig. 15-16 (*b*) shows the diagrammatic representation.

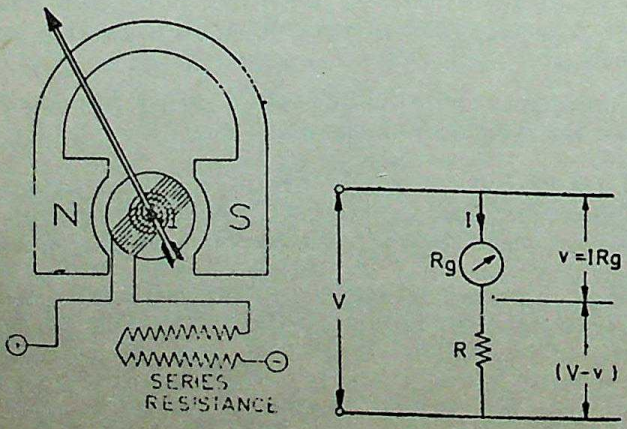
(*ii*) *As voltmeter*

The range of this instrument, when used as a voltmeter can be increased by using a high resistance in series with it (Fig. 15-17).

Let I = full-scale deflection current
R_g = its resistance
$v = IR_g$ = full-scale p.d. across it
V = voltage to be measured
R = series resistance required.

Then, it is seen that the voltage drop across R is $= V - v$

$$\therefore \quad R = \frac{V-v}{I} \quad \text{or} \quad IR = V - v$$



(*a*) Fig. 15-17 (*b*)

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Dividing both sides by v, we get

$$\frac{IR}{v}=\frac{V}{v}-1$$

or

$$\frac{IR}{I.R_g}=\frac{V}{v}-1$$

$$\therefore \quad \frac{V}{v}=\left(1+\frac{R}{R_g}\right)$$

$$\therefore \quad \text{Voltage multiplication}=\left(1+\frac{R}{R_g}\right)$$

Example 15-5. *In a moving-coil instrument, the moving coil has 40 turns and is of square shape with a mean length of 4 cm along each side. The coil hangs in a uniform radial field of 0·08 Wb/m². Find the turning moment of the coil when it is carrying a current of 10 mA.*

Solution. Force on each side of the coil is

$$F=NBIl \text{ newton}$$

Turning moment $=F\times$ width

$$=NBIl\times l=NBIl^2 \text{ N-m}$$

$$=40\times 0{\cdot}08\times 10\times 10^{-3}\times 0{\cdot}04^2 \text{ N-m}$$

$$=512\times 10^{-7} \text{ N-m}$$

Example 15-6. *A moving-coil ammeter is wound with 40 turns and gives full-scale deflection with 5 A. How many turns would be required on the same bobbin to give full-scale deflection with 20 A?*

Solution. Since full-scale deflection ampere-turns have to be the same in both cases,

$$\therefore \quad \text{No. of turns} \quad =\frac{5}{20}\times 40=10$$

Example 15-7. *Two moving-coil meters differ only in that the moving-coil in one has 60 turns and a resistance of 10 Ω, while in the other it has 750 turns and 700 Ω resistance. Find the ratios of their deflections when*

(*a*) *each is connected in turn across a battery of 1·5 V e.m.f. and 50 Ω internal resistance.*

(*b*) *the two meters are connected in series across this battery.*

Solution. The readings of the two meters depend on their deflecting torque which itself depends on the ampere-turns of the meter coil.

(*a*) Current through the first meter $=1{\cdot}5/(10+50)=1/40$ A

Ampere-turns of the first meter $=60/40=1{\cdot}5$ AT

Current through the second meter

$$=1{\cdot}5/(700+50)=1/500 \text{ A}$$

Ampere-turns of the second meter

$$=750\times 1/500=1{\cdot}5 \text{ AT}$$

Since their ATs are equal, their readings would also be equal.

(*b*) When connected in series, same current passes through the two meters. Hence, their deflections would be in the ratio of their turns *i.e.* 60 : 750 or **1 : 12·5.**

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Example 15-8. *A moving-coil instrument has a resistance of 10 ohms and gives a full scale deflection when carrying 50 mA. Show how it can be adopted to measure voltages upto 500 volts and currents upto 50 amperes.* (*A.M.I.E. Winter 1983*)

Solution. (*i*) **As Ammeter [Fig. 15-8 (a)]**

Instrument resistance $= 10\ \Omega$

Full-scale deflection current $= 50\ \text{mA} = 0{\cdot}05\ \text{A}$

Current through shunt $= 50 - 0{\cdot}05 = 49{\cdot}95\ \text{A}$

Since the shunt and the instrument coil are in parallel, the p.d. across both is the same.

$$\therefore \quad 10 \times 0{\cdot}05 = 49{\cdot}95 \times S \qquad \therefore \quad S = 0{\cdot}01\ \Omega$$

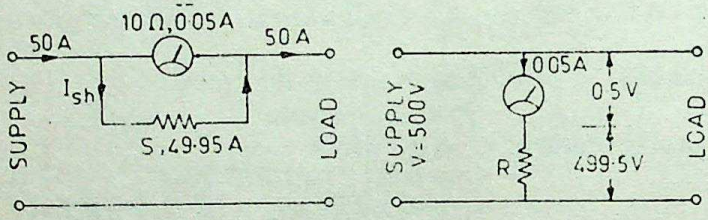

Fig. 15-8

(*ii*) **As voltmeter [Fig. 15-8 (b)]**

Full-scale deflection voltage across the instrument is

$$v = 10 \times 0{\cdot}05 = 0{\cdot}5\ \text{V}$$

Voltage to be dropped across series resistance R is

$$(V - v) = 500 - 0{\cdot}5 = 499{\cdot}5\ \text{V}$$

$$\therefore \quad R = (V - v)/I = 499{\cdot}5/0{\cdot}5 = 999\ \Omega$$

Example 15-9. *A moving-coil instrument has a resistance of 5 Ω between terminals and full-scale deflection is obtained with a current of 0·015 A. This instrument is to be used with a manganin shunt to measure 100 A full scale. Calculate the error caused by a 10°C rise in temperature.*

(*a*) *when the internal resistance of 5 Ω is due to copper only*

(*b*) *when a 4-Ω manganin swamping resistor is used in series with a copper resistor of 1 Ω.*

The temperature-resistance coefficients of copper and manganin are 0·4% per °C and 0·015% per °C respectively.

Solution. Current through instrument $= 0{\cdot}015$ A

Current through shunt $= 100 - 0{\cdot}015 = 99{\cdot}985$ A

Voltage across shunt $= 5 \times 0{\cdot}015 = 0{\cdot}075$ V

$\therefore$ Shunt resistance $= 0{\cdot}075/99{\cdot}985 = 0{\cdot}00075\ \Omega$

Shunt resistance after a rise of 10°C

$$= 0{\cdot}00075(1 + 10 \times 0{\cdot}00015) = 0{\cdot}000751\ \Omega$$

In case (*a*), the instrument resistance (which is wholly copper) after a rise of 10°C

$$= 5(1 + 10 \times 0{\cdot}004) = 5{\cdot}2\ \Omega$$

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Hence, current through the instrument corresponding to 100 A in the line

$$= \frac{0{\cdot}000751}{5{\cdot}20075} \times 100 = 0{\cdot}01444 \text{ A}$$

Reading of the instrument

$$= 0{\cdot}01444 \times 100/0{\cdot}015 = 96{\cdot}3 \text{ A}$$

$\therefore$ Percentage error $= 100 - 96{\cdot}3 = 3{\cdot}7$

In case (*b*) the instrument resistance after a rise of 10°C

$$= 1(1 + 10 \times 0{\cdot}004) + 4(1 + 10 \times 0{\cdot}00015)$$
$$= 5{\cdot}046 \ \Omega$$

Instrument current with a line current of 100 A

$$= \frac{0{\cdot}000751}{5{\cdot}046751} \times 100 = 0{\cdot}001488 \text{ A}$$

$\therefore$ Instrument reading $= 0{\cdot}001488 \times 100/0{\cdot}015 = 99{\cdot}2$

Percentage error $= 100 - 99{\cdot}2 = 0{\cdot}8$

Tutorial Problems—23

1. The flux density in the gap of a 1 mA (full scale) moving-coil ammeter is 0·1 Wb/m^2. The rectangular moving coil is 8 mm wide by 1 cm deep and is wound with 50 turns. Calculate the full-scale torque which must be provided by the springs. [4×10^{-7} N-m]

2. A moving-coil instrument has 100 turns of wire with a resistance of 10 Ω, an active length in the gap of 3 cm and a width of 2 cm. A p.d. of 45 mV produces full-scale deflection. The control spring exerts a torqe of $490{\cdot}5 \times 10^{-7}$ N-m at full-scale deflection. Calculate the flux density in the gap. [0·1817 Wb/m^2]

3. A moving-coil instrument has a resistance of 75 Ω and gives a full-scale deflection of 100 scale divisions for a current of 1 mA. The instrument is connected in parallel with a shunt of resistance 25 Ω and the combination is then connected in series with a load and a supply. What is the current in the load when the instrument gives an indication of 80 scale divisions ? [3·2 A]

4. The coil of a moving-coil meter has a resistance of 15 Ω and gives a full-scale deflection with 5 mA. Show how could this meter be adopted to measure 2 A ? If full-scale deflection were 90° of arc, what current would be required to give a deflection of 60° ? [375·9 mΩ ; 1·33 A]

5. A moving-coil instrument which has a resistance of 12·5 Ω gives a full-scale deflection with 8 mA. It is required to adopt this instrument to read 25 A. Calculate the value of the resistor required.

The resistor is to be made of three manganin strips, connected in parallel, each strip being 0·5 mm thick and 1 cm wide. Calculate the length of each strip. The resistivity of manganin is 50 μΩ-cm. [4·001 mΩ ; 12·003 cm]

6. A moving-coil voltmeter of range 0—0·5 V has the following details :

Moving-coil resistance of copper, 5 Ω

Series resistor of maganin, 128·3 Ω

The meter was calibrated correctly at 18·5°C. What would be the error in the full-scale deflection (expressed as a percentage of full-scale deflection) if used at a temperature of 65·5°C ?

What would have been the error in the full-scale deflection if both resistors had been made of copper (resistance tempt. coeff. for copper 1/234·5 per °C and for manganin assume zero) [−0·71 % ; −15·8%]

7. A moving-coil instrument, which gives full-scale deflection with 0·015 A, has a copper coil having a resistance of 1·5 Ω at 15°C and a temperature coefficient of 1/234·5 at 0°C in series with a swamp resistance of 3·5 Ω having a negligible temperature coefficient.

Determine (*a*) the resistance of shunt required for a full-scale deflection of 20 A and (*b*) the resistance required for a full-scale deflection of 250 V.

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If the instrument reads correctly at 15°C, determine the percentage error in each case when the temperature is 25°C.

[(*a*) 0·00376 Ω ; 1·3 % (*b*) 16,662 Ω, negligible]

8. The following data refer to a moving-coil voltmeter ; resistance =10,000 Ω, dimensions of coil=3×3 cm., number of turns of coil=100, flux density in air-gap=0·08 Wb/m², stiffness of springs =3×10⁻⁶ N-m per degree. Find the deflection produced by 110 V. [48·2°]

15-16. Electrodynamic or Dynamometer type Instruments

An electrodynamic instrument is a moving-coil instrument in which the operating field is produced, not by a permanent magnet, but by another fixed coil. This instrument can be used either as an ammeter or as voltmeter but is generally used as a wattmeter.

As shown in Fig. 15-19 and 15-20, the fixed coil is usually arranged in two equal sections *F* and *F* placed close together and

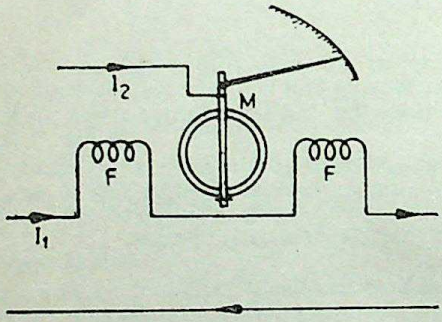

Fig. 15-19

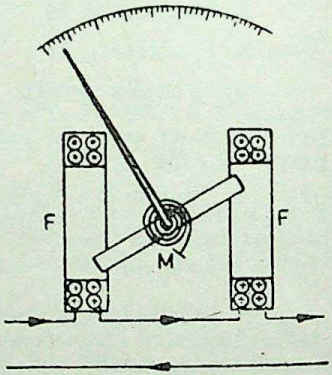

Fig. 15-20

parallel to each other. The two fixed coils are air-cored to avoid hysteresis effects when used on a.c. circuits. This has the effect of making the magnetic field in which moves the moving coil *M*, more uniform. The moving coil is spring-controlled and has a pointer attached to it as shown.

Deflecting Torque

The production of the deflecting torque can be understood from Fig. 15-21. Let the current passing through the fixed coil be I_1 and that through the moving-coil be I_2. Since there is no iron, the field strength and so the flux density is proportional to I_1.

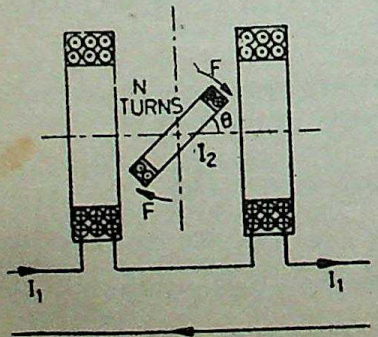

Fig. 15-21

$\therefore \quad B = KI_1$ where K is a constant.

Let us assume, for simplicity, that the moving coil is rectangular (it can be circular also) and of dimensions $l \times b$. Then force on each side of the coil having N turns is $=(NBI_2l)$ newton

The turning moment or deflecting torque on the coil is given by

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$$T_d = (NBI_2lb) = (NKI_1I_2lb) \text{ N-m}$$

Now putting $NKlb = K_1$, we have

$$T_d = K_1 I_1 I_2 \text{ where } K_1 \text{ is another constant*.}$$

It shows that the deflecting torque is proportional to the product of the currents flowing in the fixed coils and the moving coil. Since the instrument is spring-controlled, the restoring or control torque is proportional to the angular deflection θ.

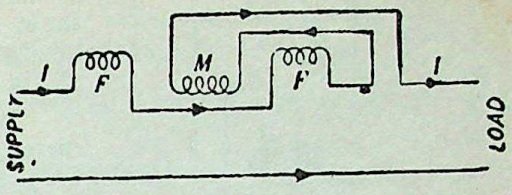

Fig. 15-22

$$\therefore \qquad T_c \propto \theta \qquad \text{or} \qquad T_c = K_2\theta$$

In the final deflected position, the two torques are equal and opposite

$$\therefore \qquad T_d = T_c$$

$$\therefore \qquad K_1 I_1 I_2 = K_2\theta$$

$$\therefore \qquad \theta \propto I_1 I_2$$

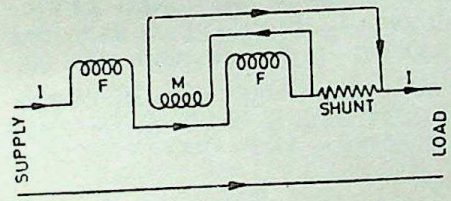

Fig. 15-23

(*i*) When the instrument is used as an ammeter, then the same current passes through both the fixed and the moving coils as shown in Fig. 15-22.

*It has been shown on page 312 that the value of torque for an instrument of inductance L and for a current of I is

$$T = \tfrac{1}{2} I^2 \frac{dL}{d\theta}$$

The equivalent inductance of the fixed and moving coils when joined in series is

$$L = L_1 + L_2 + 2M$$

where M is the mutual inductance between them and L_1 and L_2 are their individual self-inductances

Since L_1 and L_2 are fixed,

$$\frac{dL}{d\theta} = 2\frac{dM}{d\theta}$$

$$\therefore \qquad T = \tfrac{1}{2} I^2 \times 2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

If the currents in the fixed and moving coils are different, say, I_1 and I_2 then

$$T = I_1 I_2 \frac{dM}{d\theta}$$

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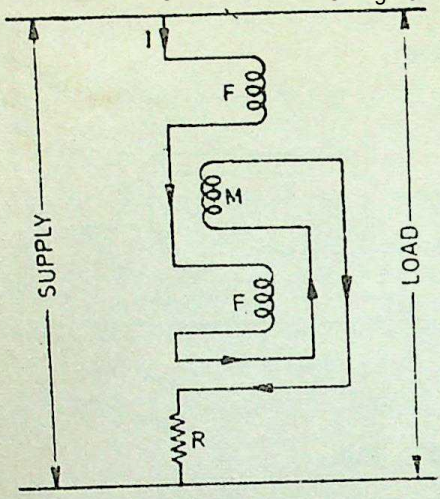

Fig. 15-24

In that case $I_1 = I_2 = I$, hence $\theta \propto I^2$ or $I \propto \sqrt{\theta}$. The connections of Fig. 15-22 are used when small currents are to be measured. In the case of heavy currents, a shunt S is used to limit current through the moving coil as shown in Fig. 15-23.

(*ii*) When used as a voltmeter, the fixed and moving coils are joined in series along with a high resistance and connected as shown in Fig. 15-24. Here again $I_1 = I_2 = I$, where $I = V/R$ in d.c. circuits and $I = V/Z$ in a.c. circuits.

$$\therefore \quad \theta \propto V \times V \quad \text{or} \quad \theta \propto V^2$$
$$\text{or} \quad V \propto \sqrt{\theta}$$

Hence, it is found that whether the instrument is used as an ammeter or voltmeter its scale is uneven throughout the whole of its range and is particularly cramped or crowded near the zero.

Damping is pneumatic since, owing to weak operating field, eddy current damping is inadmissible. Such instruments can be used for both a.c. and d.c. measurements. But it is more expensive and inferior to a moving-coil instrument for d.c. measurements.

As mentioned earlier, the most important application of electrodynamic principle is for wattmeters and is discussed in detail further on.

Errors

Since the coils are air-cored, the operating field produced is small. For producing an appreciable deflecting torque, a large number of turns is necessary for the moving coil. The magnitude of the current is also limited because the two control springs are used both for leading in and for leading out the current. Both these factors lead to a heavy moving system resulting in frictional losses which are somewhat larger than in other types and so frictional errors tend to be relatively higher. The current in the field coils is limited for the fear of heating the coils which results in the increase of their resistance. A good amount of screening is necessary to avoid the influence of stray fields.

Advantages and Disadvantages

(1) Such instruments are free from hysteresis and eddy-current errors.

(2) Since (torque/weight) ratio is small, such instruments have low sensitivity.

Example 15-10. *The mutual inductance of a 25-A electrodynamic ammeter changes uniformly at the rate of 0·0035 μH/degree.*

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The torsion constant of the control spring is 10^{-6} N-m/degree. Determine the angular deflection at full scale.

Solution. By torsion constant is meant the torque per degree of deflection. Hence, if full-scale deflection is $\theta°$, then torque on full scale $= 10^{-6} \times \theta$ N-m

Now, for an electrodynamic instrument $T = I^2\, dM/d\theta$

Here $I = 25$A, $dM/d\theta = 0{\cdot}0035 \times 10^{-6}$ H/degree

$$= 0{\cdot}0035 \times 10^{-6} \times \frac{180}{\pi} \text{ H/radian}$$

$$\therefore \quad 10^{-6} \times \theta° = 25^2 \times 0{\cdot}0035 \times 10^{-6} \times \frac{180}{\pi}$$

$$\therefore \quad \theta = 25^2 \times 0{\cdot}0035 \times \frac{180}{\pi} \text{ degrees} = 125{\cdot}4°$$

15-17. Hot-wire Instruments (*A.M.I.E. Nov. 1970*)

The working parts of the instrument are shown in Fig. 15-25. It is based on the heating effect of current. It consists of a platinum-iridium (it can withstand oxidation at high temperatures) wire *AB* stretched between a fixed end *B* and a tension adjusting screw at *A*. When current is passed through *AB*, it expands according to I^2R formula. Due to heating, *AB* expands or sags. This sag in *AB* produces a slack in phosphor-bronze tension wire *CD* attached to the centre of *AB*. This slack in *CD* is taken up by the silk fibre which after passing round the pulley *W* is attached to a spring *S*. As the silk thread is pulled by *S*, the pulley rotates, thereby deflecting the pointer. It would be noted that even a small sag in *AB* is magnified very much and is conveyed to the pointer. Expansion of *AB* is magnified by *CD* which is further magnified by the silk thread.

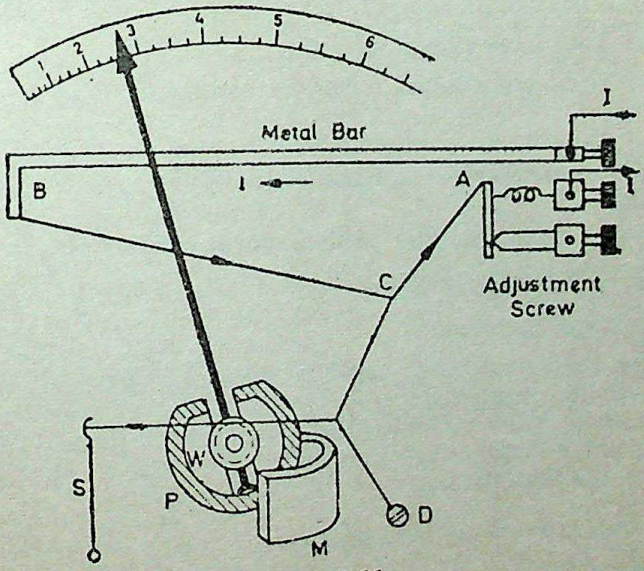

Fig. 15-25

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It will be seen that the deflection of the pointer is proportional to the extension of AB which is itself proportional to I^2. Hence, deflection is $\propto I^2$. If spring control is used, then $T_c \propto \theta$.

Hence, $\theta \propto I^2$

So, these instruments have a 'square-law' type scale. They read the r.m.s. value of current and are independent of its form and frequency.

Damping

A thin light aluminium disc P is attached to the pulley such that its edge moves between the poles of a permanent magnet M. Eddy currents produced in this disc give the necessary damping.

These instruments are primarily meant for being used as ammeters but can be adopted as voltmeters by connecting a high resistance in series with the instrument. These instruments are suited both for alternating and direct currents.

Advantages of Hot-wire Instruments :

(*i*) As their deflection depends on the r.m.s. value of the alternating current, they can be used on direct current also.

(*ii*) Their readings are independent of waveform and frequency.

(*iii*) Their calibration is the same both for a.c. as well d.c. measurements.

(*iv*) They are unaffected by stray fields.

Disadvantages :

(*i*) They are unable to withstand overload without burning out.

(*ii*) They have a cramped scale.

(*iii*) They are sluggish owing to the time taken up for the wire to heat up.

(*iv*) They have a high power consumption as compared to moving-coil instruments. Current consumption is about 200 mA at full load.

(*v*) Their zero position needs frequent adjustment.

(*vi*) They are fragile.

15-18. Range Extension

The current-carrying capacity of such instruments is limited owing to the fineness of the wire. Normally, they can carry a current of 5 A without shunt. But for higher currents, a shunt is essential. Its range as a voltmeter can be extended by connecting a high resistance in series with the hot wire.

15-19. Induction Type Instruments

Induction type instruments are used only for a.c. measurement and can be used either as ammeter, voltmeter or wattmeter. How-

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ever, the induction principle finds its widest application as a watt-hour or energy meter. In such instruments, the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux. Before discussing the two types of most commonly-used induction instruments, we will first discuss the underlying principle of their operation.

Principle

The operation of all induction instruments depends on the production of torque due to the reaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy currents induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy currents also depends on the flux producing them, the *instantaneous* value of deflecting torque is proportional to the square of the current or voltage under measurement and the value of *mean* deflecting torque is proportional to the mean square of the current or voltage.

Consider a thin aluminium or Cu disc D free to rotate about an axis passing through its centre as shown in Fig. 15-26. Two a.c. magnetic poles P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 which cut this disc. Consider any annular portion of the disc around P_1 with centre on the axis of P_1. This portion will be linked by Φ_1 and so an alternating e.m.f. e_1 will be induced in it. This e.m.f e_1 will circulate the eddy current i_1 which, as shown in Fig. 15-26 (*b*) will pass under P_2. Similarly, Φ_2 will induce an e.m.f. e_2 which will further induce the eddy current i_2 in the annular portion of the disc around P_2. This eddy current i_2 flows under pole P_1.

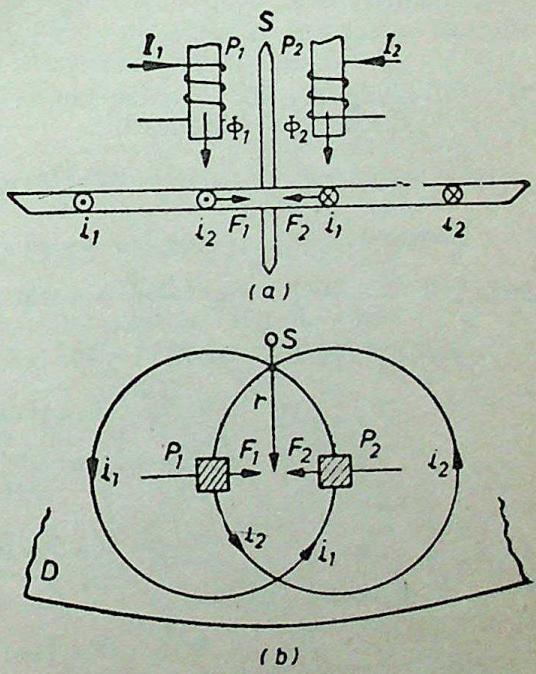

Fig. 15-26

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law the directions of the induced currents i_1 and i_2 can be found and are as indicated in Fig. 15-26 (*b*).

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The portion of the disc which lies in flux Φ_1 and carries current i_2 experiences a force F_1 along the direction as indicated ($\because F=BIl$). The force $F_1 \propto \Phi_1 i_2$. Similarly, the portion of the disc lying in flux Φ_2 and carrying current i_1 experiences a force $F_2 \propto \Phi_2 i_1$

$$\therefore \quad F_1 \propto \Phi_1 i_2 = K\Phi_1 i_2$$

and $$F_2 \propto \Phi_2 i_1 = K\Phi_2 i_1$$

It is assumed that constant K is the same in both cases due to the symmetrical positions of P_1 and P_2 with respect to the disc.

If r is the effective radius at which these forces act, then the instantaneous torque T acting on the disc which is the *difference of* the two torques, is given by

$$T = r(K\Phi_1 i_2 - K\Phi_2 i_1)$$
$$= K_1(\Phi_1 i_2 - \Phi_2 i_1) \qquad \ldots(i)$$

Let the alternating flux Φ_1 be given by $\Phi_1 = \Phi_{1m} \sin \omega t$. The flux Φ_2 which is assumed to lag Φ_1 by an angle α radian, is given by $\Phi_2 = \Phi_{2m} \sin (\omega t - \alpha)$

Induced e.m.f., $$e_1 = \frac{d\Phi_1}{dt} = \frac{d}{dt}\left(\Phi_{1m} \sin \omega t\right)$$
$$= \omega \Phi_{1m} \cos \omega t$$

Assuming the eddy current path to be purely resistive and of value R*, the value of eddy current is

$$i_1 = \frac{e_1}{R} = \frac{\omega \Phi_{1m}}{R} \cos \omega t$$

Similarly, $$e_2 = \omega \Phi_{2m} \cos (\omega t - \alpha)$$

and $$i_2 = \frac{\omega \Phi_{2m}}{R} \cos (\omega t - \alpha)\dagger$$

Substituting these values of i_1 and i_2 in Eq. (i) above, we get

$$T = \frac{K_1\omega}{R}\Big[\Phi_{1m} \sin \omega t.\ \Phi_{2m} \cos (\omega t - \alpha) - \Phi_{2m} \sin (\omega t - \alpha)\ \Phi_{1m} \cos \omega t\Big]$$
$$= \frac{K_1\omega}{R}\ \Phi_{1m}\Phi_{2m}[\sin \omega t \cos (\omega t - \alpha) - \cos \omega t \sin (\omega t - \alpha)]$$
$$= \frac{K_1\omega}{R}\ \Phi_{1m}\Phi_{2m} \sin \alpha$$
$$= K_2\, \omega\, \Phi_{1m}\Phi_{2m} \sin \alpha \qquad (\text{put } K_1/R = K_2)$$

---

*If it has a reactance of X, then impedance Z should be taken whose value is given by $Z = \sqrt{R^2 + X^2}$.

†It being assumed that both paths have the same resistance.

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It is obvious that

(*i*) if $\alpha=0$ *i.e.* if the two fluxes are in phase, then net torque is zero. If, on the other hand, $\alpha=90°$, then torque is maximum for the given values of Φ_{1m} and Φ_{2m}.

(*ii*) the torque is in such a direction as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(*iii*) since the expression for the torque does not involve 'ωt', it has the same value at all times.

(*iv*) the torque is inversely proportional to R—the resistance of the eddy current path. Hence, for large torques, the disc material should have low resistivity. Usually, it is made of Cu or, more often, of aluminium.

15-20. Induction Ammeters

It has been shown above, that

$$T=K_2\omega\ \Phi_{1m}\ \Phi_{2m}\ \sin\alpha$$

Obviously, if both fluxes are produced by the alternating current (of maximum value I_m) to be measured, then

$$T=K_3\omega I_m^2\ \sin\alpha$$

Hence, for a given frequency and angle α, the torque is proportional to the square of the current. If the disc has spring control, it will take up a steady deflected position where controlling torque becomes equal to the deflecting. torque. By attaching a suitable pointer to the disc, the device can be used as an ammeter.

Two possible arrangements by which the operational requirements of induction ammeters can be met are discussed below.

15-21. Disc Ammeter with Split-phase Windings

In this arrangement, the windings on the two laminated a.c. magnets P_1 and P_2 are connected in series. But the winding of P_2

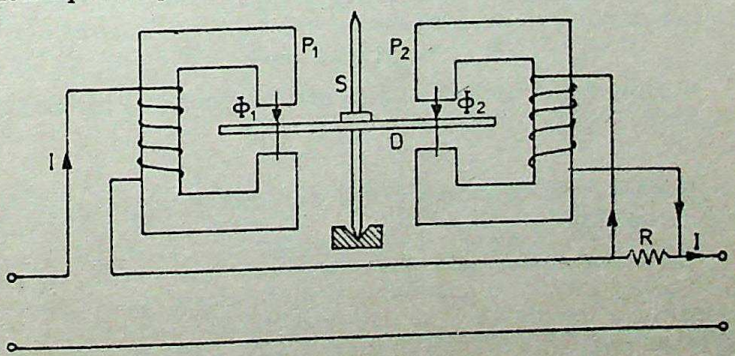

Fig. 15-27

is shunted by a resistance R with the result that the current in this winding lags with respect to the total line current. In this way, the necessary phase angle α is produced between two fluxes Φ_1 and Φ_2 produced by P_1 and P_2 respectively. This angle is of the order of

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60°. If the hysteresis effects etc. are neglected, then each flux will be proportional to the current to be measured *i.e.* line current I.

$$\therefore \quad T_d \propto \Phi_{1m}\, \Phi_{2m} \sin \alpha$$

or $$T_d \propto I^2$$ —where I is the r.m.s. value

If spring control is used, then

$$T_c \propto \theta$$

In the final deflected position, $T_d = T_c$

$$\therefore \quad \theta \propto I^2$$

Eddy current damping is employed in this instrument. When the disc rotates, it cuts the flux in the air-gap of the magnet and has eddy currents induced in it which provide efficient damping.

15-22. Shaded-pole Induction Ammeters

In the shaded-pole disc type induction ammeter (Fig. 15-28) only one single flux-producing winding is used. The flux Φ produced by this winding is split up into two fluxes Φ_1 and Φ_2 which are made to have the necessary phase difference of α by the device shown in Fig. 15-28. The portions of the upper and lower poles near the disc D are divided by a slot into two halves, one of which carries a closed 'shading' winding or ring. This shading winding or ring acts as a short-circuited secondary and the main winding as the primary. The current induced in the ring by transformer action retards the phase of flux Φ_2 with respect to that of Φ_1 by about 50° or so. The fluxes Φ_1 and Φ_2 passing through the unshaded and the shaded part, react with eddy currents i_2 and i_1 respectively and so produce the driving torque whose value is

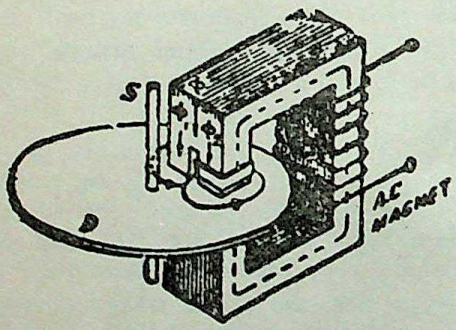

Fig. 15-28

$$T_d \propto \Phi_{1m}\Phi_{2m} \sin \alpha$$

Assuming that both Φ_1 and Φ_2 are proportional to the current I, we have

$$T_d \propto I^2$$

This torque is balanced by the controlling torque provided by the spiral springs.

The actual shaded-pole type induction instrument is shown in Fig. 15-29. It consists of a suitably-shaped aluminium or copper disc mounted on a spindle which is supported by jewelled bearings. The spindle carries a pointer and has a control spring attached to it. The edge or periphery of the disc moves in the air-gap of a laminated a.c. electromagnet which is energised either by the current to be measured (as ammeter) or by the current proportional to the voltage to be measured (as a voltmeter). Damping is by eddy currents induced by a permanent magnet embracing another portion of the

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same disc. As seen, the disc serves both for damping as well as operating purposes. The main flux is split into two component fluxes by shading one-half of each pole. These two fluxes have a phase difference of 40° to 50° between them and they induce two eddy

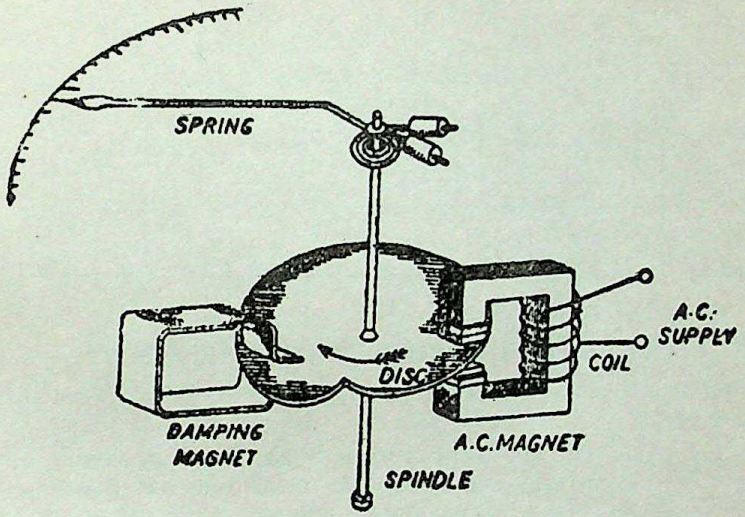

Fig. 15-29

currents in the disc. The fluxes and eddy currents produce a resultant torque which *deflects* the disc—continuous rotation being prevented by the control spring and the deflection produced is proportional to the square of current or voltage being measured.

As seen above, for a given frequency

$$T_d \propto I^2 = KI^2$$

For spring control $T_c \propto \theta$ or $T_c = K_1$

For steady deflection, we have $T_c = T_d$

$$\therefore \quad \theta \propto I^2$$

Hence, such instruments have uneven scales *i.e.* scales which are cramped at their lower ends. A more even scale can, however, be obtained by using a cam-shaped disc as shown in Fig. 15-29.

15-23. Induction Voltmeters

Their construction is similar to that of the ammeters except for this difference that their windings are wound with a large number of turns of fine wire. Since they are connected across the lines they carry very small currents (5—10 mA), the number of turns of wire has to be large for producing an adequate amount of m.m.f. Split-phase windings are obtained by connecting a high resistance in series

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with the winding of one magnet and an inductive coil in series with the winding of the other magnet as shown in Fig. 15-30.

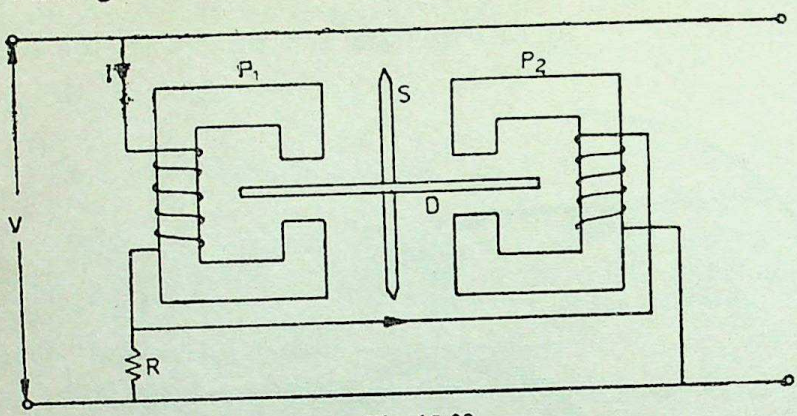

Fig. 15-30

15-24. Errors in Induction Ammeters and Voltmeters

There are two types of errors (*i*) *frequency* error and (*ii*) *temperature* error.

(*i*) Since deflecting torque depends on frequency, hence unless the alternating current to be measured has the same frequency with which the instrument was calibrated, there will be large error in its readings. Frequency error can be compensated for by the use of a non-inductive shunt in the case of ammeters. In voltmeters, such errors are not large and, to a greater extent, are self-compensating.

(*ii*) Serious errors may occur due to the variations of temperature, because the resistances of eddy current paths depend on the temperature. Such errors can, however, be compensated for by shunting in the case of ammeters and by a combination of shunt and swamping resistance in the case of voltmeters.

15-25. Advantages and Disadvantages

(*i*) A full-scale deflection of over 200° can be obtained with such instruments. Hence, they have long open scales.

(*ii*) Damping is very efficient.

(*iii*) They are not much effected by external stray fields.

(*iv*) Their power consumption is fairly large and cost relatively high.

(*v*) They can be used for a.c. measurements only.

(*vi*) Unless compensated for frequency and temperature variations, serious errors may be introduced.

15-26. Electrostatic Voltmeters

Electrostatic instruments are almost always used as voltmeters and that too more as a laboratory rather than as industrial instru-

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ments. The underlying principle of their operation is the force of attraction between electric charges on neighbouring plates between which a p.d. is maintained. This force gives rise to a deflecting torque. Unless the p.d. is sufficiently large, the force is small. Hence, such instruments are used for the measurement of very high voltages.

There are two general types of such instruments :

(*i*) *the quadrant type*—used up to 20 kV

(*ii*) *the attracted-disc type*—used up to 500 kV

15-27. Attracted-disc Type Voltmeter *(A.M.I.E. Winter 1981)*

As shown in Fig. 15-31, it consists of two discs or plates *C* and *D* mounted parallel to each other. Plate *D* is fixed and is earthed while *C* is suspended by a coach spring, the support for which carries a micrometer head for adjustment. Plate *C* is connected to the positive end of the supply voltage. When a p.d. (whether direct or alternating) is applied between the two plates, plate *C* is attracted towards *D*, but may be returned to its original position by the micrometer head. The movement of this head can be made to indicate the force *F* with which *C* is pulled downwards. For this purpose, the instrument can be calibrated by placing known weights, in turn, on *C* and observing the movement of micrometer head necessary to bring *C* back to its original position. Alternately, this movement of plate *C* is balanced by a control device which actuates a pointer attached to it that sweeps over a calibrated scale.

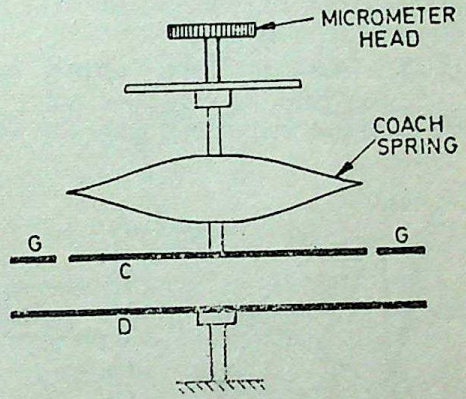

Fig. 15-31

There is a guard ring *G* surrounding the plate *C* and separated from it by a small air-gap. The ring is connected electrically to plate *C* and helps to make the field uniform between the two plates. The effective area of plate *C*, in that case, becomes equal to its actual area plus half the area of the air-gap.

Theory

In Fig. 15-32 are shown two parallel plates separated by a distance of *x* metres. Suppose the lower plate is fixed and carries a charge of $-Q$ coulombs whereas the upper plate is movable and carries a charge of $+Q$ coulombs. Let the mutual force of attraction between the two plates be *F* newtons. Suppose the upper plate is moved apart by a distance *dx*. Then mechanical work done during this movement is $F \times dx$ joules. Since charge

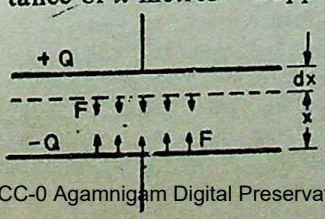

Fig. 15-32

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on the plate is constant, no electrical energy can move into the system from outside. This work is done at the cost of the energy stored in the parallel-plate capacitor formed by the two plates.

$$\therefore \quad F \times dx = d(\tfrac{1}{2}CV^2)$$

Now $C = \epsilon A/x$ where x is the plate separation

$$\therefore \quad F \times dx = d\left(\frac{1}{2} \quad \frac{\epsilon A}{x} \cdot V^2\right)$$

$$= \frac{\epsilon A V^2}{2} \cdot d\left(\frac{1}{x}\right) = -\frac{1}{2} \quad \frac{\epsilon A V^2}{x^2} \cdot dx$$

$$\therefore \quad F = -\frac{1}{2} V^2 \frac{\epsilon A}{x^2}$$

Hence, we find that force is directly proportional to the square of the voltage to be measured. The negative sign merely shows that it is a force of attraction.

15-28. Quadrant Type Voltmeters

The working principle and basic construction of such instruments can be understood from Fig. 15-33. A light aluminium vane

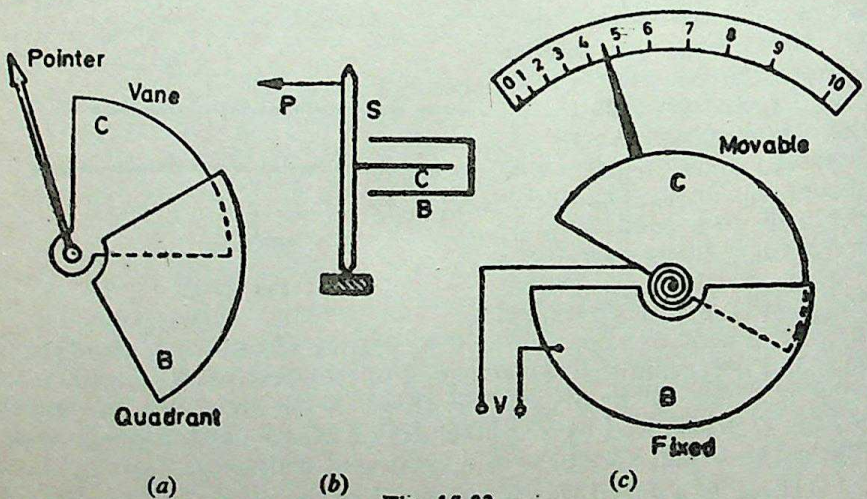

Fig. 15-33

C is mounted on a spindle S and is situated partially within a hollow metal quadrant B. Alternately, the vane may be suspended in the quadrant. When the vane and the quadrant are oppositely charged by the voltage under measurement, then the vane is further attracted inwards into the quadrant thereby causing the spindle and hence the pointer to rotate. The amount of rotation and hence the deflecting torque is found proportional to V^2. The deflecting torque in the case of arrangement shown in Fig. 15-33 is very small unless V is extremely large. The force on the vane may be increased by using a greater number of quadrants and a double-ended vane. Such voltmeters have an uneven scale [Fig. 15-33(*c*)] Controlling torque is produced by torsion of the suspension strip or by the spring (used in pivoted type voltmeters). Damping is by a disc or vane immersed in oil in the case of suspended type or by air friction in the case of pivoted type instruments.

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Theory

With reference to Fig. 15-33, suppose the quadrant and vane are connected across a source of V volts and let the resulting deflection be θ. If C is the capacitance between the quadrant and vane in the deflected position, then the charge on the instrument will be CV coulombs. Suppose the voltage is changed from V to $(V+dV)$, then as a result, let θ, C and Q change to $(\theta+d\theta)$, $(C+dC)$ and $(Q+dQ)$ respectively. Then the energy stored in the electrostatic field is increased by $dE=d(\frac{1}{2}CV^2)=\frac{1}{2}V^2.\ dC+CV.dV$ joules

If T newton-metre is the value of controlling torque corresponding to a deflection of θ, then the additional energy stored in the control system will be $T\times d\theta$ joules.

Total increase in stored energy

$$=T\times d\theta+\tfrac{1}{2}V^2\ dC+C\ V\ dV \text{ joules}$$

It is seen that during this change the source supplies a charge dQ at potential V. Hence, the value of energy supplied is

$$=V\times dQ=V\times d(CV)=V^2\times dC+CV.dV$$

Since the energy supplied by the source must be equal to the extra energy stored in the field and the control,

$$\therefore \quad T\times d\theta+\tfrac{1}{2}V^2\ dC+CV.dV=V^2.dC+CVdV$$

$$T\times d\theta=\tfrac{1}{2}V^2.dC \quad \therefore \quad T=\tfrac{1}{2}\ V^2\ \frac{dC}{d\theta} \text{ N-m}$$

The torque is found to be proportional to the square of the voltage to be measured whether that voltage is alternating or direct. However, on alternating circuits the scale will read r.m.s. value.

15-29. Kelvin's Multicellular Voltmeter

As shown in Fig. 15-34, it is essentially a quadrant type instrument as described above but with the difference that instead of four quadrants and one vane it has a large number of fixed quadrants and vanes mounted on the same spindle. In this way, the deflecting torque for a given voltage is increased many times. Such voltmeters can be used to measure voltages as low as 30 V. As said above, this reduction in the minimum limit of voltage is due to the increase in the operating force in proportion to the number of component units. Such an instrument has a torsion head for zero adjustment and a coach spring for protection against accidental fracture of suspension due to vibration etc. There is a pointer and scale of edgewise pattern and damping is by a vane immersed in an oil dashpot.

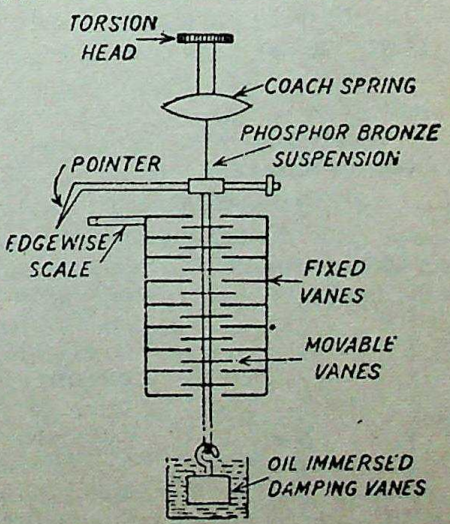

Fig. 15-34

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15-30. Advantages and Limitations of Electrostatic Voltmeters

Some of the main advantages and uses of electrostatic voltmeters are as follows :

(*i*) They can be manufactured with first grade accuracy.

(*ii*) They give correct reading on both d.c. and a.c. circuits. On a.c. circuits, the scale will, however, read r.m.s. values whatever the wave-form.

(*iii*) Since no iron is used in their construction, such instruments are free from hysteresis and eddy current losses and temperature errors.

(*iv*) They do not draw any continuous current on d.c. circuits and that drawn on a.c. circuits (due to the capacitance of the instrument) is extremely small. Hence, such voltmeters do not cause any disturbance to the circuits to which they are connected.

(*v*) Their power loss is negligibly small.

(*vi*) They are unaffected by stray magnetic fields although they have to be guarded against any stray electrostatic field.

(*vii*) They can be used up to 1000 kHz without any serious loss of accuracy.

However, their main limitations are :

(*i*) Low-voltage voltmeters (like Kelvin's Multicellular voltmeter) are liable to friction errors.

(*ii*) Since torque is proportional to the square of the voltage, their scales are not uniform although some uniformity can be obtained by suitably shaping the quadrants of the voltmeters.

(*iii*) They are expensive and cannot be made robust.

15-31. Range Extension of Electrostatic Voltmeters

The range of such voltmeters can be extended by the use of multipliers which are in the form of a resistance potential divider or capacitance potential divider. The former method can be used both for direct and alternating voltages.

(*i*) Resistance Potential Divider

This divider consists of a high non-inductive resistance across a small portion of which is attached the electrostatic voltmeter as shown in Fig. 15-35. Let R be the resistance of the whole of the potential divider across which is applied the voltage V under measurement. Suppose v is the maximum value of the voltage which the voltmeter can measure without the multiplier. If r is the resistance of the portion of the divider across which voltmeter is connected, then the multiplying factor is given by

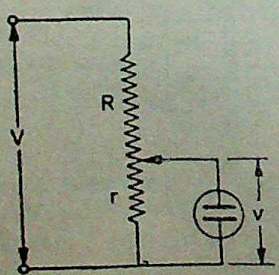

Fig. 15-35

$$\frac{V}{v} = \frac{R}{r}$$

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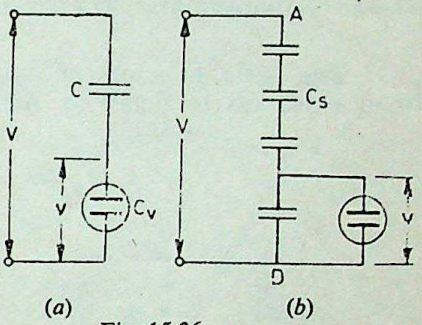
The above expression is true for d.c. circuits, but for a.c. circuits, the capacitance of the voltmeter (which is in parallel with r) has to be taken into account. Since this capacitance is variable, it is advisable to calibrate the voltmeter along with its multiplier.

(*ii*) **Capacitance Potential Divider**

In this method, the voltmeter may be connected in series with a single capacitor C and put across the voltage V which is to be measured [Fig. 15-36 (*a*)] or a number of capacitors may be joined in series to form the potential divider and the voltmeter may be connected across one of the capacitors as shown in Fig. 15-36 (*b*).



(*a*) (*b*)

Fig. 15-36

Consider the connection shown in Fig. 15-36 (*a*). It is seen that the multiplying factor is given by

$$\frac{V}{v} = \frac{\text{reactance of total circuit}}{\text{reactance of voltmeter}}$$

Now, capacitance of the total circuit is $\frac{C_v C}{C+C_v}$ and its reactance is

$$= \frac{1}{\omega \times \text{capacitance}} = \frac{C+C_v}{\omega C C_v}$$

Reactance of the voltmeter $= \frac{1}{\omega C_v}$

$$\therefore \quad \frac{V}{v} = \frac{(C+C_v)/\omega C C_v}{1/\omega C_v} = \frac{C+C_v}{C}$$

$$\therefore \text{ Multiplying factor} = \frac{C+C_v}{C} = 1 + \frac{C_v}{C}^{*} .$$

Example 15-11. *The reading '100' of a 120-V electrostatic voltmeter is to represent 10,000 volts when its range is extended by the use of a capacitor in series. If the capacitance of the voltmeter at the above reading is 70 $\mu\mu$F, find the capacitance of the capacitor multiplier required.*

Solution. Multiplying factor $= \frac{V}{v} = 1 + \frac{C_v}{C}$

Here, $V = 10{,}000$ volt ; $v = 100$ volt

C_v = capacitance of the voltmeter = 70 $\mu\mu$F

C = capacitance of the multiplier

*It is helpful to compare it with a similar expression in Art. 15-15 for permanent-magnet moving-coil instruments.

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$$\therefore \quad \frac{10{,}000}{100} = 1 + \frac{70}{C}$$

$$\therefore \quad 70/C = 99 \qquad \therefore \quad C = 70/99 = 0{\cdot}7 \ \mu\mu\text{F (approx)}$$

15-32. Wattmeters

We will discuss the two main types of wattmeters in general use, that is, (*i*) the dynamometer or electrodynamic type and (*ii*) the induction type.

15-33. Dynamometer Wattmeter

(A.M.I.E. Summer 1980)

The basic principle of dynamometer instrument has already been explained in detail in Art. 15-16. The connections of a dynamometer type wattmeter are shown in Fig. 15-37. The fixed circular coil which carries the main circuit current I_1 is wound in two halves positioned parallel to each other. The distance between the two halves can be adjusted to give a uniform magnetic field. The moving coil which is pivoted centrally carries a current I_2 which is proportional to the voltage V. Current I_2 is led into the moving coil by two springs which also supply the necessary controlling torque.

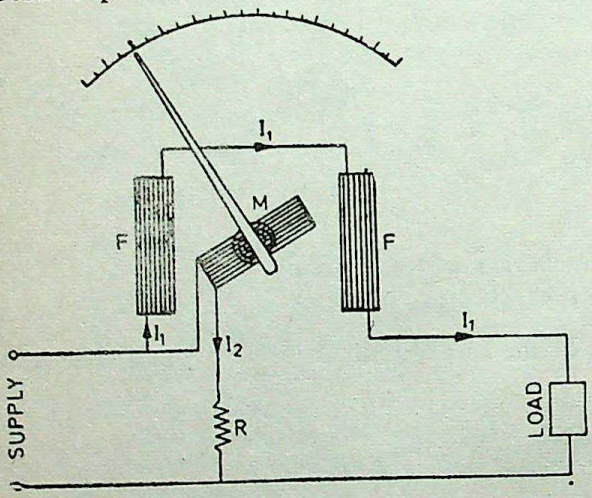

Fig. 15-37

Deflecting torque

Since coils are air-cored, the flux density produced is directly proportional to the current I_1.

$$\therefore \quad B \propto I_1 \quad \text{or} \quad B = K_1 I_1$$

$$\text{Current} \quad I_2 \propto V \quad \text{or} \quad I_2 = K_2 V$$

$$\text{Now} \quad T_d \propto BI_2 \propto I_1 V$$

$$\therefore \quad T_d = KVI_1 = K \times \text{power}$$

In d.c. circuits, power is given by the product of voltage and current in amperes, hence, the torque is directly proportional to power.

Let us see how this instrument indicates true power on a.c. circuits.

For alternating currents, the value of instantaneous torque is given by

$$T_{inst} \propto vi = Kvi$$

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where v = instantaneous value of voltage across moving coil
i = instantaneous value of circuit through *fixed* coils.

However, owing to the large inertia of the moving system, the instrument indicates the mean or average power.

$\therefore$ Mean deflecting torque $T_{mean} \propto$ average value of vi

Let $v = V_{max} \sin \theta$

$i = I_{max} \sin (\theta - \phi)$

$$\therefore \quad T_{mean} \propto \frac{1}{2\pi}\int_0^{2\pi} V_{max} \sin \theta \times I_{max} \sin (\theta - \phi)\, d\theta$$

$$\propto \frac{V_{max}\, I_{max}}{2\pi}\int_0^{2\pi} \sin \theta \sin (\theta - \phi)\, d\theta$$

$$\propto \frac{V_{max}\, I_{max}}{2\pi}\int_0^{2\pi} \frac{\cos \phi - \cos (2\theta - \phi)}{2}\, d\theta$$

$$\propto \frac{V_{max}\, I_{max}}{4\pi}\left[\theta \cos \phi - \frac{\sin (2\theta - \phi)}{2}\right]_0^{2\pi}$$

$$\propto \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos \phi \propto VI \cos \phi$$

where V and I are the r.m.s. values.

$\therefore$ $T_{mean} \propto VI \cos \phi \propto$ true power.

Hence, we find that in the case of alternating current also, the deflection is proportional to the true power in the circuit.

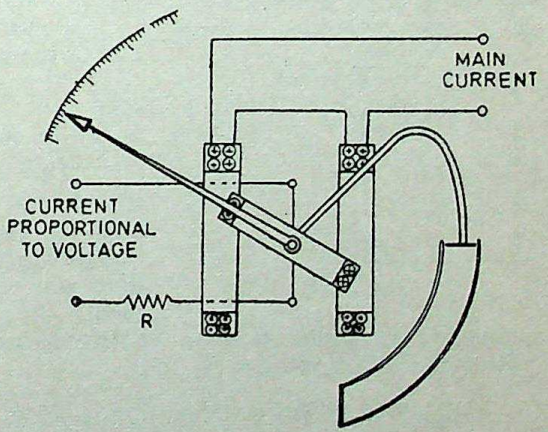

Fig. 15-38

Scales of dynamometer wattmeters are more or less uniform because the deflection is proportional to the average power and for spring control, controlling torque is proportional to the deflection, hence $\theta \propto$ power. Damping is pneumatic *i.e.* with the help of a piston moving in an air chamber as shown in Fig. 15-38.

Errors

The inductance of the moving or voltage coil is liable to cause error but the high non-inductive resistance connected in series with the coil swamps, to a great extent, the phasing effect of the voltage-coil inductance.

Another possible error in the indicated power may be due to (*i*) some voltage drop in the current circuit or (*ii*) the current taken

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by the voltage coil. In standard wattmeters, this defect is overcome by having an additional compensating winding which is connected in series with the voltage coil but is so placed that it produces a field in opposite direction to that of the fixed or current coils.

Advantages and Disadvantages

By careful design, such instruments can be built to give a very high degree of accuracy. Hence, they are used as a standard for calibration purposes. They are equally accurate on d.c. as well as a.c. circuits.

However, at low power factors, the inductance of the voltage coil causes serious error unless special precautions are taken to reduce this effect.

Example 15-12. *The current coil of a dynamometer wattmeter has a resistance of 1·5 Ω and its pressure coil circuit has a resistance of 10,000 Ω. It is connected into a d.c. circuit to measure the power taken by a load from a 200-V supply. The pressure coil terminals of the wattmeter are connected directly across the supply and the wattmeter reading is 600 W. Calculate the true power taken by the load.*

Solution. The circuit connections are shown in Fig. 15-38.
Load current = 600/200 = 3 A

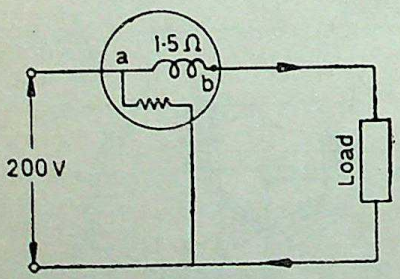

Fig. 15-39

Power consumed by the current coil of the wattmeter is

$$= I^2R = 3^2 \times 1{\cdot}5 = 13{\cdot}5 \text{ W}$$

Total wattmeter reading = 600 W

This includes power consumed by the load and the current coil (and not by the pressure coil because being connected at point *a* its current does not flow through the instrument. (If, however, its one end were connected to point *b*, then wattmeter reading would include its power consumption as well).

$\therefore$ load power consumption is = 600 − 13·5 = **586·5 W**

15-34. Induction Wattmeters

Principle of induction wattmeters is the same as that of induction ammeters and voltmeters. They can be used on a.c. circuits only in contrast with dynamometer wattmeters which can be used both on d.c. and a.c. circuits. Induction wattmeters are useful only when the frequency and supply voltage are constant.

Since, both a current and a pressure element are required in such instruments, it it not essential to use the shaded-pole principle. Instead of this, two separate a.c. magnets are used which produce two fluxes which have the required phase difference.

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Construction

The wattmeter has two laminated electromagnets one of which is excited by the current in the main circuit—its exciting winding being joined in series with the circuit, hence it is also called a *series* magnet. The other is excited by current which is proportional to the voltage of the circuit. Its exciting coil is joined in parallel with the circuit, hence this magnet is sometimes referred to as *shunt* magnet.

A thin aluminium disc is so mounted that it cuts the fluxes of both magnets. Hence, two eddy currents are produced in the disc. The deflection torque is produced due to the interaction of these eddy currents and the inducing fluxes. Two or three copper rings are fitted on the central limb of the shunt magnet and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by a suitable angle.

Two most common forms of the electromagnets are shown in Fig. 15-40 and 15-41. It is seen that in both cases one magnet is placed above and the other below the disc. The magnets are so positioned and shaped that their fluxes are cut by the disc.

In Fig. 15-40 the two pressure coils are joined in series and are so wound that both send the flux through the central limb in the

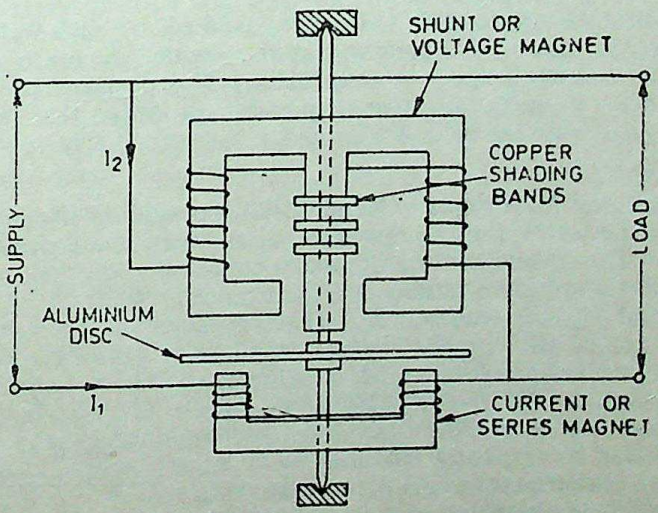

Fig. 15-40

same direction. The series magnet carries two coils joined in series and so wound that they magnetise their respective cores in the same direction. Correct phase displacement between the shunt and series magnet fluxes can be obtained by adjusting the position of the copper shading bands as shown.

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In the type of instrument shown in Fig. 15-41, there is only one pressure winding and one current winding. The two projecting

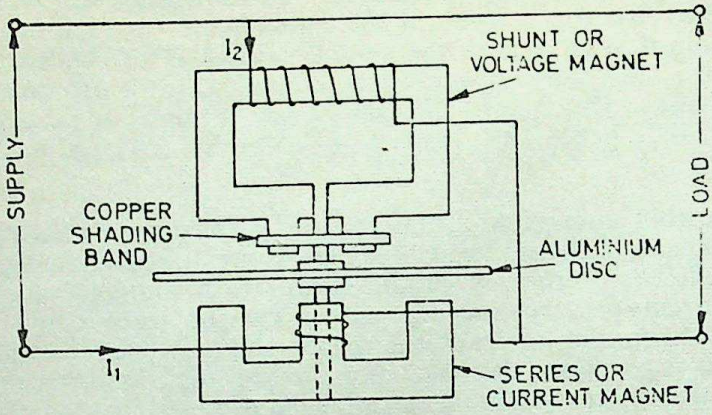

Fig. 15-41

poles of the shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with respect to the voltage.

Both types of induction wattmeters, shown above, are spring-controlled, the springs being fitted to the spindle of the moving system which also carries the pointer. The scale is uniformly even and extends over 300°.

Currents up to 100 A can be handled by such wattmeters directly, but for currents greater than this value, they are used in conjunction with current transformers. The pressure coil is purposely made as much inductive as possible in order that the flux through it should lag behind the voltage by a suitable value.

Theory

As seen from Fig. 15-40 and 15-41, the winding of one magnet carries line current I_1 so that $\Phi_1 \propto I_1$ and is in phase with I_1 (Fig. 15-42). The other coil *i.e.* pressure coil is made highly inductive having an inductance of L and negligible resistance. This is connected across the supply voltage V. The current in the pressure coil I_2 is, therefore, proportional to $V/\omega L = V/\omega L$. Hence $\Phi_2 \propto I_2 \propto V/\omega L$ and Φ_2 lags behind the voltage by 90°. Let the load current I_1 lag behind V by ϕ *i.e.* the load power factor angle ϕ. As shown in Fig. 15-42, the phase angle between Φ_1 and Φ_2 is $\alpha = (90 - \phi)$.

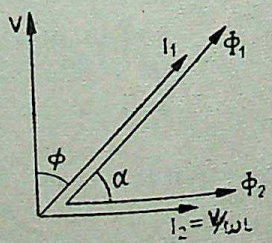

Fig. 15-42

The value of the torque acting on the disc is

$$T = K\,\omega\,\Phi_{1m}\,\Phi_{2m}\sin\alpha \qquad \text{...Art. 15-19}$$

or
$$T \propto \omega . I . \frac{V}{\omega L} \sin(90 - \phi)$$

$$\propto VI\cos\phi$$

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Hence, the torque is proportional to the power in the load circuit. For spring control, the controlling torque $T_c \propto \theta$

$$\therefore \qquad \theta \propto \text{power}$$

Hence, the scale is even.

15-35. Advantages and Limitations of Induction Wattmeters

These wattmeters possess the advantage of fairly long scale (extending over 300°), are free from the effects of stray fields and have good damping. They are practically free from frequency errors. However, they are subject to (sometime) serious temperature errors because the main effect of temperature is on the resistance of the eddy current paths (Art. 15-24).

15-36. Energy Meters

Energy meters are integrating instruments used to measure the quantity of electric energy supplied to a circuit in a given time. They give no direct indication of power *i.e.* as to the rate at which energy is being supplied because their registrations are independent of the rate at which a given quantity of electric energy is being consumed. We will consider the following energy meters.

(*i*) *Electrolytic meters*—their operation, depends on electrolytic action and (*ii*) *Motor meters*—they are really small electric motors.

15-37. Electrolytic Meter

It is used on d.c. circuits* only and is essentially an ampere-hour meter and not a true watthour meter. However, its registrations are converted to watt-hour by multiplying them by the voltage (assumed constant) of the circuit in which it is used. Such instruments are usually calibrated to read kWh directly at the declared voltage. Their readings would, obviously, be incorrect when used on any other voltage. *Because of the question of power factor, such instruments cannot be used on a.c. circuits.*

The advantages of simplicity, cheapness and of low power consumption of ampere-hour meters are, to a large extent, discounted by the fact that variations in supply voltage are not taken into account by them. As an example, suppose that the voltage of a supply whose nominal value is 220 V, has an average value of 216 volts in one hour during which a consumer draws a current of 100 A. Quantity of electricity as indicated by the instrument which is calibrated on 220 V, is $220 \times 100/1000 = 22$ kWh. Actually the energy consumed by customer is only $216 \times 100/1000 = 21{\cdot}6$ kWh. Obviously, the consumer is being overcharged to the extent of the cost of $22 - 21{\cdot}6 = 0{\cdot}4$ kWh of energy per hour. A true wattmeter-hour would have taken into account the decrease in the supply voltage and would have, therefore, resulted in a saving to the customer. If the supply voltage would have been higher by that

*Recently such instruments have been marketed for measurement of kilovolt-ampere hours on a.c. circuits using a small rectifier unit which consists of a current transformer and full-wave copper oxide rectifier.

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amount then the supply company would have been the loser (see Ex. 15-14).

In this instrument, the operating current is passed through a suitable electrolyte contained in a voltameter. Due to electrolysis, a deposit of mercury is given or a gas is liberated (depending on the type of meter) in proportion to the quantity of electricity passed (Faraday's Law of Electrolysis). The quantity of electricity passed is indicated by the level of mercury in a graduated tube. Hence, such instruments are calibrated in ampere-hours or if constancy of supply voltage is assumed, are calibrated in watt-hours or kWh.

Such instruments are cheap, simple and are accurate even at very small loads. They are not affected by stray magnetic fields and due to the absence of any moving parts, are free from friction errors.

15-38. Motor Meters

Most commonly used instruments of this type are :

(*i*) *Mercury motor* meter and (*ii*) *induction motor* meter.

Of these, mercury motor meter is normally used on d.c. circuits whereas the induction type instrument is used only on a.c. circuits.

Instruments used for d.c. work can be either in the form of amp-hour meters or watt-hour meters. In both cases, the moving system is allowed to revolve continuously instead of being merely allowed to deflect or rotate through a fraction of a revolution as in indicating instruments. The speed of rotation is directly proportional to the current in the case of amp-hour meter and to power in the case of watt-hour meter. Hence, the number of revolutions made in a given time is proportional, in the case of an amp-hour meter, to the quantity of electricity ($Q = I \times t$) and in the case of Wh meter to the quantity of energy supplied to the circuit. The number of revolutions made are registered by a counting mechanism consisting of a train of gear wheels and dials.

The control of speed of the rotating system is brought about by a permanent magnet (known as braking magnet) which is so placed as to set up eddy currents in some part of the rotating system. These eddy currents produce a retarding torque which is proportional to their magnitude—their magnitude itself depending on the speed of rotation of the rotating system. The rotating system attains a steady speed when the braking torque exactly balances the driving torque which is produced either by the current or power in the circuit.

The essential parts of motor meters are :

(*i*) An operating system which produces an operating torque proportional to the current or power in the circuit and which causes the rotation of the rotating system.

(*ii*) A retarding or braking device usually a permanent magnet, which produces a braking torque in proportion to the speed of

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rotation. Steady speed of rotation is achieved when braking torque becomes equal to the operating torque.

(*iii*) A registering mechanism for the revolutions of the rotating system. Usually, it consists of a train of wheels driven by the spindle of the rotating system. A worm which is cut on the spindle engages a pinion and so drives a wheel-train.

15-39. Errors in Motor Meters

The two main errors in such instruments are : (*i*) friction error and (*ii*) braking error. Friction error is of much more importance in their case than the corresponding error in indicating instruments because (*a*) it operates continuously and (*b*) it affects the speed of the rotor. The braking action in such meters corresponds to damping in indicating instruments. The braking torque directly affects the speed for a given driving torque, and also the number of revolutions made in a given time.

Friction torque can be compensated for by providing a small constant driving torque to be applied to the moving system independent of the load.

As said earlier, steady speed of such instruments is reached when driving torque is equal to the braking torque. The braking torque is proportional to the flux of the braking magnet and the eddy current induced in the moving system due to its rotation in the field of brake magnet.

$$\therefore \qquad T_B \propto \Phi i$$

where Φ is the flux of the braking magnet and i the induced current. Now $i = e/r$ where e is the induced e.m.f. and r the resistance of the eddy current path. Also $e \propto \Phi n$ where n is the speed of the moving part of the instrument.

$$\therefore \qquad T_B \propto \Phi \times \frac{\Phi n}{r} \propto \frac{\Phi^2 n}{r}$$

The torque T_B' at the steady speed of N is given by

$$T_B' \propto \Phi^2 N/r$$

Now $T_B' = T_D$ ...the driving torque

$$\therefore \qquad T_D \propto \Phi^2 N/r$$

or $$N \propto T_D\, r/\Phi^2$$

Hence, for a given driving torque, the steady speed is directly proportional to the resistance of the eddy current path and inversely to the square of the flux.

Obviously, it is very important that the strength of the field of the brake magnet should be constant throughout the time the meter is in service. The constancy of field strength can be assured by careful design and treatment during the manufacture of the brake magnet. Variations in temperature will affect the braking torque, since the resistance of the eddy current path will change. This error is difficult to fully compensate for.

15-40. Quantity or Ampere-hour (Ah) Meters

The use of such meters is mostly confined to d.c. circuits. Their operation depends on the production of two torques (*i*) a driv-

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ing torque which is proportional to the current I in the circuit and (*ii*) a braking torque which is proportional to the speed N of the spindle. This speed attains a steady value when these two torques become numerically equal. In that case, speed becomes proportional to current *i.e.* $N \propto I$. Over a certain period of time, the total number of revolutions $\int Ndt$ will be proportional to the quantity of electricity $\int Idt$ passing through the meter. A worm cut in the spindle at its top, engages gear wheels or the recording mechanism which has suitably-marked dials reading directly in ampere-hours. Since electric supply charges are based on watt-hours, rather than on ampere-hours the dials of ampere-hour meters are frequently marked in corresponding watt-hours at the normal supply voltage. Hence, their indications of watt-hours are correct only when the supply voltage remains constant, otherwise readings will be wrong.

15-41. Ampere-hour Mercury Motor Meter (*A.M.I.E. Nov. 1972*)

It is one of the best and most popular form of mercury Ah meter used for d.c. work.

Construction

It consists of a thin Cu disc D, mounted at the base of a spindle working in jewelled cup bearings and revolving between a pair of *permanent* magnets M_1 and M_2. One of the two magnets *i.e.* M_2 is used for driving purposes whereas M_1 is used for braking. In between the poles of M_1 and M_2 is a hollow circular box B in which rotates the Cu disc and the rest of the space is filled up with mercury which exerts a considerable upward thrust on the disc, thereby reducing the pressure on the bearings.

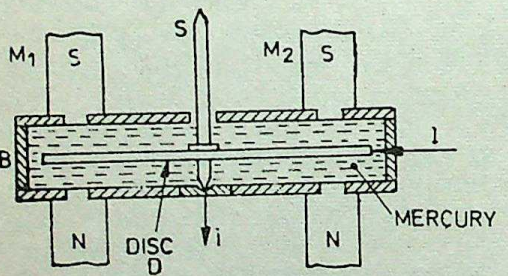

Fig. 15-43

Principle of Action

Its principle of action can be understood from Fig. 15-44 which shows a separate line drawing of the motor element.

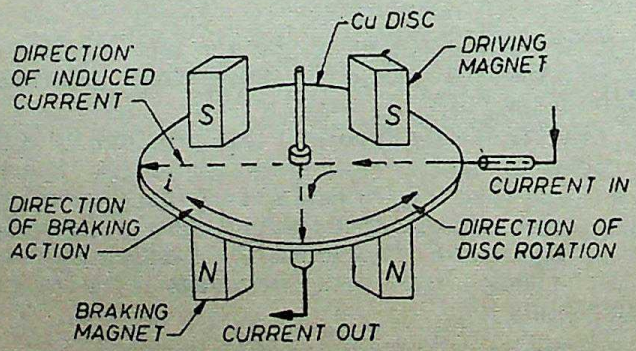

Fig. 15-44

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The current to be measured is led in the disc through the mercury, at a point at its circumference on the right-hand side. As shown by arrows, it flows radially to the centre of the disc where it passes out to the external circuit through the spindle and its bearings. It is worth noting *that current flow takes place only under the right-hand side magnet* M_2 and not under the left-hand side magnet M_1. The field of M_2 will, therefore, exert a force on the right side portion of the disc which carries the current (motor action). The direction of the force, as found by Fleming's Left-hand rule, is as shown by the arrow. The magnitude of the force depends on the flux density and current ($\because F=BIl$). The driving or motoring torque T_d so produced is given by the product of the force and the distance from the spindle at which this force acts. When the disc rotates under the influence of this torque, it cuts through the field of left-hand side magnet M_1 and hence eddy currents are produced in it which results in the production of braking torque. The magnitude of the retarding or braking torque is proportional to the speed of rotation of the disc.

Theory

Driving torque $T_d \propto$ force on the disc

$$\propto BI$$

If the flux density of M_2 remains constant, then

$$T_d \propto I$$

The braking torque T_B is proportional to the flux Φ of braking magnet M_1 and the eddy current i induced in the disc due to its rotation in the field of M_1.

$$\therefore \quad T_B \propto \Phi i$$

Now $i=e/r$ where e is the induced e.m.f. and r the resistance of eddy-current path.

Also $e \propto \Phi n$ where n is the speed of the disc

$$\therefore \quad T_B \propto \Phi \times \frac{\Phi n}{r} \propto \frac{\Phi^2 n}{r}$$

The speed of the disc will attain a steady value N when the driving and braking torques become equal. In that case

$$T_B \propto \Phi^2 N/r$$

Since $T_d = T_B$ —numerically

$$\therefore \quad I \propto \Phi^2 N/r$$

If Φ and r are constant, then

$$I \propto N$$

The total number of revolutions in any given time t *i.e.* $\int_0 N.dt$ will become proportional to $\int_0^t I.dt$ *i.e.* to the total quantity of electricity passed through the meter.

15-42. Friction Compensation

There are two types of friction in this ampere-hour meter.

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(*i*) *Bearing Friction.* The effect of this friction is normally negligible because the disc and spindle float in mercury. Due to upward thrust, the pressure on bearings is considerably reduced which results in freedom from wear as well as a great reduction in the bearing friction.

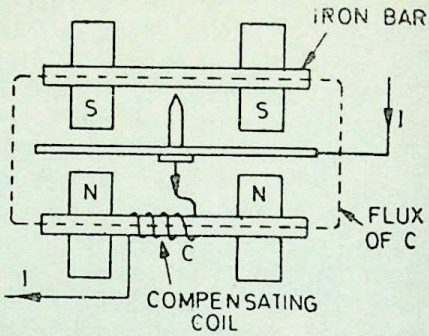

Fig. 15-45

(*ii*) *Mercury Friction.* Since the disc revolves in mercury, there is friction between mercury and the disc which gives rise to a torque approximately proportional to the square of the speed of rotation. Hence, this friction causes the meter to run slow on heavy loads. It can be compensated for in the following two ways :

(*a*) a coil of few turns is wound on one of the poles of the driving magnet M_2 and the meter current is passed through it in a suitable direction so as to increase the strength of M_2. The additional driving torque so produced can be made just sufficient to compensate for the mercury friction.

(*b*) In the other method, two iron bars are placed across the two permanent magnets, one above and one below the mercury chamber as shown in Fig. 15-45. The lower bar carries a small compensating coil through which is passed the load current. The local magnetic field set up by this coil strengthens the field of driving magnet M_2 and weakens that of the braking magnet M_1 thereby compensating for mercury friction.

15-43. Mercury Meter Modified as Watt-hour Meter

If the permanent magnet M_2 of the amp-hour meter, used for producing the driving torque, is replaced by a wound electromagnet, connected across the supply, the result is a watt-hour meter. The exciting current of this electromagnet is proportional to the voltage of the supply. The driving torque is exerted on the aluminium disc immersed in the mercury chamber below which is placed this electromagnet. The aluminium disc has radial slots cut in it for ensuring the radial flow of current through it—the current being led into and out of this disc through mercury contacts situated at diametrically opposite points. These radial slots, moreover, prevent the same disc being used for braking purposes. Braking is by a separate aluminium disc mounted on the same spindle and revolving in the air-gap of a separate braking magnet.

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15-44. Induction Type Single-phase Watthour Meter

(A.M.I.E. Summer 1986)

Induction type meters are, by far, the most common form of a.c. meters met with in every-day domestic and industrial installations. These meters measure electric energy in kilowatthours. The principle of these metres is practically the same as that of the induction wattmeters. Constructionally, the two are similar except that the control spring and the pointer of the watt-meter are replaced, in the case of watthour meter, by a brake magnet and by a spindle of the meter.

The brake magnet induces eddy currents in the disc which revolves continuously instead of rotating through only a fraction of a revolution as in the case of wattmeters.

Construction

The meter consists of two a.c. magnets of laminated construction as shown in Fig. 15-46 (*a*), one of which *i.e.* M_1 is excited by line current and is known as 'series' or 'current' magnet. The alternating flux Φ_1 produced by it is directly proportional to and in phase with current provided the effects of hysteresis and saturation are neglected. The winding of the other magnet M_2 is connected across the supply lines and so it carries a current I_2 which is proportional to the supply voltage V. Hence, it is called 'voltage' or 'shunt' magnet. The flux Φ_2 produced by magnet M_2 lags behind

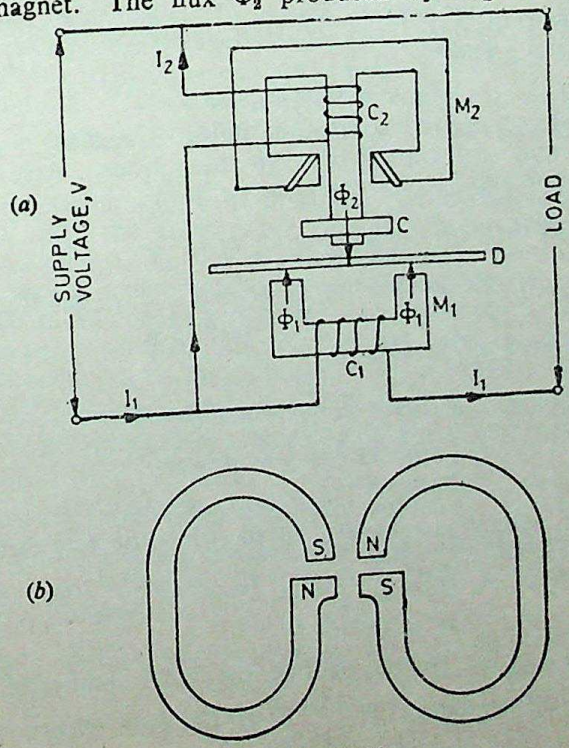

Fig. 15-46

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the voltage by 90° (Fig. 15-42). This phase displacement of exact 90° is achieved by the adjustment of the copper shading band C (also known as power factor compensator) on the shunt magnet M_2. Major portion of Φ_2 crosses the narrow gap between the centre and side limbs of M_2 but a small amount, which is the useful flux, passes through the disc D. The two fluxes Φ_1 and Φ_2 induce e.m.fs in the disc which further produce the circulatory eddy currents. The reaction between these fluxes and eddy currents produces the driving torque on the disc in a manner similar to that explained in Art 15-34. The braking torque is produced by a pair of magnets [Fig. 15-46 (*b*)] which are mounted diametrically opposite to the magnets M_1 and M_2. This arrangement minimizes the interaction between the fluxes of M_1 and M_2 and that of the braking magnet. When the peripheral portion of the rotating disc passes through the air-gap of the braking magnet, the eddy currents are induced in it which give rise to the necessary torque. The braking torque $T_B \propto \Phi^2 N/r$ where Φ is the flux of braking magnet, N the *steady* speed of the rotating disc and r the resistance of the eddy current path. If Φ and r are constant, then $T_B \propto N$.

The register mechanism is either of pointer type or cyclometer type. In the former type, the pinion on the rotor shaft drives, with the help of a suitable train of reduction gears, a series of five or six pointers rotating on dials marked in kWh.

Theory

As shown in Art. 15-34 and with reference to Fig. 15-42 the driving torque is given by

$$T_d \propto \omega \Phi_{1m} \Phi_{2m} \sin \alpha$$

where Φ_{1m} and Φ_{2m} are the maximum fluxes produced by magnets M_1 and M_2 and α the angle between these fluxes. Assuming that fluxes are proportional to the currents, we have

Current I_2 through the winding of $M_2 = \dfrac{V}{\omega L}$

Current through the winding of $M_1 = I_1$ .....the line current

$$\alpha = 90 - \phi$$

where ϕ is the load p.f. angle.

$$T_d \propto \omega . I . \frac{V}{\omega L} \cos (90 - \phi)$$

$$\propto VI \cos \phi \propto \text{power}$$

Also, $T_B \propto N$

The disc achieves a steady speed when the two torques are equal *i.e.* when

$$T_d = T_B$$

$$\therefore \quad N \propto \text{power } W$$

Hence, in a given period of time, the total number of revolutions $\int_0^t N\,dt$ is proportional to $\int_0^t W\,dt$ *i.e.* the electric energy consumed.

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15-45. Errors in Induction Watthour Meters

(1) *Phase Errors* (*A.M.I.E. Summer 1983*)

Because, ordinarily the flux due to shunt magnet does not lag behind the supply voltage by exactly 90° owing to the fact that the coil has some resistance, the torque is not zero at zero power factor. This is compensated for by means of an adjustable shading ring placed over the central limb of the shunt magnet. That is why this shading ring is known as *power factor compensator.*

The supply voltage, the full-load current and the correct number of revolutions per killowatthour are indicated on the name plate of the meter.

(2) *Friction Compensation and Creeping Error*

Frictional forces at the rotor bearings and in the register mechanism give rise to an unwanted braking torque on the disc rotor. This can be reduced to an unimportant level by making the ratio of the shunt magnet flux Φ_2 and series magnet flux Φ_1 large with the help of two shading bands. These bands embrace the flux contained in the two outer limbs of the shunt magnet and so eddy currents are induced in them which cause phase displacement between the enclosed flux and the main-gap flux. As a result of this, a small driving torque is exerted on the disc rotor solely by the pressure coil and independent of the main driving torque. The amount of this corrective torque is adjusted by the variation of the position of the two bands, so as to exactly compensate for frictional torque in the instrument. Correctness of friction compensation is achieved when the rotor does not run on no-load *with only the supply voltage connected.*

By '*creeping*' is meant the slow but continuous rotation of the rotor when only the pressure coils are excited but with no current flowing in the circuit. It may be caused due to various factors like incorrect friction compensation, to vibration, to stray magnetic fields or due to the voltage supply being in excess of the normal. In order to prevent creeping on no-load, two holes are drilled in the disc on a diameter *i.e.* on the opposite sides of the spindle.

This causes sufficient distortion of the field to prevent rotation when one of the holes comes under one of the poles of the shunt magnet

(3) *Errors Due to Temperature Variations*

The errors due to temperature variations of the instruments are usually small, because the various effects produced tend to neutralise one another.

Example 15-13. *The name-plate of a meter reads "1 kWh= 15,000 revolutions." In a check-up, the meter completed 150 revolutions during 50 seconds. Calculate the power in the circuit.*

Solution. Power metered in 150 revolutions

$= 1 \times 150/15,000 = 1/100$ kWh

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If P kilowatt is the power in the circuit, then energy consumed in 50 seconds is $= P \times 50/3600$ kWh

Equating the two amounts of energy, we have

$P \times 50/3600 = 1/100$; $P = 0{\cdot}72$ kW $=$ **720 W**

Example 15-14. *An ampere-hour meter is calibrated to give reading of energy in kilowatt-hours on 220-volt circuit. When used on 250-V circuit, it records 845 kWh in a certain time. Calculate the energy actually supplied.*

Solution. As explained in Art. 15-38, ampere-hour meters are calibrated to read direct in kWh *at the declared voltage.* Obviously, their readings would be incorrect when used on any other voltage.

Reading on the assumed 220 V $= 845$ kWh

Actual reading on 250-V circuit $= 845 \times 250/220 =$ **960**

Example 15-15. *A 5-ampere, 230-V meter on full-load unity power factor test makes 60 revolutions in 360 seconds. If the normal disc speed is 520 revolutions per kWh, what is the percentage error ?*

Solution. Testing time $= 360$ second $= 1/10$ hour

Energy-consumed in 1/10 hour $= 230 \times 5 \times 1/10 = 115$ Wh

$= 115/1000 = 0{\cdot}115$ kWh

If the meter were running correctly *i.e.* at 520 rev/kWh, then revolutions for 0·115 kWh would have been $= 0{\cdot}115 \times 520 = 59{\cdot}8$.

Actual revolutions made during this test time are 60.

$\therefore$ error $= 60 - 59{\cdot}8 = 0{\cdot}2$

percentage error $= 0{\cdot}2 \times 100/60 =$ **0·33%**

Example 15-16. *A 230-V ampere-hour type meter is connected to a 230-V d.c. supply. If the meter completes 225 revolutions in 10 minutes when carrying 14 A, calculate :*

(a) the kWh registered by the meter and

(b) the percentage error of the meter above or below the original calibration.

The timing constant of the meter is 40 A-s/revolution.

Solution. The declared constant of the meter is 40 A-s/rev. During 225 revolutions, it would register 40×225 A-s or coulomb. Since time taken is 10 minutes or 600 seconds it corresponds to a current of $40 \times 225/600 = 15$ A

(*a*) $\therefore$ energy as registered by the meter is

$$= \frac{230 \times 15}{1000} \times \frac{1}{6} = \mathbf{0{\cdot}575\ kWh}$$

(*b*) Actual energy consumed $= \frac{230 \times 14}{1000 \times 6} = 0{\cdot}5367$ kWh

$$\text{percentage error} = \frac{0{\cdot}575 - 0{\cdot}5367}{0{\cdot}575} \times 100 = \mathbf{6{\cdot}66\%}$$

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Alternately, we may find the speed thus :

Ampere-seconds passed in 10 minutes $= 14 \times (10 \times 60)$

No. of revolutions that should have been made at the rate of 40 A-s/rev is

$$= 14 \times 600/40 = 210$$

Revolutions actually made during that period is 225.

$$\therefore \quad \text{error} \quad = \frac{225 - 210}{225} \times 100 = 6{\cdot}66\%$$

Obviously, the meter reads high.

Example 15-17. *A wattmeter and watt-hour meter are connected in series with a loading resistance across a 200-V d.c. supply. The reading of the watthour meter 1½ hours after the supply was switched on was 4,320 Wh, the initial reading being zero. Find*

(*a*) *the wattmeter reading*

(*b*) *the load resistance*

If the supply voltage is increased to 300 V, what will be the readings on the instruments after a further hour ? (Neglect any instrument errors)

Solution. (*a*) Wattmeter reading $= 4{,}320 \text{ Wh}/1{\cdot}5 \text{ h} = \mathbf{2{,}880\ W}$

(*b*) $V^2/R = W$ or $R = V^2/W = 200^2/2880 = \mathbf{13{\cdot}9\ \Omega}$

When voltage is increased, wattmeter reading would increase in the ratio of $(300/200)^2$ because $W = V^2/R$

$\therefore$ Wattmeter reading with 300 V $= 2{,}880 \times (300/200)^2$

$$= \mathbf{6{,}480\ W}$$

Similarly, watthour reading would increase by an amount

$$= 2880 \times (300/200)^2 \text{ Wh} = 6{,}480 \text{ kWh}$$

Being an integrating instrument, the watthour meter readings go on adding over the length of time.

total reading of the watthour meter is

= previous reading + reading in the next hour

$= 4{,}320 + 6{,}480 = 10{,}800 \text{ W} = \mathbf{10{\cdot}8\ kWh}$

15-46. Megger

(A.M.I.E. Summer 1986)

It is a portable instrument used for testing the insulation resistance of a circuit and for measuring resistances of the order of megohms which are connected across the outside terminals XY in Fig. 15-47 (*b*).

Working Principle

The working principle of a 'cross-coil' type megger may be understood from Fig. 15-47 (*a*) which shows two coils A and B mounted rigidly at right angles to each other on a common axis and free to rotate in a magnetic field. When currents are passed through

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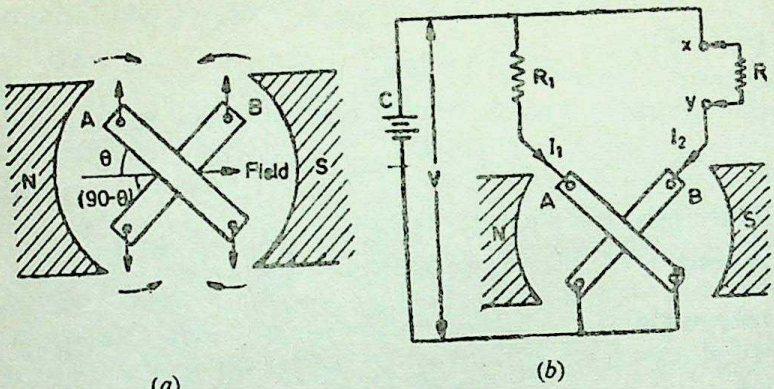
them, the two coils are acted upon by torques which are in opposite directions. The torque of coil A is proportional to $I_1 \cos \theta$ and



(a) (b)

Fig. 15-47

that of B is proportional to $I_2 \cos (90-\theta)$ or $I_2 \sin \theta$. The two coils come to a position of equilibrium where the two torques are equal and opposite *i.e.*

$$I_1 \cos \theta = I_2 \sin \theta$$

or

$$\tan \theta = I_1/I_2$$

In practice, however, by modifying the shape of pole faces and the angle between the two coils, the ratio I_1/I_2 is made proportional to θ instead of $\tan \theta$ in order to achieve a linear scale.

Suppose the two coils are connected across a common source of voltage *i.e.* battery C as shown in Fig. 15-47 (*b*). Coil A which is connected directly across V is called the voltage (or control) coil. Its current $I_1 = V/R_1$. The coil B called current or deflecting coil carries the current $I_2 = V/R$ where R is the external resistance to be measured. This resistance may vary from infinity (for good insulation or open circuit) to zero (for poor insulation or a short-circuit). The two coils are free to rotate in the field of a permanent magnet. The deflection θ of the instrument is proportional to I_1/I_2 which is equal to R/R_1. If R_1 is fixed, then the scale can be calibrated to read R directly (in practice, a current-limiting resistance is connected in the circuit of coil B, but the presence of this resistance can be allowed for in scaling). The value of V is immaterial so long as it remains constant and is large enough to give suitable currents with the high resistances to be measured.

Construction

The essential parts of the megger are shown in Fig. 15-48. Instead of battery C of Fig. 15-47 (*b*), there is a hand-driven d.c. generator. The crank turns the generator armature through a clutch mechanism which is designed to slip at a pre-determined speed. In this way, the generator speed and voltage are kept constant and at their correct values when testing.

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The generator voltage is applied across the voltage coil A through a fixed resistance R_1 and across deflecting coil B through a current limiting resistance R' and the external resistance connected across the testing terminals XY. The two coils, in fact, constitute a moving-coil voltmeter and an ammeter combined into one instrument.

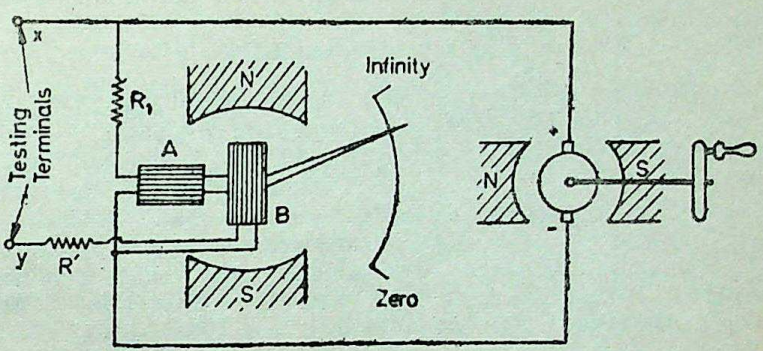

Fig. 15-48

(*i*) Suppose the terminals XY are open-circuited. Now, when crank is operated, the generator voltage so produced is applied across coil A and current I_1 flows through it but no current flows through coil B. The torque so produced rotates the moving element of the megger until the scale points to 'infinity', thus indicating that the resistance of the external circuit is too large for the instrument to measure.

(*ii*) When the testing terminals XY are closed through a low resistance or are short-circuited, then a large current (limited only by R') passes through the deflecting coil B. The deflecting torque produced by coil B overcomes the small torque of coil A and rotates the moving element until the needle points to 'zero' thus showing that the external resistance is too small for the instrument to measure.

Although, the megger can measure all resistances lying between zero and infinity, essentially it is a high-resistance measuring device. Usually, zero is the first mark and 10 kΩ is the second mark on its scale, so one can appreciate that it is impossible to accurately measure small resistances with the help of a megger.

The instrument described above is simple to operate, portable, very robust and independent of the external supplies.

15-47. Wheatstone Bridge *(A.M.I.E. Summer 1986)*

It is a four-arm bridge and is extensively used for the measurement of medium-range resistances (1 to 1,00,000 Ω). Its wire form

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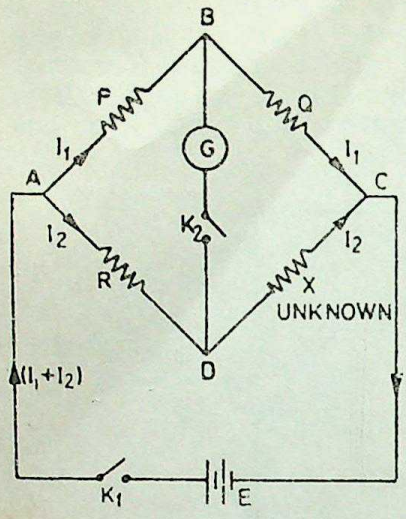

Fig. 15-49

the basis from which many other 'bridge' circuits have been developed both for direct and alternating currents. This method is, however, a comparative one, because the value of the unknown resistance is obtained in terms of a known resistance.

Layout

As shown in Fig. 15-49, four resistances are connected to form a square *ABCD*. A sensitive detector or galvanometer is connected across *BD*. The arms *P* and *Q* are known as ratio arms and are generally adjustable to one of the four values 1, 10, 100 and 1000 Ω in steps of 1·0 Ω.

First, the values of *P* and *Q* are set at 10 Ω each. The unknown resistance *X* is connected in the arm *CD*. Next the battery key K_1 is closed, causing current to flow through the two sets of bridge arms. Now, when galvanometer key K_2 (spring type) is closed, there will be some deflection shown by the galvanometer. The resistance *R* is changed suitably and K_2 is pressed again. If still the galvanometer shows some deflection, then K_2 is opened. *R* is changed again in this manner, till on pressing K_2, there is no deflection shown by the galvanometer. The bridge is then said to be balanced and this method is known as "null" deflection method.

Theory

Since there is no galvanometer deflection, no current flows along *BD*. It means that (*i*) points *B* and *D* are at the same potential (*ii*) that current through arm *BC* is the same as through *AB i.e.* I_1 and (*iii*) that current through arm *DC* is the same as through *AD i.e.* I_2

Since drop across *AB* is the same as across *AD*

$$\therefore \quad I_1P=I_2R \qquad \ldots(i)$$

Similarly $$I_1Q=I_2X \qquad \ldots(ii)$$

Dividing one equation by the other, we get

$$\frac{P}{Q}=\frac{R}{X} \quad \text{or} \quad PX=QR^*$$

$$\therefore \text{ Unknown resistances } \quad X=R\times\frac{Q}{P}$$

Hence, if *P*, *Q* and *R* are known, *X* can be easily found. The experiment is again repeated by keeping *Q* at 10 Ω as before and

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*Products of the resistances of the opposite arms are equal.

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changing P to 100 Ω and then 1000 Ω. Again, balance is obtained for each ratio and X found in each case by using the above relation. Mean of the three readings gives the value of X.

It may be noted that

(*i*) position of the battery and galvanometer could be interchanged in their connection to the square *ABCD*. However, with the connections as shown in Fig. 15-49, the bridge has greater sensitivity.

(*ii*) since galvanometer is merely an indicator of zero current, it need not be calibrated in any particular units.

The Wheatstone bridge principle has been used in the construction of Post-office box, slide-wire bridge and many loop-tests like Murray's and Varley's which are used for finding the position of earth faults on telegraph and telepone lines.

Example 15-18. *In a Wheatstone bridge experiment for determining the resistance of a wire, a balance point was obtained when $R=30$ Ω, $P=Q=10$ Ω. Find the resistance of the wire. Next, the length of wire was found to be 110 cm and its thickness as measured by a screw gauge 0·014 cm. Calculate the specific resistance of the material of the wire.*

Solution. If X is the unknown resistance, then

$$\frac{P}{Q}=\frac{R}{X}$$

$$\therefore \quad X=\frac{R\times Q}{P}=\frac{30\times 10}{10}=30\ \Omega$$

Now $$R=\rho\frac{l}{A} \qquad \therefore \quad \rho=A\,R/l$$

$$\rho=\frac{\pi(0\,014)^2\times 30\times 100}{4}=42\times 10^{-6}\ \text{ohm-cm}$$

15-48. D.C. Potentiometer *(A.M.I.E. Winter 1985)*

A potentiometer is used for measuring and comparing the e.m.fs. of different cells and for calibrating and standardizing voltmeters, ammeters and wattmeters etc. In its simplest form, it consists of a German silver or manganin wire usually one meter long and stretched between two terminals as shown in Fig. 15-50.

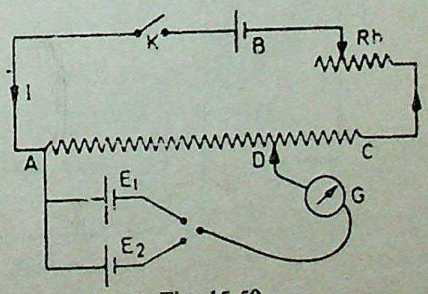

Fig. 15-50

This wire is connected in series with a suitable rheostat *RH* and battery *B* which sends a steady current through the resistance wire *AC*. As the wire is of uniform cross-section throughout, the fall in potential across it is uniform and the drop between any two points is proportional to the distance between

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them. As seen, the battery voltage is spread over the rheostat and the resistance wire AC. As we go along AC, there is a progressive fall of potential. If ρ is the resistance/cm of this wire, l its length, then for a current of I amperes, the fall of potential over the whole length of wire is ρlI volts.

The two cells whose e.m.fs are to be compared are joined as shown in Fig. 15-50, always remembering that *positive terminals of the cells and the battery must be joined together*. The cells can be joined with the galvanometer in turn through a two-way key. The other end of the galvanometer is connected to a movable contact on AC. By this movable end, a point like D, is found where there is no current in and hence no deflection of G. Then it means that the e.m.f. of the cell just balances the potential fall on AD due to the battery current passing through it.

Suppose that the balance or null point for first cell of e.m.f. E_1 occurs at length l_1 as measured from point A. Then

$$E_1 = \rho l_1 I$$

Similarly, if the balance point is at l_2 for the other cell then

$$E_2 = \rho l_2 I$$

Dividing one equation by the other, we hove

$$\frac{E_1}{E_2} = \frac{\rho l_1 I}{\rho l_2 I} = \frac{l_1}{l_2}$$

If one of the cells is a standard cell, the e.m.f. of the other cell can be found.

15-49. Direct-reading Potentiometer *(A.M.I.E. Winter 1980)*

The simple potentiometer described above is used for educational purposes only. But in its commercial form, it is so calibrated that the readings of the potentiometer give the voltage directly, thereby eliminating tedious arithmetical calculations and hence saving appreciable time.

Such a direct-reading potentiometer is shown in Fig. 15-51. The resistance R consists of 14 equal resistances joined in series, the

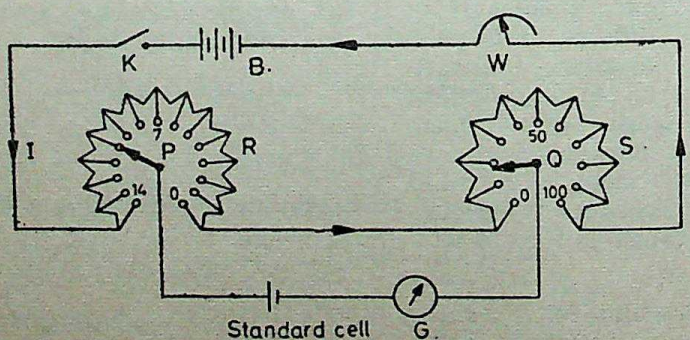

Fig. 15-51

resistance of each unit being equal to that of the whole slide wire S (which is divided into 100 equal parts). The battery current is controlled by slide wire resistance W.

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15-50. Standardizing the Potentiometer (*A.M.I.E. Winter 1980*)

A standard cell *i.e.* Weston Cadmium cell of e.m.f. 1·0183 V is connected to sliding contacts P and Q through a sensitive galvanometer G. First, P is put on stud No. 10 and Q on 18·3 division on S and then W is adjusted for zero deflection on G. In that case, potential difference between P and Q is equal to cell voltage *i.e.* 1·0183 V so that potential drop on each resistance of R is $1/10 = 0{\cdot}1$ V and every division of S represents $0{\cdot}1/100 = 0{\cdot}001$ V. After standardizing this way, *the position of W is not to be changed in any case* otherwise the whole adjustment would go wrong. After this, the instrument becomes direct reading. Suppose in a subsequent experiment, for obtaining balance, P is moved to stud No. 7 and Q to 84 division, then voltage would be $(7 \times 0{\cdot}1) + (84 \times 0{\cdot}001) = 0{\cdot}784$ V.

It should be noted that since most potentiometers have fourteen steps on R it is usually not possible to measure p.ds. exceeding 1·5 V. For measuring higher voltages, it is necessary to use a volt box.

15-51. Calibration of Ammeters (*A.M.I.E. Winter 1984*)

The ammeter to be calibrated is connected in series with a variable resistance and standard resistance F, say, of 0·1 Ω across a battery B_1 of ample current capacity as shown in Fig. 15-52. Obviously, the resistance of F should be such that with maximum current flowing through the ammeter A, the potential drop across it should not exceed 1·5 V. Some convenient current, say, 6 amperes (as indicated by A) is passed through the circuit by adjusting the rheostat RH.

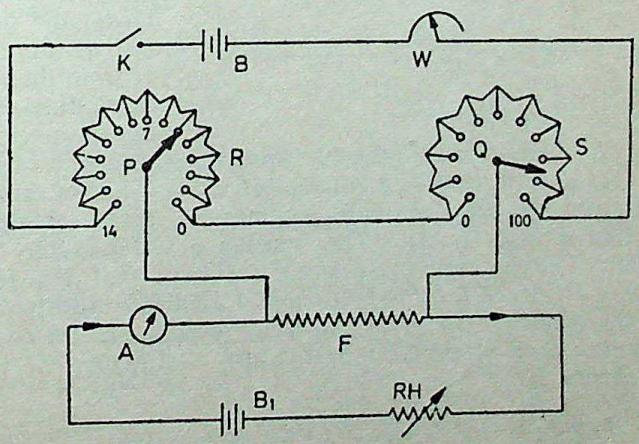

Fig. 15-52

The potential drop across F is applied between P and Q as shown. Next, the sliding contacts P and Q are adjusted for zero deflection on G. Suppose P reads 5 and Q reads 86·7. Then, it means that p.d. across F is 0·5867 V and since F is of 0·1 Ω, hence, *true* value of current through F is $0{\cdot}5867/0{\cdot}1 = 5{\cdot}867$ ampere. Hence, the

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ammeter reads high by 6—5·867=0·133 A. The test is repeated for various values of the current over the entire range of the ammeter*.

15-52. Calibration of Voltmeters *(A.M.I.E. Winter 1980)*

As pointed out in Art. 15-50, a voltage higher than 1·5 V cannot be measured by the potentiometer directly, the limit being set by the standard cell and the type of the potentiometer (as it has only 14 resistances on R as in Fig. 15-51). However, with the help of a volt-box which is nothing else but a voltage reducer, measurements of voltages up to 150 V or 300 V can be made, the upper limit of voltage depending on the design of the volt-box.

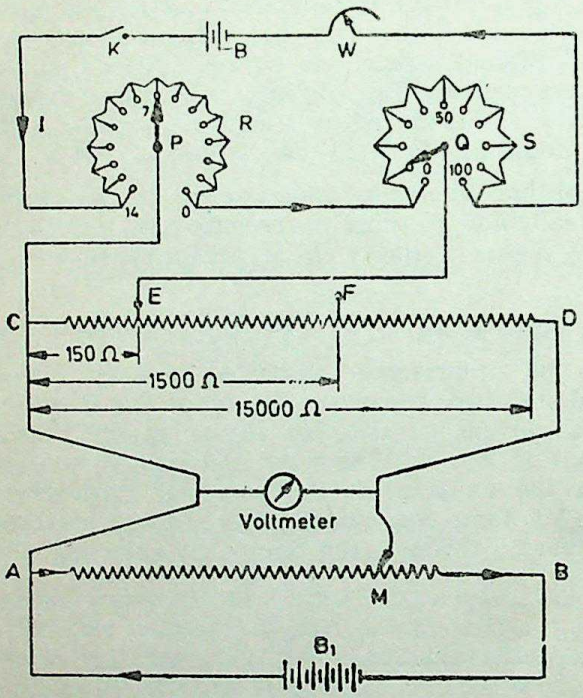

Fig. 15-53

The diagram of connections for calibration of voltmeters, is shown in Fig. 15-53. By calibration is meant the determination of the extent of error in the reading of the voltmeter throughout its range. A high resistor AB is connected across the supply terminals of a high-voltage battery B_1 and it acts as a voltage divider. The volt-box consists of a high resistance CD with tappings at accurately-determined points like E and F etc. The resistance CD is usually 15,000 to 30,000 Ω. The two tappings F and E are such that the resistances of portions CF and CE are 1/10th and 1/100th the resistance of CD. Obviously, whatever the potential drop across CD, the corresponding potential drop across CF is 1/10th and that across CE is 1/100th of that across CD.

If supply voltage is 150 V, then p.d. across AB is also 150 V, and if M coincides with B, then p.d. across CD is also 150 V, so across CF is 15 volts and across CE is 1·5 V. The p.d. across CE can be balanced over the potentiometer as shown in Fig. 15-53. Various

*This method may also be used for finding the value of unknown resistance. In that case, p.d. across the resistance is measured and then divided by current to give the value of the unknown resistance.

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voltages can be applied across the voltmeter by moving the contact point M on the resistance AB.

Suppose that M is so placed that voltmeter V reads 70 V and p.d. across CE is balanced by adjusting P and Q. If the readings on P and Q, to give balance, are 7 and 8·4 respectively, then p.d. across CE is 0·7084 V.

Hence, the true p.d. across AM or CD or voltmeter is $0{\cdot}7084 \times 100 = 70{\cdot}84$ V (because resistance of CD is 100 times greater than that of CE). In other words, the reading of the voltmeter is low by 0·84 V.

By shifting the position of M and then balancing the p.d. across CE on the potentiometer, the voltmeter can be calibrated throughout its range. By plotting the errors on a graph, a calibration curve of the instrument can also be drawn.

Example 15-19. *A standard cell of 1·0185 V, when used in a one metre long slide-wire potentiometer, balances at 60 cm. Calculate (i) the percentage error in a voltmeter which balances at 65 cm when reading 1·1 V and (ii) the percentage error in an ammeter that reads 0·345 A when balance is obtained at 40 cm with the potential drop across a 2-ohm resistance in the ammeter circuit.*

(A.M.I.E. Winter 1974)

Solution. It is obvious that 1·0185 V drops over 60 cm length of the slide-wire. Hence, potential drop across one cm length of the potentiometer wire is $= 1{\cdot}0185/60$ V/cm.

(*i*) Voltage drop across 65 cm of slide-wire

$$= 65 \times 1{\cdot}0185/60 = 1{\cdot}1035 \text{ V}$$

Reading indicated by voltmeter $= 1{\cdot}1$ V

Obviously, the voltmeter reads low by 0·0035 V.

$$\% \text{ error} = \frac{0{\cdot}0035}{1{\cdot}1035} \times 100 = \mathbf{0{\cdot}31}$$

(*ii*) Voltage drop across 40 cm of wire

$$= 40 \times 1{\cdot}0185/60 = 0{\cdot}679 \text{ V}$$

Obviously, this is the potential drop across the 2-ohm resistance.

Current through 2 ohm resistance

$$= 0{\cdot}679/2 = 0{\cdot}3395 \text{ A}$$

Ammeter reading $= 0{\cdot}345$ A

Absolute error $= 0{\cdot}345 - 0{\cdot}3395 = 0{\cdot}0055$ A

Percentage error $= 0{\cdot}0055 \times 100/0{\cdot}3395 = \mathbf{0{\cdot}14}$

It is obvious that the ammeter reads a bit high.

Example 15-20. *A potentiometer consists of 14 equal decade resistors and slide wire with 100 scale divisions, the total resistance of the slide-wire being equal to that of one of the decade resistors.*

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The potentiometer is calibrated to read voltages up to 1·5 V and is connected in turn across a standard 5 Ω resistor and an unknown resistor which are both in series with an ammeter carrying a current. The readings of the potentiometer at balance and the ammeter are as follows :

Across standard resistor : decade 12, slide wire 73

Across unknown resistor : decade 5, slide wire 56.

Ammeter reading ; 0·25 A

Calculate :

(*a*) *the value of the unknown resistor*

(*b*) *the error in the ammeter reading.*

Solution. As explained in Art. 15-50, drop across each decade resistor is 0·1 V and each division of the slide wire represents $0·1/100 = 0·001$ V.

Voltage drop across 5 Ω standard resistor is

$$= (12 \times 0·1) + (73 \times 0·001) = 1·273 \text{ V}$$

Correct value of circuit current $= 1·273/5 = 0·2546$ A

It is obvious that the ammeter is reading less.

(*b*) ammeter reading $= 0·25$ A

$$\text{error} = 0·25 - 0·2546 = -0·0046 \text{ A}$$

$$\text{percentage error} = \frac{-0·0046}{0·25} \times 100 = \mathbf{-1·84}\ \%$$

(*a*) p.d. across unknown resistance

$$= (5 \times 0·1) + (0·001 \times 56) = 0·556 \text{ V}$$

actual value of circuit current

$$= 0·2546 \text{ A}$$

$$\text{unknown resistance} = 0·556/0·2546 = \mathbf{2·184}\ \Omega$$

Tutorial Problems 23 A

1. A 230-V, 1-ϕ energy meter has a constant load current of 10 A at unity p.f. If the meter disc makes 1150 revolutions during 2 hours, calculate the meter constant. If the p.f. were 0·8, what would be the number of revolutions made by the disc in thattime ?

[250 rev/kWh ; 920] (*A.M.I.E. Summer 1976*)

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A.C. FUNDAMENTALS 16

1. Generation of Alternating Voltages and Currents 2. Equations of the Alternating Voltages and Currents 3. Alternate Method for the Equations of Alternating Voltages and Currents 4. Simple Waveforms 5. Cycle 6. Time period 7. Frequency 8. Amplitude 9. Different Forms of E.M.F. Equation 10. Phase 11. Phase Difference 12. Root Mean Square (R.M.S.) Value 13. Mid-ordinate Method 14. Analytical Method 15. Average Value 16. Form Factor 17. Peak Factor 18. R.M.S. Value of Half-wave Rectified A.C. 19. Average Value 20. Form Factor 21. Vector Representation of Alternating Quantities 22. Vector Diagrams Using R.M.S. Values 23. Vector Diagrams of Sine Waves of Same Frequency 24. Addition of Two Alternating Quantities 25. Addition and Subtraction of Vectors 26. A.C. Through Resistance, Inductance and Capacitance 27. A.C. Through Pure Ohmic Resistance Only 28. A.C. Through Inductance Only 29. A.C. Through Capacitance Alone 30. Highlights

16-1. Generation of Alternating Voltages and Alternating Currents

Referring to Art. 10-2 and Figs. 10-1 and 10-2, it is clear that alternating voltage may be generated by rotating a coil in a magnetic field as shown again in Fig. 16-1 (*a*) or by rotating a magnetic field within a stationary coil as shown in Fig. 16-1 (*b*).

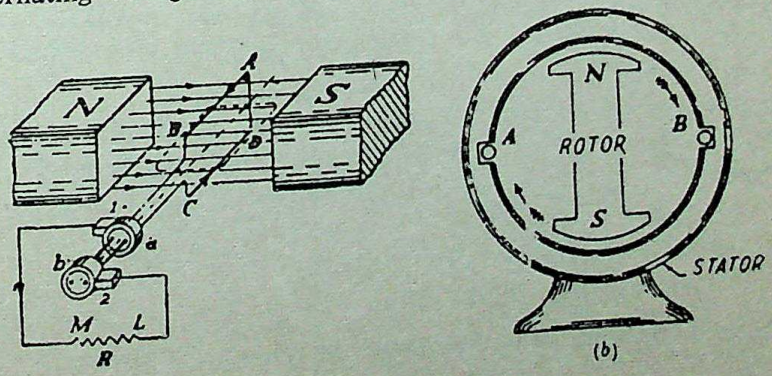

Fig. 16-1

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The value of the voltage generated depends, in each case, upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. Alternating voltage may be generated in either of the two ways shown above although rotating field method is the one which is mostly used in practice.

16-2. Equations of the Alternating Voltages and Currents

Consider a rectangular coil having N turns rotating in a uniform magnetic field with an angular velocity of ω radian/second as shown in Fig. 16-2. Let time be measured from the X-axis. Maximum flux Φ_m is linked with the coil when its plane coincides with the X-axis. In time t seconds, this coil rotates through an angle $\theta=\omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of the coil is $\Phi=\Phi_m \cos \omega t$. Hence 'flux-linkages' of the coil in this deflected position are

$$N\Phi=N\Phi_m \cos \omega t$$

According to Faraday's Laws of Electromagnetic Induction, the e.m.f. induced in the coil is given by the rate of change of flux-linkages of the coil. Hence the value of the induced e.m.f. at this instant (*i.e.* when $\theta=\omega t$) or the instantaneous value of the induced e.m.f. is

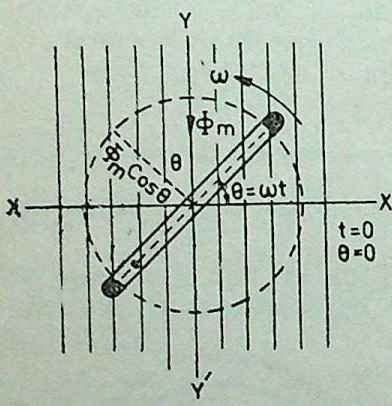

Fig. 16-2

$$e=-\frac{d}{dt}(N\Phi) \text{ volt}$$

$$=-N.\frac{d}{dt}(\Phi_m \cos \omega t) \text{ volt}$$

$$=-\omega N\Phi_m(-\sin \omega t) \text{ volt}$$

$$=\omega N\Phi_m \sin \omega t \text{ volt}$$

$$=\omega N\Phi_m \sin \theta \text{ volt} \quad ...(i)$$

When the coil has turned through 90° *i.e.* when $\theta=90°$, then $\sin \theta=1$, hence e has maximum value, say E_m. Therefore, from Eq. (*i*) we get

$$E_m=\omega N\Phi_m \text{ volt}$$

$$=\omega N B_m A \text{ volt}$$

$$=2\pi f N B_m A \text{ volt} \quad ...(ii)$$

where B_m = maximum flux density in Wb/m²

A = area of the coil in m²

f = frequency of rotation of the coil in rev/s

Substituting this value of E_m in Eq. (*i*) we get

$$e=E_m \sin \theta=E_m \sin \omega t \quad ...(iii)$$

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Similarly, the equation of the induced alternating current is

$$i = I_m \sin \omega t \qquad \ldots(iv)$$

provided the coil circuit has been closed through a resistive load.

Since $\omega = 2\pi f$, where f is the frequency of rotation of the coil, the above equations of the voltage and current can be written as

$$e = E_m \sin 2\pi ft = E_m \sin \left(\frac{2\pi}{T}\right) t$$

$$i = I_m \sin 2\pi ft = I_m \sin \left(\frac{2\pi}{T}\right) t$$

where T = time-period of the alternating voltage or current

$= 1/$

It is seen that the induced e.m.f. varies as sine function of the time angle ωt and when e.m.f. is plotted against time, a curve similar to one shown in Fig. 16-3 is obtained. This curve is known as sine curve and the e.m.f. which varies in this manner is known as *sinusoidal* e.m.f. Such a sine curve can be conveniently drawn as shown in Fig. 16-4. A vector equal in length to E_m is drawn. It rotates in the counter clockwise direction with the velocity of ω radian/second making one revolution while the generated e.m.f. makes two loops or one cycle. The projection of this vector on Y-axis gives the instantaneous value of the induced e.m.f. *i.e.* $E_m \sin \omega t$.

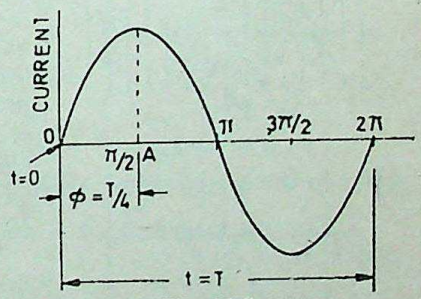

Fig. 16-3

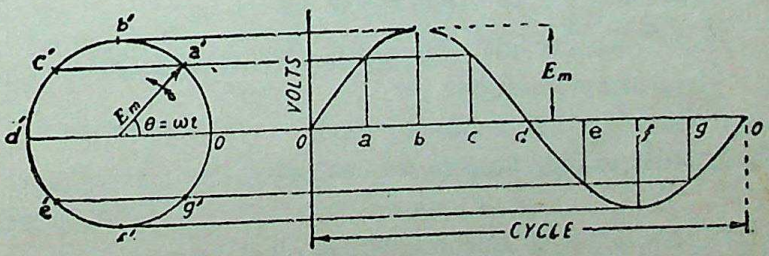

Fig. 16-4

To construct the curve, lay off along X-axis equal angular distances oa, ab, bc, cd, etc. corresponding to suitable angular displacements of the rotating vector. Now, erect ordinates at the

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poins a, b, c and d etc. (Fig. 16-4) and then project the free ends of the vector E_m at the corresponding positions a', b', c' etc. to meet these ordinates. Next draw a curve passing through these intersecting points. The curve so obtained is the graphic representation of equation (*iii*).

16-3. Alternate Method for the Equations of Alternating Voltages and Currents

In Fig. 16-5 is shown a rectangular coil AC having N turns and rotating in the magnetic field of flux density B Wb/m². Let the length of each of its sides A and C be l metres and their peripheral velocity v metres/second. Let the angle be measured from the horizontal position *i.e.* from the X-axis. When in horizontal position, the two sides A and C move parallel to the lines of magnetic flux. Hence, no flux is cut and so no e.m.f. is generated in the coil.

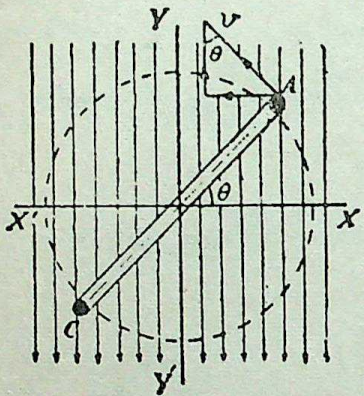

Fig. 16-5

When the coil has turned through angle θ, its velocity can be resolved into two mutually perpendicular components (*i*) $v \cos \theta$ component parallel to the direction of the magnetic flux and (*ii*) $v \sin \theta$ component perpendicular to the direction of the magnetic flux.

The e.m.f. generated is due entirely to the perpendicular component *i.e.* $v \sin \theta$.

Hence, the e.m.f. generated in one side of the coil which contains N conductors as seen from Art. 8-8 is given by

$$e = N \times Bl\, v \sin \theta$$

Total e.m.f. generated in both sides of the coil is

$$e = 2BNl\, v \sin \theta \text{ volt} \qquad \ldots(i)$$

Now, e has maximum value of E_m (say) when $\theta = 90°$

Hence, from Eq. (*i*), we get

$$E_m = 2BNl\, v \text{ volt}$$

Therefore, Eq. (*i*) can be rewritten as

$$e = E_m \sin \theta \qquad \ldots\text{as before}$$

If b = breadth of the coil in metres

f = frequency of rotation of coil in rev/s

then $v = \pi\, b\, f$

$$E_m = 2BNl \times \pi bf \text{ volt}$$

$$= 2\pi fNBA \text{ volts} \qquad \ldots\text{as before.}$$

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16-4. Simple Waveforms

The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa is called its *waveform* or *wave shape*.

An alternating voltage or current may not always take the form of a symmetrical or smooth wave such as that shown in Fig. 16-3. Thus Fig. 16-6 also represents alternating waves. But while it is scarcely possible for the manufacturers to produce sine-wave generators or alternators, yet sine wave is the ideal form sought by the designer and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves.

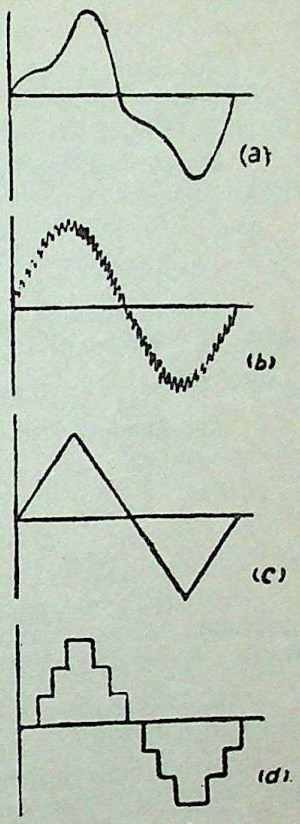

Fig. 16-6

In general, however, *an alternating current or voltage is one, the circuit direction of which reverses at regularly recurring intervals.*

16-5. Cycle

One complete set of positive and negative values of an alternating quantity is known as a cycle. Hence, each diagram of Fig. 16-6 represents one complete cycle.

A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radians.

16-6. Time period

(A.M.I.E. Summer 1979)

The time taken by an alternating quantity to complete one cycle is called its time period T. For example, a 60 Hz alternating current has a time period of 1/60 second.

16-7. Frequency

(A.M.I.E. Summer 1980

The number of cycles/second is called the frequency of the alternating quantity.

In the simple 2-pole alternator of Fig. 16-1 (*b*), one cycle of alternating current is generated in one revolution of the rotating field. However, if there were 4-poles, then two cycles would have been produced in each revolution. In fact, the frequency of the alternating voltage produced is a function of the speed and the

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number of poles of the alternator. The relation connecting the above three quantities is given by

$$f = PN/120$$

where N = revolutions in r.p.m.

P = number of poles.

For example, an alternator having 20 poles and running at 300 r.p.m. will generate alternating voltage and current whose frequency is $20 \times 300/120 = 50$ Hz.

It may be noted that the frequency is given by the reciprocal of the time period of the alternating quantity.

$$\therefore \quad f = 1/T \qquad \text{or} \qquad T = 1/f$$

16-8. Amplitude

The maximum value, positive or negative, of an alternating quantity is known as its amplitude.

16-9. Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already given in Art. 16-2 is

$$e = E_m \sin \theta = E_m \sin \omega t$$

$$= E_m \sin 2\pi ft = E_m \sin \frac{2\pi}{T} t$$

By closely looking at the above equations, we find that (*i*) the maximum value or peak value or amplitude of an *alternating voltage is given by the coefficient of the sine function.*

(*ii*) *the frequency f is given by dividing the coefficient of time in the angle by* 2π.

For example if the equation of an alternating voltage is given by

$$e = 50 \sin 314\, t$$

then its maximum value is 50 V and its frequency is

$$314/2\pi = 50 \text{ Hz}$$

Similarly, if the equation is of the form

$$e = I_m \sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\omega t$$

then its maximum value is $E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$ and the frequency is $2\omega/2\pi$ or ω/π

Example 16-1. (*a*) *What is the equation of a 25-Hz current sine wave, having an r.m.s. value of 30 A ?*

(*b*) *A 60-Hz engine-driven alternator has a speed of 1200 r.p.m. How many poles has it ?*

Solution. (*a*) $I_{r.m.s.} = 30$ A

$$\therefore \quad I_{max} = 30 \times \sqrt{2} = 42{\cdot}42 \text{ A} \qquad \text{...Art. 16-12}$$

$$\omega = 2\pi f = 2\pi \times 25 = 157 \text{ rad/s}$$

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Hence, the equation of the alternating current is

$$i = 42{\cdot}42 \sin 157t$$

(*b*) frequency, $f = PN/120$ Hz

$\therefore \quad 60 = 1200\,P/120 \quad \therefore \quad P = 6$

The alternator has 6 poles.

Example 16-2. *A square coil of 10 cm side and with 100 turns is rotated at the uniform speed of 500 r.p.m. about an axis at right angles to a uniform field of 0·5 Wb/m². Calculate the instantaneous value of the induced e.m.f. when the plane of the coil is (a) at right angles to the plane of field (b) at 30° to the plane of the field and (c) in the plane of the field.*

Solution. The instantaneous value of the e.m.f. is

$$e = 2\pi f NBA \sin\theta \text{ volt}$$

where θ is measured from the position of the coil *when its plane is at right angles to the flux* (Fig. 16-2)

Here $f = 500/60 = 25/3$ r.p.s.

$N = 100$; $B = 0{\cdot}5$ Wb/m²

$A = 10 \times 10 = 100$ cm² $= 10^{-2}$ m²

(*a*) $\theta = 0$, $\sin\theta = 0 \quad \therefore \quad e = 0$

(*b*) $\theta = 60°$, $\sin 60° = 0{\cdot}866$

$e = 2\pi \times (25/3) \times 100 \times 10^{-2} \times 0{\cdot}5 \times 0{\cdot}866$

$= \mathbf{22{\cdot}6\ V}$

(*c*) $\theta = 90°$, $\sin 90° = 1$

$\therefore \quad e = 2\pi \times (25/3) \times 100 \times 0{\cdot}5 \times 10^{-2} = \mathbf{26{\cdot}18\ V}$

Example 16-3. *An alternating current of frequency 60 Hz has a maximum value of 120 A. Write down the equation for its instantaneous value. Reckoning time from the instant the current is zero and is becoming positive find (a) the instantaneous value after 1/360 second and (b) the time taken to reach 96 A for the first time.*

Solution. The instantaneous equation is

$$i = 120 \sin 2\pi ft = 120 \sin 120\pi t$$

Now, when $t = 1/360$ second, then

(*a*) $i = 120 \sin (120 \times \pi \times 1/360)$ ...angle in radians

$= 120 \sin (120 \times 180 \times 1/360)$ ...angle in degrees

$= 120 \sin 60° = \mathbf{103{\cdot}9\ A}$

(*b*) $96 = 120 \times \sin 360 \times 60 \times t$ ...angle in degrees

or $\sin(360 \times 60 \times t) = 4/5 = 0{\cdot}8$

$\therefore \quad 360 \times 60 \times t = \sin^{-1} 0{\cdot}8 = 53°$ (approx.)

$\therefore \quad t = 53/360 \times 60 = \mathbf{0{\cdot}00245}$ **second**

Example 16-4. *The e.m.f. produced by an alternator is given by*

$$e = E_m \sin (314t + \pi/2)$$

If the rotor runs at 75 r.p.m. find the number of poles of alternator. Also find the angle through which rotor turns in T/4 second.

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Solution. As seen from its equation, the e.m.f. has a frequency of $314/2\pi = 50$ Hz.

Now, $f = PN/120$

$\therefore \quad 50 = P \times 75/120 \quad \therefore \quad \mathbf{P = 80}$

Obviously, the number of pair of poles is $= 80/2 = 40$. With one pair of poles, rotor turns through 90° (electrical) in one-quarter time period *i.e.* in $T/4$ second. With 40 pairs of poles, angle turned would be 40 times smaller *i.e.* it would be $= 90/40 = 2{\cdot}25°$

(A.M.I.E. May 1972)

16-10. Phase

By phase of an alternating current is meant the fraction of the time period of that A.C. that has elapsed since it last passed through the zero position of reference. For example, the phase of current at point A is $T/4$ second where T is the time period or expressed in terms of angle, it is $\pi/2$ radian (Fig. 16-7). Similarly, the phase of the rotating coil at the instant shown in Fig. 16-2 is ωt which is, therefore, called its phase angle.

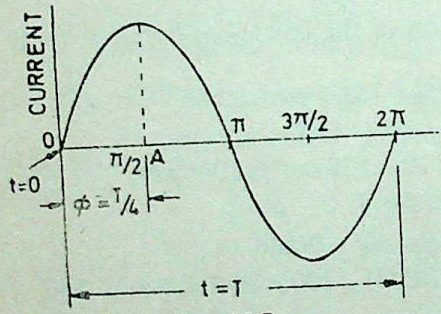
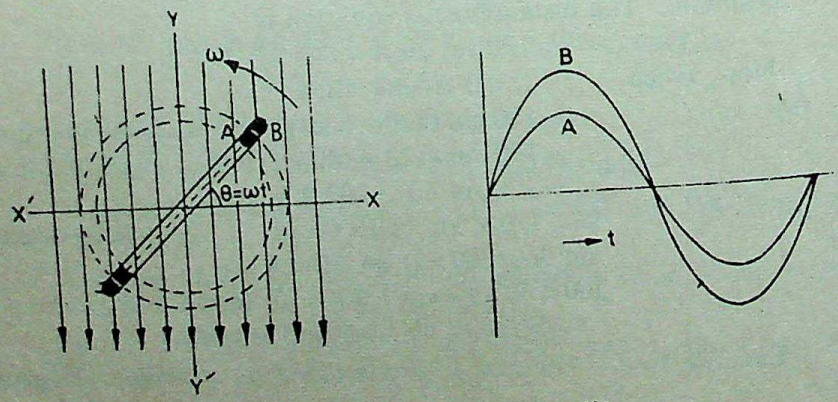


Fig. 16-7

In electrical engineering, we are, however, more concerned with relative phases *i.e.* phase differences between different alternating quantities rather than with their absolute phases. Consider two single-turn coils of different sizes [Fig. 16-8 (*a*)] arranged radially in the same plane and rotating with the same angular velocity, in a common magnetic field of uniform intensity. The e.m.fs. induced in both coils will be of the same frequency and of sinusoidal shape, although the values of instantaneous e.m.fs. induced would be different. However, the two alternating e.m.fs. would reach their maximum and zero values at the same time as shown in Fig. 16-8(*b*). Such alternating voltages (or



(*a*) (*b*)

Fig. 16-8

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currents) are said to be in phase with each other. The two voltages will have the equations $e = E_{m1} \sin \omega t$ and $e_2 = E_{m2} \sin \omega t$.

16-11. Phase Difference

Now, consider three similar single-turn coils displaced from each other by angles α and β and rotating in a uniform field with the same angular velocity [Fig. 16-9 (*a*)].

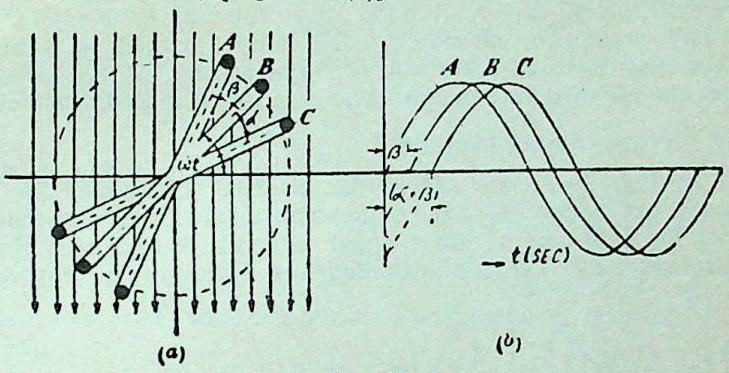

Fig. 16-9

In this case, the values of induced e.m.fs. in the three coils are the same but there is one important difference. The e.m.fs. in these coils do not reach their maximum or zero values simultaneously but one after another. The three sinusoidal waves are shown in Fig. 16-9 (*b*). It is seen that curves *B* and *C* are displaced from curve *A* by angles β and $(\alpha+\beta)$ respectively. Hence, it means that phase difference between *A* and *B* is β and between *B* and *C* is α but between *A* and *C* is $(\alpha+\beta)$. The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the term '*lag*' or '*lead*'.

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared with the other quantity.

Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity. For example, in Fig. 16-9 (*b*), *B* lags behind *A* by β and *C* lags behind *A* by $(\alpha+\beta)$.

The three equations for the instantaneous induced e.m.fs. are :

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin (\omega t - \beta)$$
$$e_C = E_m \sin [\omega t - (\alpha+\beta)]$$

In Fig 16-10, quantity *B* *leads* *A* by an angle ϕ. Hence, their equations are

$$v_A = V_m \sin \omega t$$
$$v_B = V_m \sin (\omega t + \phi)$$

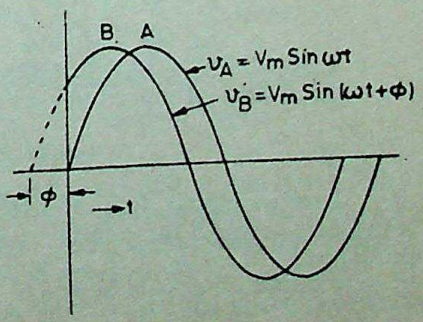

Fig. 16-10

A plus (+) sign when used in connection with phase difference denotes 'lead' whereas a minus (−) sign denotes 'lag'.

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16-12. Root-Mean-Square (R.M.S.) Value (*A.M.I.E. Winter 1986*)

The r.m.s. value of an alternating current *is given by that steady current* (*d.c.*) *which when flowing through a given circuit for a given time produces the same* **heat** *as produced by the alternating current when flowing through the same circuit for the same time.* It is also known as the *effective* or *virtual* value of a.c., the former term being used more extensively. For finding the r.m.s. value of symmetrical sinusoidal alternating current either mid-ordinate method or analytical method may be used, although for symmetrical but non-sinusoidal waves, the mid-ordinate method would be found more convenient.

16-13. Mid-ordinate Method

In Fig. 16-11 are shown the positive half-cycles for both symmetrical sinusoidal and a non-sinusoidal alternating current. Divide the time base 't' into n equal intervals of time each of duration t/n seconds. Let the instantaneous values of current during

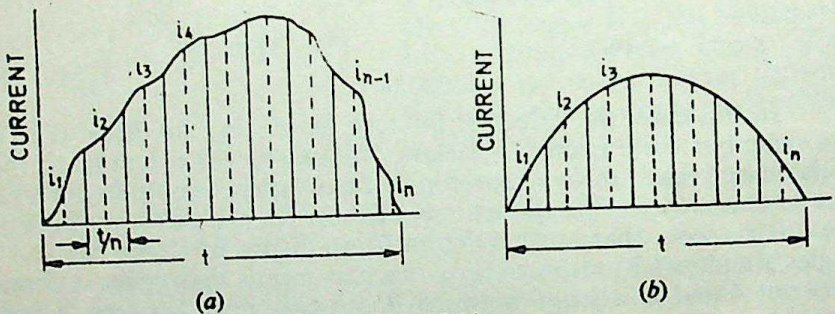

Fig. 16-11

these intervals be respectively i_1, i_2, i_3.....i_n, *i.e.* mid-ordinates in Fig. 16-11. Suppose this alternating current is passed through a circuit of resistance R ohm. Then,

Heat produced in 1st interval $= 0{\cdot}24\ i_1^2\ R\ t/n$ cal

" " 2nd $= 0{\cdot}24\ i_2^2\ R\ t/n$ cal

: : : : :

: : : : :

: : : : :

: : : : :

Heat produced in nth interval $= 0{\cdot}24\ i_n^2\ R\ t/n$ cal

Total heat produced in t seconds is

$$= 0{\cdot}24\ R\ t\left(\frac{i_1^2+i_2^2+\ldots+i_n^2}{n}\right)$$

Now, suppose that a direct current of value I produces the same heat through the same resistance during the same time t. Heat produced by it is $= 0{\cdot}24\ I^2\ R\ t$ cal. By definition, the two amounts of heat produced should be equal.

$$\therefore \quad 0{\cdot}24\ I^2\ R\ t = 0{\cdot}24\ R\left(\frac{i_1^2+i_2^2+\ldots+i_n^2}{n}\right)$$

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$$\therefore \quad I^2 = \frac{i_1^2 + i_2^2 + \ldots\ldots i_n^2}{n}$$

$$\therefore \quad I = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots\ldots i_n^2}{n}\right)}$$

= square root of the mean of squares of the instantaneous currents

Similarly, the r.m.s. value of the alternating voltage is given by the expression

$$V = \sqrt{\left(\frac{v_1^2 + v_2^2 + \ldots\ldots v_n^2}{n}\right)}$$

16-14. Analytical Method

The standard form of a sinusoidal alternating current is

$$i = I_m \sin \omega t = I_m \sin \theta$$

The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle will do),

$$= \int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$$

The square root of this value is

$$= \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$$

Hence, the r.m.s. value of the alternating current is

$$I = \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2 \theta \, d\theta\right)}$$

(Putting $i = I_m \sin \theta$)

Now $\cos 2\theta = 1 - 2\sin^2\theta$ or $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$

$$\therefore \quad I = \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{2\pi} (1 - \cos 2\theta) \, d\theta\right)}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi} \times 2\pi\right)} = \frac{I_m^2}{2}$$

$$\therefore \quad I = I_m/\sqrt{2} \quad \text{or} \quad I = 0{\cdot}707 \, I_m$$

Hence, we find that for a symmetrical sinusoidal current,

r.m.s. value of current $= 0{\cdot}707 \times$ *Max. value of current*

The r.m.s. value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the r.m.s. value of alternating current and voltages respectively. In electrical engineering work, *unless indicated otherwise, the values of the given current and voltage are always taken as the r.m.s. values.*

It should be noted that the average heating effect produced during one cycle is

$$= I^2 R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R = \frac{1}{2} I_m^2 R$$

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Example 16-5. *An alternating current, when passed through a resistance immersed in water for 5 minutes just raised the temperature of the water to boiling point. When a direct current of 4 A was passed through the same resistance under identical conditions, it took 8 minutes to boil the water. Find the r.m.s. value of the alternating current. Neglect factors other than heat given to the water.*

Solution. Let I be the r.m.s. value of the alternating current and R the value of the resistance.

Heat produced in the first case $= I^2Rt/4{\cdot}2 = 0{\cdot}24\ I^2R\ (5\times 60)$ cal

Heat produced in the second case $= 4^2 \times R \times (8\times 60)/4{\cdot}2$ cal

By definition of the r.m.s. value, the two quantities of heat should be equal

$$\therefore \quad 0{\cdot}24\ I^2\ R\times 300 = 0{\cdot}24\times 16\ R\times 480 \quad \therefore \quad I = \mathbf{5{\cdot}06\ A}$$

(*A.M.I.E. Winter 1985*)

16-15. Average Value

The average value I_{av} of an alternating current is expressed *by that steady current which transfers across any circuit the same charge as is transferred by that alternating current.* In the case of a symmetrical alternating current (*i.e.* one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over half-cycle only. *But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.*

(*i*) Mid-ordinate Method

With reference to Fig. 16-11

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n}$$

This method may be used both for sinusoidal and non-sinusoidal waves, although it is specially convenient for the latter.

(*ii*) Analytical Method

The standard equation of an alternating current is

$$i = I_m \sin\theta$$

$$\therefore \quad I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\,d\theta$$

(put value of i)

$$= \frac{I_m}{\pi}\Big|-\cos\theta\Big|_0^{\pi} = \frac{I_m}{\pi}\Big|+1-(-1)\Big| = \frac{I_m}{\pi/2} = \frac{2I_m}{\pi}$$

$$\therefore \quad I_{av} = \frac{\text{twice the maximum current}}{\pi}$$

or $I_{av} = I_m/2 = 0{\cdot}637\ I_m$

$\therefore$ *average value of current* $= 0{\cdot}637 \times$ *maximum value*

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16-16. Form Factor (*A.M.I.E. Winter 1986*)

It is defined as the ratio.

$$K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0{\cdot}707\ I_m}{0{\cdot}637\ I_m} = 1{\cdot}11$$

(for sinusoidal A.C. only)

In the case of sinusoidal alternating voltage also

$$K_f = \frac{0{\cdot}707\ E_m}{0{\cdot}637\ E_m} = 1{\cdot}11$$

Obviously, the knowledge of form factor will enable the r.m.s. value to be found from the arithmetic mean value and *vice versa*.

16-17. Crest or Peak or Amplitude Factor

(*A.M.I.E. Winter 1986*)

It is defined as the ratio

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}}$$

$$\therefore \quad K_a = \sqrt{2} = 1{\cdot}414 \quad \text{(for sinusoidal A.C. only)}$$

For sinusoidal alternating voltage also

$$K_a = \frac{E_m}{E_m/\sqrt{2}} = 1{\cdot}414$$

Knowledge of this factor is of importance in dielectric insulation testing because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. This knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

Example 16-6. *The graph in Fig. 16-12 shows the variation of voltage with time. Use this graph to calculate the r.m.s. value of the voltage. What is the frequency of the voltage ? Calculate the r.m.s. current which would flow through a 5 Ω resistor if this voltage were connected across it.*

What would be the r.m.s. voltage of a sinusoidal wave if it had the same peak value as the above voltage ?

Solution. Since the graph (Fig. 16-12) is symmetrical about the time axis, we may consider only the positive half-cycle.

Mean value of v^2 is

$$= \frac{0^2 + 10^2 + 20^2 + 30^2 + 20^2 + 10^2}{6} = \frac{950}{3}$$

$\therefore$ R.M.S. value, $V = \sqrt{950/3} = \mathbf{17{\cdot}8\ V}$

Time period $T = 12$ millisecond $= 12 \times 10^{-3}$ second

Frequency, $f = 1/T = 1/12 \times 10^{-3} = \mathbf{83{\cdot}3\ Hz}$

R.M.S. current $= 17{\cdot}8/5 = 3{\cdot}56$ A

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R.M.S. value of a sinusoidal voltage of peak value 30 V is

$$= 30/\sqrt{2} = 21{\cdot}2 \text{ V}$$

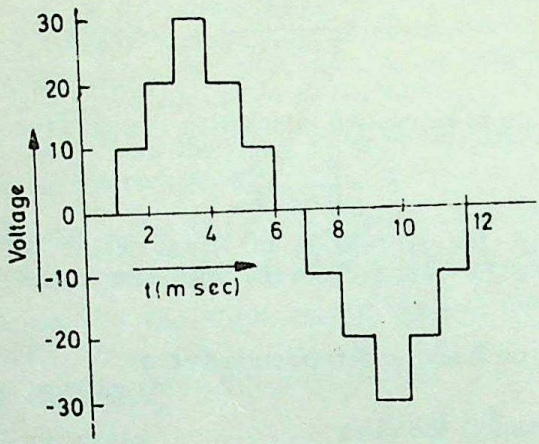


Fig. 16-12

Example 16-7. *A sinusoidal alternating voltage of 50 Hz has an r.m.s. value of 200 V. Write down the equation for the instantaneous value and find this value 0·0125 second after passing through a positive maximum value.*

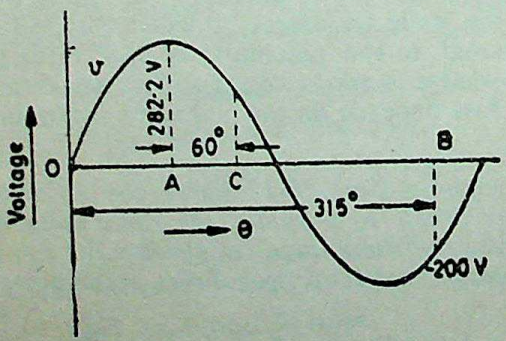
At what time measured from a positive maximum value will the instantaneous voltage be 141·4 V ? (*A.M.I.E. May 1970*)

Solution.

$V_m = 200\sqrt{2} = 282{\cdot}2$ V

$\omega = 2\pi \times 50$

$= 314$ rad/second



The instantaneous equation of the voltage wave with reference to point 0 as zero time point in Fig. 16-13 is

$$v = 282{\cdot}2 \sin 100\pi t$$

Fig. 16-13

Since time values are given from point *A* where voltage has positive and maximum value, the equation may itself be referred to point *A*. In that case, the equation becomes

$$v = 282{\cdot}2 \cos 100\pi t$$

Now, $t = 0{\cdot}0125$ second —given

$\therefore$ $v = 282{\cdot}2 \cos 100\pi \times 0{\cdot}0125$ —angle in radians

$= 282{\cdot}2 \cos 100 \times 180 \times 0{\cdot}0125$ —angle in degrees

$= 282{\cdot}2 \cos 225° = 282{\cdot}2 \times (-1/\sqrt{2})$

200 V ...point B

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Here $v = +141{\cdot}4$ V

$\therefore \quad 141{\cdot}4 = 282{\cdot}8 \cos 100 \times 180\, t$

$\therefore \quad \cos 100 \times 180\, t = 0{\cdot}5$

$\therefore \quad 100 \times 180\, t = \cos^{-1}(0{\cdot}5) = 60°$

$\therefore \quad t = 1/300$ **second** **...point C**

16-18 R.M.S. Value of Half-wave Rectified A.C.

Half-wave (H.W.) rectified alternating current is one whose one-half cycle has been suppressed *i.e.* one which flows for half the time during one cycle. It is shown in Fig. 16-14 where suppressed half-cycle is shown dotted.

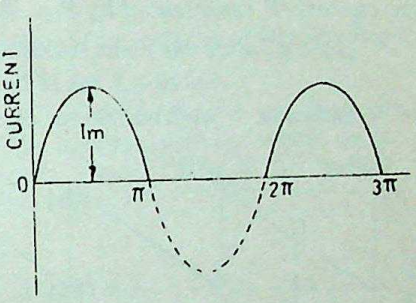

Fig. 16-14

As said earlier, for finding r.m.s. value of such an alternating current, summation should be carried over the period for which current *actually* flows *i.e.* from 0 to π, though it would be averaged for the wnole cycle *i.e.* from 0 to 2π.

$\therefore$ R.M.S. current

$$I = \sqrt{\left(\int_0^{\pi} \frac{i^2\, d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta\right)}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{\pi} (1 - \cos 2\theta)\, d\theta\right)}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}\right)}$$

$$= \sqrt{\left(\frac{I_m^2}{4\pi} \times \pi\right)} = \sqrt{\left(\frac{I_m^2}{4}\right)}$$

$$I = I_m/2 = 0{\cdot}5\, I_m$$

16-19. Average Value of Half-wave Rectified A.C.

For the same reasons as given in Art. 16-18, integration would be carried over from 0 to π.

$$\therefore \quad I_{av} = \int_0^{\pi} \frac{i\, d\theta}{2\pi} = \frac{I_m}{2\pi}\int_0^{\pi} \sin\theta\, d\theta \quad (\because i = I_m \sin\theta)$$

$$= \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{2\pi} \times 2 = \frac{I_m}{\pi}$$

16-20. Form Factor of Half-wave Rectified A.C.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{I_m/2}{I_m/\pi} = \frac{\pi}{2} = 1{\cdot}57$$

Example 16-8. *Find the average value of the periodic function shown in Fig. 16-4 (a).* **(A.M.I.E. Summer 1982)**

Solution. Since in parts *A* and *C*, function changes linearly, average value in *A* as well as in *C* is $V_m/2$. Since in part *B* function remains constant, average value $= V_m$.

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Average value of the three parts or for one half cycle is

$$= \frac{V_m/2 + V_m + V_m/2}{3} = \frac{2V_m}{3}.$$

Same will be the average for the whole cycle.

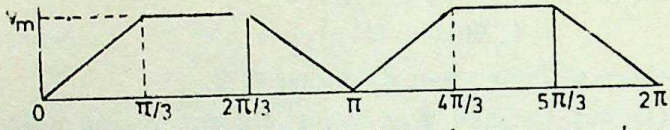

Fig. 16-4 (*a*)

Example 16-9. (*a*) *Calculate the r.m.s. and average value of the current 'i' represented by Fig. 16-15.*

(*b*) *Calculate the r.m.s. value of a current given by*

$$i = 10 + 5 \cos (628\ t + 30°)$$

Solution. (*a*) The slope of the curve *AB* of Fig. 16-15 is

$$\frac{BC}{AC} = \frac{1}{2\pi}$$

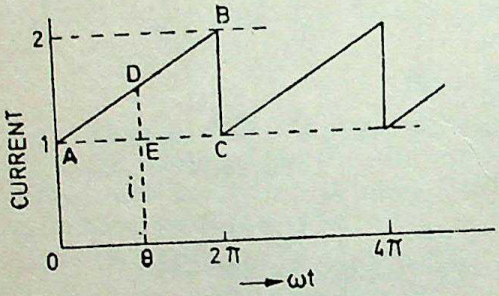

Fig. 16-15

Consider the current i at any angle θ. It is seen that

$$\frac{DE}{AE} = \frac{BC}{AC} = \frac{1}{2\pi}$$

or $$\frac{DE}{AE} = \frac{1}{2\pi}$$

$$\therefore \quad \frac{i-1}{\theta} = \frac{1}{2\pi}$$

or $$i = 1 + \frac{\theta}{2\pi}$$

This gives us the equation of the current for one cycle.

Average current $$= \frac{1}{2\pi}\int_0^{2\pi} i\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\left(1 + \frac{\theta}{2\pi}\right) d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(d\theta + \frac{\theta . d\theta}{2\pi}\right) = \frac{1}{2\pi}\left|\theta + \frac{1}{4\pi}\theta^2\right|_0^{2\pi}$$

$$= \frac{1}{2\pi}(2\pi + \pi) = \frac{3\pi}{2\pi} = \frac{3}{2} = 1{\cdot}5^*$$

Mean square value $$= \frac{1}{2\pi}\int_0^{2\pi} i^2\, d\theta = \frac{1}{2\pi}\int_0^{2\pi}\left(1 + \frac{\theta}{2\pi}\right)^2 d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(1 + \frac{\theta^2}{4\pi^2} + \frac{\theta}{\pi}\right) d\theta$$

$$= \frac{1}{2\pi}\left|\theta + \frac{\theta^3}{12\pi^2} + \frac{\theta^2}{2\pi}\right|_0^{2\pi} = \frac{7}{3}$$

$\therefore$ R.M.S. value $= \sqrt{7/3}$

(*b*) The given current is a mixture of a d.c. component of 10 A and an alternating current of maximum value of 5 A.

*This average value could be found by taking the mean of the initial and final values. Mean value $= \frac{1+2}{2} = 1{\cdot}5$ A

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R.M.S. value $= \sqrt{(\text{d.c.})^2 + (\text{r.m.s. value of a.c.})^2}$

$= \sqrt{(10)^2 + (5/\sqrt{2})^2} = \sqrt{225/2} = 15/\sqrt{2}$ **A***

Example 16-10. *Four alternating currents of peak value 200 A have the following waveforms :*

(a) sinusoidal (b) full-wave rectified sinusoidal (c) rectangular (d) triangular.

If these currents are passed, in turn, through

(i) a moving-coil ammeter

(ii) a moving-iron ammeter connected in series, find the readings of the instrument in each case.

Solution. It should be kept in mind that a moving-coil ammeter reads the average value of the current over the whole cycle. However, moving-iron ammeter reads r.m.s. value of the current. Various wave-forms are shown in Fig. 16-16.

(*a*) *M.C.* ammeter reading $= 0$

M.I. ammeter reading $= 200/\sqrt{2} =$ **141·4 A**

(*b*) Average value of current over one cycle is [Fig. 16-16 (*b*)]

$$= \frac{2\int_0^{\pi} I_m \sin\theta \, d\theta}{2\pi} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{2I_m}{\pi} = \frac{2 \times 200}{\pi} = 127{\cdot}4 \text{ A}$$

$\therefore$ *M.C.* ammeter reads **127·4 A**

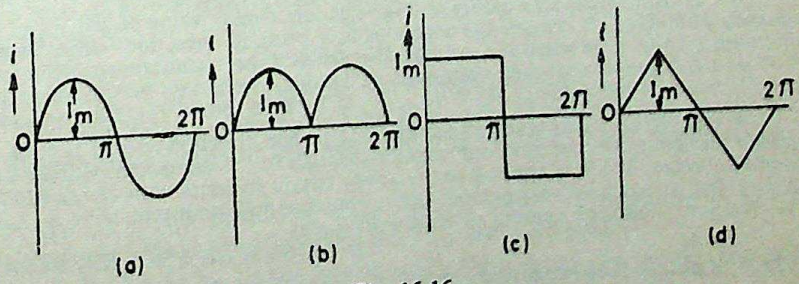

Fig. 16-16

R.M.S. value of full-wave rectified current is

$$= 2\int_0^{\pi} \frac{i^2 \, d\theta}{(2\pi - 0)} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} (1 - \cos 2\theta) \, d\theta = \frac{I_m^2}{2}$$

*The phase difference of 30° and the fact that it is a cosine function makes no difference to the r.m.s. or average values.

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$\therefore \quad I = I_m/\sqrt{2} = 200/\sqrt{2} = 141{\cdot}4$ A

M.I. ammeter reading $= 141{\cdot}4$ A

(*c*) Average value of current over one cycle is zero.

$\therefore$ *M.C.* ammeter reading $= 0$

R.M.S. value $= 200$ A

$\therefore$ *M.I.* ammeter reading $= 200$ A

(*d*) For a triangular wave-form

Average value over one cycle $= 0$

R.M.S. value $= I_m/\sqrt{3}$

$\therefore$ *M.C.* ammeter reading $= 0$

M.I. ammeter reading $= 200/\sqrt{3} = 115{\cdot}4$ A

Tutorial Problems—24

1. Calculate the maximum value of the e.m.f. generated in a coil which is rotating at 50 rev/second in a uniform magnetic field of 0·8 Wb/m². The coil is wound on a square former having sides 5 cm in length and is wound with 300 turns. **[188·5 V]**

2. (*a*) What is the peak value of a sinusoidal alternating current of 4·78 r.m.s. amperes ?

(*b*) What is the r.m.s. value of a rectangular voltage wave with an amplitude of 9·87 V ?

(*c*) What is the average value of sinusoidal alternating current of 31 A maximum value ?

(*d*) An alternating current has a periodic time of 0·03 second. What is its frequency ?

(*e*) An alternating current is represented by $i = 70{\cdot}7 \sin 520\,t$. Determine (*i*) the frequency (*ii*) the current 0·0015 second after passing through zero and increasing positively.

[6·76 A ; 9·87 V ; 19·75 A ; 33·3 Hz ; 82·8 Hz ; 49·7 A]

3. A sinusoidal alternating voltage has an r.m.s. value of 200 V and a frequency of 50 Hz. It crosses the zero axis in a positive direction when $t = 0$. Determine (*i*) the time when the voltage first reaches the instantaneous value of 200 V and (*ii*) the time when voltage after passing through its maximum positive value reaches the value of 141·4 V. **[(*i*) 0·0025 second (*ii*) 1/300 second]**

4. A circuit offers a resistance of 10 Ω in one direction and 50 Ω in the opposite direction to the flow of current through it. A sinusoidal voltage of maximum value 100 V is applied to the above circuit in series with (*i*) an M.C. ammeter (*ii*) a moving-iron ammeter (*iii*) a moving-coil ammeter with a full wave rectifier. Find the reading of each instrument.

[(*i*) 5·1 A (*ii*) 2·55 A (*iii*) 3·82 A]

16-21. Vector Representation of Alternating Quantities

It has already been pointed out that an attempt is made to obtain alternating voltages and currents having sine waveforms. In any case, a.c. computations are based on the assumption of sinusoidal voltages and currents. It is, however, cumbersome continuously to handle the instantaneous values in the form of equations of waves like $e = E_m \sin \omega t$ etc. A conventional method is to employ vector method of representing these sine waves. These vectors may then be manipulated instead of the sine functions to achieve the desired result. *In fact, vectors are a short-hand for the*

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representation of alternating voltages and currents and their use greatly simplifies the problems in a.c. work.

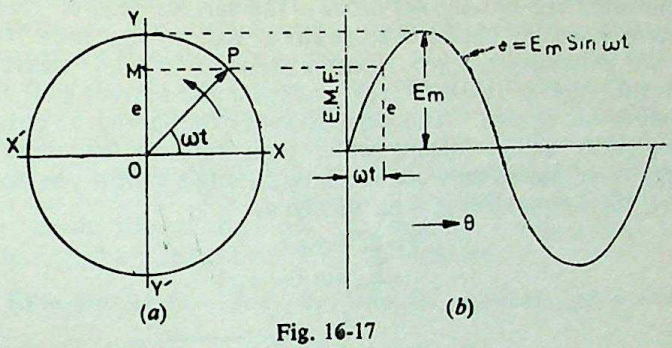

Fig. 16-17

A vector quantity is a physical quantity which has both magnitude and direction. Such vector quantities are completely known only when particulars of their magnitude, direction and the sense in which they act, are given. They are graphically represented by straight lines called vectors. The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.

The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity. In Fig. 16-17 (*a*), *OP* is such a vector which represents the maximum value of the A.C. and its angle with *X*-axis gives its phase. Let the alternating current be represented by the equation $e = E_m \sin \omega t$. It will be seen that the projection of *OP* on *Y*-axis at any instant gives the instantaneous value of that alternating current.

$$OM = OP \sin \omega t$$

or
$$e = OP \sin \omega t = E_m \sin \omega t$$

It should be noted that a line like *OP* can be made to represent an alternating voltage or current only if it satisfies the following conditions :

(*i*) Its length should be equal to the peak or maximum value of the sinusoidal alternating current to a suitable scale.

(*ii*) It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing positively.

(*iii*) Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

16-22. Vector Diagrams Using R.M.S. Values

Instead of using maximum values as above, it is more common practice to draw vector diagrams using r.m.s. values of alternating quantities. But it should be understood that in that case, the projection of the rotating vector on the *Y*-axis does not give the instantaneous value of that alternating quantity.

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16-23. Vector Diagrams of Sine Waves of Same Frequency

Two or more sine waves of the same frequency can be shown on the same vector diagram because the various vectors representing different waves all rotate counter clock-wise at the same frequency and so maintain a fixed position relative to each other. This is illustrated in Fig. 16-18 where a voltage e and current i of the same frequency are shown. The current wave is supposed to pass upward through zero at the instant when $t=0$ while at the same time the voltage wave has already advanced by an angle α from its zero value. Hence, their equations can be written as

$$i = I_m \sin \omega t$$

and

$$e = E_m \sin (\omega t + \alpha)$$

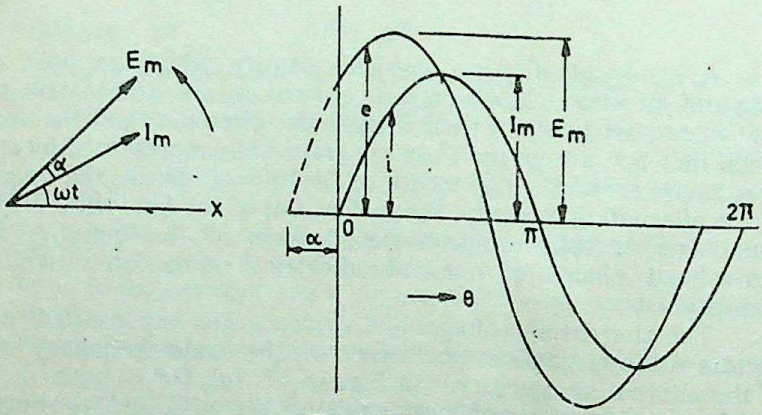
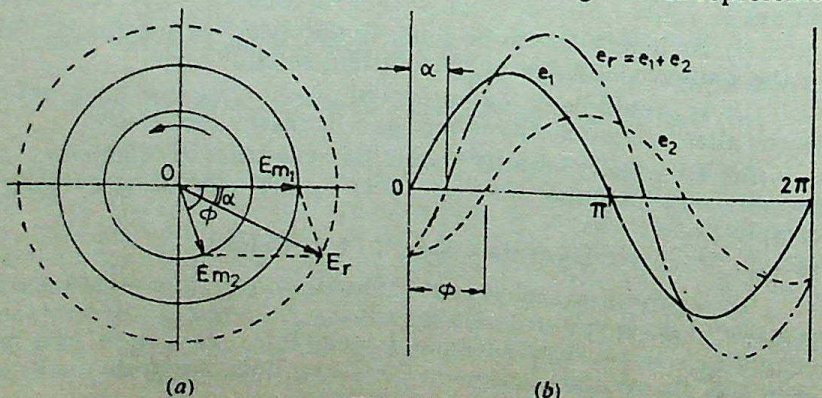


Fig. 16-18

Sine waves of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.

16-24. Addition of Two Alternating Quantities

In Fig. 16-19 (*a*) are shown two rotating vectors representing the maximum values of two sinusoidal voltage waves represented



(*a*) (*b*)

Fig. 16-19

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by $e_1 = E_{m1} \sin \omega t$ and $e_2 = E_{m2} \sin (\omega t - \phi)$. It is seen that the sum of the two sine waves of the same frequency is another sine wave of the same frequency but of a different maximum value and phase. The value of the instantaneous resultant voltage e_r at any instant is obtained by algebraically adding the projections of the two vectors on the Y-axis. If these projections are e_1 and e_2 then $e_r = e_1 + e_2$ at that time. The resultant curve has been drawn in this way by adding the ordinates. It is found that the resultant wave is a sine wave of the same frequency as the component waves but lagging behind E_{m1} by an angle α. The vector diagram of Fig. 16-19 (*a*) can be very easily drawn. Lay off E_{m2} lagging ϕ behind E_{m1} and then complete the parallelogram. So E_r is obtained.

Example 16-11. *Add the following currents as waves and as vectors* :

$$i_1 = 7 \sin \omega t$$

and $$i_2 = 10 \sin (\omega t + \pi/3)$$

Solution. As waves

$$i_r = i_1 + i_2$$
$$= 7 \sin \omega t + 10 \sin (\omega t + 60°)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos 60° + 10 \cos \omega t \sin 60°$$
$$= 12 \sin \omega t + 8{\cdot}66 \cos \omega t$$

Dividing and multiplying both sides by $\sqrt{(12^2 + 8{\cdot}66^2)} = 14{\cdot}8$, we get

$$i_r = 14{\cdot}8 \left(\frac{12}{14{\cdot}8} \sin \omega t + \frac{8{\cdot}66}{14{\cdot}8} \cos \omega t \right)$$
$$= 14{\cdot}8 (\cos \alpha \sin \omega t + \sin \alpha \cos \omega t)$$

where $\cos \alpha = \dfrac{12}{14{\cdot}8}$ and $\sin \alpha = \dfrac{8{\cdot}66}{14{\cdot}8}$ as shown in Fig. 16-20.

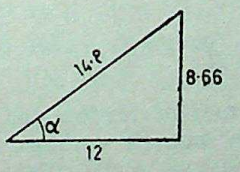

Fig. 16-20

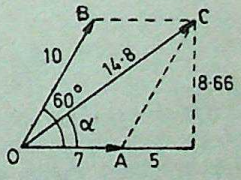

Fig. 16-21

$\therefore$ $$i_r = 14{\cdot}8 \sin (\omega t + \alpha)$$

where $$\tan \alpha = 8{\cdot}66/12 \quad \text{or} \quad \alpha = \tan^{-1} (8{\cdot}66/12) = 35{\cdot}8°$$

$$\mathbf{i_r = 14{\cdot}8 \sin (\omega t + 35{\cdot}8°)}$$

As Vectors

Vector diagram is shown in Fig. 16-21. Resolving the current vectors into their horizontal and vertical components, we have

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X-component $= 7 + 10 \cos 60° = 12$

Y-component $= 0 + 10 \sin 60° = 8{\cdot}66$

Resultant $= \sqrt{(12^2 + 8{\cdot}66^2)} = 14{\cdot}8$ A

and $\alpha = \tan^{-1}(8{\cdot}66/12) = 35{\cdot}8°$

Hence, the resultant equation can be written as

$$i_r = 14{\cdot}8 \sin(\omega t + 35{\cdot}8°)$$

16-25. Addition and Subtraction of Vectors

(*i*) **Addition.** In a.c. circuit problems, we may be concerned with a number of alternating voltages or currents of the same frequency but of different phases and it may be required to obtain the resultant voltage or current. As explained earlier (Art. 16-21)

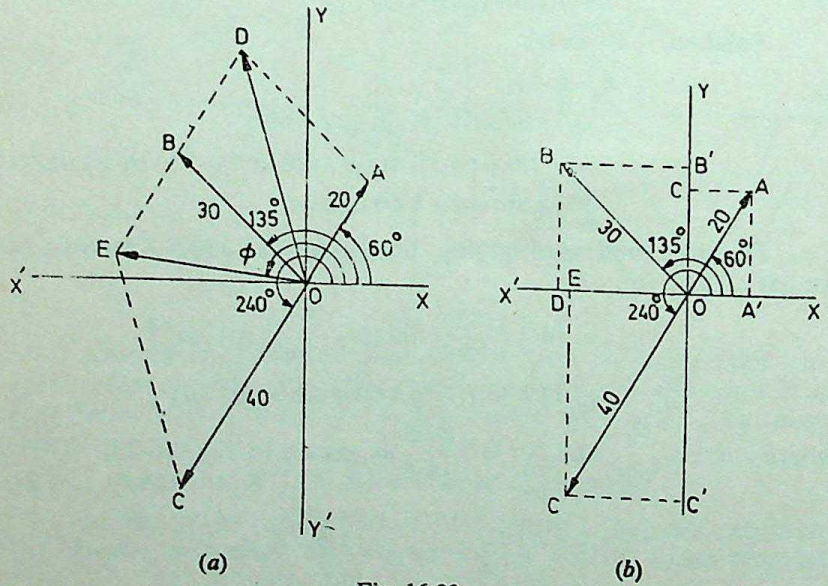

Fig. 16-22

if the quantities are sinusoidal, they may be represented by a number of rotating vectors having common axis of rotation and displaced from one another by fixed angles which are equal to the phase differences between the respective alternating quantities. The instantaneous value of the resultant voltage is given by the algebraic sum of the projections of the different vectors on Y-axis. The maximum value (or r.m.s. value if vectors represent that value) is obtained by compounding the several vectors by using the parallelogram and polygon laws of vector addition.

However, another easier method is to resolve the various vectors into their X- and Y-components and then to add them up as shown in Example 16-12.

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Suppose we are given the following three alternating e.m.fs. and it is required to find the equation of the resultant e.m.f.

$$e_1 = 20 \sin (\omega t + \pi/3)$$
$$e_2 = 30 \sin (\omega t + 3\pi/4)$$
$$e_3 = 40 \sin (\omega t + 4\pi/3)$$

Then the vector diagrams can be drawn as explained before and solved in any of the following three ways :

(*i*) By compounding according to parallelogram law as in Fig. 16-22 (*a*).

(*ii*) By resolving the various vectors into their *X*- and *Y*-components as in Fig. 16-22 (*b*).

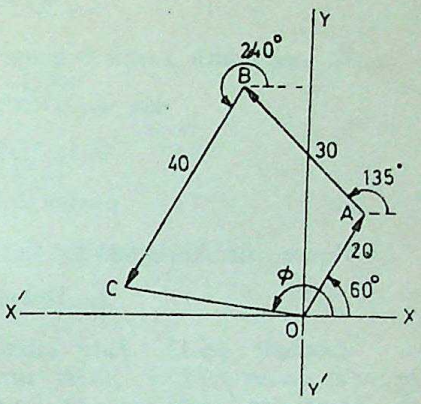

Fig. 16-23

(*iii*) By laying the various vectors end on end at their proper phase angles and then measuring the closing vector as shown in Fig. 16-23.

Knowing the magnitude of the resultant vector and its inclination ϕ with *X*-axis, the equation of the resultant e.m.f. can be written as

$$e = E_m \sin (\omega t \pm \phi)$$

Example 16-12. *In a parallel circuit with three branches, the instantaneous branch currents are represented by*

$$i_1 = 10 \sin \omega t$$
$$i_2 = 20 \sin (\omega t + \pi/3)$$
$$i_3 = 12 \sin (\omega t - \pi/6)$$

Write down the expression for the total instantaneous current in the form of $i = I_m \sin (\omega t + \phi)$

Solution. The vectors representing the maximum values of the three currents in their proper phases are shown in Fig. 16-24 (*a*). Resolving them into their *X*- and *Y*-components, we have

$$X\text{-component} = 10 + 20 \cos 60° + 12 \cos 30° = 30{\cdot}39 \text{ A}$$
$$Y\text{-component} = 20 \sin 60° - 12 \sin 30° = 11{\cdot}32 \text{ A}$$

As shown in Fig. 16-24 (*b*), the maximum value of the resultant current is

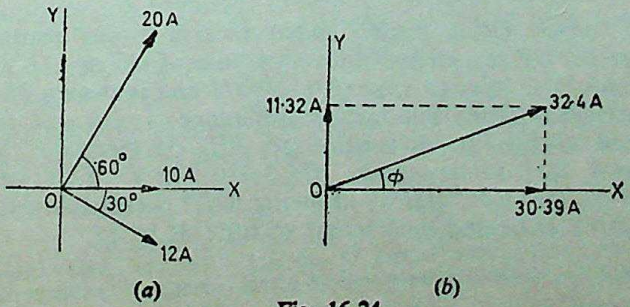

Fig. 16-24

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$$I_m = \sqrt{(X\text{-comp.})^2 + (Y\text{-comp.})^2}$$

$$= \sqrt{30{\cdot}39^2 + 11{\cdot}32^2} = 32{\cdot}4 \text{ A}$$

Its angle with X-axis is given by

$$\tan \phi = Y\text{-comp.}/X\text{-comp.}$$

$$= 11{\cdot}32/30{\cdot}39 = 0{\cdot}3726$$

$$\phi = \tan^{-1} (0{\cdot}3726) = 20° \ 26' = 0{\cdot}357 \text{ radian}$$

Hence, the expression for the total instantaneous current is

$$\mathbf{i = 32{\cdot}4 \sin (\omega t + 0{\cdot}357)}^*$$

Example 16-13. *Four circuits, A, B, C and D are connected in series across a 240-V, 50-Hz supply. The voltage across three of the circuits and their phase angles relative to the current through them are : V_A, 80 V at 50° leading ; V_B, 120 V at 65° lagging ; V_C, 135 V at 80° leading. If the supply voltage leads the current by 15°, find from a vector diagram drawn to scale the voltage V_D across the circuit D and its phase angle.*

Solution. The circuit is shown in Fig. 16-25

(*a*) The vector diagram is shown in Fig. 16-26. The current vector OM is drawn horizontally and is taken as reference vector. Taking a scale of 1 cm = 20 V, vector OA is drown 4 cm in length

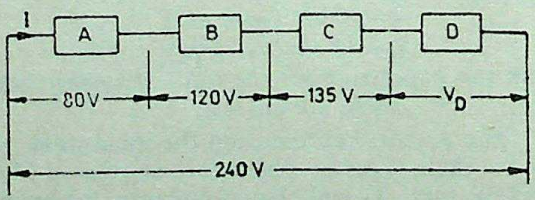

Fig. 16-25

and leading OM by an angle of 50°. Vector OB represents 120 V and is drawn lagging behind OM by 65°. The vector sum, as found by Parallelogram Law of Vectors, is given by vector OG.

Next, vector OC is drawn ahead of OM by 80° representing 135 V. Vector OF represents the vector sum of OG and OC. Vector OE represents the applied voltage of 240 V and is drawn 15° ahead of current vector OM. The vector difference of OE and OF gives the required voltage V_D. It is equal to FE. It measures 5·45 cm which means that it represents $20 \times 5{\cdot}45 = 109$ V. This vector is transferred to position OH by drawing OH parallel to EF. It is seen that OH lags behind the current vector OM by 12·4°.

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*The angle could be given in degrees also instead of radians.

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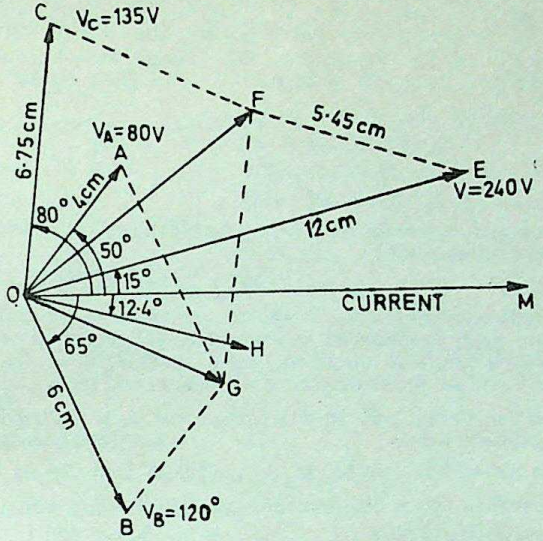
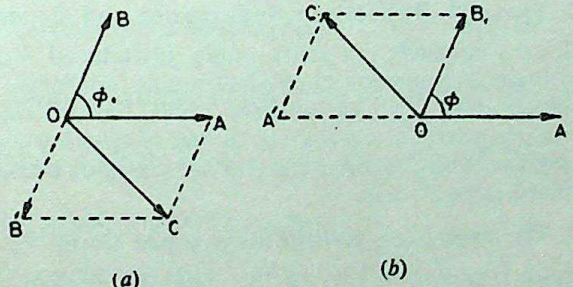


Fig. 16-26

Hence, V_D=**109** volts lagging behind the current by **12·4°**

(*ii*) **Subtraction of Vectors**



(*a*) (*b*)

Fig. 16-27

If difference of two vectors is required, then one of the vectors is reversed and this reversed vector is then compounded with the other as usual.

Suppose it is required to subtract vector *OB* from vector *OA*. Then *OB* is reversed as shown in Fig. 16-27 (*a*) and compounded with *OA* according to parallelogram law. The vector difference (**A—B**) is given by vector *OC*.

Similarly the vector *OC* in Fig. 16—17 (*b*) represents (**B—A**) *i.e.* the subtraction of *OA* from *OB*.

Tutorial Problems—25

1. The currents taken by two parallel circuits are 12 A in phase with the applied voltage and 20 A lagging 30° behind the applied voltage respectively. Determine the current taken by the combined circuit and its phase with respect to the applied voltage. **[31 A ; 18·8°]**

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2. Three circuits, A, B and C are connected in series across a 200 V supply. The voltage across circuit A is 50 V lagging the supply voltage by 45° and the voltage across circuit C is 100 V leading the supply voltage by 30°. Determine graphically, or by calculation, the voltage across circuit B and its phase displacement from its supply voltage. **[79·4 V; 10° 38′ lagging]**

3. Three alternating currents are given by

$$i_1 = 141 \sin(\omega t + \pi/4)$$
$$i_2 = 30 \sin(\omega t + \pi/2)$$
and $$i_3 = 20 \sin(\omega t - \pi/6)$$

and are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its r.m.s. value.

[$i = 167{\cdot}4 \sin(\omega t + 0797)$; $I_{r.m.s.} = 118{\cdot}4$ A]

4. Four e.m.fs. $e_1 = 100 \sin \omega t$, $e_2 = 80 \sin(\omega t - \pi/6)$, $e_3 = 120 \sin(\omega t + \pi/4)$ and $e_4 = 100 \sin(\omega t - 2\pi/3)$ are induced in four coils connected in series so that the vector sum of four e,m.fs. is obtained. Find graphically or by calculation the resultant e.m.f. and its phase difference with (*a*) e_1 and (*b*) e_2.

If the connections to the coil in which the e.m.f. e_3 is induced are reversed, find the new resultant e.m.f.

[208 sin (ωt—0·202 ; (*a*) 11° 34′ lag (*b*) 18° 26′ lead ; 76 sin (ωt+0·528)]

5. Draw to scale a vector diagram showing the following voltages :

$v_1 = 100 \sin 500t$ $\qquad$ $v_3 = -50 \cos 500\ t$

$v_2 = 200 \sin(500t + \pi/3)$ $\qquad$ $v_4 = 150 \sin(500t - \pi/4)$

Obtain graphically or otherwise, their vector sum and express this in the form $V_m \sin(500t \mp \theta)$, using v_1 as the reference vector. Give the r.m.s. value and frequency of the resultant voltage. **[306·5 sin (500t+0·056) ; 217 V ; 79·6 Hz]**

16-26. A.C. Through Resistance, Inductance and Capacitance

We will now consider the phase angle introduced between an alternating voltage and current when the circuit contains resistance only, inductance only and capacitance only. *In each case, we will assume that we are given the alternating voltage of equation* $v = V_m \sin \omega t$ and will proceed to find the equation and the phase of the alternating current produced in each case.

16-27. A.C. Through Pure Ohmic Resistance Only

The circuit is shown in Fig. 16-28. Let the applied voltage be given by the equation

$$v = V_m \sin \theta = V_m \sin \omega t \qquad \ldots(i)$$

Let R = ohmic resistance

i = instantaneous current

Obviously, the applied voltage has to overcome ohmic voltage drop only. Hence, for equilibrium

$$v = iR$$

Putting the value of 'v' from above, we get

$$V_m \sin \omega t = iR$$

or
$$i = \frac{V_m}{R} \sin \omega t \qquad \ldots(ii)$$

Current 'i' is maximum, when $\sin \omega t$ is unity.

$\therefore$ $\qquad I_m = V_m/R$

Hence, Eq. (*ii*) becomes $i = I_m \sin \omega t$ $\qquad \ldots(iii)$

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Comparing Eq. (i) and (iii), we find that the alternating voltage and current are in phase with each other as shown in Fig. 16-29. It is also shown vectorially by vectors V_R and I in Fig. 16-28.

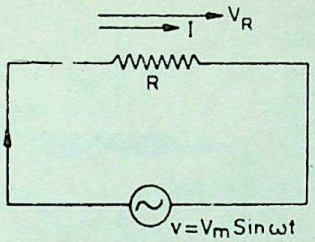

Fig. 16-28

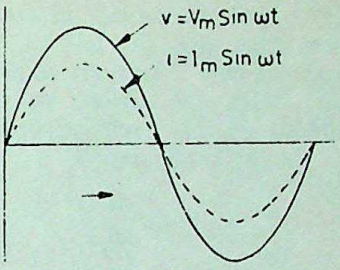

Fig. 16-29

Power

Instantaneous power $p = vi$ ...(Fig. 16-30)

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{V_m I_m}{2} (1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos \omega t$$

Power consists of a constant part $\frac{V_m I_m}{2}$ and fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current waves. For a complete cycle, the average value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero.

Hence, power for whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $P = V \times I$ watts

where V = r.m.s. value of applied voltage

I = r.m.s. value of the current

It is seen from Fig. 16-30 that no part of the power cycle becomes negative at any time. In other words, in a purely resistive circuit power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.

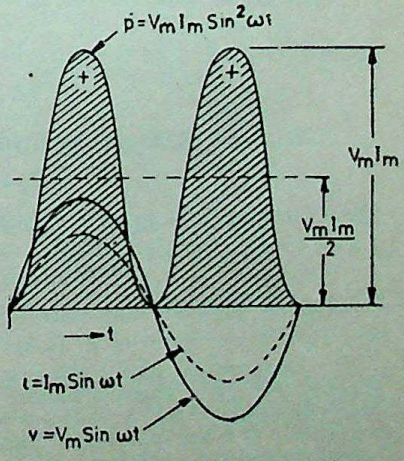

Fig. 16-30

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16-28. A.C. Through Inductance Only

Whenever an alternating voltage is applied to a pure inductive coil*, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only. So, at every step

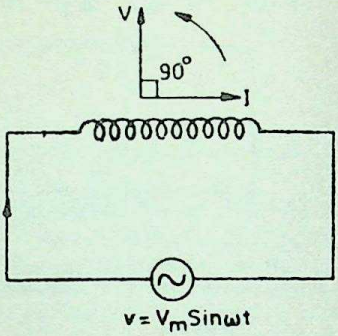

Fig. 16-31

$$v = L\frac{di}{dt}$$

Now $v = V_m \sin \omega t$

$$\therefore V_m \sin \omega t = L\frac{di}{dt}$$

$$\therefore \quad di = \frac{V_m}{L} \sin \omega t \, dt$$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin \omega t \, dt$$

$$= \frac{V_m}{\omega L}(-\cos \omega t)$$

...(constant of integration = 0)

$$= -\frac{V_m}{\omega L} \cos \omega t$$

$$\therefore \quad i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Maximum value of i is $I_m = \frac{V_m}{\omega L}$ when $\sin\left(\omega t - \frac{\pi}{2}\right)$ is unity.

Hence, equation of the current becomes

$$i = I_m \sin (\omega t - \pi/2)$$

So, we find that if applied voltage is represented by

$$v = V_m \sin \omega t$$

then, current flowing in a *purely* inductive circuit is given by

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

---

*By pure inductive coil is meant one that has no ohmic resistance and hence no I^2R loss. Pure inductance is actually not attainable though it is very nearly approached by a coil wound with such thick wire that its resistance is negligible. If it has some actual resistance then it is represented by a separate equivalent resistance joined in series with it.

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Clearly, the current lags behind the applied voltage by a quarter cycle (Fig. 16-32) or the phase difference between the two is $\pi/2$ with voltage leading. Vectors are shown in Fig. 16-31.

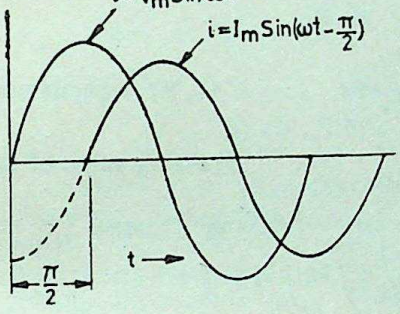

Fig. 16-32

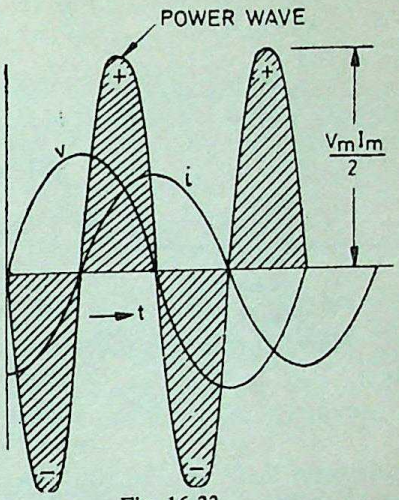

Fig. 16-33

We have seen that

$$I_m = \frac{V_m}{\omega L}$$

Here, 'ωL' plays the part of resistance. It is called the *reactance* (inductive) of the coil and is given in ohms, if L is in henrys and ω in radian/second. It is denoted by X_L.

Power

Instantaneous power $p = vi$

$$= V_m I_m \sin \omega t \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Power for the whole cycle is

$$P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

It is also clear from Fig. 16-33 that the average demand of power from the supply for a complete cycle is zero.

Here again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $\frac{V_m I_m}{2}$

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16-29. A.C. Through Capacitance Alone

When an alternating voltage is applied to the plates of a capacitor, it is charged first in one direction and then in the opposite direction. With reference to Fig. 16-34, let

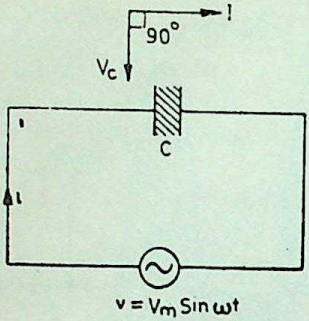

Fig. 16-34

v = p.d. between plates at any instant

q = charge on plates at that instant.

Then,

$$q = Cv \quad \text{...}C \text{ is the capacitance}$$
$$= C\,V_m \sin \omega t \quad \text{...(putting value of } v\text{)}$$

Now, current i is given by the rate of flow of charge.

$$\therefore \quad i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_m \sin \omega t)$$
$$= \omega C V_m \cos \omega t$$

or
$$i = \frac{V_m}{1/\omega C} \cos \omega t = \frac{V_m}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Obviously,
$$I_m = \frac{V_m}{1/\omega C} \quad \text{...(i)}$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The denominator '$1/\omega C$' is known as capacitive *reactance* and is in ohms if C is in farads and ω in radian/second. It is denoted by X_C.

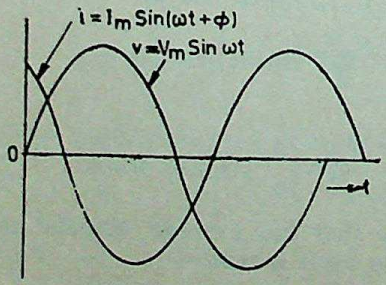

Fig. 16-35

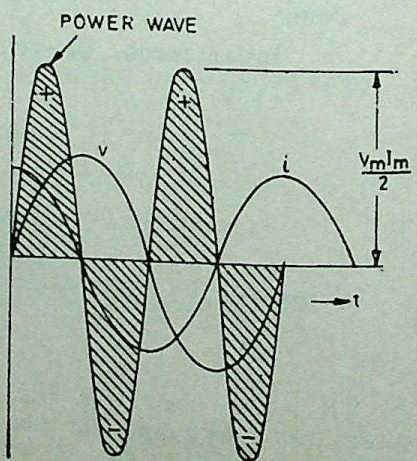

Fig. 16-36

It is seen that if the applied voltage is given by

$$v = V_m \sin \omega t$$

then current is given by $i = I_m \sin(\omega t + \pi/2)$

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Hence, we find that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 16-35 or phase difference between its voltage and current is $\pi/2$ with the current leading. Vector representation is given in Fig. 16-34. Note that V_C is along the *negative* direction of Y-axis.

Power

Instantaneous power $p = vi$

$$= V_m \sin \omega t \cdot I_m \sin (\omega t + 90°)$$

$$= V_m I_m \sin \omega t \cos \omega t = \tfrac{1}{2} V_m I_m \sin 2\omega t$$

Power for the whole cycle

$$P = V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$$

This fact is graphically illustrated in Fig. 16-36. We find that in a purely capacitive circuit*, the average demand of power from supply is zero (as in a purely inductive circuit). Again it is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of the instantaneous power is $V_m I_m/2$.

Example 16-14. *Two capacitors of 80 μF and 50 μF respectively are connected in series. Find (i) the current and (ii) the maximum energy stored in the circuit when 200-V at 50 Hz are applied across the series circuit.*

Solution. The combined or equivalent capacitance of the two series-connected capacitors is

$$C = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{80 \times 50}{80 + 50} = \frac{400}{13} \ \mu\text{F}$$

Equivalent reactance is

$$X_C = 1/\omega C = 1/2\pi fC$$

$$= \frac{10^6}{2\pi \times 50 \times 400/13} = 103{\cdot}4 \ \Omega.$$

(*i*) r.m.s. value of current is

$$I = V/X_C = 200/103{\cdot}4 = \mathbf{1{\cdot}93 \ A}$$

(*ii*) Maximum energy $E = \frac{1}{2} C V_m^2$

Now $V_m = 200 \times \sqrt{2} = 283 \text{ V}$

$$\therefore \quad E = \tfrac{1}{2} \times \frac{400}{13} \times 10^{-6} \times 283^2 = \mathbf{1{\cdot}23 \ J}$$

Example 16-15. *Two similar capacitors are connected in series and a voltage with an instantaneous value of $v = 100 \sin 314t$ is applied across their terminals. Calculate the capacitance of each capacitor if the r.m.s. current taken by the series combination is*

---

*By pure capacitor is meant one that has neither resistance nor dielectric loss. If there is loss in a capacitor, then it may be represented by loss in (*a*) a comparatively high resistance joined in parallel with the pure capacitor or (*b*) by a comparatively low resistance in series with the capacitor. But out of the two alternatives, usually (*a*) is chosen.

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0·4 A. Calculate also the maximum energy stored in the above combination.

Solution. If C is the capacitance of each capacitor, then their equivalent capacitance is $C/2$.

Now, comparing the given voltage equation with the standard equation (Art. 16-9) we have

$$V_m = 100 \text{ V}; \qquad \therefore V = 100/\sqrt{2} \text{ V}$$

$$\omega = 314 \text{ rad/s} \qquad \therefore f = 314/2\pi = 50 \text{ Hz}$$

Now $X_C = 1/2\pi f \times \text{capacitance}$

$$= \frac{1}{2\pi \times 50 \times C/2} = \frac{1}{50\pi C}\,\Omega$$

$$I_{r.m.s.} = 0{\cdot}4 \text{ A}$$

Since $I_{r.m.s.} = \dfrac{V_{r.m.s.}}{X_C}$

$$\therefore \qquad 0{\cdot}4 = \frac{100/\sqrt{2}}{1/50\pi C} \qquad \therefore C = \mathbf{36\ \mu F}$$

$$\text{Maximum energy} = \tfrac{1}{2}\left(\frac{C}{2}\right) V_m^2$$

$$= \frac{1}{2} \times \left(\frac{36}{2} \times 10^{-6}\right) \times 100^2 = \mathbf{0{\cdot}09\ J}$$

HIGHLIGHTS

1. The equation of the e.m.f. induced in a rectangular coil of N turns and area A m² rotating in a field of flux density B Wb/m² with a frequency of f revolutions/second is,

$$e = 2\pi f NBA \sin\theta$$
$$= E_m \sin\theta$$
$$= E_m \sin\omega t$$
$$= E_m \sin 2\pi f t$$

2. Similarly, the equation of the induced current is

$$i = I_m \sin\theta$$
$$= I_m \sin\omega t$$
$$= I_m \sin 2\pi f t$$

3. Those waveforms which depart from the ideal sine wave are known as *complex* waveforms. They are made up of pure sine wave (fundamental) and its harmonics.

4. The general equation of an alternating voltage and current is

$$v = V_m \sin(\omega t \pm \phi)$$
$$i = I_m \sin(\omega t \pm \phi)$$

where ϕ is the phase difference (in radians or degrees)

5. R.M.S. value of an alternating current is measured in terms of the direct current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

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$$I_{r.m.s.} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots\ldots + i_n^2}{n}}$$

$= I_m/\sqrt{2}$ ...for sinusoidal A.C.

6. The average value of an alternating current is expressed by that steady current (direct current) which transfers across any circuit the same charge as is transferred by that alternating current

$$I_{av} = \frac{i_1 + i_2 + \ldots\ldots i_n}{n}$$

$$I_{av} = \frac{2I_m}{\pi}$$ ...for sinusoidal A.C.

7. Form Factor $= \dfrac{\text{r.m.s. value}}{\text{average value}}$

$= \dfrac{I_m/\sqrt{2}}{2I_m/\pi} = 1{\cdot}11$ ...sinusoidal A.C.

8. Peak Factor $= \dfrac{\text{maximum value}}{\text{r.m.s. value}}$

$= \dfrac{I_m}{I_m/\sqrt{2}} = 1{\cdot}414$ ...sinusoidal A.C.

9. When an alternating voltage given by $v = V_m \sin \omega t$ is applied across a purely resistive circuit, the alternating current is given by

$$i = I_m \sin \omega t$$

where $I_m = V_m/R$

power $= V \times I$ watt

$=$ r.m.s. volts $\times$ r.m.s. amperes

10. In a purely inductive circuit, if the applied voltage is

$$v = V_m \sin \omega t$$

then $i = I_m \sin (\omega t - \pi/2)$

where $I_m = V_m/\omega L$

and $\omega L =$ reactance offered by the coil

Power absorbed $= 0$

11. In a purely capacitive circuit, if

$$v = V_m \sin \omega t$$

then $i = I_m \sin (\omega t + \pi/2)$

where $I_m = \dfrac{V_m}{1/\omega C} = \omega C V_m$

and $\dfrac{1}{\omega C} =$ reactance offered $= X_C$

Power absorbed $= 0$

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SERIES A.C. CIRCUITS 17

1. Alternating Current Through Resistance and Inductance 2. Power Factor 3. Active and Reactive Components of Current 4. Power in an Iron-cored Choking Coil 5. A.C. Through Resistance and Capacitance 6. Resistance, Inductance and Capacitance in Series 7. Resonance in R-L-C Circuit 8. Graphic Representation of Resonance 9. Resonance Curve 10. Points to remember 11. Q-factor of a Series Circuit 12. Highlights

17-1. A.C. Through Resistance and Inductance

A pure resistance of R ohms and a pure inductive coil of inductance L henrys are shown connected in series in Fig. 17-1.

Let, V = r.m.s. value of applied voltage

I = r.m.s. value of resultant current

$V_R = IR$ = voltage drop across R (in phase with I)

$V_L = IX_L$ voltage drop over coil (at right angles to I)

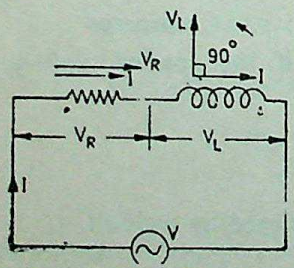

Fig. 17-1

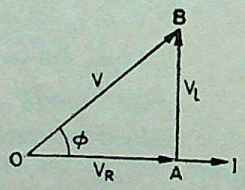

Fig. 17-2

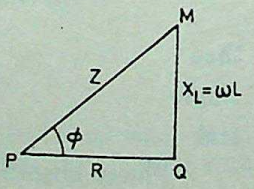

Fig. 17-3

These voltage drops are shown in the voltage triangle OAB in Fig. 17-2. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. The applied voltage V is the *vector* sum of the two *i.e.* it equals OB.

$$\therefore \quad V = \sqrt{(V_R^2 + V_L^2)}$$

$$= \sqrt{[(IR)^2 + (IX_L)^2]} = I\sqrt{(R^2 + X_L^2)}$$

$$I = \frac{V}{\sqrt{(R^2 + X_L^2)}}$$

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The quantity $\sqrt{(R^2+X_L^2)}$ is known as the *impedance* (Z) of the circuit. As seen from the impedance triangle PQM (Fig. 17-3)

$$Z^2=R^2+X_L^2$$

i.e. $$Impedance^2=Resistance^2+Reactance^2$$

From Fig. 17-2, it is clear that the current I lags behind the applied voltage by angle ϕ such that

$$\tan\phi=\frac{X_L}{R}=\frac{\omega L}{R}=\frac{\text{Reactance}}{\text{Resistance}}$$

$$\therefore \qquad \phi=\tan^{-1}(X_L/R)$$

The same fact is illustrated graphically in Fig. 17-4.

Hence, if applied voltage is given by $v=V_m \sin \omega t$, then the current equation is,

$$i=I_m \sin(\omega t-\phi)$$

where $I_m=V_m/Z$.

Power (*A.M.I.E. Winter 1974*)

In Fig. 17-5, I has been resolved into two mutually perpendicular components, $I\cos\phi$ along the applied voltage V and $I\sin\phi$ in quadrature with it (*i.e.* prependicular to V).

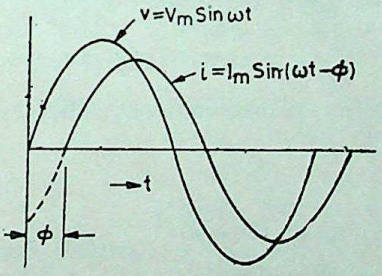

Fig. 17-4

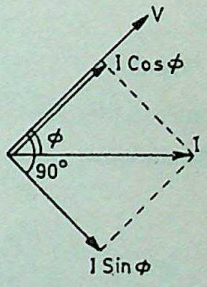

Fig. 17-5

The mean power consumed by the circuit is given by the product of V and *that part of the current I which is in phase with V.*

or $$P=V\times I\cos\phi$$

$$=\text{r.m.s. volts}\times\text{r.m.s. current}\times\cos\phi$$

The term '$\cos\phi$' is called the power factor of the circuit.

It should be remembered that in an a.c. circuit, the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (VA) and NOT true power in watts.

True power (W) = volt-amperes (VA) × power factor

or $$\textbf{Watts}=\textbf{VA}\times\cos\phi$$

Also $$\textbf{kW}=\textbf{kVA}\times\cos\phi$$

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It should be noted that power consumed is due to ohmic resistance only, because pure inductance does not consume any power. (Refer to Art. 16-28).

$$P = VI \cos \phi = VI \times (R/Z) \qquad \ldots(\cos \phi = R/Z)$$
$$= (V/Z) \times I \times R = I \times R$$
$$\therefore \qquad P = I^2R \text{ watt}$$

Graphical representation of the power consumed is shown in Fig. 17-6.

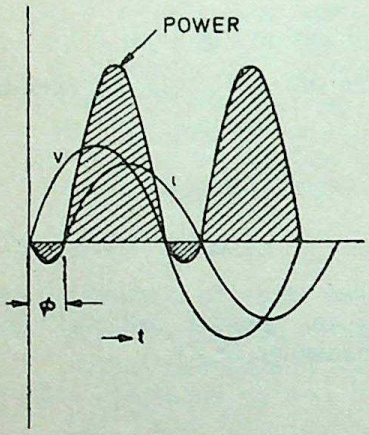

Fig. 17-6

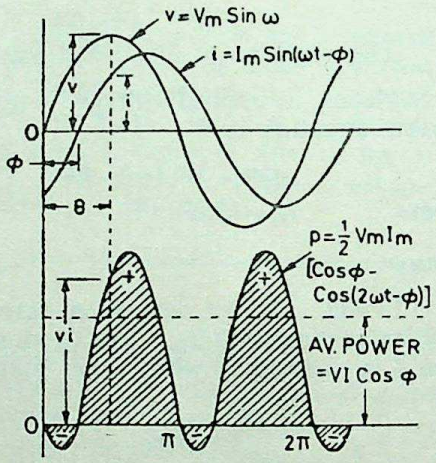

Fig. 17-7

Let us calculate power in terms of instantaneous values.

Instantaneous power is

$$p = vi$$
$$= V_m \sin \omega t \times I_m \sin (\omega t - \phi)$$
$$= V_m I_m \sin \omega t \sin (\omega t - \phi)$$
$$= \tfrac{1}{2} V_m I_m [\cos \phi - \cos (2\omega t - \phi)]$$

Obviously, this power consists of two parts (Fig. 17-7)

(*i*) a constant part $\frac{1}{2} V_m I_m \cos \phi$ which contributes to real power.

(*ii*) a pulsating component $\frac{1}{2} V_m I_m \cos (2\omega t - \phi)$ which has a frequency twice that of the voltage and current. It does not contribute to actual power since its average value over a complete cycle is zero.

Hence, average power consumed

$$= \tfrac{1}{2} V_m I_m \cos \phi$$
$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos \phi = VI \cos \phi$$

where V and I represent the r.m.s. values.

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17-2. Power Factor

(*A.M.I.E. May 1973*)

It may be defined as

(*i*) cosine of angle of lead or lag

(*ii*) the ratio $\frac{R}{Z}=\frac{\text{resistance}}{\text{impedance}}$ ...(Fig. 17-3)

(*iii*) the ratio $\frac{\text{watts}}{\text{volt-amperes}}=\frac{W}{VA}$

17-3. Active and Reactive Components of Current

(*i*) Active component is that which is in phase with the applied voltage *i.e.* $I\cos\phi$. It is also known as 'wattful' component.

(*ii*) Reactive component is that which is in quadrature with V *i.e.* $I\sin\phi$. It is also known as 'wattless' or 'idle' component.

It should be noted that the product of volts and amperes in an a.c. circuit gives volt-amperes (VA). Out of this, the actual power is VA $\cos\phi$=W and reactive power is VA $\sin\phi$. Expressing the values in kVA, we find that it has two rectangular components (*i*) active component which is obtained by multiplying kVA by $\cos\phi$. It gives power in kW.

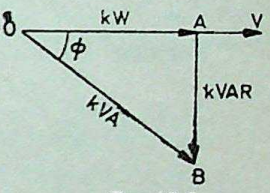

Fig 17-8

(*ii*) The other is known as reactive component and is obtained by multiplying kVA by $\sin\phi$. It is written as kVAR (kilovar).

The following relations can be easily deduced

$$\text{kVA}=\sqrt{\text{kW}^2+\text{kVAR}^2}$$
$$\text{kW}=\text{kVA}\cos\phi$$
$$\text{kVAR}=\text{kVA}\sin\phi$$

These relations can be easily understood by referring to kVA triangle of Fig. 17-8 where *it should be specially noted that lagging kVAR have been taken as negative.*

For example, suppose a circuit draws a current of 100 A at a voltage of 20,000 V and its power factor is 0·8. Then

$$\text{kVA input}=\frac{100\times 20{,}000}{1000}=2000$$

$$\cos\phi=0{\cdot}8\ ;\ \sin\phi=0{\cdot}6$$

Hence,
$$\text{kW}=2000\times 0{\cdot}8=1600$$
$$\text{kVAR}=2000\times 0{\cdot}6=1200$$

Obviously, $\sqrt{1600^2+1200^2}=2000$

i.e.
$$\text{kVA}=\sqrt{\text{kW}^2+\text{kVAR}^2}$$

Example 17-1 *An e.m.f. given by 326 sin 418 t is applied to a certain circuit and the current is 2 sin (418 t—1·3736). Find (i) the*

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frequency of the voltage (ii) phase angle between the voltage and current in degrees and (iii) the resistance of the circuit.

(A.M.I.E. Winter 1976)

Solution. (*i*) $2\pi f = 418$, $f = 418/2\pi = \mathbf{66{\cdot}5\ Hz}$

(*ii*) By comparing the voltage and current equations, the phase difference is

$$\phi = 1{\cdot}3736 \text{ radian} = 1{\cdot}3736 \times 180/\pi = \mathbf{78{\cdot}7^\circ}$$

(*iii*) $R = Z \cos \phi$

Now, $Z = V_m/I_m = 326/2 = 163\ \Omega$

$\cos \phi = \cos 78{\cdot}7^\circ = 0{\cdot}1977$

$\therefore \quad R = 163 \times 0{\cdot}1977 = \mathbf{32{\cdot}2\ \Omega}$

Example 17-2. *A single-phase motor operating from 400-V, 50-Hz supply is developing 7·46 kW output with an efficiency of 84% and a p. f. of 0·7 lagging. Calculate (i) the input kVA (ii) the active and reactive components of current and (iii) the reactive kilovoltamperes.*

(A.M.I.E. Summer 1981)

Solution. Motor output $= 7{\cdot}46$ kW $= 7{,}460$ W

Motor input $= 7{,}460/0{\cdot}84 = 8880$ W

Now, $VI \cos \phi =$ input

Motor input in volt-amperes $= 8{,}880/0{\cdot}7 = 12{,}685$ VA

(*i*) kVA drawn by motor $= 12{,}685/1000 = \mathbf{12{\cdot}685\ kVA}$

(*ii*) current drawn by motor $I = 12{,}685/400 = 31{\cdot}7$ A

Active component $= I \cos \phi = 31{\cdot}7 \times 0{\cdot}7 = \mathbf{22{\cdot}2\ A}$

Reactive component $= I \sin \phi = 31{\cdot}7 \times 0{\cdot}7 = \mathbf{22{\cdot}2\ A}$

(*iii*) reactive kilovoltamperes $= \text{kVA} \times \sin \phi$

$= 12{\cdot}685 \times 0{\cdot}7 = \mathbf{8{\cdot}88\ kVAR.}$

Example 17-3 *A resistor R in series with a capacitor C is connected to a 50-Hz, 240-V supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum energy stored in C.* *(A.M.I.E. Summer 1986)*

Solution. $100^2/R = 300$; $R = 100/3\ \Omega$

$I^2R = 300$; $I = 3$A, $Z = 240/3 = 80\ \Omega$

$X_C = \sqrt{Z^2 - R^2} = \sqrt{80^2 - (100/3)^2} = 72{\cdot}7\ \Omega$

$X_C = 1/2\pi fC$; $C = 1/2\pi f X_C = 1/2\pi \times 50 \times 72{\cdot}7$

$= 43{\cdot}7\ \mu\text{F}$

$$Q_{max} = C\,V_{max} = 43{\cdot}7 \times 10^{-6} \times 240 \times \sqrt{2}$$

$$= \mathbf{0{\cdot}0148\ J}$$

$$E_{max} = \tfrac{1}{2} C\,V^2_{max} = \tfrac{1}{2} \times 43{\cdot}7 \times 10^{-6} \times (240\sqrt{2})^2$$

$$= \mathbf{2{\cdot}5\ J.}$$

Example 17-4. *An alternating voltage of $v = 100 \sin 376{\cdot}8t$ is applied to a circuit consisting of a coil having a resistance of 6 Ω and an inductance of 21·22 mH.*

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(a) Express the current flowing through the circuit in the form

$$i = I_m \sin (376{\cdot}8t \pm \phi)$$

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(*b*) *If a moving-iron voltmeter, a wattmeter and a frequency meter are connected in the circuit, what would be the respective readings on the instruments.*

Solution. $X_L = \omega L = 376{\cdot}8 \times 21{\cdot}22 \times 10^{-3} = 8\ \Omega$

$Z = \sqrt{6^2 + 8^2} = 10\ \Omega$

$I_m = V_m/Z = 100/10 = 10\ \text{A}$

$\phi = \tan^{-1}(8/6) = 53{\cdot}1°$ (lag)

(*a*) The circuit current equation is

$$i = \mathbf{10 \sin (376{\cdot}8t - 53{\cdot}1°)}$$

(*b*) A moving-iron voltmeter measures the r.m.s. voltage.

Hence, its reading would be $= 100/\sqrt{2} = \mathbf{70{\cdot}7\ V}$

Wattmeter reading $= I^2R = \frac{1}{2} I_m^2 R = \frac{1}{2} \times 10^2 \times 6 = \mathbf{300\ W}$

Frequency-meter reading $= 376{\cdot}8/2\pi = \mathbf{60\ Hz}$

Example 17-5. *A series combination of a resistance and inductance when connected across 100-V, d.c. supply dissipates 500 W of power. When connected across 100 V (r.m.s.) a.c. supply of frequency 50 Hz, it dissipates 200 W of power. Calculate the values of resistance and inductance of the combination.*

Solution. D.C. Supply

$$I = 500/100 = 5\ \text{A};\quad R = 100/5 = \mathbf{20\ \Omega}$$

A.C. Supply

I^2R = power dissipated

$$I^2 \times 20 = 200 \qquad \therefore \qquad I = \sqrt{10} = 3{\cdot}16\ \text{A}$$

$$\therefore \qquad Z = 100/3{\cdot}16 = 31{\cdot}64\ \Omega$$

Now $\qquad Z = \sqrt{R^2 + X_L^2}$

or $\qquad X_L = \sqrt{Z^2 - R^2} = \sqrt{31{\cdot}64^2 - 20^2} = 24{\cdot}5\ \Omega$

Now, $\qquad X_L = 2\pi f L$

$$\therefore \qquad L = 24{\cdot}5/50 \times 2\pi = \mathbf{0{\cdot}078\ H}$$

Example 17-6. *A 100-V, 60-W lamp is to be operated off 220-V, 50-Hz mains. What (i) pure resistance and (ii) pure inductance, placed in series with the lamp, will enable it to be used without being over-run ? Which method would be more economical ?* (*A.M.I.E. Winter 1983*)

Solution. Rated current of the bulb $= 60/100 = 0{\cdot}6$ A

(*i*) A resistor R has been shown connected in series with the lamp in Fig. 17-9.

p.d. across R is

$$V_R = 220 - 100 = 120\ \text{V}$$

It is in phase with the applied voltage

$$\therefore \quad R = 120/0{\cdot}6 = \mathbf{200\ \Omega}$$

(*ii*) p.d. across bulb $= 100$ V

p.d. across pure inductance

$$V_L = \sqrt{220^2 - 100^2} = 196\ \text{V}$$

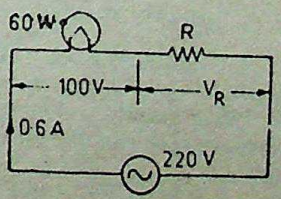

Fig. 17-9

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(Remember that V_L is in quadrature with V_R —Fig. 17-2)

Now, $V_L = IX_L = 0{\cdot}6 \times X_L$

or $196 = 0{\cdot}6 \times 2\pi \times 50 \times L, \quad \therefore \quad L = \mathbf{1{\cdot}04\ H}$

Method (*ii*) is preferrable to (*i*) becames in method (*ii*) there is no loss of power. Ohmic resistance of 200 Ω dissipates large power $= 120 \times 0{\cdot}6 = 72$ W—more than the bulb itself.

Example 17-7. *A circuit consists of a pure resistance and a coil connected in series. Powers dissipated in the resistance and in the coil are 1000 W and 250 W respectively. Voltage drops across the resistance and the coil are 200 V and 300 V respectively. Determine the reactance of the coil and the supply voltage.*

(*A.M.I.E. Summer 1975*)

Solution. The ohmic resistance R and coil have been shown connected in series in Fig. 17-10 (*a*). The voltage vector diagram is shown in Fig. 17-10 (*b*).

Now, $V^2/R = 1000$; $R = 40\ \Omega$

Also $IR = 200$, $\therefore \quad I = 200/40 = 5$ A

Now, $I^2 R_L = 250$; $R_L = 250/5^2 = 10\ \Omega$

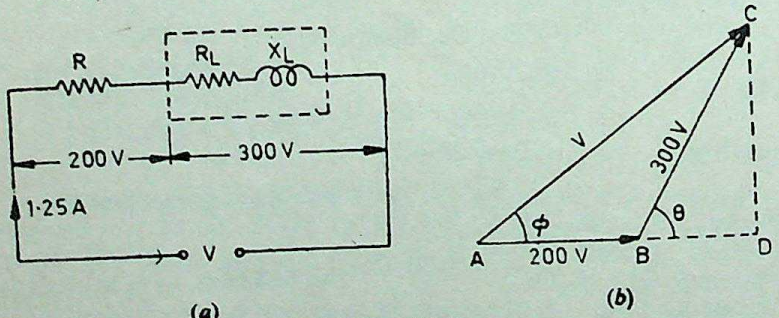

Fig. 17-10

Coil impedance $Z = 300/5 = 60\ \Omega$

$$X_L = \sqrt{60^2 - 10^2} = \mathbf{59{\cdot}2}\ \Omega$$

Combined resistance $= R + R_L = 40 + 10 = 50\ \Omega$

Combined impedance $= \sqrt{50^2 + 59{\cdot}2^2} = 77{\cdot}5\ \Omega$

Supply voltage $= IZ = 5 \times 77{\cdot}5 = \mathbf{387{\cdot}5\ V}$

Example 17-8. *A resistance R_1 in series with an impedance Z consisting of a resistance R_L and a reactance X_L is connected across a 250-V supply. The voltages across R_1 and Z are 125 V and 200 V respectively and the current is 5 A. Calculate R_L, X_L and Z.*

Solution. The phasor diagram is shown in Fig. 17-11 (*b*)

$Z = 200/5 = \mathbf{40}\ \Omega$

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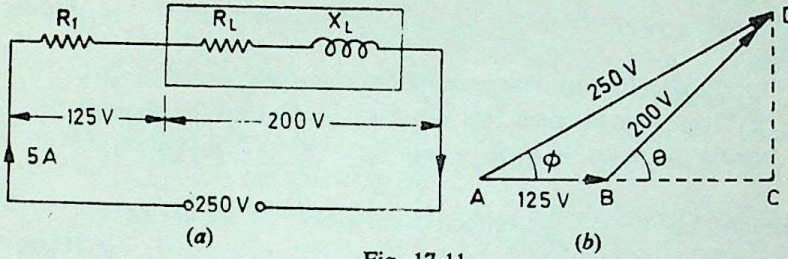
From $\triangle BCD$, we get $BC^2+CD^2=200^2$ ...(*i*)

From $\triangle ACD$, we get $(125+BC)^2+CD^2=250^2$

From (*i*) and (*ii*), we get $(125+BC)^2-BC^2=250^2-200^2$

$\therefore$ $BC=27{\cdot}5$ V ; $CD=\sqrt{200^2-27{\cdot}5^2}=198{\cdot}1$ V



(*a*) (*b*)

Fig. 17-11

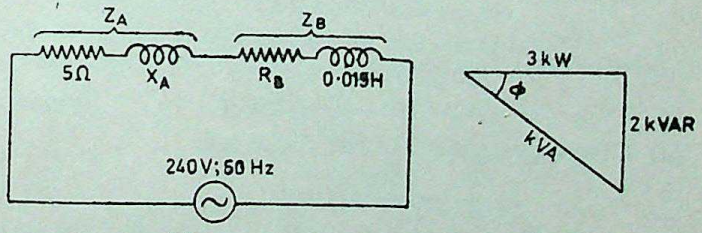
Now $BC=I.R_L$ $\therefore$ $R_L=27{\cdot}5/5=\mathbf{5{\cdot}5\ \Omega}$

$CD=I.X_L$ $\therefore$ $X_L=198{\cdot}1/5=\mathbf{39{\cdot}6\ \Omega}$

Example 17-9. *Two coils A and B are connected in series across a 240-V, 50-Hz-supply. The resistance of A is 5 Ω and the inductance of B is 0·015 H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.*

Solution. The kVA triangle is shown in Fig. 17-12 (*b*) and the circuit in Fig. 17-12 (*a*). The circuit kVA is given by

$$\text{kVA}=\sqrt{3^2+2^2}=3{\cdot}606 \quad \text{or} \quad \text{VA}=3606 \text{ voltamperes}$$



(*a*) (*b*)

Fig. 17-12

Circuit current $=3606/240=15{\cdot}03$ A

$\therefore$ $15{\cdot}03^2\,(R_A+R_B)=3000$

$\therefore$ $R_A+R_B=3000/15{\cdot}03^2=13{\cdot}3\ \Omega$

$\therefore$ $R_B=13{\cdot}3-5=\mathbf{8{\cdot}3\ \Omega}$

Now, impedance of the whole circuit is given by

$$Z=240/15{\cdot}03=15{\cdot}97\ \Omega$$

$\therefore$ $X_A+X_B=\sqrt{Z^2-(R_A+R_B)^2}$

$=\sqrt{15{\cdot}97^2-13{\cdot}3^2}=8{\cdot}843\ \Omega$

Now $X_B=2\pi\times 50\times 0{\cdot}015=4{\cdot}713\ \Omega$

$\therefore$ $X_A=8{\cdot}843-4{\cdot}713=4{\cdot}13\ \Omega$

$\therefore$ $2\pi\times 50\times L_A=4{\cdot}13$ $\therefore$ $L_A=\mathbf{0{\cdot}0132\ H\ (approx)}$

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Now $Z_A = \sqrt{R_A^2 + X_A^2} = \sqrt{5^2 + 4 \cdot 13^2} = 6 \cdot 485\ \Omega$

p.d. across coil $A = IZ_A = 15 \cdot 03 \times 6 \cdot 485 = \mathbf{97 \cdot 5\ V}$

$Z_B = \sqrt{8 \cdot 3^2 + 4 \cdot 713^2} = 9 \cdot 545\ \Omega$

$\therefore$ p.d. across coil $B = IZ_B = 15 \cdot 03 \times 9 \cdot 545 = \mathbf{143 \cdot 5\ V}$

17-4. Power in an Iron-cored Choking Coil

Total power, W taken by an iron-cored choking coil is used to supply,

(i) Power loss in ohmic resistance *i.e.* I^2R

(ii) Iron-loss in core, W_i

$\therefore \quad W = W_i + I^2R \quad \text{or} \quad \frac{W}{I^2} = \frac{W_i}{I^2} + R$

Now $\frac{W}{I^2}$ = effective resistance of the choke

$\frac{W_i}{I^2}$ = equivalent resistance of the iron loss

$\therefore$ Effective resistance = true resistance R + equivalent resistance $\frac{W_i}{I^2}$.

Example 17-10. *An iron-cored choking coil takes 5 A when connected to a 20-V d.c. supply and takes 5 A at 100-V a.c. and consumes 250 W. Determine* (a) *impedance* (b) *the power factor* (c) *the iron loss* (d) *the inductive reactance of the coil.*

Solution. (a) $Z = 100/5 = \mathbf{20\ \Omega}$

(b) $W = VI \cos \phi \quad \text{or} \quad 250 = 100 \times 5 \times \cos \phi$

$\cos \phi = 250/500 = \mathbf{0 \cdot 5}$

(c) Total loss = loss in resistance + iron loss

$\therefore \quad 250 = 20 \times 5 + W_i$

$\therefore \quad W_i = 250 - 100 = \mathbf{150\ W}$

(d) Effective resistance of the choke is

$$\frac{W}{I^2} = \frac{250}{25} = 10\ \Omega$$

$$X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(400 - 100)} = \mathbf{17 \cdot 32\ \Omega}$$

17-5. A.C. Through Resistance and Capacitance

The circuit is shown in Fig. 17-13 (*A.M.I.E. Summer 1985*)

Here, $V_R = IR$ = drop across R (in phase with I)

$V_C = IX_C$ = drop across capacitor (lagging I by $\pi/2$)

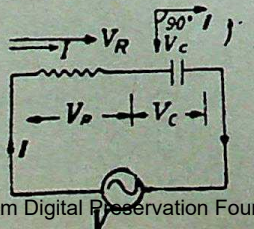

Fig. 17-13

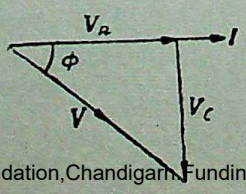

Fig. 17-14

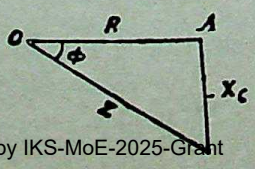

Fig. 17-15

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As capacitive reactance (X_C) is taken negative V_C is shown along negative direction of Y-axis as shown in voltage triangle (Fig. 17-14).

Now
$$V=\sqrt{V_R^2+V_C^2}=\sqrt{(IR)^2+(-IX_C)^2}$$
$$=I\sqrt{R^2+X_C^2}$$

or
$$I=\frac{V}{\sqrt{R^2+X_C^2}}=\frac{V}{Z}$$

The denominator is called the *impedance* of the circuit.

Impedance triangle is shown in Fig. 17-15

From Fig. 17-14 it is found that I leads V by angle ϕ such that
$$\tan\phi=-X_C/R$$

Hence, it means that if the equation of the applied alternating voltage is
$$v=V_m \sin \omega t$$
the equation of the resultant current in the R-C circuit is
$$i=I_m \sin(\omega t+\phi)$$
so that current leads the applied voltage by an angle ϕ. This fact is shown graphically in Fig. 17-16.

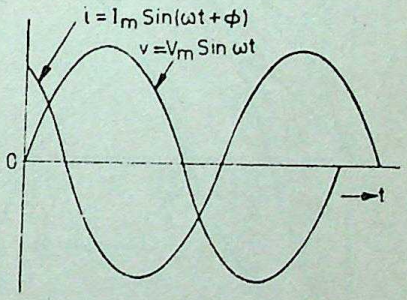

Fig. 17-16

Power consumed is $VI\cos\phi$.

Example 17-11. *A capacitor having a capacitance of 10-μF is connected in series with a non-inductive resistance of 120 Ω across a 100-V, 50-Hz supply. Calculate (a) the current (b) the phase difference between the current and the supply voltage and (c) the power.*

Solution. $R=120\ \Omega$

$$X_C=\frac{10^6}{2\pi\times 50\times 10}=318{\cdot}3\ \Omega$$

$$Z=\sqrt{120^2+318{\cdot}3^2}=340\ \Omega$$

(*a*) $I=100/340=\mathbf{0{\cdot}294\ A}$

(*b*) $\tan\phi=318{\cdot}3/120$

$\therefore$ $\phi=\tan^{-1}(318{\cdot}3/120)=\mathbf{69°\ 20'}$

(*c*) $W=I^2R=0{\cdot}294^2\times 120=\mathbf{10{\cdot}4\ W}$

Example 17-12. *The current in a circuit, consisting of a resistance of 318 Ω in series with a capacitor of 10 μF, is given by the equation $i=\sin 314\ t$ where i is the instantaneous value of the current in amperes and t is the time in seconds.*

(*i*) *Find the equation for the instantaneous voltage across the resistor*

(*ii*) *Find the equation for the instantaneous voltage across the capacitor*

(*iii*) *Find the equation for the instantaneous voltage across the entire circuit*

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(*iv*) *Draw a phasor diagram of the circuit to a suitable scale*

(*v*) *Find the energy stored in the capacitor when the current in the circuit is zero.* (*A.M.I.E. Nov. 1970*)

Solution. $X_C = 10^6/314 \times 10 = 318\ \Omega$

$Z = \sqrt{R^2 + X_C^2} = 318\sqrt{2} = 450\ \Omega$

(*i*) $v_R = iR = 318 \sin 314\ t$

(*ii*) Remembering that voltage across the capacitor lags behind the current by $\pi/2$ radian, the equation for instantaneous voltage across the capacitor becomes

$$v_C = iX_C = 318 \sin \left(314\ t - \frac{\pi}{2} \right)$$

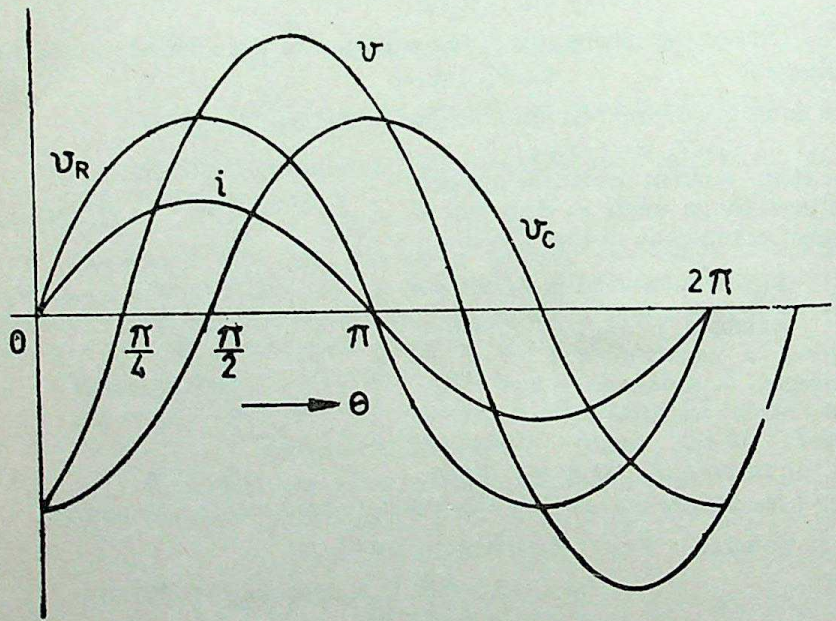

Fig. 17-17

(*iii*) In an $R-C$ circuit, current leads the voltage by an angle ϕ such that $\tan \phi = X_C/R = 318/318 = 1$ or $\phi = 45°$ (lead). In other words, the applied voltage across the entire circuit lags behind the circuit current by 45°. Hence, equation of the applied voltage is

$$v = iZ = 450 \sin \left(314\ t - \frac{\pi}{4} \right)$$

(*iv*) Phasor diagram is shown in Fig. 17-17.

(*v*) When circuit current is zero, p.d. across the capacitor is maximum and equals 450 V.

Energy stored $= \frac{1}{2} \times 10 \times 10^{-6} \times 450^2 = 1{\cdot}01$ J.

Example 17-13. *A voltage of 125 V at 60 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current in the circuit is 2·2 A. The power loss in the resistor is 96·8 watts and that in the capacitor is negligible. Calculate the resistance and the capacitance. Draw the vector diagram.*

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Solution. I^2R = power loss $\therefore \quad 2{\cdot}2^2 \times R = 96{\cdot}8$

$\therefore \quad R = 96{\cdot}8/2{\cdot}2^2 = \mathbf{20}\ \Omega$

Now $\quad Z = 125/2{\cdot}2 = 56{\cdot}82\ \Omega$

$\therefore\ X_C = \sqrt{Z^2 - R^2} = \sqrt{(55{\cdot}82^2 - 20^2)} = \mathbf{53{\cdot}2}\ \Omega\ ;\ X_C = 1/\omega C = 53{\cdot}2\ \Omega$

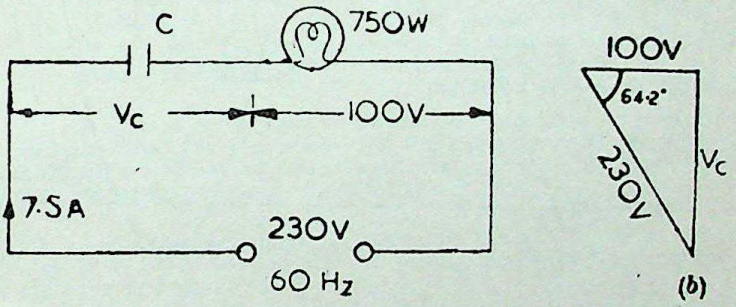
$C = 1/2\pi \times 60 \times 53{\cdot}2 = 0{\cdot}00005\ \text{F} = \mathbf{50}\ \mu\text{F}$

Example 17-14. *A metal filament lamp, rated at 750 W—100 V is to be connected in series with a capacitance across a 230- V, 60 hertz supply. Calculate (i) the capacitance required and (ii) the phase angle between the current and the supply voltage when the lamp obtains its rated voltage.* (*A.M.I.E. May 1973*)

Solution. (*i*) The circuit diagram is shown in Fig. 17-18 (*a*).

$V_C = \sqrt{230^2 - 100^2} = 207{\cdot}1\ \text{V}$

Rated lamp current $= 750/100 = 7{\cdot}5\ \text{A}$



(*a*)

Fig. 17-18

$$I = \frac{V_C}{X_C} = \frac{V_C}{1/\omega C} = \omega C V_C$$

$\therefore \quad 7{\cdot}5 = 2\pi \times 60 \times C \times 207{\cdot}1 \qquad \therefore \quad C = \mathbf{96}\ \mu\text{F}$

(*ii*) As seen from Fig. 17-18 (*b*), $\tan\phi = 207{\cdot}1/100 = 2{\cdot}071$; $\phi = \tan^{-1}(2{\cdot}071) = \mathbf{64{\cdot}2°}$.

Example 17-15. *A voltage $v = 100 \sin 314\ t$ is applied to a circuit consisting of a 25 Ω resistor and an 80 μF capacitor in series. Determine :*

(*a*) *an expression for the value of the current flowing at any instant*

(*b*) *the power consumed*

(*c*) *the p.d. across the capacitor at the instant when the current is one half of its maximum value.*

Solution. $X_C = 10^6/314 \times 80 = 39{\cdot}8\ \Omega$

$\phi = \tan^{-1}(39{\cdot}8/25) = 57°\ 52' = 1{\cdot}01$ radian (lead)

$Z = \sqrt{25^2 + 39{\cdot}8^2} = 47\ \Omega$

$I_m = V_m/Z = 100/47 = 2{\cdot}13\ \text{A}$

(*a*) $\quad i = I_m \sin(\omega t + \phi) = \mathbf{2{\cdot}13 \sin (314\ t + 1{\cdot}01)}$

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(*b*) $P = I^2R = (2{\cdot}13/\sqrt{2})^2 \times 25 = 56{\cdot}7$ W

(*c*) Now, maximum voltage across the capacitor is

$= I_m . X_C = 2{\cdot}13 \times 39{\cdot}8 = 84{\cdot}4$ V

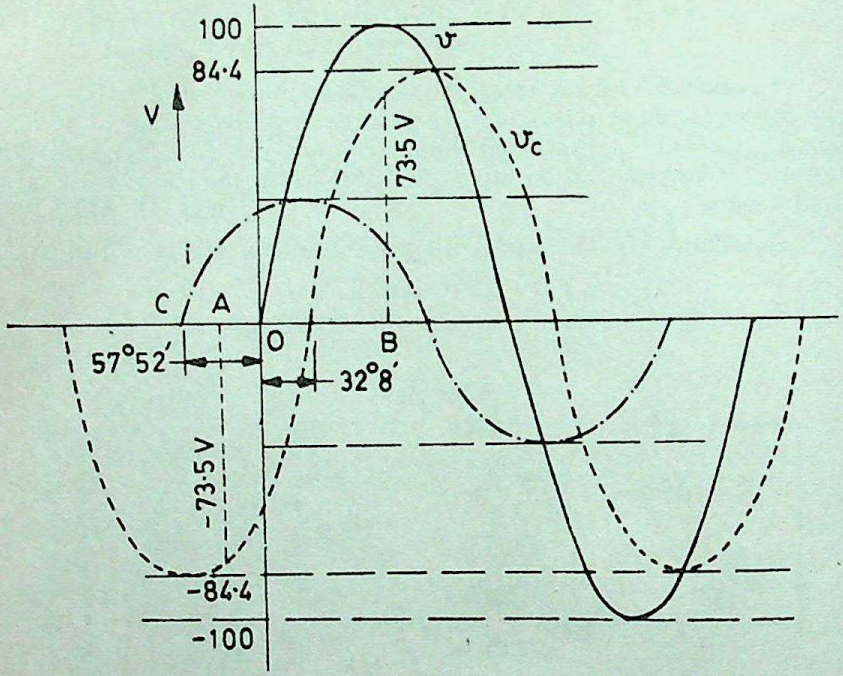

Fig. 17-19

Since capacitor voltage lags behind the circuit *current* by $\pi/2$ radian, the instantaneous voltage across the capacitor is

$$v_C = 84{\cdot}4 \sin (314\, t + 1{\cdot}01 - \pi/2)$$

As shown in Fig. 17-19, the applied voltage has been taken as the reference voltage. The current wave *leads* the applied voltage wave by 57°52′. The capacitor voltage wave lags behind the *current* wave by 90° but lags behind voltage wave by (90—57° 52′)=32° 8′.

When current is half its maximum value, we have

$$0{\cdot}5 \times 2{\cdot}13 = 2{\cdot}13 \sin (314\, t + 1{\cdot}01)$$

$$\therefore \quad 314\, t + 1{\cdot}01 = \sin^{-1}(0{\cdot}5) = \frac{\pi}{2} \text{ radian or } \frac{5\pi}{6} \text{ radian}$$

It means that current is half its maximum value at angles of 30° or 150° as measured from *point C**. It means at points *A* and *B* respectively.

At point A

$$v_C = 84{\cdot}4 \sin (\pi/6 - \pi/2)$$
$$= 84{\cdot}4 \sin (-\pi/3) = -73{\cdot}5 \text{ V}$$

*Or at angles of (30—57° 52′)=—27° 52′ (point *A*) and (150—57° 52′)=92° 8′ (point *B*) as measured from reference point *O*.

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At point B

$$v_C = 84{\cdot}4\ (5\pi/6 - \pi/2) = 84{\cdot}4 \sin(\pi/3) = 73{\cdot}5 \text{ V}$$

Hence, p.d. across the capacitor when current is half its maximum value is 73·5 V.

Tutorial Problems—26

1. When a coil is connected to a 200-V, 50-Hz supply, it takes a current of 1·5 A and the power consumption is 70 W. Calculate the resistance and inductance of the coil. **[31·15 Ω ; 0·413 H]**

2. A voltage of 260 V at 50 Hz applied to a coil produced a current of 2 A. The application of 2 V (d.c.) was necessary to produce 2 A of direct current through the coil. Calculate the inductance of the coil. **[382·5 mH]**

3. An inductor having a resistance of 10 Ω is connected to a 240-V, 50 Hz alternating current supply. The current flowing through the coil is found to be 12 A.

(*a*) Calculate the impedance, inductive reactance and inductance of the inductor

(*b*) Determine the phase angle between the current and the applied voltage

(*c*) Draw a vector diagram showing the relation between the current and various voltages across the circuit components.

[(*a*) 20 Ω ; 17·32 Ω ; 0·055 H (*b*) 60°]

4. A voltage represented by 230 sin 314 t is applied to a series circuit, the resulting current is represented by the equation $i = 10 \sin(314\ t + \pi/6)$. Sketch the current and voltage waveforms over a complete cycle. What is the frequency, resistance and reactance of the circuit ?

[50 Hz ; 19·92 Ω ; 11·5 Ω]

5. An inductive coil takes a current of 2 A and consumes 160 W when connected to a 240-V a.c. supply. A second coil when connected across the same supply takes 3 A and 500 W.

Find the current and total power when the two coils are connected in series to this supply. **[1·23 A, 151 W]**

6. A 1 henry inductor having a resistance of 50 Ω is connected in series with a 250 Ω resistor. A 20-V, 50 Hz supply is connected across this series circuit. Determine the voltage across the 250 Ω resistor. **[11·5 V]**

7. A certain iron-cored coil when connected to a 400-V, 50 Hz supply takes a current of 10 A and power of 3 kW while when it is connected to a 100 V d.c. supply, the current is 5 A. Calculate for operation on the a.c. supply :

(*a*) the power factor (*b*) the iron loss and (*c*) coil resistance.

[(*a*) 0·75 lag (*b*) 1000 W (*c*) 26·46 Ω]

8. A 100 ohm resistor is connected in series with an inductor across a sinusoidal. 400-V, 50 Hz supply. If the p.ds. across the resistor and inductor are respectively 200 V and 300 V, calculate the resistance, inductance and p.f. of the inductor. **[37·5 Ω, 0·462 H ; 0·25 lag]**

9. A 110-V, 40-W lamp is connected to a 240-V, 50 Hz supply with a capacitor in series with the lamp so that it operates at its normal voltage. Determine the capacitance of the capacitor used and the p.f. of the circuit.

[5·42 μF ; 0·458 lead]

10. An 80-W, 100-V lamp is to be used on a 240-V, 50 Hz supply. Calculate the value of ;

(*a*) the inductance of an iron-cored choke of effective resistance of 12·5 Ω

(*b*) the capacitance of a capacitor ;

either of which could be connected in series with the lamp so that it operates at its correct voltage. **[0·85 H ; 11·7 μF]**

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11. A 1-phase load takes 8 A at 50 Hz when the supply is 220 V. The power factor of the load is then 0·9. Calculate the effective resistance and reactance of the circuit and the active and reactive components of the current.
[10·98 Ω ; 5·3 Ω, 16·2 A, 7·85 A]

12. Coil *A* draws 5 A and absorbs 300 W when connected to a 110-V a.c. supply. Similar figures for coil *B* are 8 A and 400 W. Determine the current, power and reactive volt-amperes when *A* and *B* are joined in series to a 220-V supply at the same frequency. **[6·16 A ; 6·95 kW ; 1,170 kVAR]**

13. An iron-cored choking coil of resistance 6 Ω takes a current of 8 A when connected to a 200-V, 50 Hz supply and the power dissipated is 750 W. Assuming the coil to be equivalent to a series impedance, calculate (*i*) the iron loss (*ii*) the inductance at the given value of the current and (*iii*) the power factor. **[(*i*) 366 W (*ii*) 70·2 mH (*iii*) 0·469**

17-6. Resistance, Inductance and Capacitance in Series

The three are shown in Fig. 17-20 joined in series across an a.c. supply of r.m.s. voltage V.

Let $V_R = IR$ = voltage drop across R (in phase with I)
$V_L = I.X_L$ = voltage drop across L (leading I by $\pi/2$)
$V_C = I.X_C$ = voltage drop across C (lagging behind I by $\pi/2$)

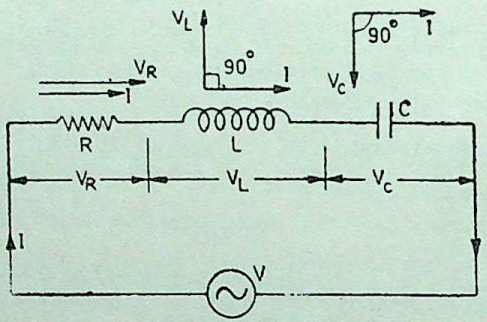

Fig. 17-20

In voltage triangle of Fig. 17-21, OA represents V_R, AB and AC represent the inductive and capacitive drops respectively. It will be seen that V_L and V_C are 180° out of phase with each other *i.e.* they are in direct opposition to each other. Moreover, it has been assumed that V_L is greater than V_C in magnitude.

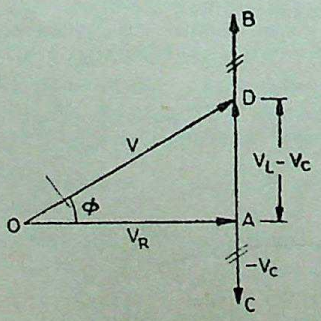

Fig. 17-21

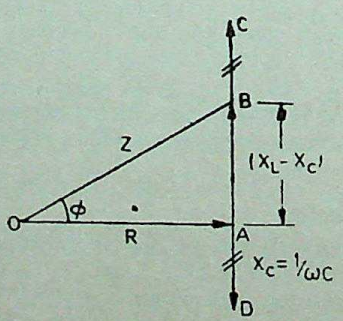

Fig. 17-22

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Subtracting $BD\ (=AC)$ from AB, we get the net reactive drop

$$AD=(V_L-V_C)=I\,(X_L-X_C)$$

The applied voltage V is represented by OD and is the vector sum of OA and AD.

$$OD=\sqrt{(OA^2+AD^2)}$$

or
$$V=\sqrt{[(IR)^2+(IX_L-IX_C)^2]}$$
$$=I\sqrt{[(R^2+(X_L-X_C)^2]}$$

or
$$I=\frac{V}{\sqrt{R^2+(X_L-X_C)^2}}=\frac{V}{Z}$$

The term $\sqrt{R^2+(X_L-X_C)^2}$ is known as the impedance of the circuit. Obviously,

$$(Impedance)^2=(Resistance)^2+(Net\ reactance)^2$$

or
$$Z^2=R^2+(X_L-X_C)^2=R^2+X^2$$

where X is the net reactance. ...(Fig. 17-22)

Remember that X_C is taken as $-$ve. Phase angle ϕ is given by

$$\tan\phi=(X_L-X_C)/R=X/R$$
$$=\text{net reactance/resistance}$$

Power factor $\cos\phi=\dfrac{R}{Z}=\dfrac{R}{\sqrt{R^2+(X_L-X_C)^2}}$

Hence, it is seen that if the equation of the applied voltage is

$$v=V_m\sin\omega t$$

then equation of the resulting current in an R-L-C circuit is given by

$$i=I_m\sin(\omega t\pm\phi)$$

The $+$ve sign is to be used when current leads *i.e.* when $X_C>X_L$. The $-$ve sign is used when current lags *i.e.* when $X_L>X_C$. In general, the current lags or leads the supply voltage by an angle ϕ such that $\tan\phi=X/R$.

Summary of Results of Series Circuit

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2+(\omega L)^2]}$	$0<\phi<90°$ lag	1 > p.f. > 0 lag
Resistance and Capacitance	$\sqrt{[R^2+(-1/\omega C)^2]}$	$0<\phi<90°$ lead	1 > p.f. > 0 lead
R—L—C	$\sqrt{[R^2+(\omega L-1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

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Example 17-16 *A coil of power factor 0·6 is in series with a 100 μF capacitor. When connected to a 50 Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.* (*A.M.I.E. Winter 1985*)

Solution. $X_C = 1/\omega C = 1/2\pi fC$

$= 10^6/2\pi \times 50 \times 100 = 31{\cdot}8\ \Omega$

Since for a supply of 50 Hz, drop across coil (IZ) equals the drop across the capacitor ($I.\ X_C$), it means that $Z = X_C = 31{\cdot}8\ \Omega$

$\therefore$ Coil $Z = 31{\cdot}8\ \Omega$

Also, coil p.f. $= \cos\phi = 0{\cdot}6$,

$\sin\phi = 0{\cdot}8$.

$\therefore$ Coil $R = Z\cos\phi$

$= 31{\cdot}8 \times 0{\cdot}6 = \mathbf{19{\cdot}1}\ \Omega$

Coil $X_L = Z\sin\phi$

$= 31{\cdot}8 \times 0{\cdot}8 = \mathbf{25{\cdot}4}\ \Omega$.

Example 17-17. *A series R-L-C circuit with a resistance of 50 Ω, a capacitance of 25 μF and an inductance of 0·15H is connected across 230-V, 50-Hz supply. Determine (i) impedance (ii) current (iii) power factor and (iv) power consumption of the circuit.* (*A.M.I.E. Summer 1983*)

Solution. $X_L = 2\pi fL = 2\pi \times 50 \times 0{\cdot}15 = 47{\cdot}1\Omega$

$X_C = -1/2\pi fC = -10^6/2\pi \times 50 \times 25 = -127{\cdot}3\ \Omega$

Net $X = 47{\cdot}1 - 127{\cdot}3 = -80{\cdot}2\ \Omega$ (capacitive)

(*i*) $Z = \sqrt{R^2 \times X^2} = \sqrt{50^2 + 80{\cdot}2^2} = \mathbf{94{\cdot}4}\ \Omega$

(*ii*) $I = V/Z = 230/94{\cdot}4 = \mathbf{2{\cdot}44}$ **A**

(*iii*) p.f. $= \cos\phi = R/Z = 50/94{\cdot}4 = \mathbf{0{\cdot}53}$ **(lead)**

(*iv*) power consumed $= VI\cos\phi = 230 \times 2{\cdot}44 \times 0{\cdot}53 = \mathbf{297}$ **W**

Example 17-18. *A coil of insulated wire of resistance 8 Ω and inductance 0·03 H is connected to an a.c. supply at 240-V, 50-Hz. Calculate :*

(a) the current, p.f. and the power

(b) the value of capacitance which when connected in series with the above coil, causes no change in the values of the current and power taken from the supply.

Solution. (*a*) $X_L = 314 \times 0{\cdot}03 = 9{\cdot}42\ \Omega$

$Z = \sqrt{8^2 + 9{\cdot}42^2} = 12{\cdot}36\ \Omega$

$I = 240/12{\cdot}36 = \mathbf{19{\cdot}4}$ **A**

$W = I^2R = 19{\cdot}4^2 \times 8 = \mathbf{3011}$ **W**

p.f. $= R/Z = 8/12{\cdot}36 = \mathbf{0{\cdot}65}$ **(lag)**

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(*b*) If the circuit is to draw the same current and at the same power factor, then the total reactance of the $R-L-C$ circuit must be 9·42 Ω. This can be achieved by selecting a capacitor which should not only neutralize the inductive reactance of 9·42 Ω of the coil but must add a further capacitive reactance of 9·42 Ω. In other words, capacitor must have a reactance of $2 \times 9{\cdot}42 = 18{\cdot}84\ \Omega$

$$1/\omega C = 18{\cdot}84 \quad \text{or} \quad C = 10^6/314 \times 18{\cdot}84 = 169\ \mu\text{F}$$

Tutorial Problems—27

1. A circuit consists of a resistance of 20 Ω in series with an inductance of 95·6 mH and a capacitor of 318 μF. It is connected to a 500-V, 25-Hz supply. Calculate the current in the circuit and the power factor. **[24·3 A ; 0·97 lead]**

2. A circuit is made up of 10 Ω resistance, 12 mH inductance and 281·5 μ F capacitance in series. The supply voltage is 10 V (constant). Calculate the value of the current when the supply frequency is (*a*) 50 Hz and (*b*) 150 Hz. **[(*a*) 8 A leading (*b*) 8 A lagging]**

3. A circuit takes a current of 3 A at a power factor of 0·6 lagging when connected to a 115-V, 50 Hz supply. Another circuit takes a current of 5 A at a power factor of 0·707 leading when connected to the same supply. If the two circuits are connected in series across a 230-V, 50-Hz supply, calculate :

(*a*) the current (*b*) the power consumed (*c*) the power factor.

[(*a*) 5·5 A (*b*) 1,188 W (*c*) 0·939 lag]

4. A series circuit consists of a choke coil, having a resistance of 10 Ω and a capacitor. When connected to a 240-V, 50-Hz supply, the current was 6 A leading the applied voltage When connected to a 240-V, 100 Hz supply, the current was 6 A lagging the applied voltage.

Calculate the inductance of the coil and the capacitance of the capacitor.

[124 mH ; 41 μF]

17-7. Resonance in R-L-C Circuit *(A.M.I.E. Winter 1981)*

A series circuit is said to be in electrical resonance when its net reactance is zero. The frequency f_0, at which this happens is known as 'resonant frequency'.

As seen in Art. 17-6 net reactance of an *R-L-C* circuit is

$$X = X_L - X_C \text{ and } Z = \sqrt{R^2 + X^2}$$

At resonance, $X = 0$ or $X_L - X_C = 0$

$\therefore$ $X_L = X_C$

or $\omega L = 1/\omega C$ or $\omega^2 = 1/LC$

$\therefore$ $(2\pi f_0)^2 = \dfrac{1}{LC}$ or $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

If L is in henry and C in farad, then f_0 is in Hz. Under resonant conditions, $X = 0$, hence $Z = R$. This is the minimum possible value of impedance. Hence, circuit current is maximum and is given by

$$I_m = V/Z = V/R$$

The circuit behaves like a pure resistive circuit and current is in phase with the applied voltage. Obviously, the power factor of the circuit is unity under resonant conditions.

As current is maximum, it produces large voltage drops across L and C. But these drops being equal and opposite, cancel each other out. Taken together, L and C form part of a circuit across

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which no voltage is developed, however large the current flowing. If it were not for the presence of R, such a circuit would act like a short circuit to currents of the frequency to which it resonates. Hence, it is sometimes called *acceptor circuit* and the series resonance is often referred to as *voltage resonance*.

17-8. Graphic Representation of Resonance

Suppose an alternating voltage of constant magnitude but of varying frequency is applied to an R-L-C circuit. The variations of resistance, inductive reactance X_L and capacitive reactance X_C with frequency are as shown in Fig. 17-23.

(*i*) R : It is independent of f, hence it is represented by a straight horizontal line.

(*ii*) $X_L = \omega L = 2\pi fL$

Inductive reactance X_L is directly proportional to f, hence its graph is a straight line through the origin. So, X_L increases linearly with f.

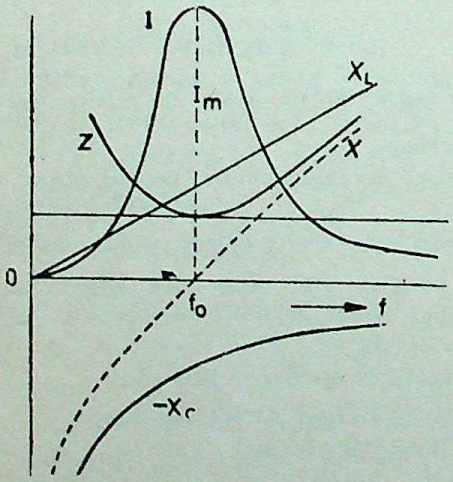

Fig. 17-23

(*iii*) $X_C = 1/\omega C = 1/2\pi fC$

Capacitive reactance X_C is inversely proportional to f. Its graph is a rectangular hyperbola which is drawn in fourth quadrant because X_L is regarded negative.

(*iv*) *Net reactance*

$$X = X_L - X_C$$

Its graph is a hyperbola (not rectangular) and crosses the X-axis at point A. The value of frequency at A is called resonant frequency f_0.

(*v*) $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

At low frequencies, Z is large (Fig. 17-23). But as $X_C > X_L$, net impedance is capacitive and the p.f. is leading. At high frequencies, Z is again large, but is inductive because $X_L > X_C$ and the p.f. is lagging. The minimum value of Z is R when $X_L = X_C$.

(*vi*) *Current I*

It has low value on both sides of resonant frequency (because Z is large) but has maximum value of $I_m = V/R$ at resonance as shown by the peaked curve (Fig. 17-23).

17-9. Resonance Curve

The curve between current and frequency is known as resonance curve. The shape of such a curve for various values of R is shown in Fig. 17-24. For smaller values of R, the current-frequency curve is sharply peaked but for larger values of R, the curve is flat.

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The variations of Z, power factor and I are separately shown in Fig. 17-25.

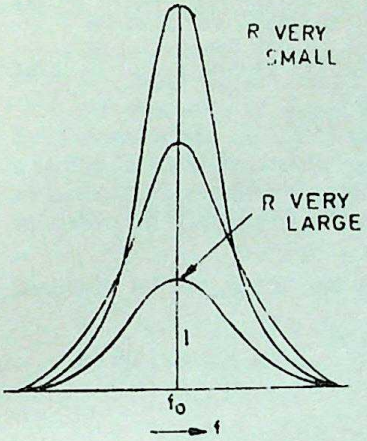

Fig. 17-24

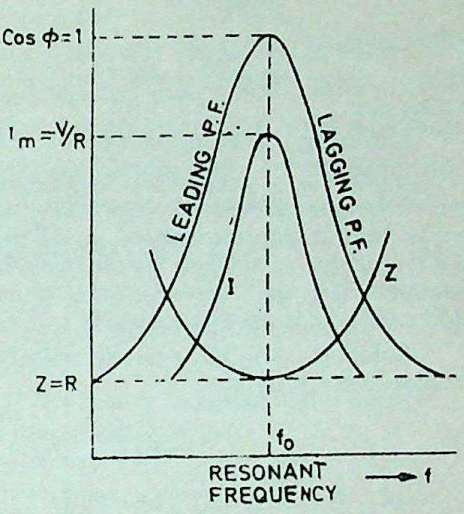

Fig. 17-25

17-10. Points to Remember

Following points about series resonance must be noted (and compared with parallel resonance later on)

(*i*) Net reactance is zero *i.e.* $X=0$ or $X_L=X_C$

(*ii*) Hence, $V_L=V_C$ or $V_L-V_C=0$

(*iii*) Impedance at resonance is minimum and $=R\ \Omega$

(*iv*) Line current at resonance is maximum and is equal to V/R and is in phase with V.

(*v*) Power factor is unity *i.e.* $\cos\phi=1$.

17-11. Q-factor of a Series Circuit

In the case of an *R-L-C* series circuit, it is defined as equal to the voltage magnification in the circuit at resonance.

We have seen that at resonance, current is maximum,

i.e. $$I_m=V/R$$

Voltage across either coil or capacitor $=I_mX_L$

Supply voltage $V=I_mR$

$\therefore$ Voltage magnification

$$=\frac{I_mX_L}{I_mR}=\frac{X_L}{R}=\frac{\omega L}{R}$$

$$\therefore \quad Q\text{-factor}=\frac{\omega L}{R}=\frac{2\pi f_0 L}{R} \qquad ...(i)$$

We have seen that resonant frequency is

$$f_0=\frac{1}{2\pi\sqrt{LC}} \quad \text{or} \quad 2\pi f_0=\frac{1}{\sqrt{LC}}$$

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Putting this value in (*i*) above, we get

$$Q\text{-factor} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

In the case of series resonance, higher Q-factor means not only higher voltage magnification, but also a higher selectivity of the tuning coil.

Obviously, *Q*-factor can be increased by having a coil of large inductance but of small ohmic resistance.

Example 17-19. *A resistance of 15 Ω, an inductance of 4 henrys and a capacitance of 25 μF are connected in series across a 230-V supply. Calculate (i) the frequency at which current shall be maximum (ii) the current at this frequency and (iii) the potential difference across the inductance.*

Solution. (*i*) The current will be maximum at the resonant frequency given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{10^3}{2\pi\sqrt{4\times 25}} = \frac{10^3}{20\pi} = 15{\cdot}9 \text{ Hz}$$

(*ii*) $I_m = V/R = 230/15 = 15{\cdot}33$ A

(*iii*) $X_L = 2\pi f_0 L = 2\pi \times 15{\cdot}9 \times 4 = 400\ \Omega$

$V_L = I_m X_L = 15{\cdot}33 \times 400 = 6{,}132$ V

Example 17-20. *A series R-L-C circuit consists of a 100 ohm resistor, an inductor of 0·318 henry and a capacitor of unknown value. When this circuit is energised by 230 ∠0° 50-Hz sinusoidal a.c. supply, the current was found to be 2·3 ∠0° amperes. Find*

(i) value of capacitor in microfarad

(ii) voltage across the inductor

(iii) total power consumed.

(A.M.I.E. Summer 1984)

Solution. Since current is 2·3∠0° *i.e.* its magnitude is 2·3 A (=230/100) and phase = 0°, it is obvious that it is a case of series resonance. In that case

(*i*) $X_L = X_C$ or $2\pi \times 50 \times 0{\cdot}318 = 1/2\pi \times 50 \times C$

$$\therefore \quad C = \frac{1}{(2\pi \times 50)^2 \times 0{\cdot}318} = 31{\cdot}4\ \mu\text{F}$$

(*ii*) $V_L = IX_L = 2{\cdot}3 \times 2\pi \times 50 \times 0{\cdot}318 = 230$ V

(*iii*) $P = I^2R = 2{\cdot}3^2 \times 100 = 529$ W

Tutorial Problems—28.

1. An a.c. series circuit has a resistance of 10 Ω, an inductance of 0·2 H and a capacitance of 60 μF. Calculate :

(*a*) the resonant frequency (*b*) the current (*c*) the power at resonance. Given that applied voltage is 200 V. [(*a*) 46 Hz (*b*) 20 A (*c*) 4 kW]

2. A resistor and a capacitor are connected in series with a variable

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inductor. When the circuit is connected to a 240-V, 50-Hz supply, the maximum current given by varying the inductance is 0·5 A. At this current the voltage across the capacitor is 250 V. Calculate the value of :

(*a*) the resistance (*b*) the capacitance (*c*) the inductance.

Neglect the resistance of the inductor. **[(*a*) 480 Ω (*b*) 6·36 μF (*c*) 1·59 H]**

3. A series circuit has the following characteristics :

$R=10\ \Omega$; $L=100/\pi$ mH ; $C=500/\pi$ μF. Find:

(*a*) the current flowing when the applied voltage is 100 V at 50 Hz

(*b*) the power factor of the circuit

(*c*) what value of supply frequency would produce series resonance ?

[(*a*) 7·07 A (*b*) 0·707 lead (*c*) 70·71 Hz]

HIGHLIGHTS

1. In an *R-L* circuit

$$Z=\sqrt{R^2+X_L^2}\ ;\quad I=V/Z \text{ and } \tan\phi=X_L/R$$

$$\text{Power}=VI\cos\phi=I^2R$$

2. kVA input of a circuit has two rectangular components

(1) active kVA or kilowatt (kW)

(2) reactive kVA or kVAR

$$\text{kVA}=\sqrt{\text{kW}^2+\text{kVAR}^2}$$

3. In an *R-C* circuit,

$$Z=\sqrt{R^2+X_C^2}\ ;\ I=V/Z \text{ and } \tan\phi=-X_C/R$$

$$\text{Power}=VI\cos\phi=I^2R$$

4. In an *R-L-C* circuit,

$$Z=\sqrt{R^2+(X_L\sim X_C)^2}=\sqrt{R^2+X^2}$$

$$I=V/Z,\ \tan\phi=X/R$$

$$\text{Power}=VI\cos\phi=I^2R$$

5. An *R-L-C* circuit is said to be in electrical resonance when $X_L=X_C$. Under resonant conditions :

(*i*) net reactance $X=X_L-X_C=0$

(*ii*) resonant frequency $f_0=\dfrac{1}{2\pi\sqrt{LC}}$ Hz

(*iii*) circuit current is maximum and equals V/R amperes.

(*iv*) voltages across the inductance and capacitance are equal in magnitude although opposite in phase *i.e.* $V_L=V_C$.

(*v*) impedance $Z=R$.

(*vi*) power factor of the resonant circuit is unity.

6. The *Q*-factor of a series circuit is given by the voltage magnification produced in the circuit under resonant conditions.

$$Q\text{- factor}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

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PARALLEL A.C. CIRCUITS 18

1. Solving Parallel Circuits 2. Vector Method 3. Admittance Method 4. Application of Admittance Method 5. Resonance in Parallel Circuits 6. Graphic Representation of Parallel Resonance 7. Points to Remember 8. Q-factor of a Parallel Circuit 9. Highlights.

18-1. Solving Parallel Circuits

When impedances are joined in parallel, then we have three methods to solve such circuits :

(*a*) *Vector Method*
(*b*) *Admittance Method*
(*c*) *Complex Algebra* (*Chapter 19*)

18-2. Vector Method

Consider the circuit shown in Fig. 18-1. Here two reactors *A* and *B* have been joined in parallel across an r.m.s. supply

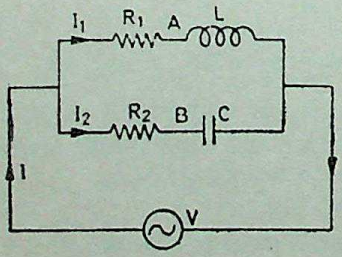

Fig. 18-1

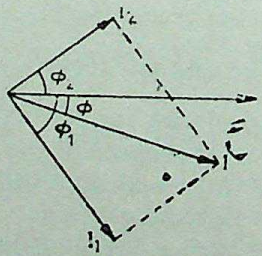

Fig. 18-2

of *V* volt. The voltage across *A* and *B* is the same, but currents through them are different.

For Branch A

$$Z_1=\sqrt{(R_1^2+X_L^2)}\ ;\ I_1=V/Z_1$$

$$\cos\ \phi_1=R_1/Z_1 \text{ or } \phi_1=\cos^{-1}(R_1/Z_1)$$

Current I_1 lags behind the applied voltage by ϕ_1 (Fig. 18-2).

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For Branch B :

$$Z_2=\sqrt{(R_2^2+X_C^2)}\ ;\ I_2=V/Z_2$$

$$\cos\ \phi_2=R_2/Z_2,\ \phi_2=\cos^{-1}(R_2/Z_2)$$

Current I_2 leads V by ϕ_2 (Fig. 18-2).

Resultant Current I

The resultant circuit current I is the vector sum of the branch currents I_1 and I_2 and can be found (*i*) by using parallelogram law

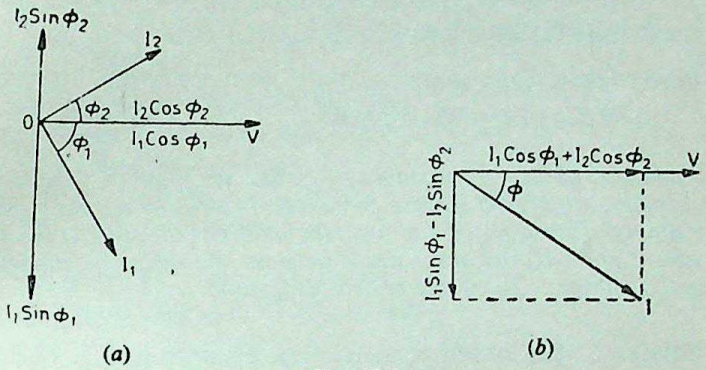

Fig. 18-3

of vectors as shown in Fig. 18-2 or (*ii*) by resolving I_1 and I_2 into their X- and Y-components (or active and reactive components) respectively and then by combining these components as shown in Fig. 18-3. Method (*ii*) is preferable as it is quick and convenient.

With reference to Fig. 18-3 (*a*), we have

Sum of the active components of I_1 and I_2

$$=I_1\cos\phi_1+I_2\cos\phi_2$$

Sum of reactive components of I_1 and I_2

$$=I_2\sin\phi_2-I_1\sin\phi_1$$

If I is the resultant current and ϕ is its phase angle, then its active and reactive components must be equal to these [Fig. 18-3 (*b*)].

$$\therefore\quad I\cos\phi=I_1\cos\phi_1+I_2\cos\phi_2$$

and

$$I\sin\phi=I_2\sin\phi_2-I_1\sin\phi_1$$

$$\therefore\quad I=\sqrt{[(I_1\cos\phi_1+I_2\cos\phi_2)^2+(I_2\sin\phi_2-I_1\sin\phi_1)^2]}$$

and

$$\tan\phi=\frac{I_2\sin\phi_2-I_1\sin\phi_1}{I_1\cos\phi_1+I_2\cos\phi_2}$$

If $\tan\phi$ is positive, then current leads and if $\tan\phi$ is negative, then current lags behind the applied voltage V.

Power factor for the whole circuit is given by

$$\cos\phi=\frac{I_1\cos\phi_1+I_2\cos\phi_2}{I}$$

Example 18-1. *An inductor of 0·5 H inductance and 90 Ω resistance is connected in parallel with a 20-μF capacitor. A voltage of*

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230 V at 50 Hz is maintained across the circuit. Determine the total power taken from the source. (*A.M.I.E. Winter 1974*)

Solution. It may be noted that power consumed by the inductor would remain the same irrespective of whether a capacitor is connected in parallel with it or not. It is so because a pure capacitor does not consume any power.

$$X_L = 2\pi \times 50 \times 0{\cdot}5 = 157\ \Omega$$

$$Z = \sqrt{90^2 + 157^2} = 181\ \Omega$$

$$I = 230/181 = 1{\cdot}27\ \text{A}$$

Power taken from supply

$$= I^2R = 1{\cdot}27^2 \times 90 = \mathbf{14{\cdot}5\ W}$$

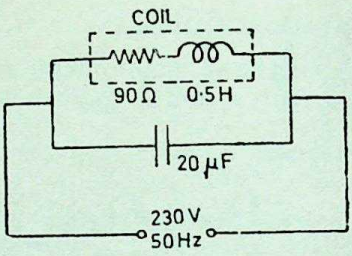


Fig. 18-4

Example 18-2. *A resistance of 50 Ω, an inductance of 0·15 H and a capacitance of 100 μF are connected in parallel across a 100-V, 50-Hz supply. Calculate (i) the current in each circuit (ii) the resultant current and (iii) the phase angle between the resultant current and the supply voltage. Draw the phasor diagram.*

(*A.M.I.E. Winter 1980*)

Solution. The circuit connections are shown in Fig. 18-5 (*a*).

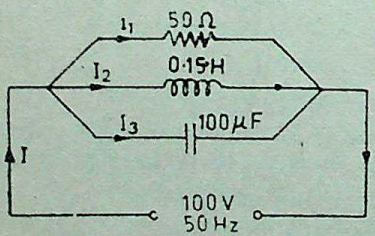
(*i*) $I_1 = 100/50 = 2$ **A** —in phase with V

$X_L = 2\pi fL = 2\pi \times 50 \times 0{\cdot}15 = 47{\cdot}1\ \Omega$

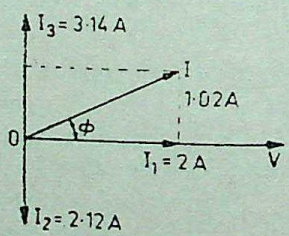
$I_2 = V/X_L = 100/47{\cdot}1 = \mathbf{2{\cdot}12}$ **A** —lagging V by 90°

$X_C = 1/2\pi fC = 10^6/2\pi \times 50 \times 100 = 31{\cdot}8\ \Omega$

$I_3 = V/X_C = 100/31{\cdot}8 = \mathbf{3{\cdot}14}$ **A** —leading V by 90°



(*a*)



(*b*)

Fig. 18-5

(*ii*) As seen from the phasor diagram of Fig. 18-5 (*b*).

$I = \sqrt{2^2 + 1{\cdot}02^2} = \mathbf{2{\cdot}245}$ **A**

(*iii*) $\tan \phi = 1{\cdot}02/2 = 0{\cdot}51$, $\phi = \mathbf{28°}$

Hence, resultant circuit current *leads* the supply voltage by 28°.

Example 18-3. *An inductive coil is connected in parallel across a non-inductive resistance of 30 Ω and this parallel circuit is connected across 1-phase, 50-Hz main. The total current taken from the mains*

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is 8 A when the current in the non-inductive resistance is 4 A and that in the inductive coil is 6 A. Calculate (a) the inductance of the coil and (b) power absorbed by the parallel circuit.

Draw a complete vector diagram for the circuit showing the voltage and currents.

Solution. The circuit is shown in Fig. 18-7 and its vector diagram in Fig. 18-8 where AB represents current drawn by 30 Ω non-inductive resistance in phase with the applied voltage and BD represents current drawn by the coil lagging behind the supply voltage by ϕ. The supply voltage is obviously $=4\times30=120$ V.

It is seen that current BD drawn by the coil has two components :

(*i*) in-phase component BC and

(*ii*) quadrature component CD.

From Fig. 18-8 we have

$$BC^2+CD^2=6^2 \qquad ...(i)$$

$$(4+BC)^2+CD^2=8^2 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$(4+BC)^2-BC^2=8^2-6^2=28$$

$$\therefore \quad 16+8BC=28 \quad \therefore \quad BC=1{\cdot}5 \text{ A}$$

$$\therefore \quad CD=\sqrt{6^2-1{\cdot}5^2}=5{\cdot}81 \text{ A}$$

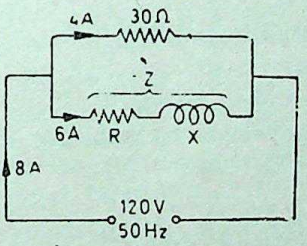

Fig. 18-7

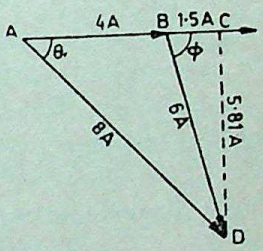

Fig. 18-8

P.f. angle of the coil is ϕ.

Now $\cos\phi=1{\cdot}5/6=0{\cdot}25$ and $\sin\phi=5{\cdot}81/6$

Impedance of coil $Z=120/6=20\ \Omega$

Now $\sin\phi=X/Z \quad \therefore \quad X=Z\sin\phi=20\times5{\cdot}81/6\ \Omega$

(*a*) $\therefore \quad 2\pi\times50\times L=20\times5{\cdot}81/6$

$$L=20\times5{\cdot}81/6\times314=\mathbf{0{\cdot}0617\ H}$$

(*b*) Power factor of the combined parallel circuit is

$$\cos\theta=5{\cdot}5/8$$

$\therefore$ Power absorbed by the parallel circuit

$$=120\times8\times5{\cdot}5/8=\mathbf{660\ W}$$

Example 18-4. *A coil having a resistance of 8 Ω and an inductance of 0·0191 H is connected in parallel with a capacitor having a capacitance of 398 μF and resistance of 5 Ω. If 100 V at 50 Hz are applied across the terminals of the above parallel circuit, calculate (a) the total current taken from the supply and (b) its phase angle with res-*

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pect to the supply voltage. Draw a complete vector diagram for the circuit showing the three currents and the supply voltage.

Solution. The circuit is shown in Fig. 18-9.

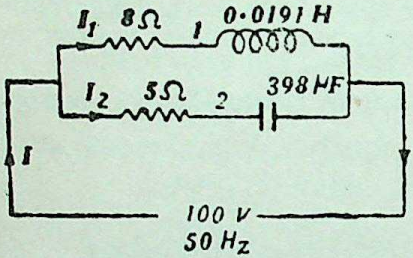

Fig. 18-9

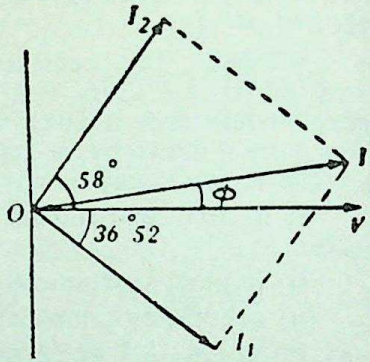

Fig. 18-10

$X_L = 2\pi f L = 2\pi \times 50 \times 0{\cdot}0191 = 6\ \Omega$

$Z_1 = \sqrt{6^2+8^2} = 10\ \Omega$; $I_1 = V/Z_1 = 100/10 = 10$ A

This current lags behind V by an angle ϕ_1 such that

$\cos\phi_1 = R_1/Z_1 = 8/10 = 0{\cdot}8$; $\phi_1 = 36°\ 52'$

$X_C = 10^6/2\pi \times 50 \times 398 = 8\ \Omega$; $R_2 = 5\ \Omega$

$Z_2 = \sqrt{5^2+8^2} = 9{\cdot}44\ \Omega$; $I_2 = 100/9{\cdot}44 = 10{\cdot}6$ A

Its angle of lead is given by

$\cos\phi_2 = R_2/Z_2 = 5/9{\cdot}44\ \Omega$; $\phi_2 = 58°$ (Fig. 18-10)

(*a*) The resultant current can be found by resolving the two branch currents into their X- and Y-components.

Total X-component $= I_1 \cos\phi_1 + I_2 \cos\phi_2$
$= 10 \times 0{\cdot}8 + 10{\cdot}6 \times 0{\cdot}53 = 13{\cdot}62$ A

Total Y-component $= I_2 \sin\phi_2 - I_1 \sin\phi_1$
$= 10{\cdot}6 \times 8/9{\cdot}44 - 10 \times 0{\cdot}6 = 3$ A

$\therefore$ resultant current $= \sqrt{13{\cdot}62^2+3^2} = \mathbf{13{\cdot}97\ A}$

(*a*) $\cos\phi = \dfrac{\text{in-phase current}}{\text{total current}} = \dfrac{13{\cdot}62}{13{\cdot}97} = 0{\cdot}975$

$\therefore$ $\phi = \cos^{-1}(0{\cdot}975) = 12°\ 48' = \mathbf{12{\cdot}8°}$

18-3. Admittance Method

Admittance of a circuit is defined as the reciprocal of its impedance. Its symbol is $\mho$.

$$Y = \frac{1}{Z} \quad \text{or} \quad Y = \frac{\text{r.m.s. amperes}}{\text{r.m.s. volts}}$$

Its unit is mho (ohm spelled backwards) and its symbol $\mho$.

One mho is defined as the admittance of a circuit having an impedance of 1 ohm.

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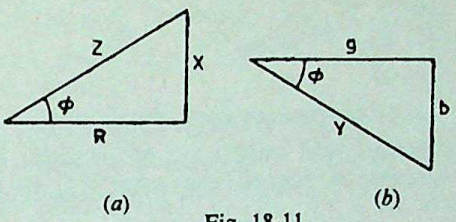
As impedance Z has two rectangular components R and X [Fig. 18-11 (*a*)] similarly, admittance Y also has two components as shown in Fig. 18-11 (*b*).



(*a*) (*b*)

Fig. 18-11

The X-component is called *conductance* (g) and the Y-component is known as *susceptance* (b)

Obviously, conductance, $g = Y \cos \phi$ ...Fig. 18-11 (*b*)

$$= \frac{1}{Z} \quad \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2} \text{ mho}$$

Similarly, susceptance

$$b = Y \sin \phi = \frac{1}{Z} \cdot \frac{X}{Z}$$

$$= \frac{X}{Z^2} = \frac{X}{R^2 + X^2} \text{ mho}$$

Admittance $Y = \sqrt{g^2 + b^2}$ as $Z = \sqrt{R^2 + X^2}$

The unit of g, b and Y is mho. We will regard *capacitive susceptance as positive and inductive susceptance as negative.*

18-4. Application of Admittance Method

Consider the 3-branched circuit of Fig. 18-12. Total conductance is found by merely adding the conductances of three branches. But total susceptance is found by *algebraically* adding the individual susceptance of each branch.

Total conductance, $G = g_1 + g_2 + g_3$

Total susceptance, $B = b_1 - b_2 + b_3$

Total admittance, $Y = \sqrt{G^2 + B^2}$

Circuit current $I = VY$

Power factor, $\cos \phi = G/Y$

Note : The branch currents are

$$I_1 = Vy_1$$

where $y_1 = \sqrt{g_1^2 + (-b_1)^2}$

$$I_2 = Vy_2$$

where $y_2 = \sqrt{g_2^2 + (-b_2)^2}$

$$I_3 = Vy_3$$

where $y_3 = \sqrt{g_3^2 + b_3^2}$

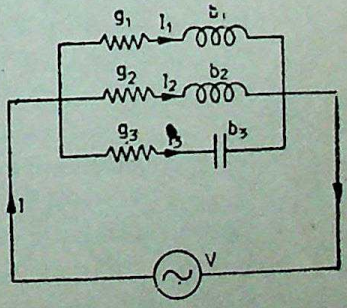


Fig. 18-12

Similarly, $\cos \phi_1 = g_1/y_1$; $\cos \phi_2 = g_2/y_2$ and $\cos \phi_3 = g_3/y_3$.

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Example 18-5. *A and B are two circuits connected in parallel across a 200-V, 50-Hz supply. Circuit A consists of a choking coil whose resistance is 5 Ω and reactance 2 Ω. Circuit B consists of a non-inductive resistor of 6 Ω connected in series with a capacitor of capacitive reactance 8 Ω. Calculate :*

(*a*) *total current*

(*b*) *power factor of the combined circuit and*

(*c*) *the resistance and reactance of a series circuit which will take the same current at the same p. f. as the parallel combination.*

Solution.

$$g_1=\frac{R_1}{R_1^2+X_1^2}=\frac{5}{5^2+2^2}=0{\cdot}1724 \text{ mho}$$

$$b_1=-\frac{X_1}{R_1^2+X_1^2}=-\frac{2}{5^2+2^2}=-0{\cdot}06897 \text{ mho}$$

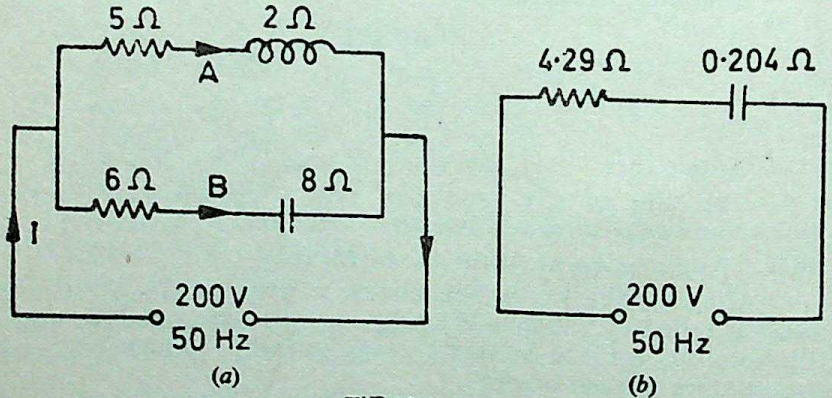

Fig. 18-13

Similarly, $$g_2=\frac{6}{6^2+8^2}=0{\cdot}06 \text{ mho}$$

$$b_2=\frac{8}{6^2+8^2}=0{\cdot}08 \text{ mho}$$

$G=g_1+g_2=0{\cdot}1724+0{\cdot}06=0{\cdot}2324$ mho

$B=-b_1+b_2=-0{\cdot}06897+0{\cdot}08=0{\cdot}01103$ mho (capacitive)

$Y=\sqrt{G^2+B^2}=\sqrt{0{\cdot}2324^2+0{\cdot}01103^2}=0{\cdot}2327$ mho

(*a*) $I=VY=200\times0{\cdot}2327=$ **46·5 A**

(*b*) p.f. $=\cos\phi=G/Y=0{\cdot}2324/0{\cdot}2327=$ **0·9987 lead**

(*c*) Equivalent series resistance is

$$R=G/Y^2=0{\cdot}2324/0{\cdot}2327^2=4{\cdot}29\ \Omega$$

Equivalent series reactance is

$$X=B/Y^2=0{\cdot}01103/0{\cdot}2327^2=0{\cdot}204\ \Omega$$

Since total susceptance is capacitive, this series reactance is capacitive in nature. Hence, the parallel combination of Fig. 18-13(*a*) is equivalent to an *R—C* circuit with $R=$ **4·29 Ω** and $X_C=$ **0·204 Ω.**

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Example 18-6. *The active and lagging reactive components of the current taken by an a.c. circuit from a 250-V supply are 50 A and 25 A respectively. Calculate the conductance, susceptance, admittance and power factor of the circuit. What resistance and reactance would an inductance coil have if it took the same current from mains at the same power factor ?*

Solution. The circuit is shown in Fig. 18-14 (*a*)

Resistance $=250/50=5\ \Omega$; Reactance $=250/25=10\ \Omega$

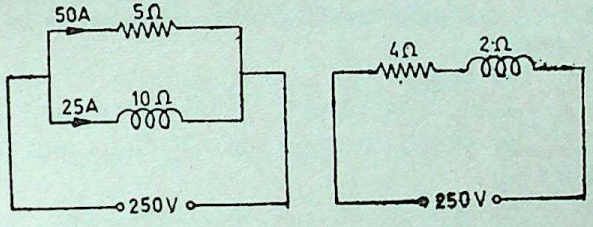
$\therefore$ Conductance $G=1/5=\mathbf{0{\cdot}2}$ ℧

Susceptance $B=-1/10=\mathbf{-0{\cdot}1}$ ℧

Admittance $Y=\sqrt{G^2+B^2}=\sqrt{0{\cdot}2^2+(-0{\cdot}1)^2}$

$=\sqrt{0{\cdot}05}=\mathbf{0{\cdot}224}\ \Omega$

$\cos\phi=G/Y=0{\cdot}2/0{\cdot}224=\mathbf{0{\cdot}894}$ **(lag)**



(*a*) (*b*)

Fig. 18-14

Equivalent resistance of series circuit is [Fig. 18-14 (*b*)]

$$R=Z\cos\phi=\frac{1}{Y}\cdot\frac{G}{Y}=\frac{G}{Y^2}$$

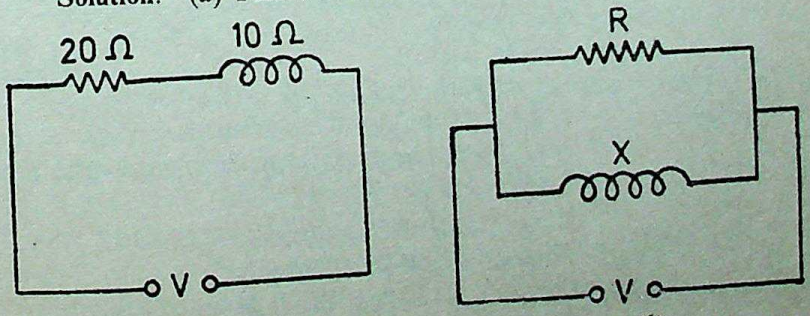
$=0{\cdot}2/0{\cdot}224^2=\mathbf{4}\ \Omega$

$X_L=B/Y^2=0{\cdot}1/0{\cdot}224^2=\mathbf{2}\ \Omega$

Example 18-7. (*a*) *Define conductance, susceptance and admittance with reference to alternating current circuit.*

(*b*) *Calculate the values of conductance and susceptance which when connected in parallel to each other will be equivalent to a circuit comprising of a resistance of 20 Ω in series with a reactance of 10 Ω.*

Solution. (*a*) Please refer to Art. 18-3.



(*a*) (*b*)

Fig. 18-15

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(*b*) Conductance of series circuit is

$$G = R/Z^2 = 20/(20^2+10^2) = 0{\cdot}04 \text{ mho}$$

Susceptance of series circuit is

$$B = -X/Z^2 = -10/(20^2+10^2) = -0{\cdot}02 \text{ mho}$$

Since parallel circuit has to have the same conductance and susceptance, the elements of the parallel circuit are :

$$R = 1/G = 1/0{\cdot}04 = \mathbf{25}\ \Omega\ ; \qquad X = 1/B = 1/0{\cdot}02 = \mathbf{50}\ \text{℧}$$

Hence, the series circuit of Fig. 18-15 (*a*) is equivalent to the parallel circuit of Fig. 18-15 (*b*).

Example 18-8. *A circuit having a resistance of 6 Ω and an inductive reactance of 8 Ω is connected across 200-V, 50-Hz mains in parallel with another circuit having a resistance of 8 Ω and a capacitive reactance of 6 Ω. Calculate* (*a*) *the admittance, conductance and susceptance of the combined circuit* (*b*) *the total current taken from the mains and its power factor. Find also* (*c*) *the value of the reactance that must be connected in parallel with this combination to raise the resultant p. f. to unity.*

Solution. **Branch A** :

$$R = 6\ \Omega,\ X_L = 8\ \Omega \qquad \therefore\ Z_1^2 = 8^2+6^2 = 100$$

$$\therefore\ g_1 = 6/100 = 0{\cdot}06\ \text{℧}\ ;\ b_1 = -8/100 = -0{\cdot}08\ \text{℧}$$

Branch B :

$$R_2 = 8\ \Omega,\ X_C = 6\ \Omega\ ;\ Z_2^2 = 8^2+6^2 = 100$$

$$g_2 = 8/100 = 0{\cdot}08\ \text{℧}\ ;\ b_2 = 6/100 = 0{\cdot}06\ \text{℧}$$

(*a*) $G = g_1+g_2 = 0{\cdot}06+0{\cdot}08 = \mathbf{0{\cdot}14}\ \text{℧}$

$B = -b_1+b_2 = -0{\cdot}08+0{\cdot}06 = \mathbf{-0{\cdot}02}\ \text{℧}$ (inductive)

$Y = \sqrt{[0{\cdot}14^2+(-0{\cdot}02)^2]} = \mathbf{0{\cdot}1414}\ \text{℧}$

(*b*) $I = VY = 200\times 0{\cdot}1414 = \mathbf{28{\cdot}28}\ \text{A}$

$$\text{p.f.} = \cos\phi = G/Y = 0{\cdot}14/0{\cdot}1414 = \mathbf{0{\cdot}99\ lag}$$

(*c*) The circuit possesses a (negative) inductive susceptance of 0·02 mho. Hence, to neutralize this we need a (positive) capacitive susceptance of an equal value. So the value of the capacitive reactance required in parallel is

$$1/X_C = 0{\cdot}02 \quad \text{or} \quad X_C = 1/0{\cdot}02 = \mathbf{50}\ \Omega$$

Example 18-9. *Applying admittance method to the series-parallel circuit shown in Fig. 18-16, find*

(*i*) *total impedance in ohms*

(*ii*) *total supply current and*

(*iii*) *power factor of the entire circuit*

Solution.

$$Z_1^2 = 3^2+4^2 = 25$$

$$g_1 = 3/25 = 0{\cdot}12 \text{ mho};\ b_1 = -4/25 = -0{\cdot}16 \text{ mho}$$

$$Z_2^2 = 4^2+3^2 = 25$$

$$g_2 = 4/25 = 0{\cdot}16 \text{ mho}\ ;\ b_2 = 3/25 = 0{\cdot}12 \text{ mho}$$

$$G_{ab} = 0{\cdot}12+0\ 16 = 0{\cdot}28 \text{ mho}$$

$$B_{ab} = -0{\cdot}16+0{\cdot}12 = -0{\cdot}04 \text{ mho (inductive)}$$

$$Y_{ab} = \sqrt{0{\cdot}28^2+(-0{\cdot}04)^2} = \sqrt{0{\cdot}08} = \mathbf{0{\cdot}283}\ \text{mho}$$

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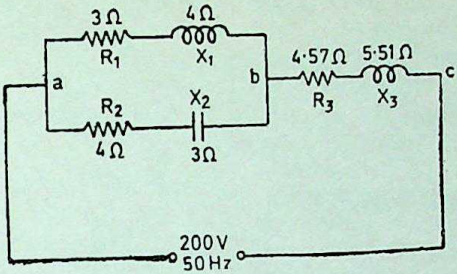

Fig. 18-16

The parallel circuit may now be reduced to an equivalent series circuit as shown in Fig. 18-17.

$$Z_{ab}=1/Y_{ab}=1/0{\cdot}283=3{\cdot}5 \text{ ohm}$$
$$R_{ab}=G_{ab}\,.\,Z_{ab}^2=0{\cdot}283\times 3{\cdot}5^2=3{\cdot}43\ \Omega$$
$$X_{ab}=B_{ab}\,.\,Z_{ab}^2=0{\cdot}04\times 3{\cdot}5^2=0{\cdot}49\ \Omega$$

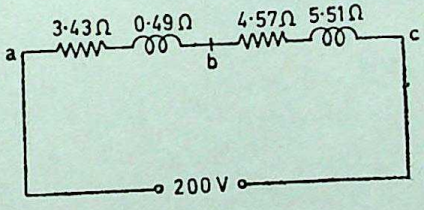

Fig. 18-17

As seen from Fig. 18-17

$$R_{ac}=3{\cdot}43+4{\cdot}57=8\ \Omega$$
$$X_{ac}=0{\cdot}49+5{\cdot}51=6\ \Omega$$

(*i*) $Z_{ac}=\sqrt{8^2+6^2}=\mathbf{10}\ \Omega$

(*ii*) $I_{ac}=200/10=\mathbf{20}\ \mathbf{A}$

(*iii*) p.f.$=\cos\phi=R_{ac}/Z_{ac}=8/10=\mathbf{0{\cdot}8}$ **(lag)**

Example 18-10. (*a*) *A coil which has 6 Ω resistance and 25·5 mH inductance is energised from a 400-V, 50-Hz supply. Calculate the current.*

(*b*) *A capacitor is then connected in parallel with the coil so that the overall p.f. is unity. Calculate the capacitance of the capacitor.*

(*c*) *What current will be flowing in the mains supply cable when the capacitor is connected in the circuit and what is capacitor current ?*

Solution. The circuit is shown in Fig. 18-18

(*a*) $X_L=2\pi\times 50\times 25{\cdot}5\times 10^{-3}=8\ \Omega$

$Z=\sqrt{6^2+8^2}=10\ \Omega$

$I_L=400/10=\mathbf{40}\ \mathbf{A}$; $\phi_L=\cos^{-1}(6/10)=53{\cdot}1°$

As shown in phasor diagram of Fig. 18-19, the coil current ags the voltage by 53·1°.

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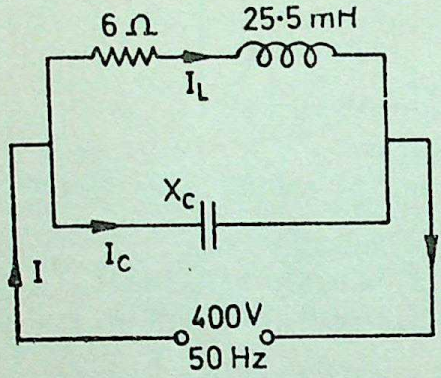
(*b*) If the combined p.f. is to be unity, then current I_C drawn by the capacitor should be such that when vectorially combined with I_L, it should give the resultant current I in phase with V. As seen from Fig. 18-19 in that case

$$I_C = I_L \sin \phi_L = 40 \times \sin 53{\cdot}1° = 40 \times 0{\cdot}8 = 32 \text{ A}$$



Flg. 18-18

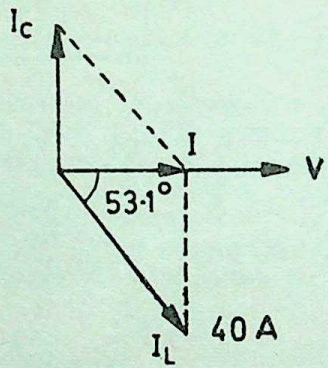


Fig. 18-19

Now $$I_C = \frac{V}{X_C} = \frac{V}{1/\omega C} = V\omega C$$

$\therefore$ $$32 = 400 \times 2\pi \times 50 \times C$$

$\therefore$ $$C = 255 \ \mu\text{F}$$

(*c*) When the capacitor is connected to the circuit, the current in the mains supply cable is

$$I = I_L \cos \phi_L = 40 \times 0{\cdot}6 = \mathbf{24 \text{ A}}$$

As found in (*b*) above, $I_C = \mathbf{32 \text{ A}}$

Note : If required, the kVA rating of the capacitor may be found. It is $= 32 \times 400 = 12{,}800$ VA $= 12{\cdot}8$ kVA. This kVA is reactive, since capacitor does not absorb any power.

Example 18-11. *With the use of a capacitor in parallel with an induction motor, it is desired to raise the power factor to 0·9. The induction motor is a 240-V single-phase motor and takes 20 A at 0·75 lagging power factor. Calculate the capacitance of the capacitor to be used.*

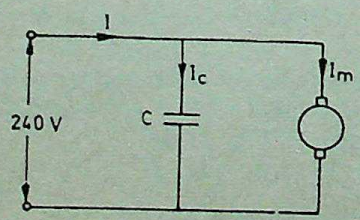


Fig. 18-20

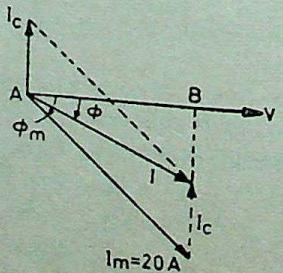


Fig. 18-21

Solution. The current taken by the motor is $I_m = 20$ A and

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lags behind the applied voltage (as shown in Fig. 18-21) by an angle ϕ_m such that $\phi_m = \cos^{-1}(0{\cdot}75) = 41° \; 24'$

When a capacitor is connected in parallel with the motor as shown, then the current I_C drawn by it should be such that when combined with the motor current I_m, it should produce a resultant current I which lags behind the applied voltage by an angle

$$\phi = \cos^{-1}(0{\cdot}9) = 25° \; 48'$$

It is obvious from the vector diagram of Fig. 18-21 that

$$I \cos \phi = I_m \cos \phi_m \qquad (\text{each} = AB)$$

or $$I \times 0{\cdot}9 = 20 \times 0{\cdot}75 \qquad \therefore \quad I = 16{\cdot}7 \text{ A}$$

Also $$I_m \sin \phi_m - I \sin \phi = I_C \qquad (\because \quad ID = BD - BI)$$

$$20 \sin 41° \; 24' - 16{\cdot}7 \sin 25° \; 48' = I_C$$

$$I_C = 13{\cdot}25 - 7{\cdot}27 = 5{\cdot}98 \text{ A}$$

Now $$I_C = \frac{V}{1/\omega C} = V\omega C = 2\pi fCV$$

Assuming a frequency of 50 Hz, we have

$$5{\cdot}98 = 2\pi \times 50 \times 240 \times C \qquad \therefore \quad C = 79{\cdot}3 \; \mu\text{F}$$

Tutorial Problems—29

1. A circuit consisting of a resistance of 30 Ω in series with an inductance of 75 mH is connected in parallel with a circuit consisting of resistance of 20 Ω in series with a capacitance of 100 μF. If the parallel combination is connected to a 240-V, 50-Hz single-phase supply, calculate (*a*) current in each branch (*b*) the total current and p.f. and (*c*) power consumed.

[(*a*) 6·28 A ; 6·38 A (*b*) 8·46 A ; 0·985 lead (*c*) 2 kW]

2. A circuit consists of three branches connected in parallel across a 100-V, 50-Hz supply. Branch (*i*) is a 200-Ω resistor. Branch (*ii*) is a 50 Ω resistor in series with a 30 μF capacitor while Branch (*iii*) is an inductor of 100 Ω resistance and 0·5 H inductance.

Calculate branch currents and total current in magnitude and phase.

[0·5 A ; 0·855 A ; 0·535 A ; 1·21 A ; 13·5° leading]

3. During a test on a circuit possessing resistance and inductance in series, the following figures were obtained :

Power 1·5 kW ; voltage 200 V ; frequency 50 Hz ; current 10 A. Find the resistance and inductance of the circuit.

If a capacitor of 105 μF capacitance is connected in parallel with this circuit, what is the p.f. of the new load ? **[15 Ω ; 42 mH ; unity]**

4. An inductive circuit in parallel with a non-inductive resistance of 20 Ω is connected across a 50-Hz supply. The currents through the inductive circuit and the non-inductive resistance are respectively 4·3 A and 2·7 A and the current taken from the supply is 5·8 A. Find :

(*a*) the power absorbed by the inductive circuit

(*b*) its inductance and

(*c*) the p.f. of the combined circuit.

[(*a*) 72·8 W (*b*) 0·038 H (*c*) 0·72]

5. A circuit consists of two branches, *A* and *B*, connected in parallel. Branch *A* is a coil having a resistance of 20 Ω and an inductance of 0·12 H. Branch *B* consists of a non-inductive resistor of 50 Ω. Determine the kW, kVA and kVAR taken by this circuit from a 230-V, 50-Hz supply.

[1·64 kW, 1·97 kVA ; 1·09 kVAR]

6. A 230-V, single-phase circuit takes 20 A at 0·7 power factor lagging. Determine the capacitance of a capacitor to be connected in parallel with the circuit in order to raise the overall p.f. to 0·95 lagging. **[134 μF]**

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7. A mercury-vapour lamp unit is connected to a 240-V, 50-Hz, single-phase supply. The unit consists of the lamp in series with an inductor of negligible resistance and having a capacitor connected across this series combination for the purpose of power factor correction. If the lamp itself is purely resistive and under normal operating conditions has a voltage across it of 175 V and a current of 2 A flowing through it, calculate :

(*a*) the inductance of the inductor

(*b*) capacitor value needed to make the supply power factor unity

(*c*) the power taken from the suppy.

[(*a*) 0·262 (*b*) 18·3 μF (*c*) 350 W]

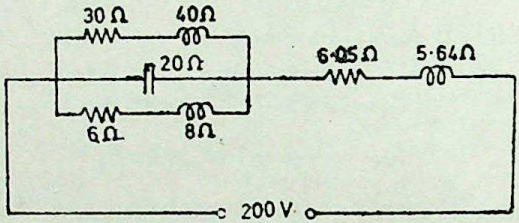

Fig. 18-22

8. Applying admittance method to the circuit shown in Fig. 18-22, find

(*i*) total circuit impedance

(*ii*) total current taken from the mains

(*iii*) p.f. of the combined circuit

[(*i*) 20 Ω (*ii*) 10 A (*iii*) 0·8 (lag)]

18-5. Resonance in Parallel Circuits

We will consider the practical case of a coil in parallel with a capacitor as shown in Fig. 18-23. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as resonant frequency.

The vector diagram for the circuit is shown in Fig. 18-24.

Net reactive or wattless component

$$= I_C - I_L \sin \phi$$

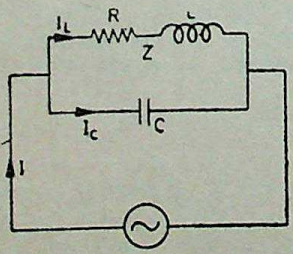

Fig. 18-23

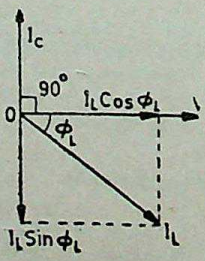

Fig. 18-24

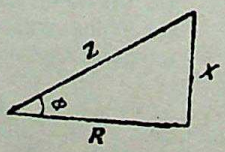

Fig. 18-25

As at resonance its value is zero,

$$\therefore \quad I_C - I_L \sin \phi = 0 \quad \text{or} \quad I_L \sin \phi = I_C$$

Now $I_L = V/Z$; $\sin \phi = X_L/Z$; $I_C = V/X_C$

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Hence, condition for resonance becomes

$$\frac{V}{Z} \times \frac{X_L}{Z} = \frac{V}{X_C} \quad \text{or} \quad X_L \times X_C = Z^2$$

Now, $X_L = \omega L, \; X_C = \frac{1}{\omega C}$

$$\therefore \quad \frac{\omega L}{\omega C} = Z^2 \quad \text{or} \quad \frac{L}{C} = Z^2 \qquad \ldots(i)$$

or $$\frac{L}{C} = R^2 + X_L^2$$

or $$\frac{L}{C} = R^2 + (2\pi f_0 L)^2 \quad \text{or} \quad (2\pi f_0 L)^2 = \frac{L}{C} - R^2$$

or $$2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{or} \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

This is the resonant frequency and is given in Hz if R is ohms, L in henrys and C in farads. If R is negligible, then

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}} \qquad \ldots\text{same as for series resonance}$$

Current at Resonance

As shown in Fig. 18-24 because the wattless component of the line current is zero, the circuit current is minimum and is

$$I_{min} = I_L \cos\phi = \frac{V}{Z} \cdot \frac{R}{Z}$$

or $$I_{min} = \frac{VR}{Z^2}$$

Putting the value of $Z^2 = L/C$ from (i) above, we get

$$I_{min} = \frac{VR}{L/C} = \frac{V}{L/CR}$$

The denominator 'L/CR' is known as the *equivalent* or *dynamic impedance* of the parallel circuit at resonance. It should be noted that this impedance is resistive only. Since current is minimum at resonance, L/CR must then represent the maximum impedance of the circuit.

Current at resonance is minimum, hence such a circuit (when used in radio work) is sometimes known as *rejector* circuit because it rejects (or takes minimum current of) that frequency to which it resonates. This resonance is often referred to as 'current resonance' also because the current circulating *between* the two branches is many times greater than the line current taken from the supply.

The phenomenon of parallel resonance is of great practical importance because it forms the basis of tuned circuits in electronics.

18-6. Graphic Representation of Parallel Resonance

We will now discuss the effect of variation of frequency on the susceptances of the two parallel branches. The variations are shown in Fig. 18-26.

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(*i*) **Inductive Susceptance.** $b = 1/X_L = 1/\omega L = 1/2\pi fL$

It is inversely proportional to the frequency of the applied voltage. Hence, it is repesented by a rectangular hyperbola drawn in the fourth quadrant ($\because$ it is assumed negative).

(*ii*) **Capacitive Susceptance** $b = 1/X_C = \omega C = 2\pi fC$

It increases with the increase in frequency of the applied voltage. Hence, it is represented by a straight line drawn in the first quadrant ($\because$ it is assumed positive)

(*iii*) **Net Susceptance B**

It is the difference of the two susceptances and is represented by the dotted hyperbola. At point *A*, net susceptance is zero.

(*iv*) **Admittance**

Variations of admittance (Y) are as shown in Fig. 18-26. It has large values for frequencies lower as well as higher than the resonant frequency f_0. For $f < f_0$, Y is inductive whereas for $f > f_0$, it is capacitive in nature. It has minimum value of $Y = G$ at $f = f_0$. Its value would have been zero if R had been zero (because in that case $G = 0$)

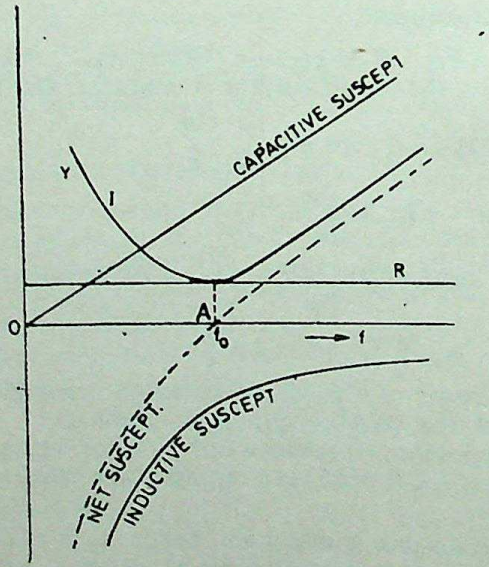

Fig. 18-26

(*v*) **Current**

It varies in exactly the same manner as Y. It has minimum value at point *A* (where $f = f_0$) and equals $VY = VG$. Its value would have been zero if R had been equal to zero.

Obviously, below resonant frequency (corresponding to point *A*), inductive susceptance predominates, hence line current lags behind the applied voltage. But for frequencies above the resonant frequency, capacitive susceptance predominates, hence line current leads.

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18-7. Points to Remember

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Following points about parallel resonance should be noted and compared with those about series resonance. For parallel resonance :

(*i*) net susceptance is zero *i.e.* $1/X_C = X_L/Z^2$ or $X_L \times X_C = Z^2$ or $Z^2 = L/C$.

(*ii*) reactive or wattless component of line current is zero.

(*iii*) dynamic impedance $= L/CR\ \Omega$

(*iv*) line current at resonance is minimum and equals $\dfrac{V}{L/CR}$ and is in phase with the applied voltage.

(*v*) power factor of the circuit is unity.

Comparison of Series and Parallel Resonant Circuits

Item	*Series circuit*	*Parallel circuit*
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum $= V/R$	Minimum $= V/(L/CR)$
Effective impedance	R	L/CR
Power factor at resonance	Unity	Unity
Resonant frequency	$1/2\pi\sqrt{(LC)}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$
It magnifies	Voltage	Current
Magnification is	$\omega L/R$	$\omega L/R$

18-8. Q factor of a Parallel Circuit

It is defined as '*the ratio of the circulating current to the line current*' or as '*the current magnification*'.

Let the circulating current between capacitor and coil be I_C.

Then Q-factor $= I_C/I$

Now $I_C = V/X_C = V/(1/\omega C) = V\omega C$

$$I = V/(L/CR)$$

$$Q\text{-factor} = V\omega C \div \frac{V}{L/CR} = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R}$$

(same as for series circuit)

Now, resonant frequency when R is negligible is, $f_0 = \dfrac{1}{2\pi\sqrt{(LC)}}$

Putting this value above, we get

$$Q\text{-factor} = \frac{2\pi L}{R} \quad \frac{1}{2\pi\sqrt{(LC)}} = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$$

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It should be noted that, in series circuits, Q-factor gives the voltage magnification, whereas in parallel circuits, it gives the current magnification.

Example 18-12. *Explain the meaning of resonance in a parallel circuit. Show that in a parallel circuit, the resistance of the inductor affects the frequency of resonance. Is this true in a series resonant circuit ?*

A resonant circuit consists of a 4 μF capacitor in parallel with an inductor of 0·25 H having a resistance of 50 Ω. Calculate the frequency of resonance.

Solution. The resonance frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It is seen that f_0 is dependent on R. When R is large, f_0 is reduced. In a series circuit $f_0 = 1/2\pi\sqrt{LC}$. Obviously, f_0 does not depend on R.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{10^6}{4\times 0{\cdot}25} - \frac{50^2}{0{\cdot}25^2}} = \mathbf{156\ Hz}$$

Example 18-13. *A coil of resistance 30 Ω and inductance 20 mH is connected in parallel with a variable capacitor across a supply of 25 V and frequency 1000/π Hz. The capacitance of the capacitor is then varied until the current taken from the supply is a minimum (i.e. until the overall p.f. of the circuit is unity). For this condition, find.*

(*a*) *the capacitance of the circuit*

(*b*) *the value of the current.*

Solution. $X_L = 2\pi f L = 2\pi \times (1000/\pi) \times 20 \times 10^{-3} = 40\ \Omega$

$$Z = \sqrt{30^2 + 40^2} = 50\ \Omega$$

(*a*) At resonance, $Z^2 = L/C$

$$\therefore \quad C = L/Z^2 = 20 \times 10^{-3}/50^2 = 8 \times 10^{-6}\ \text{F} = \mathbf{8\ \mu F}$$

(*b*) $$I_{min} = \frac{V}{L/CR}$$

Dynamic impedance $= 20 \times 10^{-3}/8 \times 10^{-6} \times 30 = 83{\cdot}3\ \Omega$

$$\therefore \quad I_{min} = 25/80{\cdot}3 = 0{\cdot}3\ \text{A}$$

Example 18-14. *A series combination of a capacitance C and resistance R is shunted by an inductive coil having resistance R and inductance L and the combination is connected to an a.c. source. Show that impedance of the circuit will be independent of frequency if* $R = \sqrt{L/C}$. *(A.M.I.E. Winter 1983)*

Solution. The impedance of the circuit of Fig. 18-27 is

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(R + j\omega L)(R - j/\omega C)}{(R + j\omega L) + (R - j/\omega C)}$$

$$\frac{R^2 + (L/C) - jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}$$

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Putting $R = \sqrt{L/C}$

or $R^2 = L/C$, we get

$$Z = \frac{R^2 + R^2 + jR(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}$$

$$= R\left[\frac{2R + j(\omega L - 1/\omega C)}{2R + j(\omega L - 1/\omega C)}\right]$$

or $Z = R$

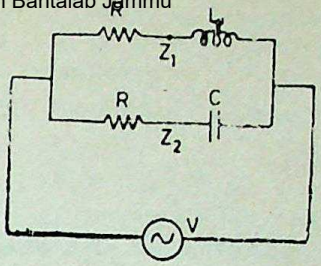

Fig. 18-27

Tutorial Problems—30

1. A circuit consisting of a coil of inductance 10 mH and resistance 300 Ω connected in parallel with a certain capacitor resonates at 48 kHz.

Determine the size of this capacitor and the dynamic impedance of the circuit. **[1090 μF ; 30·6 kΩ]**

2. A coil of inductance 31·8 mH and resistance of 10 Ω is connected in parallel with a capacitor across a 250-V. 50-Hz supply. Determine the value of the capacitance if no reactive current is taken from the supply. **[159 μF]**

3. A series circuit, consisting of a coil (R=30 Ω, L=0·5 H) and a capacitor resonates at a frequency of 48 Hz. Calculate the capacitance of a capacitor which, when connected in parallel with this circuit, will increase the resonant frequency to 60 Hz. Calculate in the latter case the total current and the current in each branch of the circuit if the line voltage is 100 V.

[32·8 μF ; 0·546 A ; 1·35 A ; 1·235 A]

4. A 100-V, 80-W lamp is to be operated on 230-V, 50-Hz a.c. supply. Calculate the inductance of the choke required to be connected in series with the lamp for this operation. The lamp can be taken as equivalent to a non-inductive resistance. If the p.f. of the lamp circuit is to be improved to unity, calculate the value of the capacitor which is to be connected across the circuit.

[L=0·823 H ; 10 μF]

5. A coil of resistance 10 Ω and inductance 0·1 H is connected in parallel with a capacitor of 200 μF.

Calculate the frequency at which the circuit behaves as non-inductive resistance and the current taken if the supply voltage is 250 V.

[31·83 Hz ; 5 A]

HIGHLIGHTS

1. Admittance of a circuit is defined as the reciprocal of its impedance. Its unit is mho.

$$Y = \sqrt{G^2 + B^2}$$

2. Admittance has two rectangular components :

(i) conductance (G) and (ii) susceptance (B).

3. $G = R/Z^2$; $B = X/Z^2$

4. Equivalent resistance and reactance of a series cir... given by

$$R = G/Y^2 \text{ and } X = B/Y^2$$

5. A parallel a.c. circuit is said to be in resonance whe... it does not draw any reactive current. Resonant frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

6. Condition for parallel resonance is

$$Z^2 = L/C$$

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COMPLEX ALGEBRA AND A.C. CIRCUITS 19

1. Mathematical Representation of Vectors 2. Symbolic Notation 2. Significance of Operator J 4. Conjugate Complex Numbers 5. Trigonometrical Form of Vector Representation 6. Exponential Form of Vector Representation. 7. Polar Form of Vector Representation 8. Addition and Subtraction of Complex Quantities 9. Multiplication and Division of Complex Quantities 10. Powers and Roots of Vectors 11. Complex Algebra Applied to Series Circuits 12. Applied to Parallel Circuits 13. Series Parallel Circuits 14. Highlights.

19-1. Mathematical Representation of Vectors

There are various forms or methods of representing vector quantities all of which enable those operations which are carried on graphically in a vector diagram, to be performed analytically. The various methods are :

(*i*) *Symbolic Notation*—according to which a vector quantity is expressed algebraically in terms of its rectangular components. Hence, this form of representation is also known as rectangular or Cartesian form of notation or representation.

(*ii*) *Trigonometrical Form*

(*iii*) *Exponential Form*

(*iv*) *Polar Form*

19-2. Symbolic Notation

A vector can be specified in terms of its X-component and Y-component. For example, the vector OV_1 (Fig. 19-1) may be completely described by stating that its horizontal component is a_1 and vertical component is b_1. But, instead of stating this verbally, we may express symbolically

$$V_1 = a_1 + jb_1$$

where symbol j, known as an operator, indicates that component b_1 is perpendicular to component a_1 and that the two terms are *not* to be treated like terms in any algebraic expression. The vector written in this way is said to be written in *complex form*. In mathe-

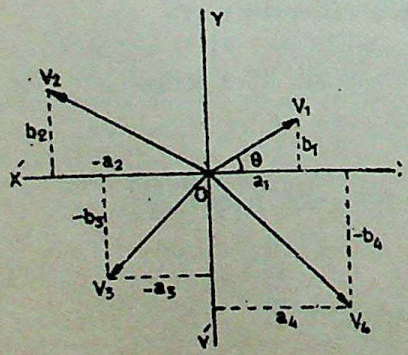

Fig. 19-1

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matics, a_1 is known as real component and b_1 as imaginary component but in electrical engineering these are known as *in-phase* (or active) and *quadrature* (or reactive) components respectively.

The other vectors OV_2, OV_3 and OV_4 can similarly be expressed in the form :

$$\mathbf{V}_2 = -a_2 + jb_2 \; ; \; \mathbf{V}_3 = -a_3 - jb_3 \; ; \; \mathbf{V}_4 = +a_4 - jb_4$$

It should be noted that in this book, a vector quantity would be represented by letters in heavy type and its numerical or scalar value by the same letter in ordinary type.* Other methods adopted for indicating a vector quantity are to put a dot above or below the letter as V or V

The numerical value of vector $\mathbf{V}_1$ is $\sqrt{a_1^2 + b_1^2}$

Its angle with X-axis is given by $\phi = \tan^{-1}(b_1/a_1)$

19-3. Significance of Operator j

The letter j used in the above expression is a symbol of an operation. Just as symbols $\times$, $+$, $\surd$, $\int$ etc. are used with numbers, for indicating certain operations to be performed on those numbers, similarly symbol j is used to indicate the counter clockwise rotation of a vector through 90°. It is assigned a value of $\sqrt{(-1)}$†. The double operation of j on a vector rotates it counter-clockwise through 180° and hence reverses its sense because

$$jj = j^2 = [\sqrt{(-1)}]^2 = -1$$

When operator j is operated on vector V, we get the new vector jV which is displaced by 90° in counter-clockwise direction from V (Fig. 19-2). Further application of j will give $j^2\mathbf{V} = -\mathbf{V}$ as shown.

If the operator j is applied to the vector j^2V, the result is $j^3\mathbf{V} = -j\mathbf{V}$. The vector j^3V is 270° counter clockwise from the reference axis and is directly opposite to jV. If the vector j^3V is, in turn, operated on by j, the result will be

$$j^4\mathbf{V} = [\sqrt{(-1)}]^4 \; \mathbf{V} = \mathbf{V}$$

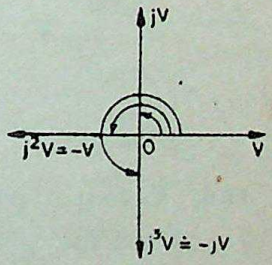

Fig. 19-2

Hence, it is seen that successive applications of the operator j to the vector V produce successive 90° steps of rotation of the vector in the counter-clockwise direction without in any way affecting the magnitude of the vector.

It will also be seen from Fig. 19-2 that the application of $-j$ to V yields $-j$V which is a vector of identical magnitude but rotated 90° *clockwise* from V .

---

*The magnitude of a vector is sometimes called 'modulus' and is represented by $|V|$ or V.

†In mathematics, $\sqrt{(-1)}$ is denoted by i but in electrical engineering j is adopted because letter i is reserved for representing current. This helps to avoid confusion.

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Summarising the above, we have

$j = 90°$ CCW rotation $= \sqrt{(-1)}$

$j^2 = 180°$ CCW rotation $= [\sqrt{(-1)}]^2 = -1$

$j^3 = 270°$ CCW rotation $= [\sqrt{(-1)}]^3 = -\sqrt{(-1)} = -j$

$j^4 = 360°$ CCW rotation $= [\sqrt{(-1)}]^4 = +1$

$j^5 = 450°$ CCW rotation $= [\sqrt{(-1)}]^5 = \sqrt{(-1)} = j$

It should also be noted that

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j.$$

19-4. Conjugate Complex Numbers

Two complex numbers are said to be conjugate if they differ only in the algebraic sign of their quadrature components. Accordingly, the numbers $(a+jb)$ and $(a-jb)$ are conjugate. The sum of two conjugate components gives in phase (or active) component and their difference gives quadrature (or reactive) component.

19-5 Trigonometrical Form of Vector Representation

From Fig. 19-3, it is seen that the X-component of V is $V \cos \theta$ and Y-component is $V \sin \theta$. Hence, we can represent the vector V in the form

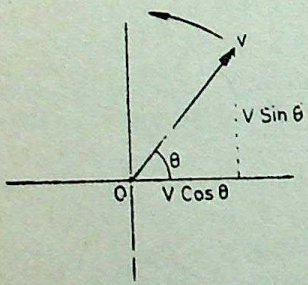

Fig. 19-3

$$\mathbf{V} = V \cos \theta + j V \sin \theta$$
$$= V (\cos \theta + j \sin \theta)$$

This is equivalent to the rectangular form $\mathbf{V} = a + jb$ because $a = V \cos \theta$ and $b = V \sin \theta$.

In general

$$\mathbf{V} = V (\cos \theta \pm j \sin \theta)$$

19-6. Exponential Form of Vector Representation

It can be proved that

$$e^{\pm j\theta} = (\cos \theta \pm j \sin \theta)$$

This equation is known as Euler's equation after the famous mathematician of 18th century, Leonard Euler.

This equation follows directly from an inspection of Maclaurin's* series expansions of $\sin \theta$, $\cos \theta$ and $e^{j\theta}$

*Functions like $\cos \theta$, $\sin \theta$ and $e^{j\theta}$ etc. can be expanded into series form with the help of Maclaurin's Theorem. The theorem states :

$$f(\theta) = f(0) + \frac{f'(0)\theta}{1} + \frac{f''(0)\theta^2}{\angle 2} + \frac{f'''(0)\theta^3}{\angle 3} + \cdots$$

where $f(\theta)$ is the function of θ which is to be expanded, $f(0)$ is the value of the function when $\theta = 0$, $f'(0)$ is the value of first derivative of $f(\theta)$ when $\theta = 0$; $f''(0)$ is the value of second derivative of the function $f(\theta)$ when $\theta = 0$ etc.

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When expanded into series form :

$$\cos\theta = 1 - \frac{\theta^2}{\angle 2} + \frac{\theta^4}{\angle 4} - \frac{\theta^6}{\angle 6} + \cdots$$

$$\sin\theta = \theta - \frac{\theta^3}{\angle 3} + \frac{\theta^5}{\angle 5} - \frac{\theta^7}{\angle 7} + \cdots$$

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{\angle 2} + \frac{(j\theta)^3}{\angle 3} + \frac{(j\theta)^4}{\angle 4} + \frac{(j\theta)^5}{\angle 5} + \frac{(j\theta)^6}{\angle 6} + \cdots$$

Keeping in mind that

$$j^2 = -1,\ j^3 = -j,\ j^4 = 1,\ j^5 = j,\ j^6 = -1$$

we get

$$e^{j\theta} = \left(1 - \frac{\theta^2}{\angle 2} + \frac{\theta^4}{\angle 4} - \frac{\theta^6}{\angle 6} + \cdots\right) + j\left(\theta - \frac{\theta^3}{\angle 3} + \frac{\theta^5}{\angle 5} - \frac{\theta^7}{\angle 7} + \cdots\cdots\right)$$

$$\therefore \quad e^{j\theta} = \cos\theta + j\sin\theta$$

Similarly, it can be shown that

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

Hence $\mathbf{V} = V(\cos\theta \pm j\sin\theta)$ or $\mathbf{V} = Ve^{\pm j\theta}$

This is known as exponential form of representing vector quantities. It represents a vector of numerical value V and having a phase angle of $\pm\theta$ with the reference axis.

19-7. Polar Form of Representation

The expression $V(\cos\theta + j\sin\theta)$ is written in the simplified form of $V \angle\theta$. In this expression, V represents the magnitude of the vector and θ its inclination (in *CCW* direction) with the X-axis. For angles in *CW* direction, the expression becomes $V \angle -\theta$. In general, the expression is written as $V \angle \pm\theta$. *It may be pointed out here that* $V \angle \pm\theta$ *is simply a short-hand or symbolic style of writing* $Ve^{\pm j\theta}$. Also, this form is purely conventional and does not possess the mathematical significance of the other various forms of vector representation given above.

Summarizing, we have the following alternate ways of representing vector quantities :

(*i*) Rectangular Form (or complex form) $\mathbf{V} = a \pm jb$

(*ii*) Trigonometrical Form $\mathbf{V} = V(\cos\theta \pm j\sin\theta)$

(*iii*) Exponential Form $\mathbf{V} = Ve^{\pm j\theta}$

(*iv*) Polar Form (conventional) $\mathbf{V} = V \angle \pm\theta$

Example 19-1. *Write the equivalent exponential and polar forms of vector* $3 + j4$. *How will you illustrate the vector by means of a vector diagram ?*

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Solution. With reference to Fig. 19-4, magnitude of the vector is

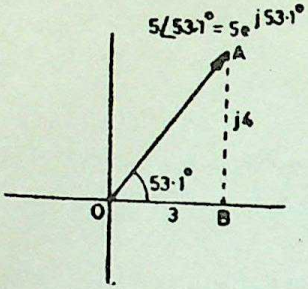

Fig. 19-4

$$= \sqrt{3^2+4^2} = 5$$

$$\tan\theta = 4/3 \; ; \; \theta = \tan^{-1}(4/3)$$

$$= 53{\cdot}1°$$

$\therefore$ Exponential form

$$= 5\, e^{j53{\cdot}1°}$$

The angle may also be expressed in radians.

Polar form $= 5 \angle 53{\cdot}1°$

Example 19-2. *A vector is represented by* $20\, e^{-j2\pi/3}$. *Write the various equivalent forms of the vector and illustrate by means of a vector diagram, the magnitude and position of the above vector.*

Solution. The vector is drawn in a direction making an angle of $2\pi/3 = 120°$ in the clockwise direction (Fig. 19-5). The clockwise direction is taken because the angle is negative.

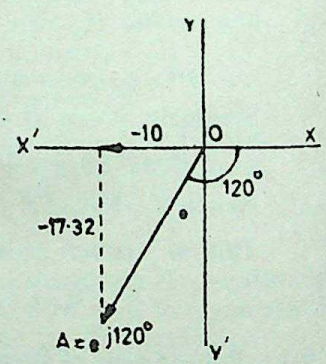

Fig. 19-5

(*i*) **Rectangular form**

$$a = 20 \cos(-120°) = -10$$

$$b = 20 \sin(-120°) = -17{\cdot}32$$

$\therefore$ expression is $= -10 - j17{\cdot}32$

(*ii*) **Polar Form** is $20 \angle -120°$

19-8. Addition and Subtraction of Complex Quantities

Rectangular form is best suited for addition and subtraction of vector quantities.

Suppose we are given two vector quantities

$$\mathbf{V}_1 = a_1 + jb_1 \quad \text{and} \quad \mathbf{V}_2 = a_2 + jb_2$$

and it is required to find their sum and difference.

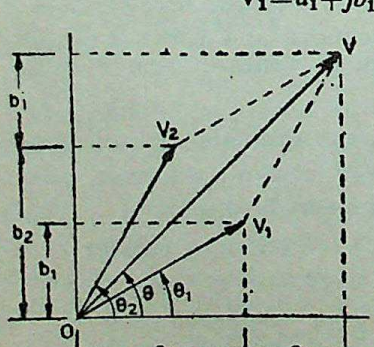

Fig. 19-6

Addition

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = a_1 + jb_1 + a_2 + jb_2$$

$$= (a_1 + a_2) + j\,(b_1 + b_2)$$

The magnitude of resultant vector **V** is

$$= \sqrt{(a_1+a_2)^2 + (b_1+b_2)^2}$$

The position of **V** with respect to *X*-axis is

$$\theta = \tan^{-1}\left(\frac{b_1+b_2}{a_1+a_2}\right)$$

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A graphic representation of the addition process is shown in Fig. 19-6.

Subtraction

$$\mathbf{V}=\mathbf{V}_1-\mathbf{V}_2$$
$$=(a_1+jb_1)-(a_2+jb_2)$$
$$=(a_1-a_2)+j(b_1-b_2)$$

Magnitude of V

$$=\sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$$

Its position with respect to X-axis is given by the angle

$$\theta=\tan^{-1}\left(\frac{b_1-b_2}{a_1-a_2}\right)$$

The graphic representation of the process of subtraction is shown in Fig. 19-7.

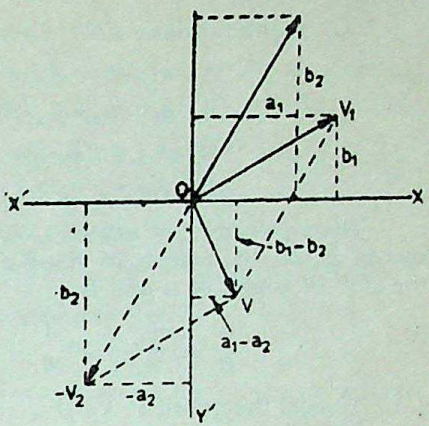

Fig. 19-7

19-9. Multiplication and Division of Complex Quantities

Multiplication and division of vectors becomes very simple and easy *if they are represented in the polar or exponential form.* As will be shown below, the rectangular form of representation is not well suited for this process.

(*i*) **Multiplication**—*Rectangular Form*

Let the two vectors be given by

$$\mathbf{A}=a_1+jb_1 \quad \text{and} \quad \mathbf{B}=a_2+jb_2$$

$$\therefore \quad \mathbf{A}\times\mathbf{B}=\mathbf{C}=(a_1+jb_1)(a_2+jb_2)$$
$$=a_1a_2+j^2b_1b_2+j(a_1b_2+b_1a_2)$$
$$=(a_1a_2-b_1b_2)+j(a_1b_2+b_1a_2) \qquad (\because \quad j^2=-1)$$

The magnitude of $\mathbf{C}=\sqrt{[(a_1a_2-b_1b_2)^2+(a_1b_2+b_1a_2)^2]}$

Its angle with respect to X-axis is given by

$$\theta=\tan^{-1}\left(\frac{a_1b_2+b_1a_2}{a_1a_2-b_1b_2}\right)$$

(*ii*) **Division**—*Rectangular Form*

$$\frac{\mathbf{A}}{\mathbf{B}}=\frac{a_1+jb_1}{a_2+jb_2}=\frac{(a_1+jb_1)(a_2-jb_2)}{(a_2+jb_2)(a_2-jb_2)}$$

Both the numerator and denominator have been multiplied by the conjugate of (a_2+jb_2) *i.e.* by (a_2-jb_2)

$$\therefore \quad \frac{\mathbf{A}}{\mathbf{B}}=\frac{(a_1a_2+b_1b_2)+j(b_1a_2-a_1b_2)}{a_2^2+b_2^2}$$

$$=\frac{a_1a_2+b_1b_2}{a_2^2+b_2^2}+j\ \frac{b_1a_2-a_1b_2}{a_2^2+b_2^2}$$

The magnitude and the angle with respect to X-axis can be found in the same way as given above.

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As will be noted, both results are somewhat awkward, but unfortunately, there is no easier way to perform multiplication in rectangular form.

(*iii*) **Multiplication**—*Polar Form*

Let $\mathbf{A}=a_1+jb_1=A\angle\alpha=Ae^{j\alpha}$ where $\alpha=\tan^{-1}(b_1/a_1)$

$\mathbf{B}=a_2+jb_2=B\angle\beta=Be^{j\beta}$ where $\beta=\tan^{-1}(b_2/a_2)$

$\therefore$ $\mathbf{AB}=A\angle\alpha\times B\angle\beta=AB\angle(\alpha+\beta)$*

or $\mathbf{AB}=Ae^{j\alpha}\times Be^{j\beta}=AB\,e^{j(\alpha+\beta)}$

Hence, product of any two vectors **A** and **B** is given by another vector equal in length to $A\times B$ and having a phase angle equal to the sum of angles of **A** and **B**.

$$\frac{\mathbf{A}}{\mathbf{B}}=\frac{A\angle\alpha}{B\angle\beta}=\frac{A}{B}\angle(\alpha-\beta)$$

Hence, the quotient $\mathbf{A}\div\mathbf{B}$ is another vector having a magnitude of $A\div B$ and having phase angle equal to the angle of **A** minus the angle of **B**.

Also $$\frac{\mathbf{A}}{\mathbf{B}}=\frac{Ae^{j\alpha}}{Be^{j\beta}}=\frac{A}{B}e^{j(\alpha-\beta)}$$

As seen, the division and multiplication become extremely simple if vectors are represented in their polar or exponential form.

Example 19-3. *Add the following vectors given in rectangular form and illustrate the process graphically.*

$\mathbf{A}=16+j\,12$, $\mathbf{B}=-6+j\,10{\cdot}4$

Solution. $\mathbf{A}+\mathbf{B}=\mathbf{C}=(16+j\,12)+(-6+j\,10{\cdot}4)$

$=10+j\,22{\cdot}4$

$\therefore$ Magnitude of $\mathbf{C}=\sqrt{(10^2+22{\cdot}4)^2}$

$=25{\cdot}5$ units

Slope of $\mathbf{C}=\theta=\tan^{-1}(22{\cdot}4/10)$

$=65{\cdot}95°$

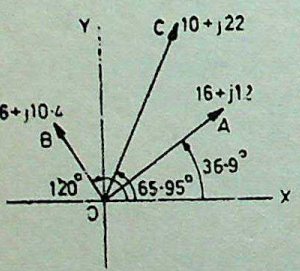

Fig. 19-8

The vector addition is shown in Fig. 19-8

$\alpha=\tan^{-1}(12/16)=36{\cdot}9°$

$\beta=\tan^{-1}(-10{\cdot}4/6)=-240°$ or $120°$

The resultant vector is found by using parallelogram law of vectors (Fig. 19-8)

---

* $\mathbf{A}=A(\cos\alpha+j\sin\alpha)$ and $\mathbf{B}=B(\cos\beta+j\sin\beta)$

$\mathbf{AB}=AB(\cos\alpha\cos\beta+j\sin\alpha\cos\beta+j\cos\alpha\sin\beta+j^2\sin\alpha\sin\beta)$

$=AB[(\cos\alpha\cos\beta-\sin\alpha\sin\beta)+j(\sin\alpha\cos\beta+\cos\alpha\sin\beta)]$

$=AB[\cos(\alpha+\beta)+j\sin(\alpha+\beta)]=AB\angle(\alpha+\beta)$

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Example 19-4. *Subtract the following given vectors from one another.*

$$A = 30 + j\,52 \text{ and } B = -39{\cdot}5 - j\,14{\cdot}36$$

Solution. $\mathbf{A} - \mathbf{B} = \mathbf{C} = (30 + j\,52) - (-39{\cdot}5 - j\,14{\cdot}36)$

$$= 69{\cdot}5 + j\,66{\cdot}36$$

$\therefore$ Magnitude of $\mathbf{C} = \sqrt{(69{\cdot}5^2 + 66{\cdot}36^2)} = 96$

Slope of $\mathbf{C} = \tan^{-1}(66{\cdot}36/69{\cdot}5) = 43{\cdot}6°$

$\therefore$ $\mathbf{C} = 96 \angle 43{\cdot}6°$

Similarly $\mathbf{B} - \mathbf{A} = -69{\cdot}5 - j\,66{\cdot}36$

$= 96 \angle 223{\cdot}6°$ or $= 96 \angle -136{\cdot}4°$

Example 19-5. *Given the following two vectors :*

$$A = 20 \angle 60°, \quad B = 5 \angle 30°$$

Perform the following indicated operations and illustrate graphically (i) $A \times B$ and (ii) A/B.

Solution. (*i*) $\mathbf{A} \times \mathbf{B} = \mathbf{C} = 20 \angle 60° \times 5 \angle 30° = 100 \angle 90°$

Vectors are shown in Fig. 19-9.

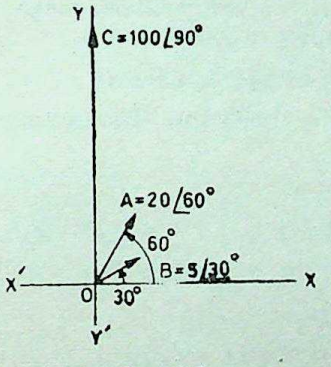

Fig. 19-9

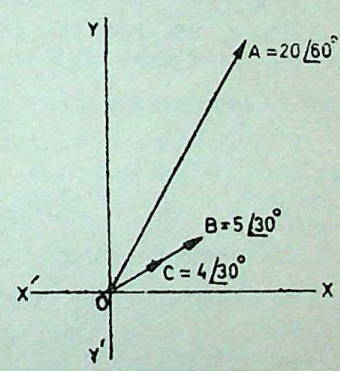

Fig. 19-10

(*ii*) $\dfrac{\mathbf{A}}{\mathbf{B}} = \dfrac{20 \angle 60°}{5 \angle 30°} = 4 \angle 30°$ (Fig. 19-10)

Example 19-6. *The following three vectors are given :*

$$A = 20 + j\,20, \; B = 30 \angle -120° \text{ and } C = 10 + j\,0$$

Perform the following indicated operations :

(*i*) $\dfrac{AB}{C}$ *and* (*ii*) $\dfrac{BC}{A}$

Solution. Rearranging all three vectors in polar form, we get

$$\mathbf{A} = 28{\cdot}3 \angle 45°, \; \mathbf{B} = 30 \angle -120°, \; \mathbf{C} = 10 \angle 0°$$

(*i*) $\dfrac{28{\cdot}3 \angle 45° \times 30 \angle -120°}{10 \angle 0°} = 84{\cdot}9 \angle -75°$

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(*ii*) $\dfrac{30\angle -120^\circ \times 10\angle 0^\circ}{28{\cdot}3\angle 45^\circ} = 10{\cdot}6\angle -165^\circ$

19-10. Powers and Roots of Vectors

(*a*) Powers

Suppose it is required to find the cube of the vector $3\angle 15^\circ$. For this purpose, the vector has to be multiplied by itself three times

$$\therefore \quad (3\angle 15)^3 = 3\times 3\times 3\angle(15^\circ+15^\circ+15^\circ) = 27\angle 45^\circ$$

In general,

$$\mathbf{A}^n = A^n \angle n\alpha$$

Hence, nth power of vector **A** is a vector whose magnitude is A^n and whose phase angle with respect to X-axis is $n\alpha$.

It is also clear that

$$\mathbf{A}^n\mathbf{B}^n = A^nB^n \angle(n\alpha+n\beta)$$

(*b*) Roots

It is clear that $\sqrt[3]{}\,(8\angle 45^\circ) = 2\angle 15^\circ$

In general, $\sqrt[n]{}\,\mathbf{A} = \sqrt[n]{}\,A\angle \alpha/n$

Hence, nth root of a vector **A** is a vector whose magnitude is $\sqrt[n]{}\,A$ and whose phase angle with respect to X-axis is α/n.

19-11. Complex Algebra Applied to Series Circuit

Consider the series circuit of Fig. 19-11 (*a*). The impedance of the circuit may be written as

$$\mathbf{Z} = R + jX_L$$

The numerical value of impedance is $|\,Z\,| = \sqrt{R^2+X_L^2}$

Its phase angle with respect to the reference axis is

$$\phi_L = \tan^{-1}(X_L/R)$$

It may be expressed in polar form as $\mathbf{Z} = Z\angle\phi$

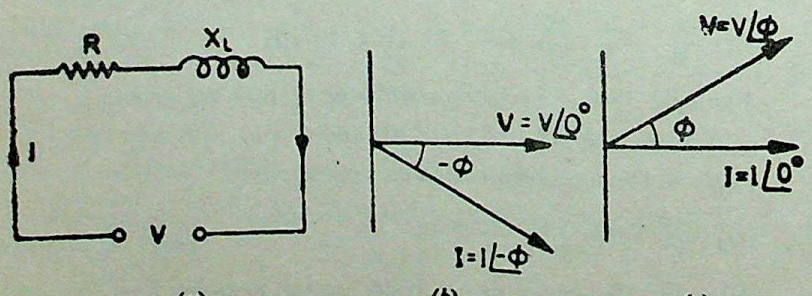

Fig. 19-11

(*i*) If we take voltage vector along the reference axis [Fig. 19-11 (*b*)] then it may be expressed as

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$$\mathbf{V}=V\angle 0^\circ$$

$$\therefore \quad \mathbf{I}=\frac{\mathbf{V}}{\mathbf{Z}}=\frac{V\angle 0^\circ}{Z\angle \phi^\circ}=\frac{V}{Z}\angle -\phi^\circ$$

It shows that current vector lags behind the voltage vector by ϕ° The numerical value of current is $|I|=V/Z$.

(*ii*) However, if current vector is taken along reference axis [Fig. 19-11 (*c*)] then it may be written as

$$\mathbf{I}=I\angle 0^\circ$$

Then $\mathbf{V}=\mathbf{IZ}=I\angle 0^\circ\times Z\angle \phi^\circ=IZ\angle \phi$

It shows that voltage vector is ϕ° ahead of the current vector in the *CCW* direction as shown.

Example 19-7. *In a given R-L circuit, $R=35\ \Omega$ and $L=1H$. Find the current flowing through the series circuit if a 50-Hz voltage $V=110\angle 30^\circ$ volts is applied across the circuit.*

Solution. $R=35\ \Omega$, $X_L=\omega L$

$$X_L=1\times 314=314\ \Omega$$

$$\mathbf{Z}=35+j314$$

$$=\sqrt{(35^2+314^2)}/\tan^{-1}(314/35)$$

$$=315{\cdot}9/83{\cdot}67^\circ$$

$$\therefore \quad \mathbf{I}=\frac{\mathbf{V}}{\mathbf{Z}}=\frac{110\angle 30^\circ}{315{\cdot}9/83{\cdot}67^\circ}$$

$$=0{\cdot}35/-53{\cdot}67^\circ$$

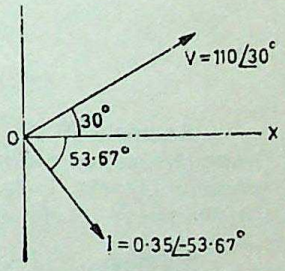

Fig. 19-12

Example 19-8. *An alternating voltage $(8+j6)$ V is applied to a series A.C. circuit and the current flowing is $(=2+j5)$ A. Find (i) the impedance (ii) the power consumed and (iii) the phase angle.*

Solution. $\mathbf{V}=8+j6=10\angle 36{\cdot}9^\circ$

$$\mathbf{I}=-2+j5=5{\cdot}39\angle \tan^{-1}-5/2$$

$$=5{\cdot}39\angle(180^\circ-68{\cdot}2^\circ)=5{\cdot}39\angle 111{\cdot}8^\circ$$

(*i*) $\therefore$ Impedance $\mathbf{Z}=\dfrac{\mathbf{V}}{\mathbf{I}}=\dfrac{10\angle 36{\cdot}9^\circ}{5{\cdot}39\angle 111{\cdot}8^\circ}=1{\cdot}86\angle -75^\circ$

$\therefore$ Impedance $=1{\cdot}86\ \Omega$

(*ii*) The method of conjugate will be used to determine the real power and reactive volt-amperes. This is a convenient way of calculating these quantities when both voltage and current are expressed in Cartesian form. If the *conjugate* of current is multiplied by the voltage in Cartesian form, the result is a complex quantity, the real part of which gives the real power and *j* part of which is the reactive volt-amperes (vars). It should, however, be noted that the real power as obtained by this method of conjugates is the same regardless of whether **V** or **I** is reversed although sign of reactive volt-amperes will depend on the choice of **V** or **I***.

*If voltage conjugate is used, then capacitive VARS are positive and inductive VARS negative. If current conjugate is used, then capacitive VARS are negative and inductive VARS are positive.

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Using the conjugate of current, we get

$$\mathbf{P}_{VA}=(8+j6)(-2-j5)=14-j52$$

$\therefore$ Power consumed = **14 W**

(*iii*) Obviously, phase angle = 111·8° − 36·9° = **75°** with current leading.

Example 19-9. *If the potential difference across a circuit is represented by (40+j25) volts and the circuit consists of a resistance of 20 Ω in series with an inductance of 0·06 henry and the frequency is 79·5 Hz, find the complex number representing the current in amperes.*

Solution. $X_L=\omega L=0{\cdot}06\times 2\pi\times 79{\cdot}5=29{\cdot}97\ \Omega$

$$\mathbf{Z}=20+j29{\cdot}9\ ; \qquad \mathbf{V}=40+j25$$

$$\mathbf{I}=\frac{\mathbf{V}}{\mathbf{Z}}=\frac{40+j25}{20+j29{\cdot}97}$$

$$=\frac{(40+j25)(20-j29{\cdot}97)}{(20+j29{\cdot}97)(20-j29{\cdot}97)}$$

$$\mathbf{I}=(1{\cdot}19-j0{\cdot}54)\ \text{A}$$

Example 19-10. *The current in a circuit is given by (4·5+j12) when the applied voltage is (100+j150). Determine (a) the complex expression for the impedance stating where it is inductive or capacitive (b) the power (c) the phase angle between the current and voltage.*

Solution. $\mathbf{I}=4{\cdot}5+j12\ ; \qquad \mathbf{V}=100+j150$

(*a*) $$\mathbf{Z}=\frac{\mathbf{V}}{\mathbf{I}}=\frac{100+j150}{4{\cdot}5+j12}=\frac{(100+j150)(4{\cdot}5-j12)}{(4{\cdot}5+j12)(4{\cdot}5-j12)}$$

$$=13{\cdot}7-j3{\cdot}2$$

Because the *j*-term is negative, hence impedance is capacitive.

(*b*) Using current conjugate

$$\mathbf{P}_{VA}=(100+j150)(4{\cdot}5-j12)=2250-j525$$

$\therefore$ real power = **2250 W**

(*c*) $$\mathbf{V}=\sqrt{(100^2+150^2)}\ \angle\tan^{-1}150/100=180{\cdot}2\angle 56°\ 26'$$

$$\mathbf{I}=\sqrt{(4{\cdot}5^2+12^2)}\ \angle\tan^{-1}12/4{\cdot}5=12{\cdot}8\ \angle 69°\ 26'$$

$\therefore$ Phase difference = 69° 26′ − 56° 26′ = **13°**

19-12. Complex Algebra Applied to Parallel Circuits

First, consider the parallel circuit of Fig. 19-13 (*a*). The two impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ have the same p.d. across them.

$\therefore$ $$\mathbf{I}_1=\frac{\mathbf{V}}{\mathbf{Z}_1}=\frac{V}{R_1+jX_1}\ \angle\tan^{-1}R_1/X_1$$

$$\mathbf{I}_2=\frac{\mathbf{V}}{\mathbf{Z}_2}=\frac{V}{R_2+jX_2}\ \angle\tan^{-1}R_2/X_2$$

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Total current $\mathbf{I}=\mathbf{V}/\mathbf{Z}$, where $\mathbf{Z}$ is the combined impedance of the parallel circuit which may be found by using law of parallel impedances.

$$\frac{1}{\mathbf{Z}}=\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2} \qquad \text{or} \qquad \mathbf{Z}=\frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}$$

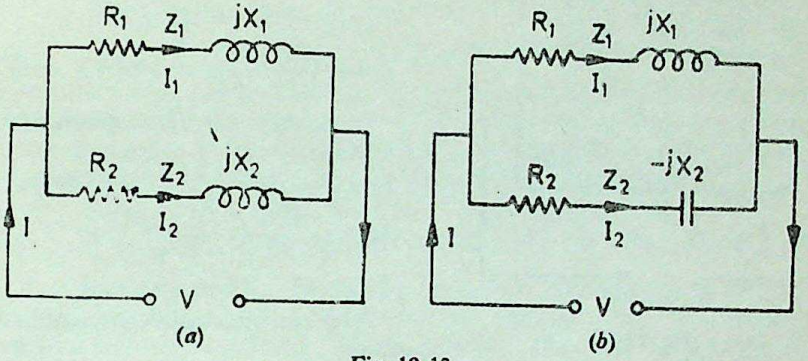

Fig. 19-13

Now, consider the circuit of Fig. 19-13 (*b*).

$$\mathbf{Z}_1=R_1+jX_1 \qquad \text{and} \qquad \mathbf{Z}_2=R_2-jX_2$$

Again $$\mathbf{Z}=\frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2} \qquad \text{and} \qquad \mathbf{I}=\frac{\mathbf{V}}{\mathbf{Z}}$$

19-13. Series—parallel Circuits

Consider the circuit of Fig. 19-14. First, equivalent impedance of parallel branches is calculated and it is then added to the series impedance to get the total circuit impedance. Then circuit current can be easily found.

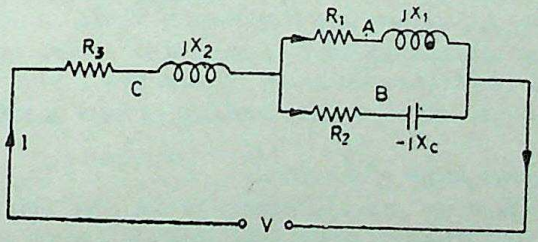

Fig. 19-14

$$\mathbf{Y}_A=\frac{1}{R_1+jX_1}\ ; \qquad \mathbf{Y}_B=\frac{1}{R_2-jX_C}$$

$$\therefore \qquad \mathbf{Y}=\frac{1}{R_1+jX_1}+\frac{1}{R_2-jX_C}$$

$$\therefore \qquad \mathbf{Z}_P=\frac{1}{\mathbf{Y}}\ ;\ \mathbf{Z}_C=R_3+jX_3\ \therefore\ \mathbf{Z}=\mathbf{Z}_P+\mathbf{Z}_C \text{ and } \mathbf{I}=\mathbf{V}/\mathbf{Z}$$

Example 19-11. *Two circuits A and B are connected in parallel to a 200-V supply. Circuit A consists of a resistance of 10 Ω in series*

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with an inductive reactance of 10 Ω and circuit B consists of a resistance of 20 Ω in series with a capacitive reactance of 10 Ω. Find the current taken by each circuit and the total current. Find also the p.f. of the circuit.

Solution. Take voltage vector as the reference vector as shown in Fig. 19-15 so that $V = 200 \angle 0°$

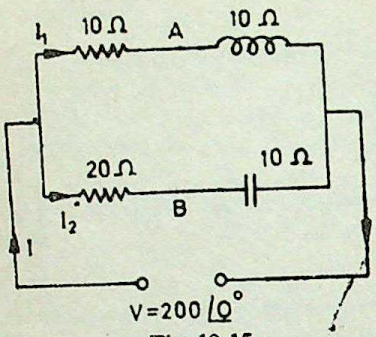

Fig. 19-15

$$Z_1 = 10 + j10 = 14{\cdot}14 \angle 45°$$

$$\therefore \quad I_1 = \frac{200 \angle 0°}{14{\cdot}14 \angle 45°} = 14{\cdot}14 \angle -45°$$

Obviously, current lags behind the voltage.

$$Z_2 = 20 - j10 = 22{\cdot}35 \angle -26{\cdot}6°$$

$$I_2 = \frac{200 \angle 0°}{22{\cdot}35 \angle -26{\cdot}6°}$$

$$= 8{\cdot}95 \angle 26{\cdot}6°$$

The current leads the applied voltage.

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= \frac{(10 + j10) \times (20 - j10)}{(10 + j10) + (20 - j10)} = \frac{(300 + j100)}{30}$$

$$= 10 - j3{\cdot}33 = 10{\cdot}54 \angle 18° 26'$$

$$I = \frac{200 \angle 0°}{10{\cdot}54 \angle 18° 26'} = 18{\cdot}98 \angle -18° 26'$$

p.f. $= \cos 18° 26' =$ **0·9487 (lag)**

Example 19-12. *The following three impedances are expressed at a frequency of 25 Hz, $Z_1 = 5 + 4j$; $Z_2 = 2 + j8$; $Z_3 = 3 - j8$. Write down the corresponding expressions at a frequency of 50 Hz.*

If the three impedances were connected in series, calculate the effective impedance of the combination. Hence, find

(a) the current taken by this combination from a 250-V, 50-Hz supply

(b) the power factor of the circuit

(c) the inductance or capacitance, as the case may be, of the parallel-connected component which would give unity power factor.

Solution. Since Z_1 and Z_2 have positive j-terms, they are inductive impedances. As inductive reactance is directly proportional to frequency, hence at 50 Hz

$$Z_1 = 5 + j8 \quad \text{and} \quad Z_2 = 2 + j16$$

Now, negative j-term in Z_3 represents capacitive reactance which is inversely proportional to frequency. Hence, at 50 Hz

$$\therefore \quad Z_3 = 3 - j4$$

Total impedance, $Z = Z_1 + Z_2 + Z_3$

$$= 5 + j8 + 2 + j16 + 3 - j4$$

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$$=10+j20$$

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(a) $$\mathbf{I}=\frac{\mathbf{V}}{\mathbf{Z}}=\frac{250+j0}{10+j20}=\frac{250(10-j20)}{(10+j20)(10-j20)}$$

$$=(5-j10)\text{ A}$$

$$I=\sqrt{5^2+10^2}=\mathbf{11{\cdot}2\ A}$$

(b) $$\text{p.f.}=\cos\phi=\frac{\text{'real' component of current}}{\text{total current}}$$

$$=5/11{\cdot}2=\mathbf{0{\cdot}447\ lag}$$

(c) For raising the p.f. to unity, the lagging reactive component—$j10$ must be neutralized by an equal but leading current. For this purpose, a capacitor should be connected in parallel with the series circuit such that it should draw a current of $j10$A.

Now, $I=V\omega C$ $\therefore$ $C=I/\omega V$

$\therefore$ $C=10/2\pi\times50\times250=127\times10^{-6}\text{ F}=\mathbf{127\ \mu F}$

Example 19-13. *A variable capacitor is connected in series with a circuit consisting of a non-inductive resistance of 50 Ω in parallel with a coil across a 200-V, 50-Hz supply. The coil has an inductance of 0·2 H and negligible resistance. Calculate :*

(a) *the capacitance of capacitor when the p. f. of the circuit is unity.*

(b) *the corresponding p.d. across the capacitor. Draw the vector diagram, not to scale, representing the voltage and currents.*

(A.M.I.E. May 1971)

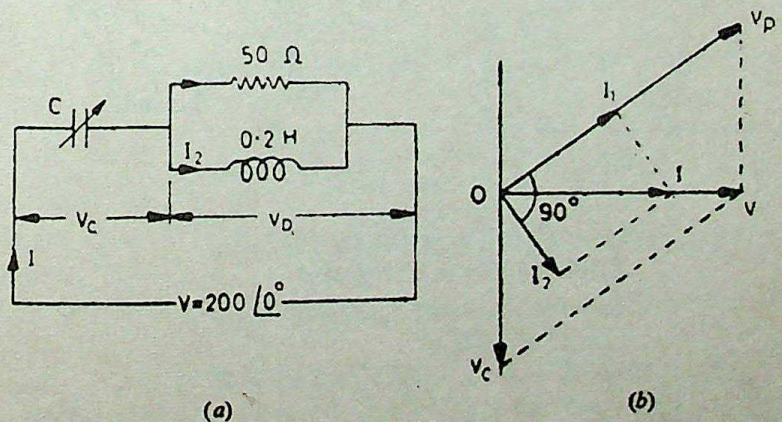
Solution. The series-parallel circuit is shown in Fig. 19-16 (a).

$$X_L=2\pi\times50\times0{\cdot}2=62{\cdot}8\ \Omega$$

The admittance of the parallel circuit is

$$\mathbf{Y}=\mathbf{Y}_1+\mathbf{Y}_2=\frac{1}{50+j0}+\frac{1}{0{\cdot}+j62{\cdot}8}$$

$$=\frac{1}{50}-\frac{j}{62{\cdot}8}=0{\cdot}02-j\,0{\cdot}0159\text{ mho}$$



(a) (b)

Fig. 19-16

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Impedance of the parallel circuit is

$$Z = \frac{1}{Y} = \frac{1}{0{\cdot}02 - j\,0{\cdot}0159}$$

$$= 30{\cdot}6 + j\,24{\cdot}35\ \Omega$$

The impedance of the capacitor required for making the power factor of the whole circuit unity should be equal in magnitude to the reactive component of the impedance of the parallel circuit.

Impedance of the capacitor $= -j\,24{\cdot}35\ \Omega$

$$\therefore \quad \frac{1}{\omega C} = 24{\cdot}35$$

or $$C = 1/2\pi \times 50 \times 24{\cdot}35 = 130{\cdot}9\ \mu F$$

Circuit current $$I = \frac{(200 + j0)}{30{\cdot}6} = \frac{200}{30{\cdot}6} \angle 0°$$

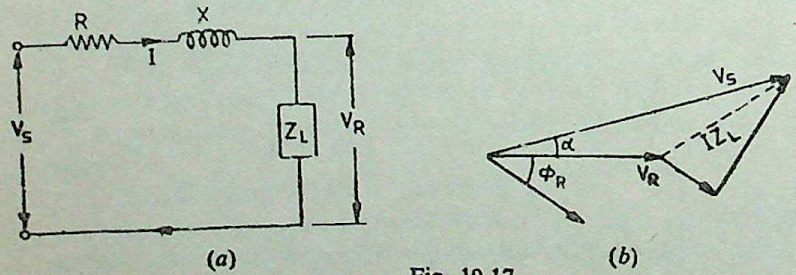
$$V_C = -j\,24{\cdot}35 \times \frac{200}{30{\cdot}6} \angle 0° = -j\,159\ V$$

The vector diagram, not to scale, is shown in Fig. 19-16 (*b*) where I_1 represents current through the resistor, I_2 through the coil and I through the capacitor.

Example 19-14. *A transmission line having a resistance R of 1·4 ohm and an inductive reactance of 1·6 ohm at 60 Hz supplies power to a load connected at the terminal at 460 volts (r.m.s.) and takes a current of 27 A (r.m.s.) at a p. f. of 0·8 lagging. Find the voltage at the generator terminal and its power factor.*

(A.M.I.E. Winter 1977)

Solution. In vector diagram of Fig. 19-17, the receiving end voltage V_R has been taken as the reference vector.



(*a*) (*b*)

Fig. 19-17

$$V_R = 460 \angle 0° = (460 + j0)$$

$$Z_L = 1{\cdot}4 + j\,1{\cdot}6 = 2{\cdot}12 \angle 48{\cdot}8°$$

$$\cos \phi_R = 0{\cdot}8\ ;\ \phi_R = \cos^{-1}(0{\cdot}8) = 36°52'$$

$$I = 27 \angle -36°52'$$

$$\text{Line drop} = IZ = 27 \angle -36°52' \times 2{\cdot}12 \angle 48°48'$$

$$= 57{\cdot}24 \angle 11°56' = 57{\cdot}4\ (\cos 11°56' + j \sin 11°56')$$

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$$= 56{\cdot}2 + j\ 11{\cdot}9$$

$$V_S = V_R + IZ$$

$$= (460 + j0) + (56{\cdot}2 + j\ 11{\cdot}9)$$

$$= 516{\cdot}2 + j\ 11{\cdot}9 = 516{\cdot}3 \angle 1{\cdot}3°$$

$$\phi_S = \phi_R + \alpha = 36°52' + 1°18' = 38°10$$

$$\cos \phi_S = \cos 38°10' = \mathbf{0{\cdot}783}$$

HIGHLIGHTS

(1) Summary of Complex Algebra

(*i*) Rectangular Form of an impedance is

$$\mathbf{Z} = R \pm jX$$

(*ii*) Polar Form

$$\mathbf{Z} = Z \angle \pm \theta \text{ where } \theta = \tan^{-1}(X/R) \text{ and } Z = \sqrt{(R^2 + X^2)}$$

(*iii*) Trigonometrical Form

$$\mathbf{Z} = Z(\cos \theta \pm j \sin \theta)$$

(*iv*) Exponential Form

$$\mathbf{Z} = Ze^{\pm j\theta}$$

(*v*) Addition

Let $\mathbf{Z}_1 = R_1 + jX_1$ and $\mathbf{Z}_2 = R_2 + jX_2$

$$\mathbf{Z}_1 + \mathbf{Z}_2 = (R_1 + R_2) + j(X_1 + X_2)$$

(*vi*) Subtraction

$$\mathbf{Z}_1 - \mathbf{Z}_2 = (R_1 - R_2) + j(X_1 - X_2)$$

(*vii*) Multiplication

$$\mathbf{Z}_1\mathbf{Z}_2 = (R_1 + jX_1)(R_2 + jX_2)$$

$$= (R_1R_2 - X_1X_2) + j(X_1R_2 + X_2R_1)$$

$$= Z_1Z_2 \angle (\theta_1 + \theta_2)$$

$$= Z_1Z_2e^{j(\theta_1 + \theta_2)}$$

$$= Z_1Z_2[\cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2)]$$

(*viii*) Division

$$\frac{\mathbf{Z}_1}{\mathbf{Z}_2} = \frac{R_1 + jX_1}{R_2 + jX_2}$$

$$= \left(\frac{R_1R_2 + X_1X_2}{R_2^2 + X_2^2}\right) + j\left(\frac{X_1R_2 - X_2R_1}{R_2^2 + X_2^2}\right)$$

$$= \frac{Z_1}{Z_2} \angle (\theta_1 - \theta_2)$$

$$= \frac{Z_1}{Z_2} e^{j(\theta_1 - \theta_2)}$$

$$= \frac{Z_1}{Z_2} [\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2)]$$

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THREE PHASE CIRCUITS 20

1. Generation of Three-phase Voltages 2. Phase Sequence 3. Numbering of Phases 4. Inter-connection of Three Phases 5. Star or Wye (Y) Connection 6. Voltages and Currents in Y-connection 7. Delta (Δ) or Mesh Connection 8. Comparison ; Star and Delta Connections 9. Comparison Between Single and Three Phase Supply Systems 10. Power Factor Improvement 11. Power Factor Correction Equipment 12. Power Measurement in 3-phase Circuits 13. Three Wattmeter Method 14. Two Wattmeter Method 15. Balanced Load 16. Variations in Wattmeter Readings 17. Leading Power Factor 18. Power Factor-Balanced Load 19. Reactive Power. 20. One Wattmeter Method 21. Highlights.

20-1. Generation of Three-phase Voltages

The kind of alternating currents and voltages discussed in chapters 16 to 18 are known as single-phase voltages and currents ; because they consist of a single alternating current and voltage wave. A single-phase alternator was diagrammatically depicted in Fig. 16-1 (*b*) and it was shown to have one armature winding only. But if the number of armature windings is increased, then it becomes three-phase alternator and it produces as many independent voltage waves as the number of windings or phases. These windings are displaced from one another by equal angles, the values of these angles being determined by the number of phases or windings.

In a two-phase alternator, the armature windings are displaced 90 electrical degrees apart. A 3-phase alternator, as the name shows, has three independent armature windings which are 120 electrical degrees apart. Hence, the voltages induced in the three windings are 120° apart in time phase. With the exception of two-phase windings, it can be stated that, in general, the electrical displacement between different phases is $360/n$ where n is the number of phases or windings.

Three-phase systems are the most common although, for certain special jobs, greater number of phases is also used. For example, almost all mercury-arc rectifiers for power purposes are either six-phase or twelve-phase and most of the rotary convertors in use are six-phase. All modern generators are practically three-phase. For transmitting large amounts of power, three-phase is invariably used. The reasons for the immense popularity of three-phase apparatus are that (*i*) it is more efficient (*ii*) it uses less material for a given capacity and (*iii*) it costs less than single-phase apparatus etc.

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In Fig. 20-1 is shown a two-pole stationary-armature, rotating-field type three-phase alternator. It has three armature coils *aa'*, *bb'* and *cc'* displaced 120° apart from one another. With the position and clockwise rotation of the poles indicated in Fig. 20-1, it is found that the e.m.f. induced in conductor '*a*' of coil *aa'* is maximum and its direction* is *away* from the reader. The e.m.f. in conductor '*b*' of coil *bb'* would be maximum and away from the reader, when the *N*-pole has turned through 120° *i.e.* when *N-S* axis lies along *bb'*. It is clear that the induced e.m.f. in conductor '*b*' reaches its maximum value 120° later than the maximum value in conductor '*a*'. In the like manner, the maximum e.m.f. induced (in the direction away from the reader) in conductor '*c*' would occur 120° later than in '*b*' or 240° later than that in '*a*'.

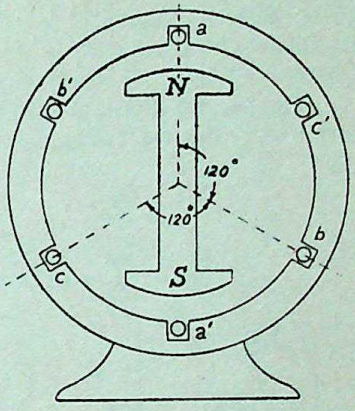

Fig. 20-1

Thus the three coils have three e.m.fs. induced in them which are similar in all respects except that they are 120° out of time

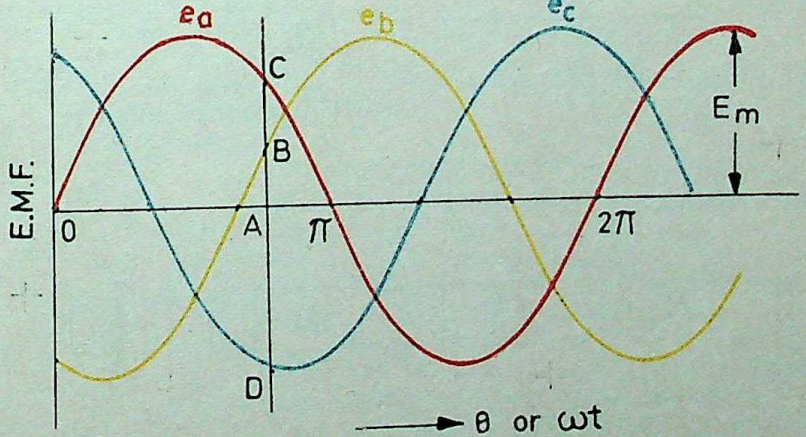

Fig. 20-2

*The direction is found with the help of Fleming's Right-hand rule. But while applying the rule, it should be remembered that the relative motion of the conductor with respect to the field is anti-clockwise although the motion of the field with respect to the conductor is clockwise as shown. Hence, thumb should point to the left.

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phase with one another as pictured in Fig. 20-2. Each voltage wave is assumed to be sinusoidal and having maximum value of E_m.

In practice, the space on the armature is completely covered and there are many slots/phase/pole.

Fig. 20-3 illustrates relative positions of the 3 phases of a 4-pole alternator and Fig. 20-4 shows the developed diagram of its armature windings. Assuming full-pitched winding and the direction of rotation as shown, phase '*a*' occupies the position under the centres of *N* and *S*-poles. It starts at S_a and ends or finishes at F_a.

The second phase '*b*' starts at S_b which is 120 electrical degrees apart from the start of phase '*a*', progresses round the armature clockwise as does '*a*' and finishes at F_b. Similarly, phase '*c*' starts at S_c which is 120 electrical degrees away from S_b, progresses round the armature and finishes at F_c. As the three circuits are exactly similar but are 120 electrical degrees apart, the e.m.f. waves generated in them (when the field rotates) are displaced from each other by 120°. Assuming these waves to be sinusoidal and counting the time from the instant when the e.m.f. in phase '*a*' is zero, the instantaneous values of the three e.m.fs. will be given by curves of Fig. 20-2. Their equations are :

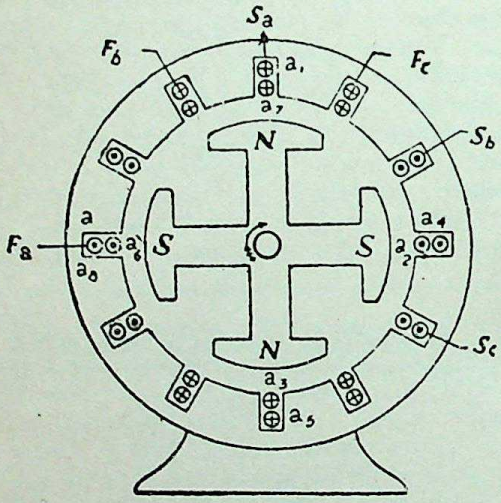

Fig. 20-3

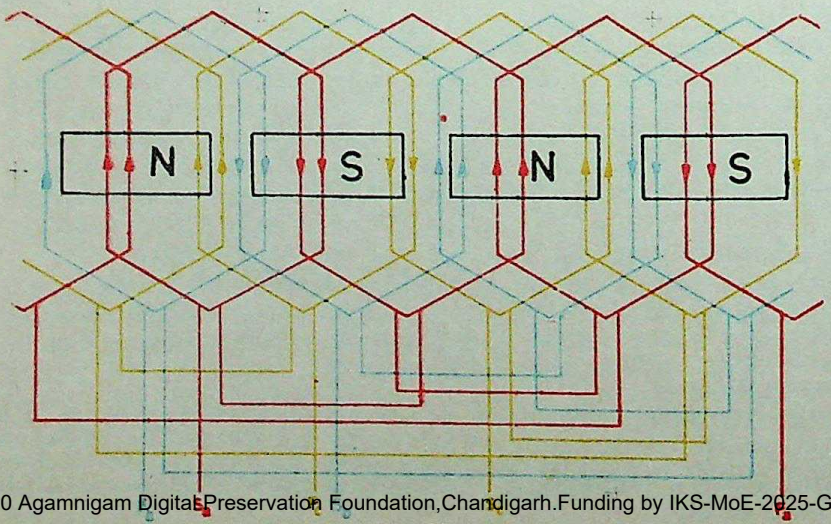

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Fig. 20-4

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$$e_a = E_m \sin \omega t \quad \ldots(i)$$
$$e_b = E_m \sin (\omega t - 120°) \quad \ldots(ii)$$
$$e_c = E_m \sin (\omega t - 240°) \quad \ldots(iii)$$

As shown in chapter 16, alternating voltages may be represented by revolving vectors which indicate their maximum values (or r.m.s. values if desired). The actual values of these voltages vary from peak positive to zero and to peak negative values in one revolution of the vectors. In Fig. 20-5 are shown the three vectors representing the r.m.s. voltages of the three phases : E_a, E_b and E_c [in the present case $E_a = E_b = E_c = E$ (say)]

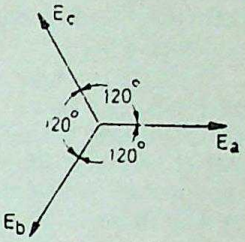

Fig. 20-5

It can be shown that the sum of the three phase e.m.fs. is zero in the following three ways :

(*i*) The sum of the above three equations (*i*), (*ii*) and (*iii*) is zero as shown below :

Resultant instantaneous e.m.f. is

$$= e_a + e_b + e_c$$
$$= E_m \sin \omega t + E_m \sin (\omega t - 120°) + E_m \sin (\omega t - 240°)$$
$$= E_m [\sin \omega t + 2 \sin (\omega t - 180°) \cos 60°]$$
$$= E_m [\sin \omega t - 2 \sin \omega t \cos 60°] = 0$$

(*ii*) The sum of ordinates of the three e.m.f. curves of Fig. 20-2 is zero. For example, taking ordinates AB and AC as positive and AD as negative, it can be shown that

$$AB + AC + (-AD) = 0$$

(*iii*) If we add the three vectors ot Fig. 20-5 either vectorially or by calculation, the result is zero.

Vector Addition

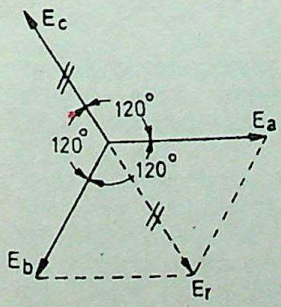

Fig. 20-6

As shown in Fig. 20-6, the resultant of E_a and E_b is E_r and its magnitude is $2E \cos 60° = E$, where

$$E_a = E_b = E_c = E$$

This resultant is equal and opposite to E_c. Hence, their resultant is zero.

By Calculation

Let us take E_a as reference voltage and assuming clockwise phase sequence,

$$\mathbf{E}_a = E \angle 0° = E + j0$$
$$\mathbf{E}_b = E \angle -120°$$
$$= E[\cos (-120°) + j \sin (-120°)]$$
$$= E(-0{\cdot}5 - j0{\cdot}866)$$
$$\mathbf{E}_c = E \angle -240° = E \angle 120° = E(-0{\cdot}5 + j0{\cdot}866)$$
$$\therefore \quad \mathbf{E}_a + \mathbf{E}_b + \mathbf{E}_c = (E + j0) + E(-0{\cdot}5 - j0{\cdot}866) + E(-0{\cdot}5 + j0{\cdot}866) = 0$$

20-2. Phase Sequence

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By phase sequence is meant the order in which the three phases

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attain their peak or maximum values. In the development of the three-phase e.m.fs. in Fig. 20-2, clockwise rotation of the field system in Fig. 20-1 was assumed. This assumption made the e.m.f. of phase '*b*' lag behind that of '*a*' by 120° and in a similar way, made that of '*c*' lag behind that of '*b*' by 120° (or that of '*a*' by 240°). Hence, the order in which the e.m.fs. of phases, *a*, *b* and *c* attain their maximum values is *a b c*. *It is called the phase order* or *phase sequence a b c*.

If, now, the rotation of the field structure of Fig. 20-1 is reversed *i.e.* made anti-clockwise, then the order in which the three phases would attain their corresponding maximum voltages would also be reversed. The phase sequence would become *a c b*. This means that e.m.f. of '*c*' would now lag behind that of phase '*a*' by 120° instead of 240° as in the previous case.

In general, the phase sequence of the voltages applied to a load is determined by the order in which the 3-phase lines are connected. The phase sequence can be reversed by interchanging any pair of lines. In the case of an induction motor, reversal of sequence results in the reversed direction of motor rotation. In the case of 3-phase unbalanced loads the effect of sequence reversal is, in general, to cause a completely different set of values of line currents. Hence, when working on such systems, it is essential that phase sequence be clearly specified otherwise unnecessary confusion will arise.

20-3. Numbering of Phases

The three phases may be numbered 1, 2, 3 or *a*, *b*, *c* or as is customary, they may be given three colours. The colours used commercially are red, yellow (or sometimes white) and blue. In that case the sequence is *RYB*.

Obviously, in any three-phase system, there are two possible sequences in which three coil or phase voltages may pass through their maximum value *i.e.* red→yellow→blue (*RYB*) or red→ blue→ yellow (*RBY*). By convention, *RYB* is regarded as positive sequence and *RBY* as negative sequence.

20-4. Interconnection of Three Phases

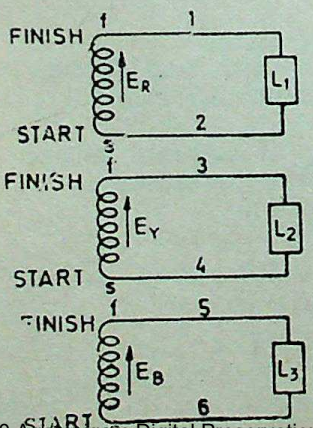

Fig. 20-7

If the three armature coils of the 3-phase alternator (Fig. 20-3) are not interconnected but are kept separate as shown in Fig. 20-7 then each phase or circuit would need two conductors, the total number of conductors, in that case, being six. It means that each transmission cable would contain six conductors which will make the whole system complicated and expensive. Hence the three phases are, generally, inter-connected which results in substantial saving of copper. The general methods of interconnection are

(*a*) Star or Wye (*Y*) connection and

(*b*) Mesh or Delta (△) connection.

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20-5. Star or Wye (Y) Connection

In this method of interconnection, the *similar* ends, say, 'start' ends of three coils (it could be 'finishing' ends also) are joined together at point N as shown in Fig. 20-8 (b).

The point N is known as *star point* or *neutral point*. The three conductors meeting at point N are replaced by a single conductor known as *neutral conductor* as shown in Fig. 20-8 (b). Such an

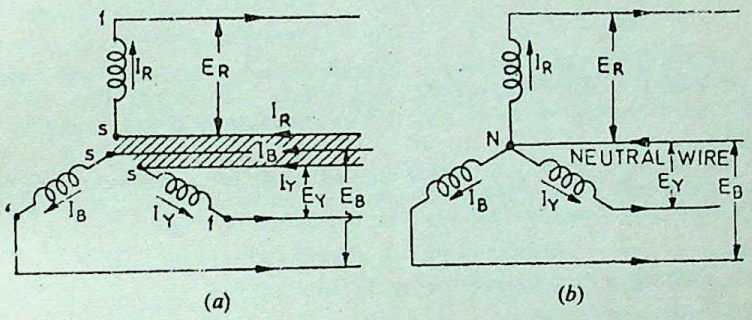

Fig. 20-8

interconnected system is known as four-wire 3-phase system and is diagrammatically shown in Fig. 20-8 (b). If this three-phase voltage is applied across a balanced symmetrical load, the neutral wire will be carrying three currents which are exactly equal in magnitude but are 120° out of phase with each other. Hence their *vector* sum is zero.

i.e. $$I_R + I_Y + I_B = 0 \quad \text{...vectorially}$$

The neutral wire, in that case, may be omitted although its retention is useful for supplying loads at low voltages (Ex. 20-18). The p.d. between any terminal (or line) and neutral (or star) point gives the *phase* or *star* voltage. But the p.d. between any two lines gives the line voltage. This connection is also referred to as 3-phase, 4-wire system.

Note. When considering the distribution of currents in a 3-phase system, it is extremely important to bear in mind that :

(i) the arrows placed alongside the currents I_R, I_Y and I_B flowing in three phases [Fig. 20-8 (b)] indicate the directions of currents when they are assumed to be *positive* and not the directions at a particular instant. It should be clearly understood *that at no instant will all the three currents flow in the same direction either outwards or inwards.* The three arrows indicate that first the current flows outwards in phase R, then after a phase-time of 120°, it will flow outwards from phase Y and after a further 120°, outwards from phase B.

(ii) the current flowing outwards in one or two conductors is alway equal to that flowing inwards in the remaining conductor or conductors. In other words, *each conductor, in turn, provides a return path for the currents of the other conductors.*

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In Fig. 20-9 are shown the three phase currents having the same peak value of 20 A but displaced from each other by 120°. At instant '*a*' the currents in phases *R* and *B* are each +10 A (*i.e.* flowing outwards) whereas the current in phase *Y* is −20 A (*i.e.* flowing inwards). In other words, at the instant '*a*', phase *Y* is acting as return path for the currents in phases *R* and *B*. At instant *b*, $I_R = +15$ A and $I_Y = +5$ A but $I_B = -20$ A which means that now phase *B* is providing the return path.

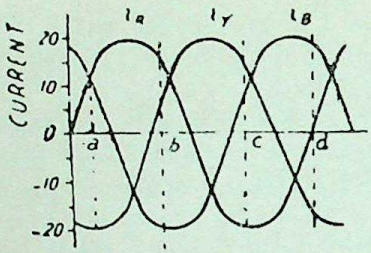

Fig. 20-9

At instant *c*, $I_Y = +15$ A and $I_B = +5$ A and $I_R = -20$ A.

Hence, now phase *R* carries current inwards whereas *Y* and *B* carry currents outwards. Similarly, at point *d*, $I_R = 0$, $I_B = 17{\cdot}3$ A and $I_Y = -17{\cdot}3$ A. In other words, current is flowing outwards from phase *B* and returning *via* phase *Y*.

In addition, it may be noted that although the distribution of current between the three lines is continuously changing, yet at any instant, the algebraic sum of the *instantaneous* value of the three currents is zero *i.e.*

$$i_R + i_Y + i_B = 0 \quad \text{—algebraically.}$$

20-6. Voltages and Currents in Y-Connection

The voltage induced in each winding is called the 'phase' voltage and current in each winding is likewise known as 'phase' current. However, the voltage available between any pair of terminals (or outers) is called *line* voltage (V_L) and the current flowing in each *line* is called line current (I_L).

As seen from Fig. 20-10 (*a*), in this form of interconnection, there are two phase windings between each pair of terminals, but

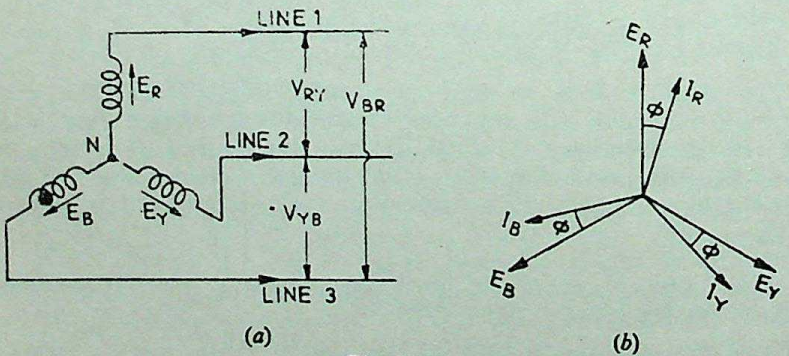

Fig. 20-10

since their *similar* ends have been joined together, they are in *opposition*. Obviously, the instantaneous value of p.d. between any two terminals is the *difference* of the two phase e.m.fs. concerned. However, the r.m.s. value of this p.d. is given by the *vector difference* of two phase e.m.fs.

The vector diagram for phase voltages and currents in a star connection is shown in Fig. 20-10 (*b*) where a balanced system has

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been assumed.* It means that $E_R = E_Y = E_B = E_{ph}$ (phase e.m.f.).

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Line voltage V_{RY} between line 1 and line 2 is vector difference of E_R and E_Y.

Line voltage V_{YB} between line 2 and line 3 is the vector difference of E_Y and E_B.

Line voltage V_{BR} between line 3 and line 1 is the vector difference of E_B and E_R.

(*i*) Line Voltages and Phase Voltages

The p.d. between lines 1 and 2 is

$$V_{RY} = E_R - E_Y \text{ (vector difference).}$$

Hence, V_{RY} is found by compounding E_R and E_Y reversed and its value is given by the diagonal of the parallelogram of Fig. 20-11. Obviously, the angle between E_R and E_Y reversed is 60°. Hence, if $E_R = E_Y = E_B$ = say, E_{ph}—the phase e.m.f., then

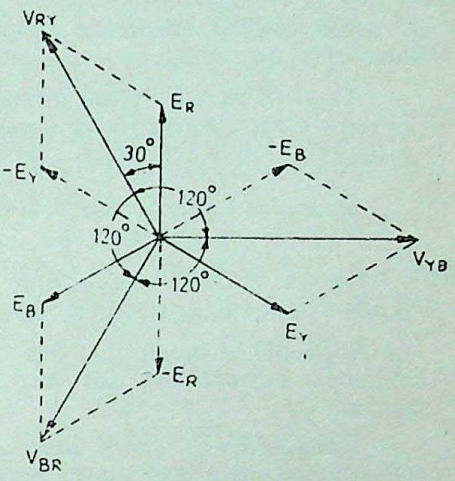

Fig. 20-11

$$V_{RY} = 2 \times E_{ph} \times \cos (60°/2)$$
$$= 2 \times E_{ph} \times \cos 30°$$
$$= 2 \times E_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3} E_{ph}$$

Similarly,

$$V_{YB} = E_Y - E_B \text{ (vector difference)}$$
$$= \sqrt{3} \cdot E_{ph}$$

and $$V_{BR} = E_B - E_R \text{ (vector difference)}$$
$$= \sqrt{3} . E_{ph}$$

Now $$V_{RY} = Y_{YB} = V_{BR}$$
$$= \text{line voltage, say, } V_L$$

Hence, in star connection

$$V_L = \sqrt{3}\, E_{ph}$$

It will be noted from Fig. 20-11 that

(*a*) Line voltages are 120° apart.

(*b*) Line voltages are 30° ahead of their respective phase voltages.

(*c*) The angle between the line currents and the corresponding line voltages is $(30 + \phi)$ with current lagging.

*A balanced system is one in which (*i*) the voltages in all phases are equal in magnitude and differ in phase from one another by equal angles, in this case, the angle=360/3=120° (*ii*) the current in the three phases are equal in magnitude and also differ in phase from one another by equal angles.

A 3-phase balanced load is that in which the loads connected across three phases are identical in magnitude and phase.

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(*ii*) **Line Current and Phase Currents.**

It is seen from Fig. 20-10 (*a*) that each line is in series with its individual phase winding, hence the line current in each line is the same as the current in the phase winding to which it is connected.

Current in line $1 = I_R$

Current in line $2 = I_Y$

Current in line $3 = I_B$

Since $I_R = I_Y = I_B =$ say, I_{ph}—the phase current

$\therefore$ line current $I_L = I_{ph}$

(*iii*) **Power**

The total power in the circuit is the sum of the three phase powers. Hence

Total power = 3 × phase power

or $$P = 3 \times E_{ph}\, I_{ph} \cos \phi$$

Now $E_{ph} = V_L/\sqrt{3}$ and $I_{ph} = I_L$

Hence, in terms of line values, the above expression becomes,

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi$$

or $$P = \sqrt{3}\, V_L\, I_L \times \cos \phi$$

It should be noted that ϕ is the angle between *phase* voltage and *phase* current and not between the line voltage and line current.

Example 20-1. *400-V (line to line) is applied to three star-connected identical impedances each consisting of a 4 Ω resistance in series with 3 Ω inductive reactance. Find (a) line current and (b) total power supplied.*

Solution. The circuit is shown in Fig. 20-12

Here $R = 4\ \Omega$; $X_L = 3\ \Omega$

$\therefore$ $Z_{ph} = \sqrt{4^2 + 3^2} = 5\ \Omega$

p.f. $= \cos \phi = R/Z = 4/5 = 0{\cdot}8$

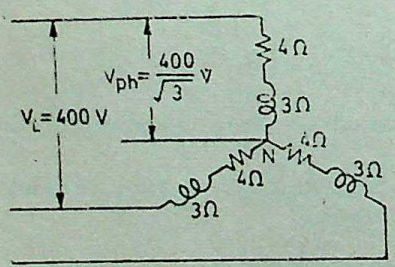

Fig. 20-12

$V_L = 400$ V

$V_{ph} = 400/\sqrt{3}$ V

$I_{ph} = V_{ph}/Z_{ph} = 400/\sqrt{3} \times 5$

$= 80/\sqrt{3}$ A

(*a*) $\therefore$ $I_L = I_{ph} = 80/\sqrt{3}$ A

(*b*) Total power

$P = \sqrt{3} V_L\, I_L \cos \phi$

$\therefore$ $P = \sqrt{3} \times 400 \times \frac{80}{\sqrt{3}} \times 0{\cdot}8$

$= 25{,}600$ W

[Note $P = 3 \times I_{ph}^2 R_{ph} = 3 \times \left(\frac{80}{\sqrt{3}}\right)^2 \times 4 = 25{,}600$ W]

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Example 20-2. *Calculate the active and reactive components of the current in each phase of a star-connected, 5000-V, 3-phase alternator supplying 3,000 kW at a power factor 0·8.*

Solution. $P=\sqrt{3}\,V_L\,I_L\cos\phi$

or $3000\times1000=\sqrt{3}\times5000\times I_L\times0{\cdot}8$

$\therefore \quad I_L=433$ A $\qquad \therefore\ I_{ph}=433$ A

Active component $=I_{ph}\times\cos\phi=433\times0{\cdot}8=\mathbf{346{\cdot}2}$ **A**

Reactive component $=I_{ph}\times\sin\phi=433\times0{\cdot}6=\mathbf{260}$ **A**

Example 20-3. *Deduce the relationship between the phase and line voltages of a 3-phase star-connected alternator. If the phase voltage of a 3-phase star-connected alternator be 200 V, what will be the line voltages (a) when the phases are correctly connected and (b) when the connections to one of the phases are reversed.*

Solution. (*a*) When phases are correctly connected, the vector diagram is as shown in Fig. 20-11. As proved in Art. 20-6 (*i*),

$$V_{RY}=V_{YB}=V_{BR}=\sqrt{3}.E_{ph}$$

Each line voltage $=\sqrt{3}\times200=\mathbf{346}$ **V**

(*b*) Suppose connections to *B*-phase have been reversed. Then voltage vector diagram for such a case is shown in Fig. 20-13. It should be noted that E_B has been drawn in the reversed direction, so that angles between the three phase voltages are 60°.

$V_{RY}=E_R-E_Y$

...vector difference

$=2\times E_{ph}\times\cos 30°$

$=\sqrt{3}\times200=\mathbf{346}$ **V**

$V_{YB}=E_Y-E_B$

...vector difference

$=2\times E_{ph}\times\cos 60°$

$=2\times200\times\frac{1}{2}=\mathbf{200}$ **V**

$V_{BR}=E_B-E_R$

...vector difference

$=2\times E_{ph}\times\cos 60°=2\times200\times\frac{1}{2}=\mathbf{200}$ **V**

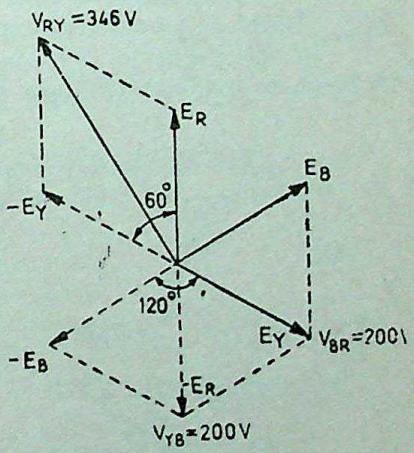

Fig. 20-13

Example 20-4. *Each phase of a star-connected load consists of a non-reactive resistance of 100 Ω in parallel with a capacitor of capacitance 31·8 μF.*

Calculate the line current, the power absorbed, the total kVA and the power factor when connected to a 416-V, 3-phase, 50-Hz supply.

Solution. The circuit is shown in Fig. 20-14.

$\mathbf{V}_{ph}=(416/\sqrt{3})\angle0°=240\angle0°=(240+j0)$

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Admittance of each phase is

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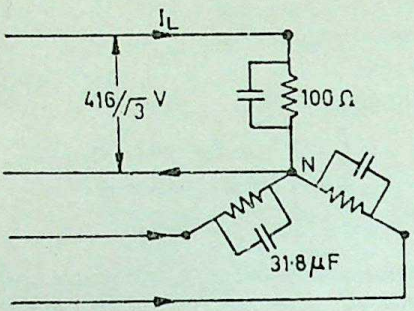

Fig. 20-14

$$Y_{ph} = \frac{1}{R} + j\omega C$$

$$= \frac{1}{100} + j2\pi \times 50 \times 31{\cdot}8 \times 10^{-6}$$

$$= 0{\cdot}01 + j0{\cdot}01$$

$$\therefore \quad I_{ph} = V_{ph} . Y_{ph}$$

$$= 240(0{\cdot}01 + j0{\cdot}01)$$

$$= 2{\cdot}4 + j2{\cdot}4 = 3{\cdot}39 \angle 45°$$

Since $I_{ph} = I_L$—for a star connection

$\therefore \quad I_L = 3{\cdot}39$ A

Power factor $= \cos 45° = 0{\cdot}707$ (leading)

Now $V_{ph} = (240 + j0)$ and $I_{ph} = (2{\cdot}4 + j2{\cdot}4)$

$\therefore \quad P_{VA} = (240 + j0)(2{\cdot}4 - j2{\cdot}4)$

$$= 240 \times 2{\cdot}4 - j2{\cdot}4 \times 240 = 576 - j576$$

$$= 814{\cdot}4 \angle -45°$$ ...per phase

Now total power $= 3 \times 576 = 1728$ W $= 1{\cdot}728$ kW

Total voltamperes $= 814{\cdot}4 \times 3 = 2443$ VA

Kilovolt-amperes $= 2{\cdot}443$ kVA

20-7. Delta (△) or Mesh Connection

In this form of interconnection, the *dissimilar* ends of the three-phase windings are joined together *i.e.* the 'starting' end of

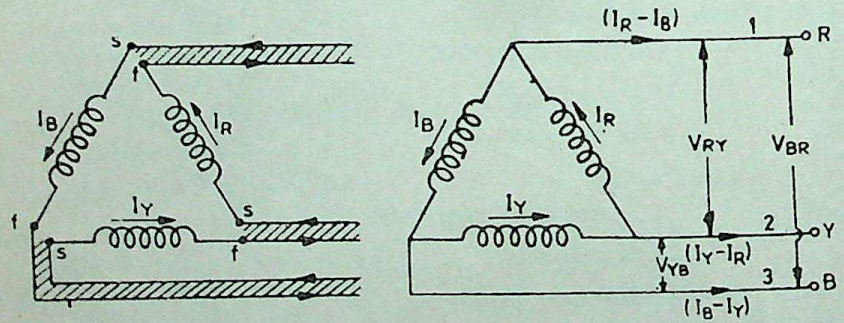

Fig. 20-15

one phase is joined to the 'finishing' end of the other phase and so on as shown in Fig. 20-15 (*a*). In other words, the three windings are joined in series to form a closed mesh as shown in Fig. 20-15 (*b*).

Three leads are taken out from the three junctions as shown and outward directions are taken as positive.

It might look as if this sort of interconnection results in short-circuiting the three windings. However, if the system is balanced, then sum of three voltages round the closed mesh is zero, hence no current of fundamental frequency can flow around the mesh when the terminals are open. It should be clearly understood that at any

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instant, the e.m.f. in one phase is equal and opposite to the resultant of those in the other two phases.

This type of connection is also referred to as 3-phase, 3-wire system.

(*i*) Line Voltages and Phase Voltages

It is seen from Fig. 20-15 (*b*) that there is only one phase winding completely included between any pair of terminals. Hence, in $\triangle$-connection, the voltage between any pair of lines is equal to the phase voltage of the phase winding connected between the two lines considered. Since phase sequence is *R*, *Y*, *B* hence the voltage having its positive direction from *R* to *Y* leads by 120° on that having its positive direction from *Y* to *B*. Calling the voltage between lines 1 and 2 as V_{RY} and that between lines 2 and 3 as V_{YB}, we find that V_{RY} leads V_{YB} by 120°. Similarly, V_{YB} leads V_{BR} by 120° as shown in Fig. 20-16. Let $V_{RY}=V_{YB}=V_{BR}=$line voltage V_L. Then it is seen that

$$V_L=V_{ph}$$

(*ii*) Line Currents and Phase Currents

It will be seen from Fig. 20-15 (*b*) that current in each line is the *difference* of the two phase currents flowing through that line. For example

Current in line 1 is $I_1=I_R-I_B$
Current in line 2 is $I_2=I_Y-I_R$ } vector difference
Current in line 3 is $I_3=I_B-I_Y$

Current in line No. 1 is found by compounding I_R with I_B reversed and its value is given by the diagonal of the parallelogram of Fig. 20-16. The angle between I_R and I_B reversed (*i.e.* $-I_B$) is 60°. If $I_B=I_R=I_Y$ =phase current I_{ph} (say) then

Current in line No. 1 is

$$I_1=2\times I_{ph}\times\cos(60°/2)$$
$$=2\times I_{ph}\times\sqrt{3}/2$$
$$=\sqrt{3}\,I_{ph}$$

Similarly, current in line No. 2 is

$$I_2=I_Y-I_R \quad \text{...vector difference}$$
$$=\sqrt{3}\,I_{ph}$$

and current in line No. 3 is

$$I_3=I_B-I_Y \text{...vector difference}$$
$$=\sqrt{3}I_{ph}$$

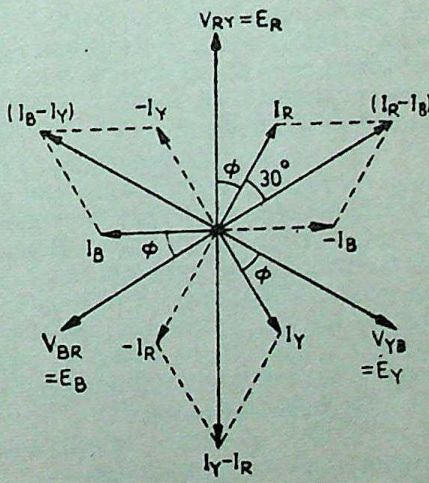

Fig. 20-16

Since all the line currents are equal in magnitude *i.e.*

$$I_1=I_2=I_3=I_L$$

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$\therefore \quad I_L = \sqrt{3}\, I_{ph}$

With reference to Fig. 20-16 it should be noted that

(*a*) line currents are 120° apart

(*b*) line currents are 30° behind the respective phase currents

(*c*) the angle between the line current and the corresponding line voltage is $(30+\phi)$ with the current lagging.

(*iii*) **Power**

Power/phase $= V_{ph}\, I_{ph} \cos\phi$

Total power $= 3 \times V_{ph}\, I_{ph} \cos\phi$

Now $V_{ph} = V_L$, but $I_{ph} = I_L/\sqrt{3}$

Hence, in terms of line values, the above expression for power becomes

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

$$= \sqrt{3}\, V_L\, I_L \cos\phi$$

where ϕ is the phase power factor angle.

20-8 Comparison : Star and Delta Connections

(*a*) **Star Connection**

1. Alternators are usually star-wound for the reason that phase voltage has to be only $1/\sqrt{3}$ of the line voltage, whereas for a delta-winding, the phase voltage has to be equal to the line voltage. Now, the number of conductors per phase in the windings of an alternator or motor, for a given frequency and flux, is directly proportional to the phase voltage. Hence, for a given line voltage, fewer turns/phase are required with *Y*-connection than with a delta-connection.

2. With star connection, the system of distribution mains can be arranged to suit both lighting and power circuits without using transformers. Heating and lighting circuits (up to 260 V), are put across neutral and any line wire (Fig. 20-20) whereas the 3-phase motors, running at higher voltages can be joined across the lines directly (Fig. 20-20).

3. Another advantage of *Y*-connection is that the neutral point of the alternators can be (and usually is) earthed. In that case, the p.d. between each line and earth is equal to the phase voltage *i.e.* $1/\sqrt{3}$ of the line voltage. Hence, if through fault, line conductor is earthed, the insulators will have to bear $1/\sqrt{3}$ of the line voltage (*i.e.* 57·7%) only. But in the case of a delta connection, if any line conductor is earthed, the insulators will have to bear full line voltage. Hence, there would be produced a correspondingly higher stress in the insulators with greater liability to breakdown.

(*b*) **Delta Connection**

1. The advantage of this connection is that transformers, in general, work more satisfactorily.

2. It is the only connection suitable for such machines as rotary converters.

3. This connection is much used for comparatively small low-voltage three-phase motors.

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20-9. Comparison Between Single- and 3-phase Supply Systems

1. Power in a single phase system is pulsating (at twice the frequency of voltage). This is not objectionable for lighting (if flicker is not appreciable) or for small motors. But with large motors, pulsating power supply causes excessive vibration.

2. Single-phase motors (except commutator type) have no starting torque, hence they need ancillary apparatus for self-starting. This is unnecessary in the case of 3-phase motors working on 3-phase supply.

3. Power factor of a single-phase motor is lower than that of a 3-phase motor of the same output and speed.

4. For a given size of frame, the output of a 3-phase machine is greater than that of a single-phase motor.

5. To transmit a given amount of power at a given voltage at a given distance, 3-phase transmission requires 3/4th weight of copper of a single-phase system.

6. Three-phase currents can produce rotating magnetic fields when passed through stationary coils (as in the case of induction and synchronous motors).

Example 20-5. *Three similar coils, each of resistance 20 Ω and inductance 0·5 H are connected (a) in star (b) in delta to a 3-phase, 50-Hz, 400-V (between lines) supply. Calculate the line current and the total power absorbed.*

Solution.

$$R_{ph} = 20\ \Omega$$

$$X_L = 2\pi f L = 2\pi \times 0{\cdot}5 \times 50 = 157\ \Omega$$

$$Z_{ph} = \sqrt{20^2 + 157^2} = 158{\cdot}2\ \Omega$$

$$\cos\phi = R_{ph}/Z_{ph} = 20/158{\cdot}2 = 0{\cdot}1264$$

(*a*) **Star Connection**

$$V_{ph} = 400/\sqrt{3} = 231\ \text{V}\ ;\ Z_{ph} = 158{\cdot}2\ \Omega$$

$$I_{ph} = 231/158{\cdot}2 = 1{\cdot}46\ \text{A}\ ;\ I_L = \mathbf{1{\cdot}46\ A}$$

$$\text{Power} = \sqrt{3}\ V_L\ I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 1{\cdot}46 \times 0{\cdot}1264 = \mathbf{127{\cdot}8\ W}$$

or $$P = 3\ I_{ph}^2\ R_{ph}$$

$$= 3 \times 1{\cdot}46^2 \times 20 = \mathbf{127{\cdot}8\ W}$$

(*b*) **Delta Connection**

$$V_{ph} = V_L = 400\ \text{V}\ ;\quad Z_{ph} = 158{\cdot}2\ \Omega$$

$$\therefore \quad I_{ph} = 400/158{\cdot}2\ \text{A}$$

$$I_L = \sqrt{3} \times I_{ph} = \sqrt{3} \times 400/158{\cdot}2 = \mathbf{4{\cdot}38\ A}$$

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(It may be noted that for the same V_L and Z_{ph}, line current for delta connection is three times the line current for star connection)

Power $= \sqrt{3} \times 400 \times 4{\cdot}38 \times 0{\cdot}1264 = \mathbf{383{\cdot}4\ W}$

(Note : Power is also three times)

Example 20-6. *Three similar resistors are connected in star across 400-V, 3-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors delta-connected ?*

Solution. Star Connection

$$I_L = I_{ph} = 5\ \text{A}$$

$$V_{ph} = 400/\sqrt{3} = 231\ \text{V}$$

$$\therefore \quad R_{ph} = 231/5 = \mathbf{46{\cdot}2\ \Omega}$$

Delta Connection

$$I_L = 5\ \text{A} \qquad \text{...(given)}$$

$$I_{ph} = 5/\sqrt{3}\ \text{A}$$

$$R_{ph} = 46{\cdot}2\ \Omega \qquad \text{...found above}$$

$$\therefore \quad V_{ph} = I_{ph} \times R_{ph} = 5 \times 46{\cdot}2/\sqrt{3} = \mathbf{133{\cdot}3\ V}$$

(Note : Voltage needed is 1/3rd the star value)

Example 20-7. *Three identical impedances are connected in delta to a 3-phase supply of 400-V. The line current is 34·65 A and the total power taken from the supply is 14·4 kW. Calculate the resistance and reactance values of each impedance.*

Solution.

$$V_{ph} = V_L = 400\ \text{V} \ ; \ I_L = \sqrt{3}\ I_{ph}$$

$$\therefore \quad I_{ph} = I_L/\sqrt{3} = 34{\cdot}65/\sqrt{3}\ \text{A}$$

$$Z_{ph} = V_{ph}/I_{ph} = 400 \times \sqrt{3}/34{\cdot}65 = 20\ \Omega$$

Now, power $P = \sqrt{3}\ V_L\ I_L \cos\phi$

$$\therefore \quad \cos\phi = \frac{P}{\sqrt{3}\ V_L\ I_L} = \frac{14{,}400}{\sqrt{3} \times 400 \times 34{\cdot}65}$$

Now $\cos\phi = R_{ph}/Z_{ph}$

$$\therefore \quad R_{ph} = Z_{ph} \cos\phi$$

$$= 20 \times \frac{14{,}400}{\sqrt{3} \times 400 \times 34{\cdot}65} = \mathbf{12\ \Omega}$$

$$X_{ph} = \sqrt{Z_{ph}^2 - R_{ph}^2} = \sqrt{20^2 - 12^2} = \mathbf{16\ \Omega}$$

Example 20-8. *The name-plate of an a.c. motor reads as follows :*

74·6 kW ; 3-ph ; 400-V ; 0·8 p.f. delta-connected. If the efficiency of the motor at the rated output is given as 80%, calculate the current drawn from the supply and the phase currents in the motor windings.

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Solution.

Motor output $= 74{\cdot}6 \times 1000$ W

Motor input $= 74{,}600/0{\cdot}88$ W

$V_L = 400$ V, $\cos\phi = 0{\cdot}8$

$\therefore$ Motor input $= \sqrt{3}\, V_L\, I_L \cos\phi$

or $74{,}600/0{\cdot}88 = \sqrt{3} \times 400 \times I_L \times 0{\cdot}8$

$\therefore$ $I_L = 153$ A ; $I_{ph} = 153/\sqrt{3} = 88{\cdot}3$ A

Example 20-9. *Three 40-Ω non-inductive resistances are connected (i) in star (ii) in mesh across 200-V, 3-phase lines. Calculate the power taken from the mains in each case. If one of the two resistors is disconnected, what would be the power taken from the mains in each case ?*

Solution. (*i*) **Star connection**

$V_{ph} = 200/\sqrt{3}$ V

$\therefore$ $I_{ph} = 200/\sqrt{3} \times 40 = 5/\sqrt{3}$ A

$P = \sqrt{3}\, V_L\, I_L \cos\phi$

$= \sqrt{3} \times 200 \times \frac{5}{\sqrt{3}} \times 1 = 1000$ W

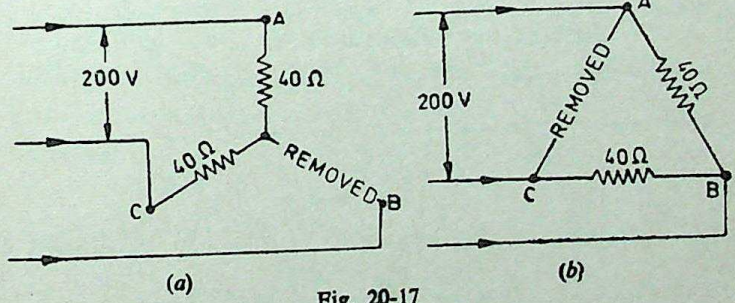

Fig. 20-17

(*ii*) **Delta connection**

$V_{ph} = 200$ V ; $R_{ph} = 40\ \Omega$

$I_{ph} = 200/40 = 5$ A ; $I_L = 5 \times \sqrt{3}$ A

$P = \sqrt{3} \times 200 \times 5 \times \sqrt{3} \times 1 = 3{,}000$ W.

When one of the resistors is disconnected

(*i*) **Star connection** [Fig. 20-17 (*a*)]

The circuit no longer remains a 3-phase circuit, but consists of two 40 Ω resistors in series across a 200-V supply. Current in lines *A* and *C* is $= 200/80 = 2{\cdot}5$ A.

Power absorbed in both $= 200 \times 2{\cdot}5 = 500$ W

Hence, by disconnecting one resistor, the power consumption is reduced by half.

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(*ii*) **Delta connection** [Fig. 20-17 (*b*)]

In this case, currents in *A* and *C* lines remain as usual 120° out of phase with each other.

Current in each phase $=200/40=5$ A

Power consumption in both $=2\times5^2\times40=$ **2,000 W**

(or $P=2\times5\times200=2000$ W)

In this case, when one resistor is disconnected, the power consumption is reduced by one-third.

Example 20-10. *Three coils are connected in delta to a 3-phase, 3-wire, 415-V, 50-Hz supply and take a line current of 5 A at 0·8 power factor lagging. Calculate the resistance and inductance of the coils.*

If the coils are star-connected to the same supply, calculate the line current and the total power.

Solution. The delta-connection is shown in Fig. 20-18 (*a*).

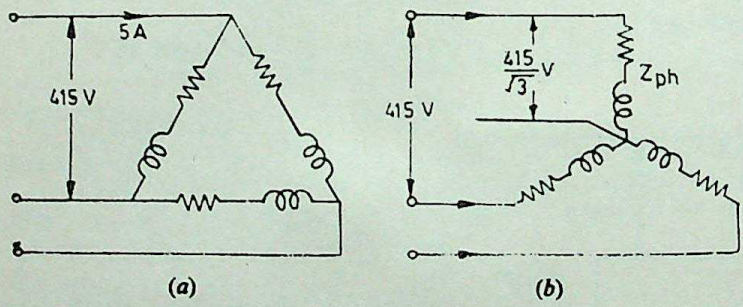

Fig. 20-18

$$V_{ph}=V_L=415 \text{ V} ; I_L=5 \text{ A} ; I_{ph}=5/\sqrt{3} \text{ A}$$

$$\therefore \quad Z_{ph}=\frac{V_{ph}}{I_{ph}}=\frac{415}{5/\sqrt{3}}=143{\cdot}7\ \Omega$$

Resistance/phase $=Z_{ph}\cos\phi=143{\cdot}7\times0{\cdot}8=$ **115 Ω**

Reactance/phase $=Z_{ph}\sin\phi=143{\cdot}7\times0{\cdot}6=86{\cdot}2\ \Omega$

$$\therefore \quad L=86{\cdot}2/2\pi\times50=\mathbf{0{\cdot}274\ H}$$

The star-connection is shown in Fig. 20-18 (*b*).

$$V_{ph}=415/\sqrt{3} \text{ V} ; Z_{ph}=143{\cdot}7\ \Omega$$

$$\therefore \quad I_{ph}=415/143{\cdot}7\times\sqrt{3}=2{\cdot}89/\sqrt{3} \text{ A}$$

$$I_L=2{\cdot}89/\sqrt{3} \text{ A}$$

Power consumed $=\sqrt{3}\,V_L\,I_L\cos\phi$

$=\sqrt{3}\times415\times(2{\cdot}89/\sqrt{3})\times0{\cdot}8=$ **960 W**

Example 20-11. *The load connected to a 3-phase supply comprises three similar coils connected in star. The line currents are 25 A and the kVA and kW inputs are 20 and 11 respectively. Find the line*

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and phase voltages, the kVAR input and the resistance and reactance of each coil.

If the coils are now connected in delta to the same three-phase supply, calculate the line currents and the power taken.

Solution. Star connection

$$\cos\phi = \text{kW/kVA} = 11/20\ ;\ I_L = 25\ \text{A}$$

$$P = 11\ \text{kW} = 11{,}000\ \text{W}$$

Now $P = \sqrt{3}\ V_L\ I_L \cos\phi$

$\therefore$ $11{,}000 = \sqrt{3} \times V_L \times 25 \times 11/20 \quad \therefore \quad V_L = \mathbf{462\ V}$

$$V_{ph} = 462/\sqrt{3} = \mathbf{267\ V}$$

$$kVAR = \sqrt{kVA^2 - kW^2} = \sqrt{20^2 - 11^2} = \mathbf{16{\cdot}7}$$

$$Z_{ph} = 267/25 = 10{\cdot}68\ \Omega$$

$\therefore$ $R_{ph} = Z_{ph} \times \cos\phi = 10{\cdot}68 \times 11/20 = \mathbf{5{\cdot}87\ \Omega}$

$X_{ph} = Z_{ph} \times \sin\phi = 10{\cdot}68 \times 0{\cdot}88 = \mathbf{8{\cdot}92\ \Omega}$

Delta connection

$$V_{ph} = V_L = 462\ \text{V}\ ; \qquad Z_{ph} = 10{\cdot}68\ \Omega$$

$\therefore$ $I_{ph} = 462/10{\cdot}68\ \text{A}\ ;\ I_L = \sqrt{3} \times 462/10{\cdot}68 = \mathbf{75\ A}$

$$P = \sqrt{3} \times 462 \times 75 \times 11/20 = \mathbf{33{,}000\ W}$$

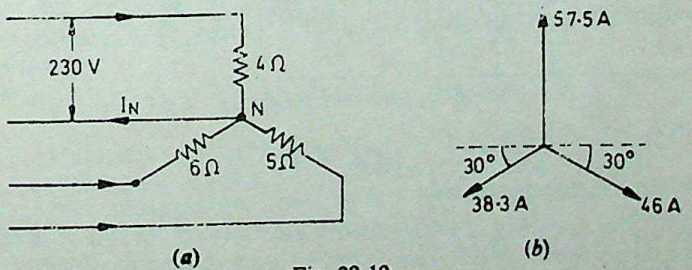
Example 20-12. *A 3-phase, star-connected system with 230 V between each phase and neutral has resistances of 4, 5 and 6 Ω respectively in the three phases. Estimate the current flowing in each phase and the neutral current. Find the total power absorbed.*

Solution. Here $V_{ph} = 230$ V [Fig. 20-19 (*a*)]

Current in 4 Ω resistor $= 230/4 = 57{\cdot}5$ A

Current in 5 Ω ,, $= 230/5 = 46$ A

,, ,, 6 Ω ,, $= 230/6 = 38{\cdot}3$ A



(*a*) (*b*)

Fig. 20-19

These currents are mutually displaced by 120°. The neutral current I_N is the vector sum* of these three currents. I_N can be obtained by splitting up these three phase currents into their

---

1. Some writers disagree with this on the ground that according to Kirchhoff's First Law, at any junction,

$$I_R + I_B + I_Y + I_B = 0 \quad \therefore \quad I_n = -(I_R + I_Y + I_B)$$

Hence, according to them, numerical value of I_n is the same but its phase is changed by 180°.

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X-components and Y-components and then by combining them together.

$$X\text{-component} = 46 \cos 30° - 38{\cdot}3 \cos 30° = 6{\cdot}64 \text{ A}$$

$$Y\text{-component} = 57{\cdot}5 - 46 \sin 30° - 38{\cdot}3 \sin 30° = 15{\cdot}3 \text{ A}$$

$$\therefore \quad I_N = \sqrt{6{\cdot}64^2 + 15{\cdot}3^2} = 16{\cdot}71 \text{ A}$$

Total power absorbed $= 230(57{\cdot}5 + 46 + 38{\cdot}3) = 32{,}610$ W

Example 20-13. *A 3-phase, 4-wire system supplies power at 400 V and lighting at 230 V. If the lamps in use require 70, 84 and 33 A in each of the lines, what should be the current in the neutral wire? If a 3-phase motor is now started, taking 200 A from the lines at a power factor of 0·2, what should be the current in each line and the neutral current? Find also the total power supplied to the lamps and the motor.*

Solution. The lamp and motor connections are shown in Fig. 20-20.

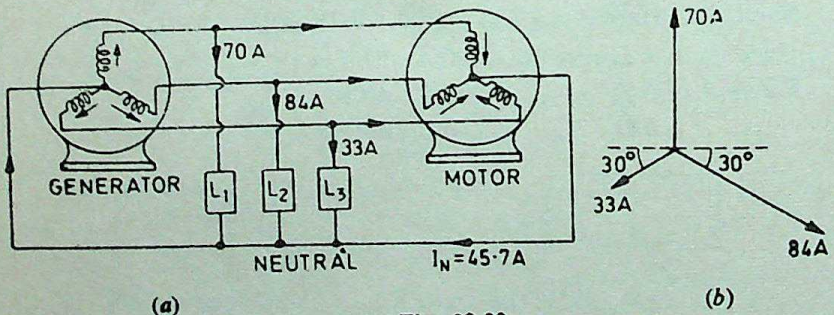
When motor is not started

The neutral current is the vector sum of lamp currents. Again splitting up the currents into their X- and Y-components, we get

$$X\text{-component} = 84 \cos 30° - 33 \cos 30° = 44{\cdot}2 \text{ A}$$

$$Y\text{-component} = 70 - 84 \sin 30° - 33 \sin 30° = 11{\cdot}5 \text{ A}$$

$$\therefore \quad I_N = \sqrt{44{\cdot}2^2 + 11{\cdot}5^2} = 45{\cdot}7 \text{ A}$$



(a) (b)

Fig. 20-20

When motor is started

A 3-phase motor is a balanced load. Hence, when it is started it will change the line currents, but being a balanced load, it contributes nothing to the neutral current. Hence, *the neutral current remains unchanged even after starting the motor.*

Now, the motor takes 200 A from the lines. It means that each line will carry motor current (which lags) as well as lamp current (which is in phase with the voltage). The current in each line would be the vector sum of these two currents.

$$\text{Motor p.f.} = \cos \phi = 0{\cdot}2$$

$$\sin \phi = 0{\cdot}9799 \quad \text{...from tables}$$

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Active component of motor current $=200\times0\cdot2=40$ A

Reactive „ „ „ „ $=200\times0\cdot9799=196$ A

(*i*) Current in first line $=\sqrt{(40+70)^2+196^2}=224\cdot8$ A

(*ii*) Current in second line $=\sqrt{(40+84)^2+196^2}=232$ A

(*iii*) Current in third line $=\sqrt{(40+33)^2+196^2}=210\cdot6$ A

Power supplied to lamps $=230(33+84+70)=43{,}000$ W

Power supplied to motor $=\sqrt{3}\times200\times400\times0\cdot2=27{,}700$ W

20-10. Power Factor Improvement (*A.M.I.E. Winter 1986*)

The heating and lighting loads supplied from 3-phase supply have power factors ranging from 0·95 to unity. But motor loads have usually low lagging power factors ranging from 0·5 to 0·9. Single-phase motors may have as low a power factor as 0·4 and electric welding units have even lower power factors of 0·2 or 0·3.

The power factor ($\cos\phi$) is given by

$$\cos\phi=\frac{kW}{kVA} \quad \text{or} \quad kVA=kW/\cos\phi$$

In the case of single-phase supply,

$$kVA=\frac{VI}{1000} \quad \text{or} \quad I=\frac{1000\,kVA}{V}$$

$$\therefore \quad I\propto kVA$$

In the case of 3-phase supply

$$kVA=\frac{\sqrt{3}V_LI_L}{1000} \quad \text{or} \quad I_L=\frac{1000\,kVA}{\sqrt{3}\times V_L}$$

$$\therefore \quad I_L\propto kVA$$

In each case, the kVA is directly proportional to current. The chief disadvantage of a low p.f. is that the current required for a given power, is very high. This fact leads to the following undesirable results :

(*i*) **Large kVA for a given amount of power**

All electric machinary like alternators, transformers, switchgears and cables are limited in current carrying capacity by the permissible temperature rise which is proportional to I^2. Hence, they may all be fully loaded with respect to their rated kVA without delivering their full power. Obviously, it is possible for an existing plant of a given kVA rating to increase its earning capacity (which is proportional to the power supplied in kW) if the overall power factor is improved *i.e.* raised.

(*ii*) **Poor voltage regulation**

When a load having a low lagging power factor is switched on, there is a large voltage drop in the supply voltage because of the increased voltage drop in the supply lines and transformers. This

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drop in voltage adversely effects the starting torques of motors and necessitates expensive voltage stabilizing equipment for keeping the consumer's voltage fluctuations within the statutory limits. Hence, all supply undertakings try to encourage consumers to have a high power factor.

Example 20-14. *A 50-MVA, 11-kV, 3-ϕ alternator supplies full load at a lagging power factor of 0·7. What would be the precentage increase in earning capacity if the power factor is increased to 0·95 ?*

Solution. The earning capacity is proportional to the power (in MW or kW) supplied by the alternator.

MW supplied at 0·7 lagging p.f. $= 50 \times 0{\cdot}7 = 35$

MW supplied at 0·95 lagging p.f. $= 50 \times 0{\cdot}95 = 47{\cdot}5$

Increase in MW $= 12{\cdot}5$

The increase in earning capacity is proportional to 12·5.

$\therefore$ Percentage increase in earning capacity

$$= (12{\cdot}5/35) \times 100 = \mathbf{35{\cdot}7}$$

20-11. Power Factor Correction Equipment

The following equipment is generally used for improving or correcting the power factor.

(*i*) Synchronous Motors (or capacitors)

These machines draw leading kVAR when they are over-excited and, especially, when they are running idle. They are employed for correcting the power factor in bulk and have the special advantage that the amount of correction can be varied by changing their excitation.

(*ii*) Static Capacitors

They are installed to improve the power factor of a group of a.c. motors and are practically loss-free (*i.e.*, they draw a current leading in phase by 90°). Since their capacitances are not variable, they tend to over-compensate on light loads unless arrangements for automatic switching off the capacitor bank are made.

(*iii*) Phase Advancers

They are fitted with individual machines.

However, it may be noted that the economic degree of correction to be applied in each case, depends upon the tariff arrangement between the consumers and the supply authorities.

Example 20-15. *A 3-phase, 37·3 kW, 440-V, 50-Hz induction motor operates on full-load with an efficiency of 89% and at a power factor of 0·85 lagging. Calculate the total kVA rating of capacitors required to raise the full-load power factor to 0·95 lagging. What will be the capacitance per phase if the capacitors are delta-connected ?*

Solution.

$$\text{Motor input} = \frac{\text{output}}{\text{efficiency}} = \frac{37{\cdot}3}{0{\cdot}89} = 41{\cdot}91 \text{ kW}$$

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Power Factor 0·85 (lag)

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$\cos \phi_1 = 0{\cdot}85$; $\sin \phi_1 = 0{\cdot}527$ ...from tables

$\therefore$ Motor $kVA_1 = kW/\cos \phi_1 = 41{\cdot}91/0{\cdot}85 = 49{\cdot}3$

Motor $kVAR_1 = kVA_1 \times \sin \phi_1 = 49{\cdot}3 \times 0{\cdot}527 = 25{\cdot}98$ (lag)

Power factor 0·95 (lag)

The power supplied [$= AB$ in Fig. 20-21 (b)] has to remain the same but the $kVAR$ is to be reduced from BD to BC such that AC makes the new power factor angle ϕ_2 with AB.

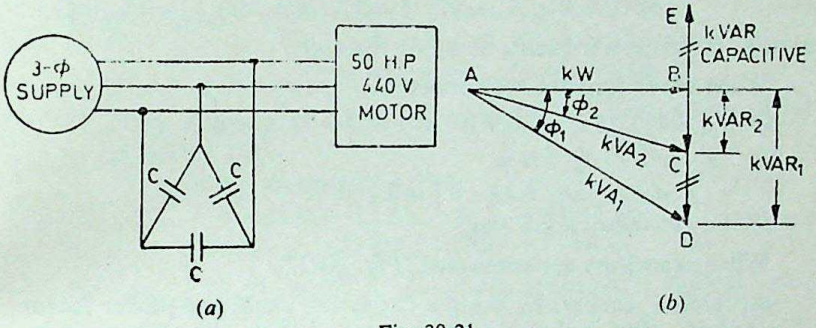

Fig. 20-21

$\cos \phi_2 = 0{\cdot}95$; $\tan \phi_2 = 0{\cdot}329$

Now, $BC/AB = \tan \phi_2$

$\therefore$ $BC = 41{\cdot}91 \times 0{\cdot}329 = 13{\cdot}79$

$\therefore$ $kVAR_2 = 13{\cdot}79$

Hence, *leading* $kVAR$ supplied by capacitors

$= kVAR_1 - kVAR_2$

$= 25{\cdot}98 - 13{\cdot}79 = \mathbf{12{\cdot}19}$

This is represented by BE in the kVA diagram of Fig. 20-21 (b) and equals DC. This is the $kVAR$ (or kVA because capacitors are assumed loss-free) of the capacitors required.

Capacitor VAR (or VA) $= 12{,}190$

$\therefore$ $12{,}190 = \sqrt{3}\, V_L I_L$

$\therefore$ Capacitor line current, $I_L = \dfrac{12{,}190}{\sqrt{3} \times 440} = 15{\cdot}99$ A

Since capacitors are delta-connected,

$\therefore$ current drawn by each capacitor

$= 15{\cdot}99/\sqrt{3} = 9{\cdot}23$ A

$\therefore$ Capacitive reactance/phase $= 440/9{\cdot}23 = 47{\cdot}65\ \Omega$

Now $X_C = \dfrac{1}{2\pi fC}$

or $47{\cdot}65 = \dfrac{1}{2\pi \times 50 \times C}$

$C = 1/314 \times 47{\cdot}65 = 66{\cdot}79 \times 10^{-6}$ F $= \mathbf{66{\cdot}8\ \mu F}$

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Example 20-16. *Three impedance coils, each having a resistance of 20 Ω and a reactance of 15 Ω, are connected in star to a 400-V, 3-ϕ, 50-Hz supply. Calculate (i) the line current (ii) power supplied and (iii) the power factor.*

If three capacitors, each of the same capacitance, are connected in delta to the same supply so as to form parallel circuit with the above impedance coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0·95 lagging.

Solution. $V_{ph}=400/\sqrt{3}$ V, $Z_{ph}=\sqrt{20^2+15^2}=25\ \Omega$

$\cos\phi_c=R_{ph}/Z_{ph}=20/25=0{\cdot}8$ (lag) ; $\sin\phi_c=0{\cdot}6$ (lag)

where ϕ_c is the power factor angle of the coils.

When capacitors are not connected

(*i*) $I_{ph}=400/25\times\sqrt{3}=9{\cdot}24$ A $\quad\therefore\quad I_L=\mathbf{9{\cdot}24\ A}$

(*ii*) $P=\sqrt{3}\,V_L\,I_L\cos\phi_c$

$=\sqrt{3}\times400\times9{\cdot}24\times0{\cdot}8=\mathbf{5{,}120\ W}$

(*iii*) Power factor $=\mathbf{0{\cdot}8}$ (lag)

When capacitors are connected (Fig. 20-22).

As stated earlier, by using a capacitor bank, the power factor can be raised. But it should be remembered that capacitors themselves do not consume any power. Their function is merely to neutralize the *lagging kVAR* of the load by their leading *kVAR*.

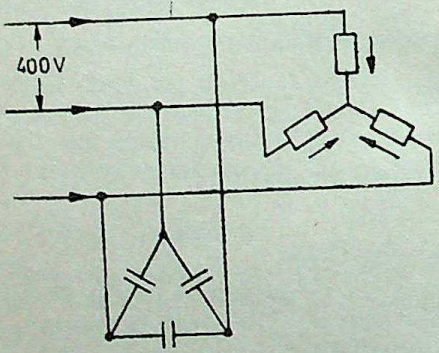

Fig. 20-22

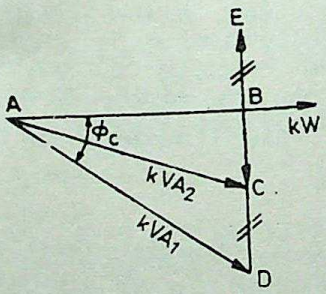

Fig. 20-23

Let ϕ be the phase angle of the combined load such that $\cos\phi=0{\cdot}95$.

Total *VA* of the load

$=\sqrt{3}\times400\times9{\cdot}24=6400$

$\therefore\quad kVA_1=6{\cdot}4$ kVA

$kW=kVA_1\times\cos\phi_c=6{\cdot}4\times0{\cdot}8$

$=5{\cdot}12$ ($=AB$ in Fig. 20-23)

$kVAR_1=kVA_1\times\sin\phi_c=6{\cdot}4\times0{\cdot}6$

$=3{\cdot}84$ ($=BD$ in Fig. 20-23)

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Since the capacitors themselves do not absorb any power, the power input of the combined load remains unaltered *i.e.* 5·12 kW.

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Now kVA input $=5{\cdot}12/\cos\phi=5{\cdot}12/0{\cdot}95=5{\cdot}39$

$$\therefore \quad kVA_2=5{\cdot}39 \ (=AC \text{ in Fig. 20-23})$$

$$kVAR_2=5{\cdot}39\times\sin\phi=5{\cdot}39\times0{\cdot}3123$$

$$=1{\cdot}684 \ (=BC \text{ in Fig. 20-23})$$

It means that before joining the capacitors, the $kVAR_1=3{\cdot}84$. Now it is 1·684. Hence, the leading $kVAR$ contributed by the capacitor bank

$$=BD-BC=kVAR_1-kVAR_2$$

$$=3{\cdot}84-1{\cdot}684=2{\cdot}156 \text{ (leading)}$$

The leading $kVAR$ of the capacitor has been shown separately by the vector BE.

If I_L is the line current, then

$$2156=\sqrt{3}\ V_L\ I_L=\sqrt{3}\times400\times I_L$$

$$\therefore \quad I_L=2156/\sqrt{3}\times400=3{\cdot}112 \text{ A}$$

Since the capacitors are delta-connected, current drawn by each capacitor

$$=3{\cdot}112/\sqrt{3}=1{\cdot}797 \text{ A}$$

$$\therefore \quad \text{Reactance/phase}=400/1{\cdot}797=222{\cdot}6\ \Omega$$

$$X_C=\frac{1}{2\pi fC} \qquad \therefore \quad 222{\cdot}6=\frac{1}{2\pi\times50\times C}$$

$$\therefore \quad C=14{\cdot}31\times10^{-6} \text{ F}=\mathbf{14{\cdot}31}\ \mu\text{F}$$

Example 20-17. *A three-phase, star-connected motor, connected across a 3-phase, star-connected supply of 400-V, 50-Hz takes a current of 20 A at 0·8 power factor lagging. Determine the capacitance of the capacitors per phase that are to be connected in delta across the terminals of the motor to raise the power factor to unity and also the new value of the supply line current with the capacitors connected.*

Solution. Motor input $kVA=\sqrt{3}.\ V_L\ I_L/1000$

$$=\frac{\sqrt{3}\times400\times20}{1000}=13{\cdot}856$$

$$kVAR=kVA\sin\phi=13{\cdot}856\times0{\cdot}6=8{\cdot}314 \text{ (lagging)}$$

$$kW=kVA\cos\phi=13{\cdot}856\times0{\cdot}8=11{\cdot}085$$

Since the resultant power factor is unity, it means that all the capacitors together should contribute as many *leading* $kVAR$ as the lagging $kVAR$ of the motor.

If I_C is the current drawn by each capacitor, then total volt-amperes taken by the three delta-connected capacitors $=3I_C\times400$

$$\therefore \quad 3I_C\times400=8314$$

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or $I_C = 6{\cdot}93$ A ; $X_C = 400/6{\cdot}93\ \Omega$

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or $$\frac{1}{2\pi \times 50 \times C} = \frac{400}{6{\cdot}93}$$

$$\therefore \quad C = 55{\cdot}17 \times 10^{-6}\text{F} = 55{\cdot}17\ \mu\text{F}$$

Since the reactive component of the motor current is neutralized by the capacitors, the new line current is the active component of the previous line current.

New line current $= 20 \times \cos\phi = 20 \times 0{\cdot}8 = 16$ A

Tutorial Problems—31

1. Three coils, each of resistance 6 Ω and inductive reactance 8 Ω are joined in delta across 400-V, 3-phase lines. Calculate the line current and power absorbed. [69·3 A ; 28,800 W]

2. Three non-inductive resistances, each of 100 Ω are connected in star across 400-V supply. Calculate the current through each.

What would be the current through each if they were connected in delta across the same supply ? [2·31 A ; 4 A]

3. Three 100 Ω resistors are connected (*i*) in star (*ii*) in delta across 440-V, 3-phase lines. Calculate the line and phase currents and the power taken from the mains in each case.

[(*i*) 2·54 A ; 1936 W (*ii*) 17·62 A ; 5808 W]

4. Three similar coils, each having a resistance of 5 Ω and an inductance of 0·021 H, are connected (*i*) in star (*ii*) in delta to a 440-V, 3-phase, 50-Hz supply. Calculate the line current and the total power absorbed in each case.

[(*i*) 30·8 A ; 14·2 kW (*ii*) 92·5 A, 42·6 kW]

5. A star-connected 3-phase load consists of three similar impedances. When the load is connected to a 3-phase, 500-V, 50-Hz supply, the line current is 28·85 A and the p.f. is 0·8 lagging.

(*a*) Calculate :

(*i*) the total power taken by the load

(*ii*) the resistance of each phase of the load

(*b*) If the load were re-connected in delta and supplied from the same three-phase system, calculate the current flowing in each line.

[(*a*) 20 kW ; 8 Ω (*b*) 86·55 A]

6. Three identical coils connected in delta to a 440-V, 3-phase supply take a total power of 50 kW and a line current of 90 A. Find (*a*) the phase currents (*b*) the power factor (*c*) the total apparent power taken by the coils.

[(*a*) 52 A (*b*) 0·73 (*c*) 68·5 kVA]

7. Three similar choke coils are connected in star to a 3-phase supply. If the line currents are 15 A, the total power consumed is 11 kW and volt-ampere input is 15 kVA, find the line and phase voltages, the kVAR input and the resistance and reactance of each coil.

[577·3 V ; 333·3 V ; 10·2 kVAR ; 16·3 Ω ; 15·1 Ω]

8. A balanced load takes 20 kVA at a p.f. of 0·8 lagging from a 250-V, 50-Hz, 3-phase supply. Calculate the values of the components in the circuit if the load is (*i*) delta-connected (*ii*) star-connected.

What would be the values of the three line currents if one of the fuses 'blows' whilst in star connection.

[(*i*) 7·5 Ω ; 17·9 mH (*ii*) 2·5 Ω ; 5·96 mH ; 40 A ; 40 A ; 0]

9. Three coils each having a resistance of 40 Ω and reactance 30 Ω are connected in star across a 415-V line, 3-phase, 50-Hz supply. Calculate :

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(*a*) the line current (*b*) the total kVA (*c*) the p.f. (*d*) the total power (*e*) the value of each of three capacitors which must be connected in star to the supply in order to bring the overall p.f. to unity.

[(*a*) 4·8 A (*b*) 3·45 kVA (*c*) 0·8 lag (*d*) 2·76 kW (*e*) 38·2 μF]

20-12. Power Measurement in 3-phase Circuits

Following methods are available for measuring power in a 3-phase load.

(*a*) Three Wattmeter Method

In this method, three wattmeters are inserted in each phase and the algebraic sum of their readings gives the total power consumed by the 3-phase load.

(*b*) Two Wattmeter Method

(*i*) This method gives true power in the 3-phase circuit without regard to balance or wave-form provided in the case of *Y*-connected load, the neutral of the load is isolated from the neutral of the source of power. Or if there is a neutral connection, the neutral wire should not carry any current. This is only possible if the load is perfectly balanced and there are no harmonics present of triple frequency or any other multiples of that frequency.

(*ii*) This method can also be used for 3-phase, 4-wire system in which the neutral wire carries the neutral current. In this method, the current coils of the wattmeters are supplied from current transformers inserted in the principal line wires in order to get the correct magnitude and phase differences of the currents in the current coils of the wattmeters, because in the 3-phase, 4-wire system, the sum of the instantaneous currents in the principal line wires is not necessarily equal to zero as in 3-phase 3-wire system.

(*c*) One Wattmeter Method

In this method, a single wattmeter is used to obtain the two readings which are obtained by two wattmeters by the two-wattmeter method. This method can, however, be used when the load is balanced.

20-13. Three Wattmeter Method

A wattmeter consists of (*i*) a low-resistance current coil which is inserted in series with the line carrying the current and (*ii*) a high-resistance pressure coil which is connected across the two points whose potential difference is to be measured.

A wattmeter shows a reading which is proportional to the product of the current through its current coil, the p.d. across its potential or pressure coil and cosine of the angle between this voltage and current.

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As shown in Fig. 20-24, in this method three wattmeters are inserted in each of the three phases of the load whether △-con-

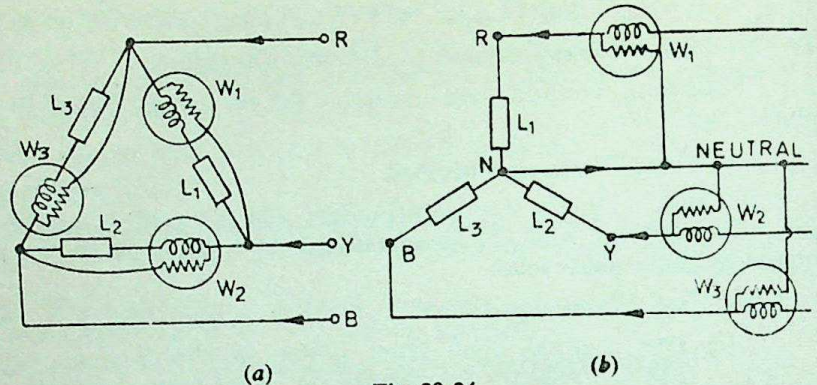

Fig. 20-24

nected or *Y*-connected. The current coil of each wattmeter carries the current of one phase only and the pressure coil measures the phase voltage of that phase. Hence, each wattmeter measures the power in a single phase. The algebraic sum of the readings of the three wattmeters must give the total power in the load.

The difficulty with this method is that under ordinary conditions, it is not generally feasible to break into the phases of a delta-connected load nor is it always possible, in the case of a *Y*-connected load, to get at the neutral point which is required for connection as shown in Fig. 20-24 (*b*). However, it is not necessary to use three wattmeters to measure power, two wattmeters can be used for the purpose as shown below.

20-14. Two Wattmeter Method—*Balanced or Unbalanced load*

As shown in Fig. 20-25, the current coils of the two wattmeters are inserted in *any* two lines and their potential coils joined to the third line. It can be proved that the sum of the instantaneous powers indicated by W_1 and W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 and L_3. The star-connected load is considered in the following discussion, although it can be equally applied

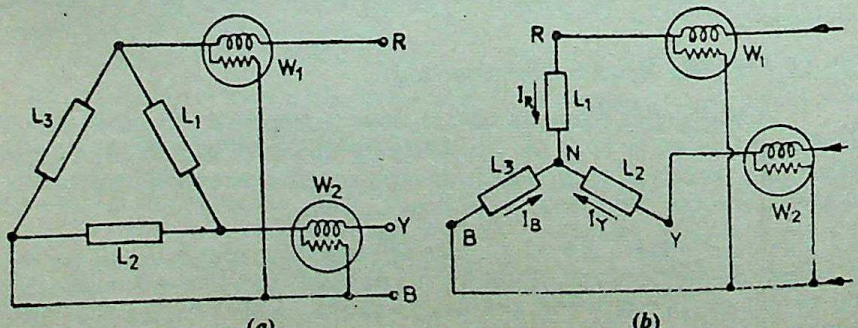

Fig. 20-25

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to $\triangle$-connected load, because a $\triangle$-connected load can always be replaced by an equivalent Y-connected load.

Now, before we consider the currents through and p.d. across each wattmeter, it may be pointed out that *it is important to take the direction of the voltage through the circuit the same as that taken for the current when establishing the readings of the two wattmeters.*

Instantaneous current through $W_1 = i_R$

,, ,, P.D. across $W_1 = e_{RB} = e_R - e_B$

,, ,, power read by $W_1 = i_R\,(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

,, ,, P.D. across $W_2 = e_{YB} = e_Y - e_B$

,, ,, power read by $W_2 = i_Y\,(e_Y - e_B)$

$$\therefore \quad W_1 + W_2 = i_R\,(e_R - e_B) + i_Y\,(e_Y - e_B)$$
$$= i_R\,e_R + i_Y\,e_Y - e_B\,(i_R + i_Y)$$

Now $i_R + i_Y + i_B = 0$ ...Kirchhoff's Point Law

$$\therefore \quad i_R + i_Y = -i_B$$

$$\therefore \quad W_1 + W_2 = i_R.e_R + i_Y.e_Y + i_B.e_B = p_1 + p_2 + p_3$$

where p_1 is the power absorbed by load L_1, p_2 that absorbed by L_2 and p_3 that absorbed by L_3.

$$\therefore \quad W_1 + W_2 = \text{total power absorbed.}$$

This proof is true whether the load is balanced or unbalanced. If the load is Y-connected, then it should have no neutral connection (*i.e.* 3-ϕ, 3-wire connected) and if it has a neutral connection (*i.e.* 3-ϕ, 4-wire connected) then it should be exactly balanced so that in each case there is no neutral current i_N otherwise Kirchhoff's Point Law will give $i_N + i_R + i_Y + i_B = 0$.

We have considered *instantaneous readings*, but, in fact, the moving system of the wattmeter, due to its inertia, cannot quickly follow the variations taking place in a cycle, hence it indicates the *average* power.

$$\therefore \quad W_1 + W_2 = \frac{1}{T}\int_0^T i_R\,e_{RB}\,dt + \frac{1}{T}\int_0^T i_Y\,e_{YB}\,dt$$

20-15. Two Wattmeter Method—*Balanced load*

(A.M.I.E. Summer 1984)

If the load is balanced, then power factor of the load can also be found from the two wattmeter readings. The Y-connected load in Fig. 20-25 (*b*) will be assumed inductive. The vector diagram for such a balanced Y-connected load is shown in Fig. 20-26. We shall now consider the problem in terms of r.m.s. values instead of instantaneous values.

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Let V_R, V_Y and V_B be the r.m.s. values of the three-phase voltages and I_R, I_Y and I_B be the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by vectors, the currents lagging behind their phase voltages by ϕ.

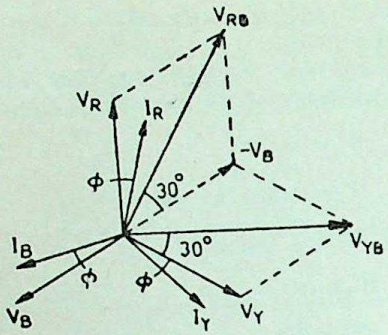

Fig. 20-26

Current through wattmeter W_1 [Fig. 20-25 (*b*)] $= I_R$, p.d. across voltage coil of W_1 is $V_{RB} = V_R - V_B$ (vectorially).

Thus V_{RB} is found by compounding V_R and V_B reversed as shown in Fig. 20-26. It is seen that phase difference between V_{RB} and $I_R = (30 - \phi)$.

$\therefore$ Reading of $W_1 = V_{RB}\, I_R \cos (30 - \phi)$

Similarly, as seen from Fig. 20-25 (*b*)

Current through $W_2 = I_Y$

p.d. across $W_2 = V_{YB} = V_Y - V_B$ ...vectorially

Again V_{YB} is found by compounding V_Y with V_B reversed as shown in Fig. 20-26. The angle between I_Y and V_{YB} is $(30 + \phi)$.

Reading of $W_2 = V_{YB}\, I_Y \cos (30 + \phi)$

Since the load is balanced,

$$V_{RB} = V_{YB} = \text{line voltage, } V_L$$

$$I_R = I_Y = \text{line current, } I_L$$

$\therefore$ $W_1 = V_L\, I_L \cos (30 - \phi)$

$W_2 = V_L\, I_L \cos (30 + \phi)$

$\therefore$ $W_1 + W_2 = V_L\, I_L \cos (30 - \phi) + V_L\, I_L \cos (30 + \phi)$

$= V_L\, I_L [\cos 30° \cos \phi + \sin 30° \sin \phi + \cos 30° \cos \phi - \sin 30° \sin \phi]$

$= V_L\, I_L (2 \cos 30° \cos \phi) = \sqrt{3} V_L\, I_L \cos \phi$

= Total power in the 3-phase load

Hence, the sum of the two wattmeter readings gives the total power consumption in the 3-phase load.

It should be noted that phase sequence *RYB* has been assumed above. Reversal of phase sequence will interchange the readings of the two wattmeters.

20-16. Variations in Wattmeter Readings

It has been shown above that for a *lagging power factor*

$$W_1 = V_L\, I_L \cos (30 - \phi)$$

$$W_2 = V_L\, I_L \cos (30 + \phi)$$

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From this it is clear that individual readings of the wattmeters not only depend on *the load but upon its power factor also.* We will consider the following cases :

(*a*) When $\phi=0$ *i.e.* power factor is unity (*i.e.* resistive load) then

$$W_1=W_2=V_L\ I_L \cos 30°$$

Both wattmeters indicate equal and positive *i.e.* up-scale readings.

(*b*) When $\phi=60°$ *i.e.* power factor$=0{\cdot}5$ (lagging)

Then $W_2=V_L\ I_L \cos (30°+60°)=0$. Hence, the power is measured by W_1 alone.

(*c*) When $90°>\phi>60°$ *i.e.* $0{\cdot}5 >$ p.f. > 0, then W_1 is still positive but reading of W_2 is reversed because the phase angle between the current and voltage is more than 90°. For getting the total power, the reading of W_2 is to be subtracted from that of W_1.

Under this condition W_2 will read 'down-scale' *i.e.* backwards. Hence, to obtain a reading on W_2, it is necessary to reverse either its pressure coil or current coil, usually the former. *All readings taken after reversal of pressure coil are to be taken as negative.*

(*d*) When $\phi=90°$ (*i.e.* p.f.$=0$ as for pure inductive or capacitive load), then

$$W_1=V_L\ I_L \cos (30°-90°)=V_L\ I_L \sin 30°$$

$$W_2=V_L\ I_L \cos (30°+90°)=-V_L\ I_L \sin 30°$$

Then two readings are equal but of opposite sign.

$\therefore \quad W_1+W_2=0$

Summarizing the above we get for a *lagging* power factor,

ϕ	0°	60°	90°
cos ϕ	1	0·5	0
W_1	+ve	+ve	+ve
W_2	+ve $W_1=W_2$	0	−ve $W_1=W_2$

20-17. Leading Power Factor

In the above discussion, lagging angles are taken positive. Now, we will see how wattmeter readings are changed if the power factor becomes leading. For $\phi=+60°$ (lag), W_1 is zero. But for $\phi=-60°$(lead), W_1 is zero. So we find that for angles of lead, the readings of the two wattmeters are interchanged. Hence for a *leading* power factor,

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$$W_1 = V_L I_L \cos (30° + \phi)$$
$$W_2 = V_L I_L \cos (30° - \phi)$$

20-18. Power Factor—*Balanced Load*

In case the load is balanced (and currents and voltages are sinusoidal) and for a *lagging* power factor :

$$W_1 + W_2 = V_L I_L \cos (30° - \phi) + V_L I_L \cos (30° + \phi) \quad ...(i)$$
$$= \sqrt{3} V_L I_L \cos \phi$$

Similarly
$$W_1 - W_2 = V_L I_L \cos (30 - \phi) - V_L I_L \cos (30 + \phi)$$
$$= V_L I_L (2 \times \sin \phi \times 1/2) = V_L I_L \sin \phi \quad ...(ii)$$

Dividing (*ii*) by (*i*), we have

$$\tan \phi = \sqrt{3} \frac{(W_1 - W_2)^*}{(W_1 + W_2)} \quad ...(iii)$$

Knowing $\tan \phi$ and hence ϕ the value of power factor $\cos \phi$ can be found by consulting the trigonometrical tables. It should, however, be kept in mind that if W_2 reading has been taken after reversing the pressure coil *i.e.* if W_2 is negative, then the above relation becomes

$$\tan \phi = \sqrt{3} \frac{W_1 - (-W_2)}{W_1 + (-W_2)} = \sqrt{3} \frac{W_1 + W_2}{W_1 - W_2}$$

We may express power factor in terms of the ratio of the two wattmeter readings as under.

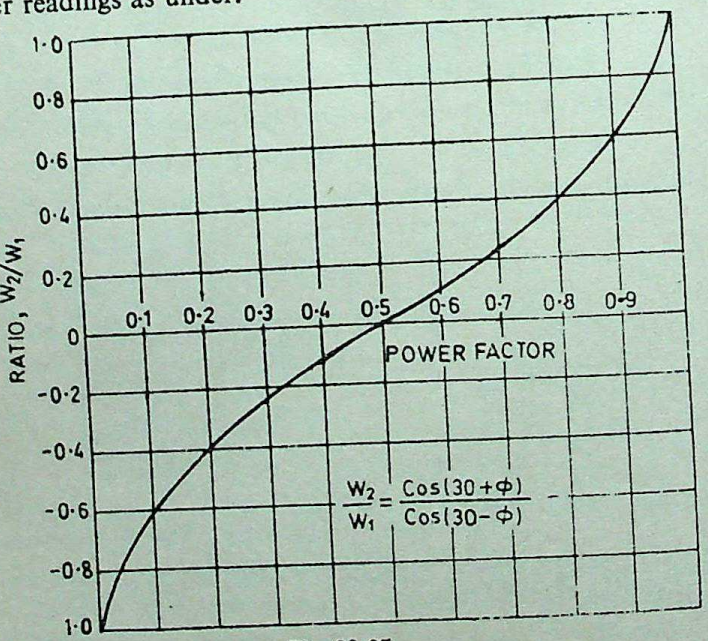

Fig. 20-27

*For a leading power factor, this expression becomes

$$\tan \phi = - \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}.$$

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Let $\frac{\text{smaller reading}}{\text{larger reading}} = \frac{W_2}{W_1}$

Then from equation (*iii*) above,

$$\tan \phi = \frac{\sqrt{3}\,[1-(W_2/W_1)]}{1+(W_2/W_1)} = \frac{\sqrt{3}\,(1-r)}{1+r}$$

Now $\sec^2 \phi = 1+\tan^2 \phi$ or $\frac{1}{\cos^2 \phi} = 1+\tan^2 \phi$

$$\cos \phi = \frac{1}{\sqrt{1+\tan^2 \phi}} = \frac{1}{\sqrt{1+3\left(\frac{1-r}{1+r}\right)^2}}$$

$$= \frac{1+r}{2\sqrt{1-r+r^2}}$$

If r is plotted against $\cos \phi$, then a curve called watt-ratio curve is obtained as shown in Fig. 20-27.

20-19. Reactive Power

We have seen that $\tan \phi = \sqrt{3}\frac{(W_1-W_2)}{W_1+W_2}$

Since the tangent of the angle of lag between phase current and phase voltage of a circuit is always equal to the ratio of the reactive power to the true power (in watts), it is clear that $\sqrt{3}\,(W_1-W_2)$ represents the reactive power (Fig. 20-28). Hence, for a balanced load, the reactive power is given by $\sqrt{3}$ times the difference of the readings of the two wattmeters used to measure the power for a 3-phase circuit by the two wattmeter method. It may also be proved mathematically as follows :

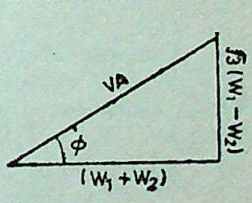

Fig. 20-28

$$\sqrt{3}\,(W_1-W_2) = \sqrt{3}\,[V_L I_L \cos(30°-\phi) - V_L I_L \cos(30°+\phi)]$$
$$= \sqrt{3}\,V_L I_L\,(\cos 30° \cos \phi + \sin 30° \sin \phi - \cos 30° \cos \phi + \sin 30° \sin \phi)$$
$$= \sqrt{3}\,V_L I_L \sin \phi$$

20-20. One Wattmeter Method

In this case, it is possible to apply two-wattmeter method by means of one wattmeter without breaking the circuit. The current coil is connected in any one line and the pressure coil is connected alternately between this and other two lines (Fig. 20-29). The two readings so obtained, for a balanced load, correspond to those obtained by normal two wattmeter method. It should be kept in mind that this method is not of as much universal application as the two wattmeter method

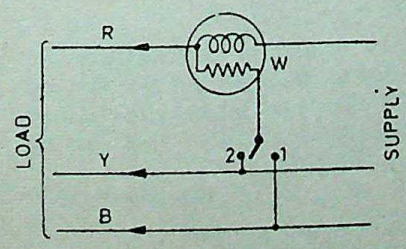

Fig. 20-29

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because it is restricted to fairly balanced loads only. However, it may be conveniently applied, for instance, when it is desired to find the power input to a factory motor in order to check the load upon the motor.

It may be pointed out here that the two wattmeters used in the two wattmeter method (Art. 20-14) are usually combined into a single instrument in the case of switchboard wattmeters which is then known as a polyphase wattmeter. The combination is affected by arranging the two sets of the coils such as to operate on a single moving system resulting in an indication of the total power on the scale.

Example 20-18. *In a 2-wattmeter method of measurement of 3-phase power, the readings of the wattmeters are 3 kW and 1 kW respectively. Calculate the power and power factor for a balanced circuit.*

Solution. $W_1 = 3$ kW ; $W_2 = 1$ kW

Total power $= W_1 + W_2 = \mathbf{4}$ kW

Now $$\tan \phi = \sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$= \sqrt{3} \times (3-1)/(3+1) = 0{\cdot}866$$

$\therefore$ $\phi = \tan^{-1}(0{\cdot}866) = 40°\ 54'$

$\therefore$ $\cos \phi = \cos 40°\ 54' = 0{\cdot}7558$

$\therefore$ power factor $= \mathbf{0{\cdot}7558}$ **(lag)**

Example 20-19. *A 500-V, 3-phase motor has an output of 37·3 kW and operates at a p. f. of 0·85 lagging with an efficiency of 90%. Calculate the reading on each wattmeter connected to measure the input.*

Solution. Motor output $= 37{\cdot}3 \times 1000 = 37{,}300$ W

Motor input $= 37{,}300/0{\cdot}9 = 41{,}440$ W $= 41{\cdot}44$ kW

$$W_1 + W_2 = 41{\cdot}44 \text{ kW}$$

Now $\cos \phi = 0{\cdot}85$

$\phi = 31°\ 48'$, $\tan 31°\ 48' = 0{\cdot}62$

$\therefore$ $$0{\cdot}62 = \sqrt{3}\ \frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\ \frac{W_1 - W_2}{41{\cdot}44}$$

or $$W_1 - W_2 = 0{\cdot}62 \times 41{\cdot}44/\sqrt{3} = 14{\cdot}86 \text{ kW} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$W_1 = \mathbf{28{\cdot}15\ kW},\ W_2 = \mathbf{13{\cdot}29\ kW}$$

Example 20-20. *Two wattmeters connected to measure the input to a balanced 3 phase circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit (a) when both readings are positive (b) when latter reading is obtained after reversing the connection to the current coil of one instrument.*

Solution. (*a*) $W_1 + W_2 = 2500$ W ; $W_1 - W_2 = 1500$ W

$$\tan \phi = \sqrt{3} \times \frac{1500}{2500} = 1{\cdot}0392$$

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$\therefore \quad \phi = 46°\ 6' \quad \text{and} \quad \cos 46°\ 6' = \mathbf{0{\cdot}6934}$

(*b*) $\tan \phi = \sqrt{3}\dfrac{(W_1 + W_2)}{(W_1 - W_2)} = \sqrt{3} \times \dfrac{2500}{1500} = 2{\cdot}887$

$\therefore \quad \phi = 70°\ 54' \quad \therefore \quad \cos 70°\ 54' = \mathbf{0{\cdot}3272}$

Example 20·21. *Three identical coils, each having reactance of 20 Ω and a resistance of 20 Ω are connected in (a) star (b) in delta across 440-V, 3-phase line. Calculate for each method of connection the line current and readings on each of the two wattmeters connected to measure the power.*

Solution. (*a*) **Star Connection**

Impedance/phase $= \sqrt{(20^2 + 20^2)} = 20\sqrt{2}\ \Omega$

$V_{ph} = 440/\sqrt{3}$ V

$I_{ph} = I_L = 440/\sqrt{3} \div 20\sqrt{2} = \mathbf{8{\cdot}98\ A}$

$\tan \phi = 20/20 = 1 \quad \therefore \quad \phi = 45°$

$\therefore$ Power factor $= \cos \phi = \cos 45° = 1/\sqrt{2}$

Total power taken $= \sqrt{3}\ V_L\ I_L \cos \phi$

$= \sqrt{3} \times 440 \times 8{\cdot}98 \times 1/\sqrt{2} = \mathbf{4{,}840\ W}$

If W_1 and W_2 are the wattmeter readings, then

$W_1 + W_2 = 4{,}840 \qquad \ldots(i)$

As $\tan 45° = 1 = \sqrt{3}\dfrac{(W_1 - W_2)}{W_1 + W_2}$

$\therefore \quad W_1 - W_2 = 4840/\sqrt{3} = 2{,}795\ \text{W} \qquad \ldots(ii)$

From equations (*i*) and (*ii*), we find

$W_1 = \mathbf{3817{\cdot}5\ W}$; $W_2 = \mathbf{1022{\cdot}5\ W}$

(*b*) **Delta Connection**

$Z_{ph} = 20\sqrt{2}$; $V_{ph} = 440$ V

$I_{ph} = 440/20\sqrt{2} = 15{\cdot}56$ A

$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 15{\cdot}56 = \mathbf{26{\cdot}95\ A}$

$\cos \phi = \cos 45° = 1/\sqrt{2}$

Total power $= \sqrt{3}\ V_L\ I_L \cos \phi$

$= \sqrt{3} \times 440 \times 26{\cdot}95 \times 1/\sqrt{2} = 14{,}520$ W

(Note : This power is 3 times the *Y*-power)

$\therefore \quad W_1 + W_2 = 14{,}520 \qquad \ldots(iii)$

Again $\tan 45° = 1 = \sqrt{3}\dfrac{(W_1 - W_2)}{W_1 + W_2}$

$\therefore \quad W_1 - W_2 = 14{,}520/\sqrt{3} = 8{,}385\ \text{W} \qquad \ldots(iv)$

From equations (*iii*) and (*iv*), we find

$W_1 = \mathbf{12{,}452{\cdot}5\ W}$ and $\mathbf{W_2 = 3067{\cdot}5\ W}$

Tutorial Problems—32

1. In a 3-phase, star-connected balanced circuit, the line voltage is 400 V, the line current 25 A and the p.f. is lagging. What will be readings of the two wattmeters connected to measure the power in the circuit ? [3,928 W ; 9,929 W]

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2. The power taken by a 3-phase, 400-V motor is measured by the 2-wattmeter method and the readings of the two wattmeters are 460 and 780 W respectively. Estimate the p.f. of the motor and the line current. [0·9129; 1·96 A]

3. Each of the two wattmeters connected to measure the input to a 3-phase circuit reads 10 kW on a balanced load when the p.f. is unity. What does each instrument read when the power factor falls to (*a*) 0·866 lagging (*b*) 0·5 lagging, the total 3-phase power remaining unchanged.

[(*a*) $W_1=13{\cdot}33$ kW ; $W_2=6{\cdot}66$ kW (*b*) $W_1=20$ kW, $W_2=0$]

HIGHLIGHTS

1. For a *Y*-connection :

$$V_{ph}=V_L/\sqrt{3}\ ;\quad I_{ph}=I_L\ ;\ P=\sqrt{3}V_L\ I_L\cos\phi$$

2. For a $\triangle$-connection

$$V_{ph}=V_L;\quad I_{ph}=I_L/\sqrt{3}\ ;\ P=\sqrt{3}\ V_L I_L\cos\phi$$

3. Both in 1-phase and 3-phase systems, the kVA is directly proportional to current *I*. The disadvantage of a low power factor is that current required for a given power is very high, which fact leads to many undesirable results. The power factor can be improved by the following :

(*a*) Synchronous capacitors—used for correcting p.f. in bulk to any desired amount.

(*b*) Static capacitors—used for a group of motors but give a fixed amount of correction.

(*c*) Phase advancers—are fitted to individual motors.

4. Three-phase power is generally measured by the two-wattmeter method. In this method, the current coils of the two wattmeters are connected in *any* two lines and their potential coils to the remaining third line. The sum of the two wattmeter readings gives the total power in the circuit. If the load is balanced, then its power factor can also be calculated from these two readings. The readings of the two wattmeters are :

$$\left.\begin{aligned}(i)\quad & W_1=V_L\ I_L\cos(30°-\phi)\\ & W_2=V_L\ I_L\cos(30°+\phi)\\ (ii)\quad & \tan\phi=\sqrt{3}\,\frac{(W_1-W_2)}{W_1+W_2}\end{aligned}\right\}\ \text{lagging p.f.}$$

$$\left.\begin{aligned}(iii)\quad & W_1=V_L\ I_L\cos(3+\phi)\\ & W_2=V_L\ I_L\cos(30-\phi)\\ & \tan\phi=-\sqrt{3}\,\frac{(W_1-W_2)}{W_1+W_2}\end{aligned}\right\}\ \text{leading p.f.}$$

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TRANSFORMER 21

1. Working Principle of a Transformer 2. Transformer Construction 3. Core-type Transformer 4. Shell-type Transformer 5. Elementary Theory of an Ideal Transformer 6. E.M.F. Equation of a Transformer 7. Voltage Transformation Ratio 8. Transformer with Losses but no Magnetic Leakage 9. Transformer with Winding Resistance but no Magnetic Leakage 10. Equivalent or Referred Resistance 11. Magnetic Leakage 12. Transformer with Resistance and Leakage Reactance 13. Total Approximate Voltage Drop in a Transformer 14. Transformer Tests 15. Open-circuit Test 16. Short-circuit or Impedance Test 17. Voltage Regulation of a Transformer 18. Losses in a Transformer 19. Efficiency 20. Condition for Maximum Efficiency 21. Load Corresponding to Maximum Efficiency 22. All-day Efficiency 23. Three-phase Transformers 24. Highlights.

21-1. Working Principle of a Transformer

(A.M.I.E. Summer 1986)

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed to electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is *mutual induction* between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separate but magnetically linked through a path of low reluctance as shown in Fig. 21-1. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction

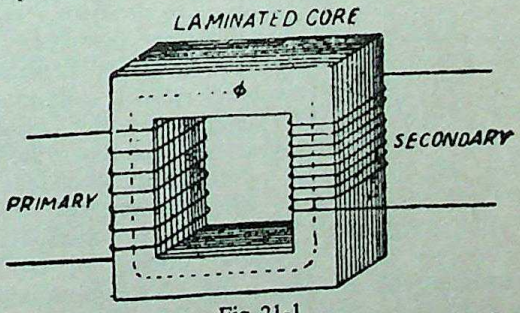

Fig 21-1

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i.e. $e = MdI/dt$). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called *primary winding* and the other, from which energy is drawn out, is called *secondary winding*. In brief, a transformer is a device that

(*i*) transfers electric power from one circuit to another

(*ii*) it does so without change of frequency

(*iii*) it accomplishes this by electromagnetic induction and

(*iv*) where the two electric circuits are in mutual inductive influence with each other.

21-2. Transformer Construction

(*A.M.I.E. Summer 1986*)

The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and from the steel core. Other necessary parts are : some suitable container for the assembled core and windings ; a suitable medium for insulating the core and its windings from its container ; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.

In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with minimum of air-gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations varies from 0·35 mm for a frequency of 50 Hz to 0·5 mm for a frequency of 25 Hz. The core laminations (in the form of strips) are joined as shown in Fig. 21-2. It is seen that the joints in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross-section of the core. Such staggered joints are said to be 'imbricated'.

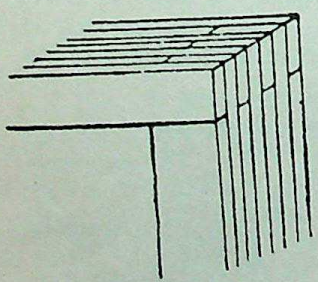

Fig. 21-2

Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated steel core. The two types are known as (*i*) **core-type** and (*ii*) **shell-type**.

Another recent development is *spiral core* or *wound-core type* the trade name being *spirakore* transformer.

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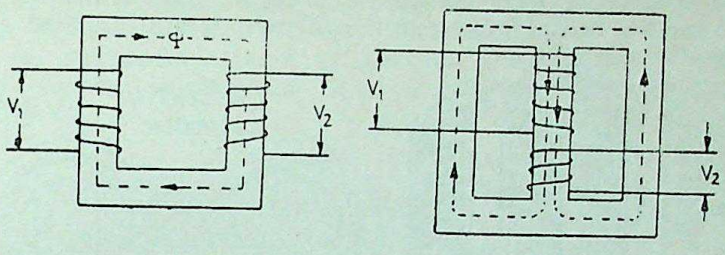


(*a*) (*b*)

Fig. 21-3

In the so-called core-type transformers, the *windings surround a considerable part of the core* whereas in shell-type transformers, the *core surrounds a considerable portion of the windings* as shown schematically in Fig. 21-3 (*a*) and (*b*) respectively.

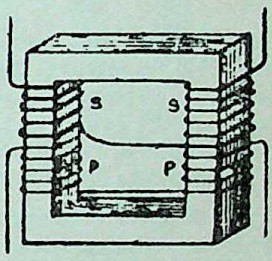
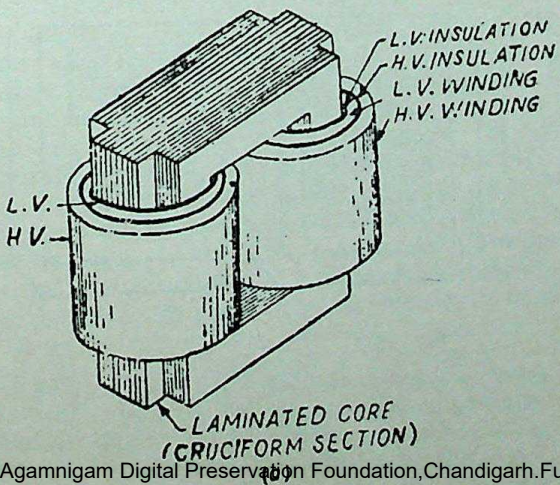


Fig. 21-4

In the simplified diagram for the core-type transformers [Fig. 21-3 (*a*)] the primary and secondary windings are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux. As shown in Fig. 21-4, half the primary and half the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or 'leg') and the secondary on the other.

21-3. Core-type Transformers

The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval rectangular. In small size core-type transformers, a simple rectangular core is useds with cylindrical coil which are either circular or rectangular in form. But for large size core-type transformers, round or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 21-5 (*a*). The circular cylindrical coils are used in most of the core-type transfor-



(*b*)

Fig. 21-5

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mers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from

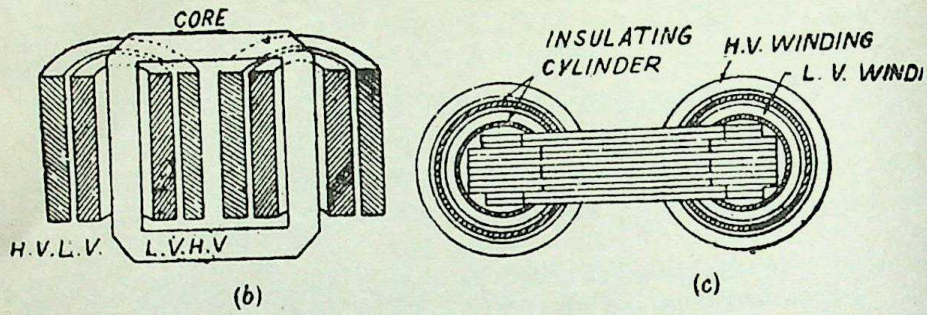

Fig. 21-5

each other by paper, cloth, micarta board or cooling ducts. Fig. 21-5 (*c*) shows the general arrangement of these coils with respect to the core. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. Since the low-voltage (*LV*) winding is easiest to insulate, it is placed nearest to the core.

Because of laminations and insulation, the net or effective core area is reduced, due allowance for which has to be made. It is found that, in general, the reduction in core sectional area due to the presence of paper, surface oxide etc. is of the order of 10% approximately.

21-4. Shell-type Transformers

In their case also, the coils are form-wound but are multi-layer disc type usually wound in the form of pancakes. The different

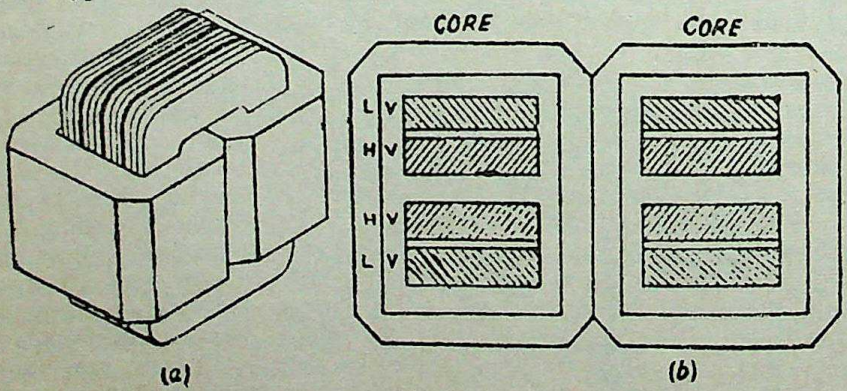

Fig. 21-6

layers of such multi-layer discs are insulated from each other by paper. The complete winding consists of stacked discs with insulation spaces between the coils—the spaces forming horizontal cooling and insulating ducts. A shell type transformer may have a simple rectangular form as shown in Fig. 21-6 (*a*) or it may have a distributed form as shown in Fig. 21-7.

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It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise—a humming sound—during operation.

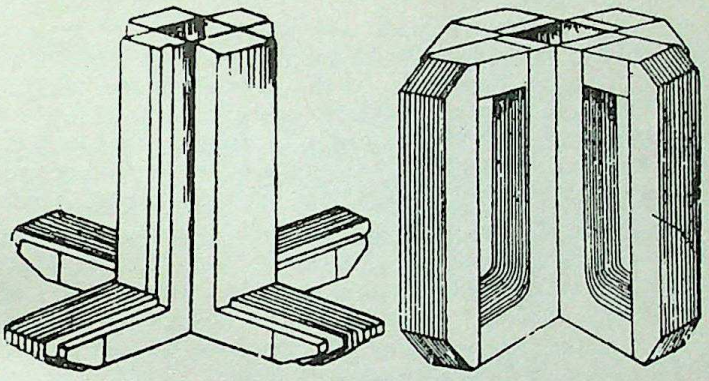

Fig. 21-7

Transformers are generally housed in tightly-fitted sheet-metal tanks filled with special insulating oil. This oil has been highly developed and its function is two-fold. By circulation, it not only keeps the coils reasonably cool, but also provides the transformer with additional insulation not obtainable when the transformer is left in the air.

In cases where a smooth tank surface does not provide sufficient cooling area, the sides of the tank are corrugated or provided with radiators mounted on the sides. Good transformer oil should be absolutely free from alkalies, sulphur and particularly from moisture. The presence of even an extremely small percentage of moisture in the coil is highly detrimental from the insulation view point because it lowers the dielectric strength of the oil considerably. The importance of avoiding moisture in the transformer oil is clear from the fact that even an addition of 8 parts of water in 1,000,000 reduces the insulating quality of the oil to a value generally recognized as below standard. Hence, the tanks are sealed air-tight in smaller units. In the case of large-sized transformers where complete air-tight construction is impossible, chambers known as *breathers* are provided to permit the oil inside the tank to expand and contract as its temperature increases and decreases. The atmospheric moisture is entrapped in these breathers and is not allowed to pass on to the oil. Another thing to avoid in the oil is sledging which is simply the decomposition of oil with long and continued use. Sledging is caused principally by exposure to oxygen during heating and results in the formation of large deposits of dark and heavy matter that eventually clogs the cooling ducts in the transformer.

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No other feature in the construction of a transformer is given more attention and care than the insulating materials, because the life of the unit almost solely depends on the quality, durability and handling of these insulating materials. All the insulating materials are selected on the basis of their high quality and their ability to preserve this high quality even after many years of normal use.

All the transformer leads are brought out of their cases through suitable bushings. There are many designs of these, their size and construction depending on the voltage of the leads. For moderate voltages, porcelain bushings are used to insulate the leads as they come out through the tank. In general, they look almost like the insulators used on the transmission lines. In high-voltage installations, oil-filled or capacitor-type bushings are employed.

The choice of core or shell-type construction is usually determined by cost because similar characteristics can be obtained with both types. For very high-voltage transformers or for multi-winding design, shell-type construction is preferred by many manufacturers. In this type, usually the mean length of coil turn is longer than in a comparable core-type design. Both core and shell forms are used and hence selection is decided by many factors such as voltage rating, kVA rating, weight, insulation stress, heat distribution etc.

Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use :

(*a*) *Oil-filled self-cooled*
(*b*) *Oil-filled water-cooled*
(*c*) *Air-blast type*

Small and medium-sized distribution transformers—so called because of their use on distribution systems as distinguished from line transmission—are of type (*a*). The assembled windings and cores of such transformers are mounted in a welded, oil-tight steel tank provided with a steel cover. After putting the core at its proper place, the tank is filled with purified, high quality insulating oil. The oil serves to convey the heat from the core and the windings to the case from where it is radiated out to the surroundings. For all sizes, the tanks are usually smooth-surfaced, but for larger sizes, the cases are frequently corrugated or fluted to get greater heat-radiation area without increasing the cubical capacity of the tank. Still larger sizes are provided with radiators or pipes.

Construction of very large self-cooled transformers is expensive, a more economical form of construction for such large transformers is provided in the oil-immersed, water-cooled type. As before, the windings and the core are immersed in the oil, but there is mounted near the surface of oil, a cooling coil through which cold water is kept circulating. The heat is carried away by this water. The largest transformers such as those used with high voltage transmission lines, are constructed in this manner.

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Oil-filled transformers are built for outdoor duty, and as these require no housing other than their own, a great saving is thereby effected. These transformers require only periodic inspection.

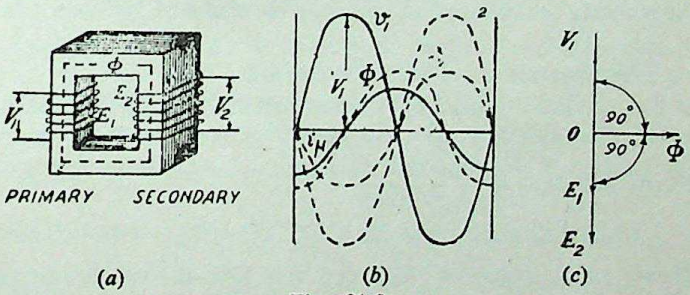
For voltages below 25,000 V, transformers can be built for cooling by means of an air blast. The transformer is not immersed in oil but is housed in a thin sheet-metal box open at both ends through which air is blown from the bottom to the top by means of a fan or blower.

21-5. Elementary Theory of an Ideal Transformer

A.M.I.E. Summer 1981

An ideal transformer is one which has no losses *i.e.* whose windings have no ohmic resistance, so that there is no I^2R loss and no core loss and in which there is no magnetic leakage. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core. It may, however, be noted *that it is impossible to realize such a transformer in practice, yet for convenience, we will start with such a transformer and step by step approach an actual transformer.*

Consider an ideal transformer [Fig. 21-8 (*a*)] whose secondary is open and whose primary is connected to a sinusoidal alternating



(*a*) (*b*) (*c*)

Fig. 21-8

voltage V_1. This potential difference causes the flow of an alternating current in primary winding. Since the primary coil is purely inductive and there is no output (secondary being open), the primary draws the magnetising current I_M only. The function of this current is merely to magnetise the core, it is small in magnitude and lags V_1 by 90°. This alternating current I_M produces an alternating flux Φ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and hence is in phase with it. This changing flux is linked both with the primary and the secondary windings. Therefore, it produces self-induced e.m.f. in the primary. This *self-induced* e.m.f. E_1 is, at every time, equal to and in opposition to V_1. It is also known as *counter* e.m.f. or *back* e.m.f. of the primary.

Similarly, there is produced in the secondary an induced e.m.f. E_2 which is known as *mutually*-induced e.m.f. This e.m.f. is anti-phase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns.

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The instantaneous values of applied voltage, induced e.m.f.s, flux and magnetising current are shown by sinusoidal waves in Fig. 21-8 (*b*). Fig. 21-8 (*c*) shows the vectorial representation of the effective values of the above quantities.

21-6. E.M.F Equation of a Transformer (*A.M.I.E., Winter 1986*)

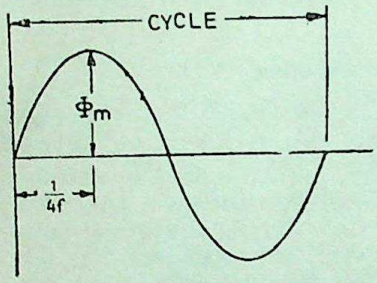

Fig. 21-9

Let N_1 = No. of turns in primary

N_2 = No. of turns in secondary

Φ_m = Maximum flux in the core in webers

$= B_m \times A$

f = frequency of a.c. input in Hz

As shown in Fig. 21-9, the core flux increases from its zero value to maximum value Φ_m in one quarter of the cycle *i.e.* in 1/4 f second.

$\therefore$ Average rate of change of flux $= \dfrac{\Phi_m}{1/4f} = 4f\Phi_m$ W volt

Now, rate of change of flux per turn means induced e.m.f. in volts.

$\therefore$ Average e.m.f. induced/turn $= 4f\Phi_m$ volt

If flux Φ varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1{\cdot}11$$

$\therefore$ r.m.s. value of e.m.f./turn $= 1{\cdot}11 \times 4f\Phi_m = 4{\cdot}44f\Phi_m$ volt

Now, r.m.s. value of induced e.m.f. in the whole of primary winding

$=$ (induced e.m.f./turn) $\times$ No. of primary turns

$\therefore$ $$E_1 = 4{\cdot}44f\, N_1\, \Phi_m = 4{\cdot}44\, fN_1 B_m A$$

Similarly, r.m.s. value of e.m.f. induced in secondary is

$$E_2 = 4{\cdot}44f\, N_2\, \Phi_m = 4{\cdot}44\, fN_2 B_m A$$

In an ideal transformer on no load,

$$V_1 = E_1 \quad \text{and} \quad E_2 = V_2$$

where V_2 is the terminal voltage (Fig. 21-10).

21-7. Voltage Transformation Ratio (K)

From the above equations we get

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

This constant K is known as voltage transformation ratio.

(*i*) If $N_2 > N_1$ *i.e.* $K > 1$, then transformer is called *step-up* transformer.

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(*ii*) If $N_2 < N_1$ *i.e.* $K < 1$, then transformer is known as *step-down* transformer

Again for an *ideal* transformer,

Input VA = Output VA

$$\therefore \quad V_1 I_1 = V_2 I_2$$

or
$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

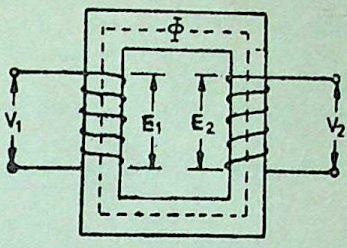

Fig. 21-10

Example 21-1. *The no-load ratio of a 50-Hz, single-phase transformer is 6,000/250-V. Estimate the number of turns in each winding if the maximum flux is 0·06 Wb in the core.*

Solution. Using the transformer e.m.f. equation, we get

$$E_1 = 4{\cdot}44 f N_1 \Phi_m$$

$$\therefore \quad 6{,}000 = 4{\cdot}44 \times 50 \times N_1 \times 0{\cdot}06 \quad \therefore \quad N_1 = \mathbf{450}$$

Similarly, $\quad 250 = 4{\cdot}44 \times 50 \times N_2 \times 0{\cdot}06 \quad \therefore \quad N_2 = \mathbf{19}$

[or $\quad N_2 = KN_1 = (250/6000) \times 450 = 19$]

Example 21-2. *A 2000/200-V, 20-kVA transformer has 66 turns in the secondary. Calculate the primary turns and the primary and secondary full-load currents, neglecting losses.*

Solution. A 2000/200-V transformer is one whose normal primary and secondary voltages are 2000 and 200 V respectively.

Here
$$K = \frac{E_2}{E_1} = \frac{200}{2000} = \frac{1}{10}$$

Now
$$\frac{N_2}{N_1} = K = \frac{1}{10}$$

$$\therefore \quad \frac{66}{N_1} = \frac{1}{10} \quad \text{or} \quad N_1 = \mathbf{660}$$

Secondary output $= 20{,}000$ VA

$$\therefore \quad I_2 = 20{,}000/200 = \mathbf{100\ A}$$

Similarly $\quad I_1 = 20{,}000/2000 = \mathbf{10\ A}$

(or $\quad I_1 = KI_2 = 100/10 = \mathbf{10\ A}$)

Example 21-3. *A 10-kVA, 1-phase transformer has 200 turns on the primary and 20 on the secondary. The primary is connected to a 1500-V, 50-Hz supply. Find (i) the secondary volts on open-circuit (ii) the approximate values of currents in the two windings on full-load and (iii) the maximum value of flux.* (*A M.I.E. Winter 1976*)

Solution. (*i*) $k = N_2/N_1 = 20/200 = 1/10$

$$E_2 = kE_1 = 1500 \times 1/10 = \mathbf{150\ V}$$

(*ii*) $\quad I_2 = 10{,}000/150 = \mathbf{200/3\ A}$

$$I_1 = kI_2 = (200/3)(1/10) = \mathbf{20/3\ A}$$

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(*iii*) $E_1 = 4{\cdot}44 f N_1 \Phi_m$

$1500 = 4{\cdot}44 \times 50 \times 200 \times \Phi_m$; $\Phi_m = 33{\cdot}8$ mWb

Example 21-4. *A single-phase, 50-Hz core-type transformer has square cores of 20 cm side. The permissible flux density in the core is 1·0 Wb/m². Calculate the number of turns per limb on the high and low voltage sides for a 3000/220-V ratio. To allow for insulation of stampings, assume the net iron length to be 0·9 × gross iron length.* (*A.M.I.E. Nov. 1970*)

Solution. Iron-length on one-side of the core (Fig. 21-11)

$= 0{\cdot}9 \times 20 = 18$ cm

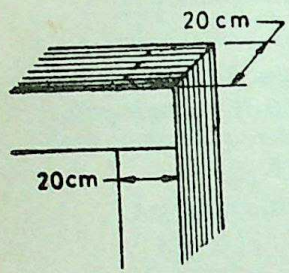

Fig. 21-11

Since, the other side of the square is unaffected by insulation, core area is

$= 18 \times 20 = 360$ cm²

$E_1 = 4{\cdot}44 f N_1 B_m$ A

$3000 = 4{\cdot}44 \times 50 \times N_1 \times 1 \times (360 \times 10^{-4})$

$N_1 = 375$ ...h.v. side

$N_2 = KN_1 = 375\ (220/3000)$

$= 17$ ...h.v. side

21-8. Transformer with Losses but No Magnetic Leakage

We will consider two cases (*i*) when such a transformer is on no-load and (*ii*) when it is loaded.

(*i*) Transformer on No-load (*A.M.I.E. Winter 1981*)

In the above discussion, we assumed an ideal transformer *i.e.* one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing theory. When an *actual* transformer is on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Hence, even when the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply (*i*) iron losses in the core *i.e.* hysteresis loss and eddy current loss and (*ii*) a very small amount of copper loss in primary (there being no Cu loss in secondary at it is open). Hence, the no-load primary input current I_0 is not 90° behind V_1 but lags it by an angle $\phi_0 < 90°$. No-load primary input power

$$W_0 = V_1 I_0 \cos \phi_0$$

where $\cos \phi_0$ is primary power factor under no-load conditions. No-load condition of an actual transformer is shown vectorially in Fig. 21-12.

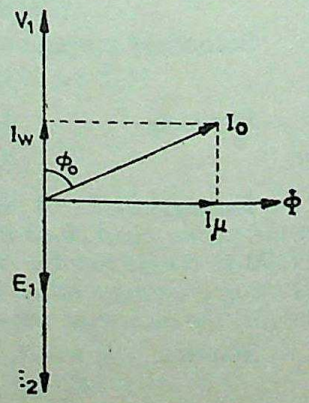

Fig. 21-12

As seen from Fig. 21-12, primary current I_0 has two components :

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(*i*) one in phase with V_1. This is known as *active* or *working* or *iron* loss component I_W, because it supplies iron loss plus a small quantity of primary Cu loss.

$$I_W = I_0 \cos \phi_0$$

(*ii*) the other component is in quadrature with V_1 and is known as idle or reactive or (more commonly) *magnetising* component I_μ because its function is to sustain the alternating flux in the core. It is wattless

$$I_\mu = I_0 \sin \phi_0$$

Obviously, I_0 is the vector sum of I_W and I_μ, hence

$$I_0 = \sqrt{(I_\mu^2 + I_W^2)}.$$

The following points should be noted carefully :

(*i*) The no-load primary current I_0 is very small as compared to the full-load primary current.

(*ii*) As I_0 is very small, no-load primary Cu loss is negligibly small which means *that no-load primary input is practically equal to the iron-loss in a transformer.*

(*iii*) As it is principally the core-loss which is responsible for shift in the current vector, angle ϕ_0 is known as *hysteresis angle of advance.*

Example 21-5. (*a*) *A 2,200/200-V transformer draws a no-load primary current of 0·6 A and absorbs 400 watts. Find the magnetising and iron-loss currents.*

(*b*) *A 2,200/250-V transformer takes 0·5 A at a p. f. of 0·3 on open circuit. Find magnetising and working components of no-load primary currents.*

Solution. Fig. 21-12 may please be referred to.

(*a*) Iron-loss current

$$= \frac{\text{No-load input in watts}}{\text{Primary voltage}}$$

$$= 400/2200 = \mathbf{0{\cdot}182\ A}$$

Now $I_0^2 = I_W^2 + I_\mu^2$

Magnetising component

$$I_\mu = \sqrt{(0{\cdot}6^2 - 0{\cdot}182^2)} = \mathbf{0{\cdot}572\ A}$$

The two components are shown in Fig. 21-12.

(*b*) $I_0 = 0{\cdot}5$ A $\quad \cos \phi_0 = 0{\cdot}3$

$\therefore$ $I_W = I_0 \cos \phi_0 = 0{\cdot}5 \times 0{\cdot}3 = \mathbf{0{\cdot}15\ A}$

$I_\mu = \sqrt{(0{\cdot}5^2 - 0{\cdot}15^2)} = \mathbf{0{\cdot}442\ A}$

Example 21-6. *The no-load current of a transformer is 5 A at 0·25 power factor when supplied at 235-V, 50-Hz. The number of turns on the primary winding is 200. Calculate :*

(a) the maximum value of flux in the core

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(*b*) *the core loss*

(*c*) *the magnetising component.*

(*A.I.M.E. May 1971*)

Solution. (*a*) $E_1 = 4{\cdot}44\, f\, N_1\, \Phi_m$

$\therefore \quad 235 = 4{\cdot}44 \times 50 \times 200\, \Phi_m$

$\therefore \quad \Phi_m = 5{\cdot}29$ **mWb**

(*b*) Primary no-load input represents the core loss

$\therefore$ Core loss $= V_1 I_0 \cos \phi_0 = 235 \times 5 \times 0{\cdot}25 = 294$ **W**

(*c*) Magnetising component is

$$I_\mu = I_0 \sin \phi_0$$

Now $\sin \phi_0 = \sqrt{1 - \cos^2 \phi_0} = \sqrt{15/16} = 0{\cdot}9682$

$\therefore \quad I_\mu = 5 \times 0{\cdot}9682 = 4{\cdot}84$ **A**

Tutorial Problems—33

1. A 3000/200-V, 50-Hz single-phase transformer is built on a core having an effective cross-sectional area of 150 cm^2 and has 80 turns in the low voltage winding. Calculate :

(*a*) the value of maximum flux density in the core

(*b*) the number of turns in the high-voltage winding

[(*a*) 0·75 Wb/m^2 (*b*) 1200]

2. A 40-kVA, 3,300/240-V, 50-Hz, single-phase transformer has 660 turns on the primary. Determine :

(*a*) the number of turns on the secondary

(*b*) the maximum value of the flux in the core

(*c*) the approximate values of primary and secondary full-load currents

Internal drops in the windings are to be ignored.

[(*a*) 48 (*b*) 22·5 mWb (*c*) 12·1 A ; 166·7 A]

3. A double-wound, single-phase transformer is required to step down from 1900 V to 240 V, 50 Hz. It is to have 1·5 V per turn. Calculate the required number of turns on the primary and secondary windings respectively. The peak value of the flux density is required to be not more than 1·2 Wb/m^2. Calculate the required cross-sectional area of the steel core.

If the output is 10 kVA, calculate the secondary current.

[1,267 ; 160 ; 56·4 cm^2 ; 41·75 A]

4. The number of turns in the low-voltage winding of a 200-kVA, 50-Hz, 2,000/250-V single-phase transformer is 25. Calculate :

(*a*) peak-value of the magnetic flux in the core

(*b*) the full-load current in the low-voltage winding

(*c*) the number of turns in the high-voltage winding.

[(*a*) 45 mWb (*b*) 800 A (*c*) 200]

5. The no-load current of a transformer is 15 A at a p.f. of 0·2 when connected to a 460-V, 50-Hz supply. If the primary winding has 550 turns, calculate

(*a*) the magnetising component of the no-load current

(*b*) the iron loss

(*c*) the maximum value of flux in the core.

[(*a*) 14·7 A (*b*) 1,380 W (*c*) 3·77 mWb]

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(*i*) Transformer on Load

When the secondary is loaded, secondary current I_2 is set up. The magnitude and phase of I_2 with respect to V_2 are determined by the characteristics of the load. Current I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. ($=N_2I_2$) and hence its own flux Φ_2 which is in opposition to the main no-load primary flux Φ_0 which is due to I_0. The secondary ampere-turns N_2I_2 are known as *demagnetising* ampere-turns. The opposing secondary flux Φ_2 weakens the primary flux Φ_0 momentarily, hence primary back e.m.f. E_1 tends to be reduced. For a moment, V_1 gains the upper hand over E_1 and hence causes more current to flow in primary.

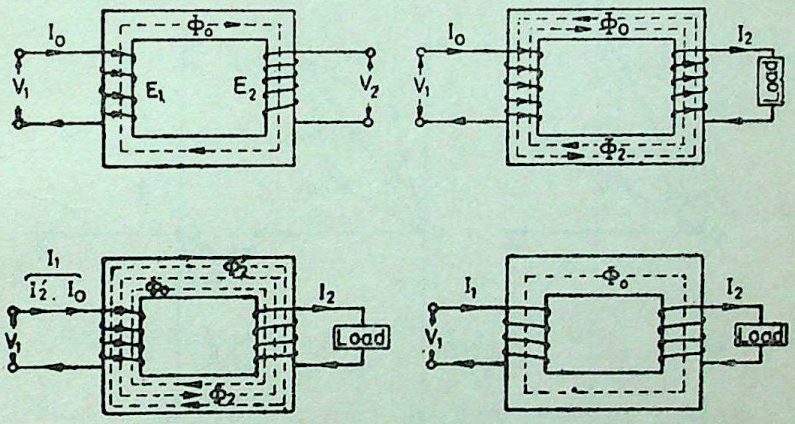

Fig. 21-13

Let the additional primary current be I_2'. It is known as *load component of primary current*. This current is antiphase with I_2. The additional primary m.m.f. N_1I_2' sets up its own flux Φ_2' which is in opposition to Φ_2 (but in the same direction as Φ_0) and is equal to it in magnitude. Hence, Φ_2 and Φ_2' cancel each other out, leaving the flux Φ_0 in the core only. So, we find that the magnetic effects of secondary current I_2 are immediately neutralised by the additional primary current I_2' which is brought into existence exactly at the same instant as I_2. The whole process is illustrated in Fig. 21-13.

Hence, whatever the load condition, *the net flux passing through the core is approximately the same as at no load.* An important deduction is that due to the constancy of core flux at all loads, *the core loss is* also practically the *same under all load conditions* (Art. 21-8).

Since $\Phi_2=\Phi_2'$

$$\therefore \quad N_2 I_2 = N_1 I_2'$$

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$$\therefore \quad I_2' = \frac{N_2}{N_1} \times I_2 = KI_2$$

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Hence, when transformer is on load, the primary winding has two currents in it ; one is I_0 and the other is I_2' which is anti-phase with I_2 and K times in magnitude. *The total primary current is the vector sum of* I_0 *and* I_2'.

In Fig. 21-14 are shown the current vector diagrams for a loaded transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for the capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Fig. 21-14 (*a*), I_2 is secondary current in phase with E_2 (strictly speaking it should be V_2). It causes primary current I_2' which is anti-phase with it and equal to its magnitude ($\because$ $K=1$). Total primary current I_1 is the vector sum of I_0 and I_2' and lags behind V_1 by an angle ϕ_1.

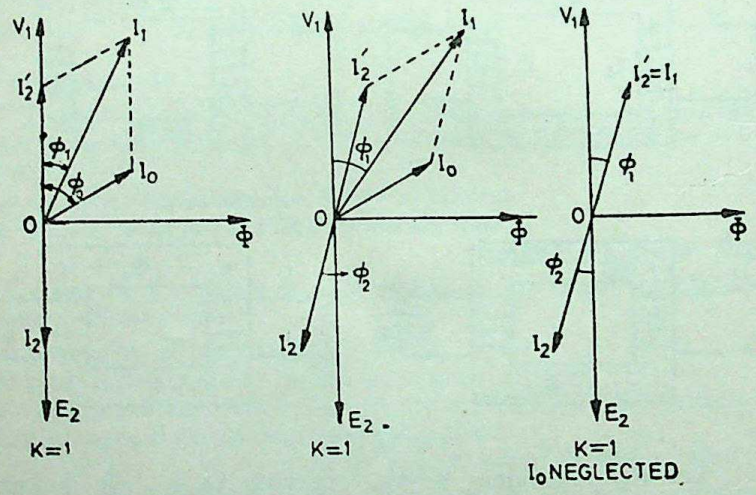

Fig. 21-14

In Fig. 21-14 (*b*), vectors are drawn for an inductive load. Here, I_2 lags E_2 (actually V_2) by ϕ_2. Current I_2' is again anti-phase with I_2 and equal to it in magnitude. As before, I_1 is the vector sum of I_2' and I_0 and lags behind V_1 by ϕ_1.

It will be observed that ϕ_1 is slightly greater than ϕ_2. But if we neglect I_0 as compared to I_2' as in Fig. 21-15 (*c*), then $\phi_1=\phi_2$. Moreover, under this assumption

$$N_1 I_2' = N_1 I_1 = N_2 I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \quad \text{or} \quad I_1 = K I_2$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant. This important relationship is made the basis of current transformer—a transformer which is used with a low-range ammeter for measuring currents in circuits where the direct connection of the ammeter is impracticable.

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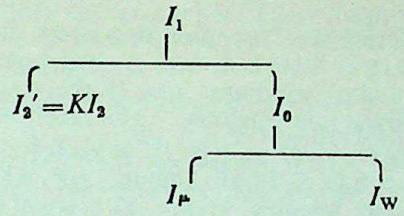
The components of the primary current are as shown below :



Example 21-7. *A single-phase transformer with a ratio of 440/110-V takes a no-load current of 5 A at 0·2 power factor lagging. If the secondary supplies a current of 120 A at a p.f. of 0·8 lagging, estimate the current taken by the primary.*

Solution. The vector diagram is shown in Fig. 21-15

$\cos \phi_2 = 0{\cdot}8$, $\phi_2 = \cos^{-1}(0{\cdot}8)$
$= 36° 54'$

$\cos \phi_0 = 0{\cdot}2$, $\phi_0 = \cos^{-1}(0{\cdot}2)$
$= 78° 30'$

Now $K = V_2/V_1 = 110/440$
$= 1/4$

$\therefore$ $I_2' = KI_2 = 120 \times 1/4$
$= 30$ A
$I_0 = 5$ A

Angle between I_0 and I_2'
$= 78° 30' - 36° 54'$
$= 41° 36'$

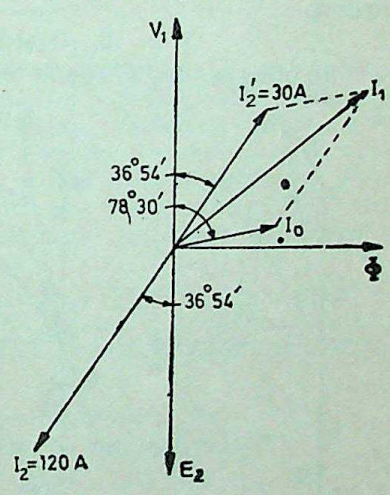


Fig. 21-15

Using parallelogram law of vectors (Fig. 21-15), we get

$I_1 = \sqrt{(5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41° 36')}$
$= \mathbf{34{\cdot}45\ A}$

The resultant current could also have been found by resolving I_2' and I_0 into their X- and Y-components.

Tutorial Problems—34

1. A 2200/200-V transformer takes 1 A at the H.T. side on no-load at a p.f. of 0·385 lagging. Calculate the iron losses. If a load of 50 A at a power factor of 0·8 lagging is taken from the secondary of the transformer, calculate the actual primary current and its p.f. **[847 W ; 5·44 A ; 0·74 lag]**

2. A transformer takes 10 A on no-load at a p.f. of 0·1. The turn ratio is 4 : 1 (step-down). If a load is supplied by the secondary at 200 A and at a p.f. of 0·8, find the primary current and power factor. **[57·2 A ; 0·717 lag]**

3. A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0·8 p.f. lagging, the primary current is 25 A at 0·707 p.f. lagging. Determine graphically or otherwise the no-load current of the transformer and its phase with respect to the voltage. **[59 A, 81°54']**

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21-9. Transformer with Winding Resistance but No Magnetic Leakage

An ideal transformer was supposed to possess no resistance, but in an actual transformer there is always present some resistance of the primary and secondary windings. Due to this resistance, there is some voltage drop in the two windings. The result is that :

(*i*) The secondary terminal voltage V_2 is vectorially less than the secondary induced e.m.f. E_2 by an amount I_2R_2 where R_2 is the resistance of the secondary winding. Hence, V_2 is equal to the vector difference of E_2 and resistive voltage drop I_2R_2.

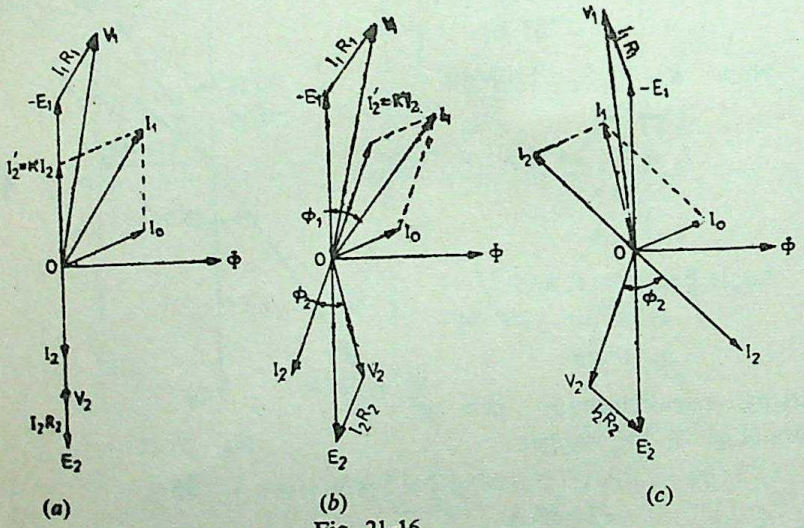
$$V_2 = E_2 - I_2R_2 \qquad \text{...vector difference}$$

In other words, E_2 is the vector sum of V_2 and I_2R_2.

(*ii*) Similarly, primary induced e.m.f. E_1 is equal to the vector difference of V_1 and I_1R_1 where R_1 is the resistance of the primary winding.

$$\therefore \qquad E_1 = V_1 - I_1R_1 \qquad \text{...vector difference}$$

In other words, V_1 equals the vector sum of $(-E_1)$ and I_1R_1



(*a*) (*b*) (*c*)

Fig. 21-16

The voltage vector diagrams for non-inductive, inductive and capacitive loads are shown in Fig. 21-16 (*a*), (*b*) and (*c*) respectively. It should be noted that vector for E_1 has been reversed so that it is written as $(-E_1)$.

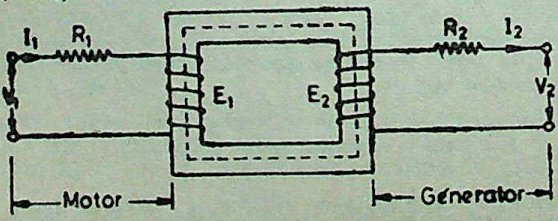


Fig. 21-17

It would be interesting to note that a transformer can be looked upon as a dual machine. Its primary behaves electrically as a motor

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and its secondary as a generator. The voltage equation of the motor applies to the primary (Art. 12-4) whereas generator equation applies to the secondary.

21-10. Equivalent or Referrred Resistances

In Fig. 21-18 is shown a transformer whose primary and secondary windings have resistances of R_1 and R_2 respectively. The resistances have been shown external to the windings.

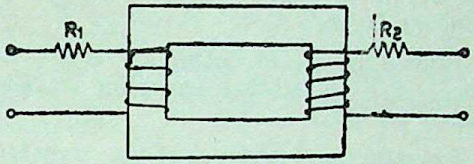

Fig. 21-18

It would now be shown that the resistances of the two windings can be transferred to any one of the two windings. The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. The value of R_2/K^2 will be denoted by R_2'—*the equivalent secondary resistance as referred to primary.*

The copper loss in secondary is $I_2^2R_2$. This loss is supplied by primary which takes a current of I_1. Hence, if R_2' is the *equivalent resistance in primary which would have caused the same loss as* R_2 in secondary, then

$$I_1^2R_2' = I_2^2R_2 \quad \text{or} \quad R_2' = (I_2/I_1)^2 R_2$$

Now, if *we neglect no-load current* I_0, then $I_2/I_1 = 1/K$*

$$\therefore \quad R_2' = R_2/K^2$$

Similarly, equivalent primary resistance as referred to secondary is

$$R_1' = K^2R_1$$

In Fig. 21-19, secondary resistance has been transferred to primary side leaving secondary circuit resistanceless. The resistance $R_1 + R_2' = R_1 + R_2/K^2$ is known as the *equivalent* or *effective resistance of the transformer as referred to primary* and may be designated as R_{01}.

$$\therefore \quad R_{01} = R_1 + R_2' = R_1 + R_2/K^2$$

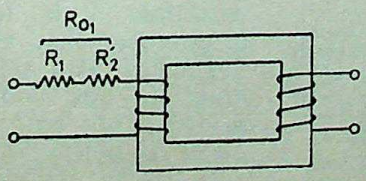

Fig. 21-19

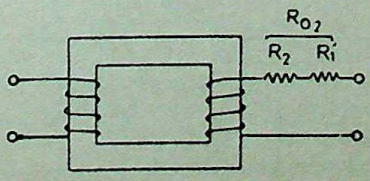

Fig. 21-20

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*Actually $I_2\ I_2' = 1/K$ =and not I_2/I_1. However, if I_0 is neglected, then $I_2' = I_1$.

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Similarly, the *equivalent resistance of the transformer as referred to secondary is*

$$R_{02} = R_2 + K^2 R_1$$

This fact is shown in Fig. 21-20 where all the resistance of the transformer has been concentrated in the secondary winding.

It will be noted that

(*i*) a resistance of R_1 in primary is equivalent to K^2R_1 in secondary. Hence, it is called *the equivalent primary resistance as referred to secondary i.e.* R_1'.

(*ii*) a resistance of R_2 in secondary is equivalent to R_2/K^2 in primary. Hence, it is called the *equivalent secondary resistance as referred to primary i.e.* R_2'.

(*iii*) Total or effective resistance of the transformer as referred to primary is

R_{01} = primary resistance + equivalent secondary resistance referred to primary

$$= R_1 + R_2' = R_1 + R_2/K^2$$

(*iv*) Similarly, total transformer resistance as referred to secondary is

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$$= R_2 + R_1' = R_2 + K^2 R_1$$

It is important to remember that

(*a*) When shifting primary resistance to the secondary, *multiply* it by K^2 *i.e.* (transformation ratio)^2.

(*b*) When shifting secondary resistance to the primary, *divide* it by K^2.

(*c*) However, when shifting voltage from one winding to another only K is used.

21-11. Magnetic Leakage

In the preceding discussion, it was assumed that all the flux linked with primary winding also links with the secondary winding. But, in practice, it is impossible to realize this condition. It is found, in practice, that all the flux linked with primary does not link with the secondary but part of it *i.e.* Φ_{L1} completes its magnetic circuit by passing through air rather than around the core, as shown in Fig. 21-21. This leakage flux is produced when the m.m.f. due ro primary ampere-turns existing between

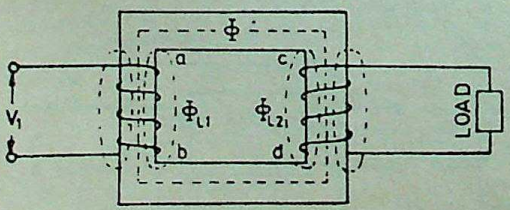

Fig. 21-21

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points *a* and *b*, acts along the leakage paths. Hence, this flux is known as *primary leakage flux* and is proportional to the primary ampere-turns alone, because the secondary turns do not link the magnetic circuit of Φ_{L1}. The flux Φ_{L1} is in time phase with I_1. It induces an e.m.f. e_{L1} in primary but none in secondary.

Similarly, secondary ampere-turns (or m.m.f.) acting across points *c* and *d* set up leakage flux Φ_{L2} which is linked with secondary winding alone (and not with primary turns). This flux Φ_{L2} is in time phase with I_2 and produces a self-induced e.m.f e_{L2} in secondary (and not in primary).

At no-load and light-loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.fs. are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux, linking with each winding, produces a self-induced e.m.f. in that winding Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drop in each series coil is equal to that produced by leakage flux. In other words, a *transformer with magnetic leakage is equivalent to an ideal transformer with an extra inductive coil connected in both primary and secondary circuits* as shown in Fig. 21-22 such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

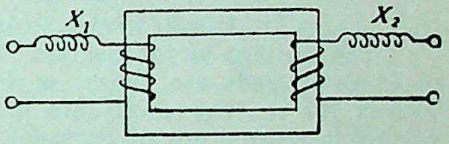

Fig. 21-22

$$\therefore \qquad X_1 = e_{L1}/I_1 \qquad \text{and} \qquad X_2 = e_{L2}/I_2$$

The terms X_1 and X_2 are known as primary and secondary *leakage* reactances respectively.

Following few points should be kept in mind :

(*i*) The leakage flux links one or the other winding but *not both*, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.

(*ii*) The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly, E_2 will have to supply I_2R_2 and I_2X_2.

(*iii*) In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 21-23 because due to their being widely-separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimised by sectionalizing and interleaving the primary and secondary windings.

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21-12. Transformer with Resistance and Leakage Reactance

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In Fig. 21-23 are shown the primary and secondary windings of a transformer with resistances and leakage reactances taken out of the windings. The primary impedance is given by

$$Z_1 = \sqrt{(R_1^2 + X_1^2)}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{(R_2^2 + X_2^2)}$$

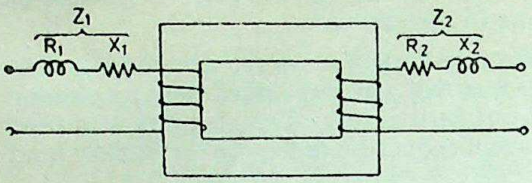

Fig. 21-23

The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1). Hence,

$$\mathbf{V}_1 = \mathbf{E}_1 + \mathbf{I}_1 (R_1 + jX_1) = \mathbf{E}_1 + \mathbf{I}_1 \mathbf{Z}_1$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2 (R_2 + jX_2) = \mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_2$$

The voltage vector diagrams for such a transformer for different kinds of loads are shown in Fig. 21-24. In these diagrams, vectors for resistive drops are drawn parallel to current vector I_1 whereas reactive drops are perpendicular to the current. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

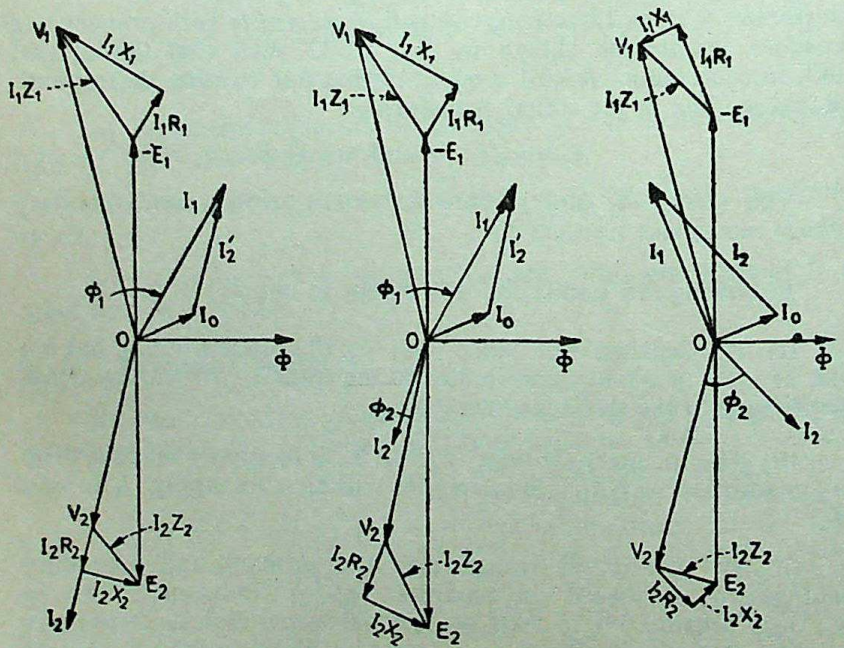

Fig. 21-24

It may be noted that leakage reactance can also be transferred from one winding to the other in the same way as resistance.

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Let X_1' be the primary reactance as referred to secondary. Voltage drop across it when in secondary $= I_2 X_1'$. But voltage drop

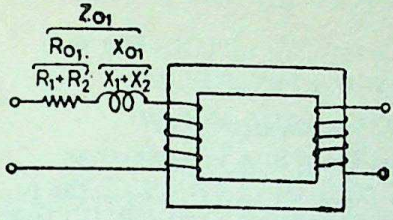

Fig. 21-25

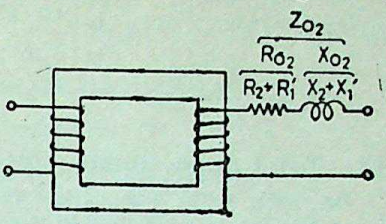

Fig. 21-26

when it is in primary $= I_1 X_1$. Remembering that voltages between the two windings are related through K, we get

$$I_2 X_1' = K I_1 X_1$$

$$\therefore \quad X_1' = K \cdot (I_1/I_2)\, X_1 = K^2 X_1$$

Similarly $\quad X_2' = X_2/K^2$

$$\therefore \quad X_{01} = X_1 + X_2' = X_1 + X_2/K^2$$

$$X_{02} = X_2 + X_1' = X_2 + K^2 X_1$$

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} \qquad \text{...Fig. 21-25}$$

and

$$Z_{02} = \sqrt{(R_{02}^2 + X_{02}^2)} \qquad \text{...Fig. 21-26}$$

Example 21-8. *A 50-kVA, 4400/220-V transformer has $R_1 = 3{\cdot}45\ \Omega$, $R_2 = 0{\cdot}009\ \Omega$. The values of reactances are $X_1 = 5{\cdot}2\ \Omega$ and $X_2 = 0{\cdot}015\ \Omega$. Calculate for the transformer (i) equivalent resistance as referred to primary (ii) equivalent resistance as referred to secondary (iii) equivalent reactance as referred to both primary and secondary (iv) equivalent impedance as referred to both primary and secondary (v) total Cu loss, first using individual resistances of the two windings and secondly, using equivalent resistances as referred to each side.*

Solution. Full-load $I_1 = 50{,}000/4400$

$= 11{\cdot}36$ A (assuming 100% efficiency)

Full-load $I_2 = 50{,}000/220 = 227$ A

$K = 220/4400 = 1/20$

(*i*) $\quad R_{01} = R_1 + R_2/K^2$

$$= 3{\cdot}45 + \frac{0{\cdot}009}{(1/20)^2} = 3{\cdot}45 + 3{\cdot}6 = \mathbf{7{\cdot}05\ \Omega}$$

(*ii*) $\quad R_{02} = R_2 + K^2 R_1 = 0{\cdot}009 + (1/20)^2 \times 3{\cdot}45$

$= 0{\cdot}009 + 0{\cdot}0086 = \mathbf{0{\cdot}0176\ \Omega}$

Also $\quad R_{02} = K^2 R_{01} = (1/20)^2 \times 7{\cdot}05 = 0{\cdot}0176\ \Omega$ (Check)

(*iii*) $X_{01} = X_1 + X_2' = X_1 + X_2/K^2 = 5{\cdot}2 + 0{\cdot}015/(1/20)^2 = \mathbf{11{\cdot}2\ \Omega}$

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1$

$$= 0{\cdot}015 + \frac{5{\cdot}2}{20^2} = \mathbf{0{\cdot}028\ \Omega}$$

Also $\quad X_{02} = K^2 X_{01} = 11{\cdot}2/400 = 0{\cdot}028\ \Omega$ (Check)

(*iv*) $Z_{01} = \sqrt{(R_{01}^2 + X_{01}^2)} = \sqrt{(7{\cdot}05^2 + 11{\cdot}2)^2} = \mathbf{13{\cdot}23\ \Omega}$

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$Z_{02}=\sqrt{(R_{02}^2+X_{02}^2)}=\sqrt{(0{\cdot}0176^2+0{\cdot}028^2)}=0{\cdot}0331\ \Omega$

Also $Z_{02}=K^2Z_{01}=13{\cdot}23/400=0{\cdot}0331\ \Omega$ (Check)

(*v*) Cu loss $=I_1^2R_1+I_2^2R_2$

$=11{\cdot}36^2\times3{\cdot}45+227^2\times0{\cdot}009=909$ W

Also Cu loss $=I_1^2R_{01}=11{\cdot}36^2\times7{\cdot}05=910$ W

$=I_2^2R_{02}=227^2\times0{\cdot}0176=910$ W

21-13. Total Approximate Voltage Drop in a Transformer

As seen from Fig. 21-27 and discussed in Art. 21-12, the primary applied voltage V_1 has to supply (*i*) resistive drop I_1R_1 (*ii*) leakage reactance drop I_1X_1 and (*iii*) has to overcome the back e.m.f. E_1 in the primary winding. In other words, V_1 is the vector sum of E_1, I_1R_1 and I_1X_1. If Z_1 is the primary impedance, then the voltage drop in the primary winding is I_1Z_1. Hence, V_1 is the vector sum of E_1 and I_1Z_1 as shown in Fig. 21-28.

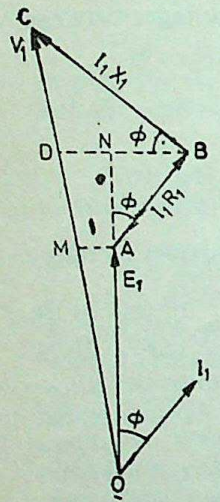

Fig. 21-27

Drop in primary $=I_1Z_1=AC=MC$ (approx.)

$=MD+DC$

From $\triangle$s ANB and DBC, we get the approximate primary voltage drop as

$=AB\cos\phi+BC\sin\phi$

$=I_1R_1\cos\phi+I_1X_1\sin\phi$

Similarly, as seen from Fig. 21-28 the voltage drop in the secondary winding is

$=I_2R_2\cos\phi+I_2X_2\sin\phi$*

The total drop in the transformer is the sum of the two. This total drop can be referred either to primary or secondary. Let us refer it to secondary. The primary drop as referred to secondary is

$=K\times$ primary drop

$=K(I_1R_1\cos\phi+I_1X_1\sin\phi)$

Hence, the total drop as referred to secondary is

$=$ Secondary drop $+K\times$ primary drop

$=(I_2R_2\cos\phi+I_2X_2\sin\phi)$

$+K(I_1R_1\cos\phi+I_1X_1\sin\phi)$

If we neglect the no-load primary current I_0, then

$I_1=KI_2$

Eliminating I_1 from the above equation, we get

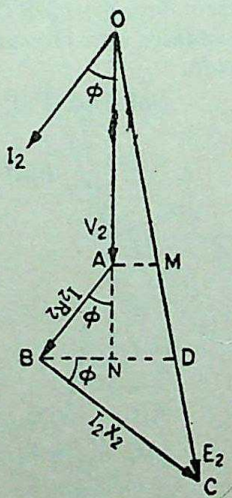

Fig. 21-28

*It being assumed that power factor angles of both primary and secondary are equal.

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Total drop $=(I_2R_2 \cos \phi + I_2X_2 \sin \phi)$
$+K(KI_2R_1 \cos \phi + KI_2X_1 \sin \phi)$

$= I_2[(R_2+K^2R_1) \cos \phi + (X_2+K^2X_1) \sin \phi]$

$= I_2(R_{02} \cos \phi + X_{02} \sin \phi)$

Hence, the total voltage drop of the transformer as referred to secondary is

$= I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$

This is the drop for a *lagging* power factor. However, if the power factor is leading, then the sine term will become negative Hence, for a *leading* power factor the expression is

$= I_2(R_{02} \cos \phi - X_{02} \sin \phi)$

Hence, if the primary voltage V_1 is kept constant, the secondary terminal voltage will decrease by this amount from no-load to full-load.

Example 21-9. *A 230/460-V transformer has a primary resistance of 0·2 Ω and a reactance of 0·5 Ω and the corresponding values for the secondary are 0·75 Ω and 1·8 Ω respectively. Find the secondary terminal voltage when supplying (a) 10 A at 0·8 power factor lagging (b) 10 A at p. f. 0·8 leading.*

Solution. $K=2, \quad R_{02}=0{\cdot}75+4(4\times0{\cdot}2)=1{\cdot}55\ \Omega$

Similarly $X_{02}=1{\cdot}8+(4\times0{\cdot}5)=3{\cdot}8\ \Omega$

$I_2=10$ A, $\cos \phi = 0{\cdot}8$, $\sin \phi = 0{\cdot}6$

(*a*) Voltage drop $=I_2(R_{02} \cos \phi + X_{02} \sin \phi)$
$=10(1{\cdot}55\times0{\cdot}8+3{\cdot}8\times0{\cdot}6)=35{\cdot}2$ V

$\therefore \quad V_2=460-35{\cdot}2=$ **424·8 V**

(*b*) Voltage drop $=10(1{\cdot}55\times0{\cdot}8-3{\cdot}8\times0{\cdot}6)=-10{\cdot}4$ V

$V_2=460-(-10{\cdot}4)=$ **470·4 V**

21-14. Transformer Tests

The transformer constants or parameters can be easily determined by two tests (*i*) *open-circuit test* and (*ii*) the *short-circuit test*. These tests are very economical and convenient, because they furnish the required information without actually loading the transformer. In fact, the testing of every large a.c. machinary consists of running two tests similar to the open and short-circuit tests of a transformer.

21-15. Open-circuit or No-load Test *(A.M.I.E. Winter 1983)*

The main purpose of this test is to determine no-load losses.

One winding of the transformer—whichever is convenient but usually high voltage winding—is kept open and the other is connected to a supply of normal voltage and frequency (Fig. 21-29). A watt-

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meter W, voltmeter V and ammeter A are connected in the low-voltage winding *i.e.* primary winding in the present case. Fig. 21-29

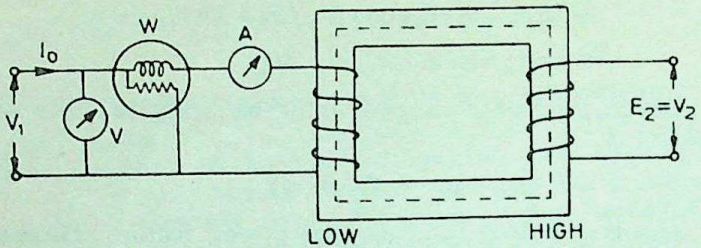

Fig. 21-29

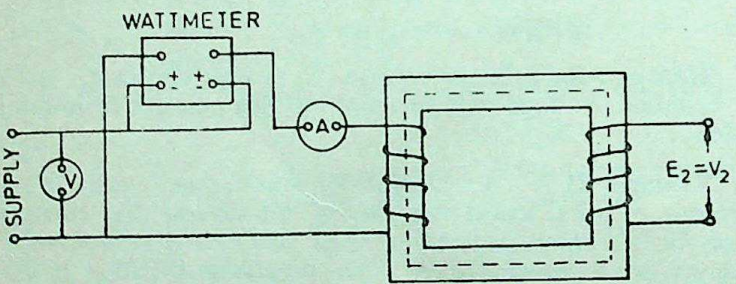

Fig. 21-30

shows the simplified diagram whereas Fig. 21-30 shows actual connections. With normal voltage applied to the primary, normal flux will be set up in the core, hence normal iron losses will occur which are recorded by the wattmeter. As the primary no load current I_0 (as measured by ammeter) is small (usually 2 to 10% of rated load current), Cu loss is negligibly small in primary and nil in secondary (being open). Hence, the *wattmeter reading represents practically the core-loss under no-load conditions* (*and which is the same for all loads as pointed out in Art. 21-8*).

Sometimes a high resistance voltmeter is connected across the secondary. The reading of the voltmeter gives the induced e.m.f. in the secondary winding. This helps in finding the transformation ratio K.

21-16. Short-circuit or Impedance Test

This is an economical method for determining the following :

(*i*) Equivalent impedance (Z_{01} or Z_{02}), leakage reactance (X_{01} or X_{02}) and resistance (R_{01} or R_{02}) of the transformer as referred to the winding in which the measuring instruments are placed.

(*ii*) Cu loss at full-load (and at any desired load). This loss is used in calculating the efficiency of the transformer.

(*iii*) Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

In this test, one winding—usually the low-voltage winding—is solidly short-circuited by a thick conductor (or through an ammeter

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which may serve the additional purpose of indicating rated load current) as shown in Fig. 21-31.

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency (though for Cu losses it is not essential) is applied to the primary and is cautiously increased till full-load currents are flowing both in primary and secondary (as indicated by the respective ammeters).

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced in the core is also a small percentage of its normal value (because flux is proportional to the voltage as shown in the e.m.f. equation of the transformer in Art. 21-6). Hence, core losses are very small with the result that the wattmeter reading represents the full-load Cu loss or I^2R loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss. If V_{SC} is the voltage required to calculate rated load current on short circuit, then

$$Z_{01}=V_{SC}/I_1$$

Also $$W=I_1^2R_{01} \quad \therefore \quad R_{01}=W/I_1^2$$

$\therefore$ $$X_{01}=\sqrt{(Z_{01}^2-R_{01}^2)}$$

Example 21-10. *The ratio of turns of a single-phase transformer is 8, the resistance of the primary and secondary windings are 0·85 Ω and 0·012 Ω respectively and the leakage reactances of these windings are 4·8 Ω and 0·07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary when the secondary terminals are short-circuited. Ignore the magnetising current.*

Solution. Turn ratio $=N_1/N_2=8 \quad \therefore \quad K=1/8$

Now $$R_{01}=R_1+R_2/K^2$$
$$=0·85+0·012\times 64=1·618\ \Omega$$
$$X_{01}=X_1+X_2/K^2$$
$$=4·8+64\times 0·07=9·28\ \Omega$$
$$Z_{01}=\sqrt{(1·618^2+9·28^2)}=9·419\ \Omega$$
$$I_1=KI_2=150/8\ \text{A}$$

Now $$V_{SC}=I_1Z_{01}=\frac{150}{8}\times 9·419=\mathbf{176·6\ V}$$

21-17. Voltage Regulation of a Transformer

It has been discussed in Art. 21-13 that when a transformer is loaded, its secondary terminal voltage falls (for a lagging p.f.) *provided the applied primary voltage V_1 is held constant.* This variation in secondary terminal voltage from no-load (N.L.) to full-load (F.L.) is called the (voltage) regulation of the transformer and is usually expressed as a percentage of the secondary no-load voltage.

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$\therefore$ Percentage regulation

$$= \frac{\text{Sec. Volt (N.L.)} - \text{Sec. Volt (F.L.)}}{\text{Sec. Volt (N.L.)}} \times 100$$

Let ${}_0V_2$ = secondary terminal voltage on no-load

V_2 = secondary terminal voltage on load

then % regulation $= \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$

Since ${}_0V_2 - V_2 = I_2 (R_{02} \cos \phi \pm X_{02} \sin \phi)$

$\therefore$ %age regn. $= \frac{I_2 (R_{02} \cos \phi \pm X_{02} \sin \phi)}{{}_0V_2}$

21-18. Losses in a Transformer *(A.M.I.E. Summer 1983)*

In a static transformer, there are no friction or windage losses. Hence, the only losses occurring are :—

(*i*) **Core or Iron Loss.** It includes both hysteresis loss and eddy current loss. Because the core flux in a transformer remains practically constant for all loads (its variation being 1 to 3% from no-load to full-load), the core loss is practically the same at all loads.

Hysteresis loss $W_h = \eta B_{max}^{1.6} f$ V watt

Eddy current loss $W_e = P B_{max}^2 f^2 t^2$ watt

These losses are minimized by using steel of high silicon content for the core and by using very thin laminations. Iron or core loss is found from the *O.C.* test. *The input of the transformer when on no-load measures the core loss.* This loss is found from the no-load test (Art. 21-16).

(*ii*) **Copper Loss.** This loss is due to the ohmic resistance of the transformer windings. Total Cu loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$. It is clear that Cu loss is proportional to (current)^2 or (kVA)^2. In other words, Cu loss at half the full-load is one-fourth of that at full-load. The value of Cu loss is found from the short-circuit test.

21-19. Efficiency of a Transformer *(A.M.I.E. Winter 1983)*

As is the case with other types of electrical machines, the efficiency of a transformer at a particular load and power factor is defined as the output divided by the input—the two being measured in the same units (either watts or kilowatts).

$\therefore$ $\text{Efficiency} = \frac{\text{output}}{\text{input}}$

But a transformer being a highly efficient piece of equipment, has very small losses, hence it is impractical to try to measure transformer efficiency by measuring input and output. These quantities

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are nearly of the same size. A better method is to determine the losses and then to calculate the efficiency from

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{output}}{\text{output} + \text{Cu loss} + \text{Iron loss}}$$

or
$$\eta = \frac{\text{Input} - \text{losses}}{\text{Input}} = 1 - \frac{\text{losses}}{\text{input}}$$

It may be noted here that efficiency is based on power output in watts and not in volt-amperes, although losses are proportional to VA. Hence, at any volt-ampere load, the efficiency depends on power factor, being maximum at a power factor of unity.

Efficiency can be computed by determining core loss from no-load or open-circuit test and Cu loss from the short-circuit test.

21-20. Condition for Maximum Efficiency

(*A.M.I.E. Winter 1983*)

$$\text{Cu loss} = I_1^2 R_{01} \quad \text{or} \quad I_2^2 R_{02} = W_c$$

$$\text{Iron loss} = \text{hysteresis loss} + \text{eddy current loss}$$

$$= W_h + W_e = W_i$$

Considering primary side,

$$\text{Primary input} = V_1 I_1 \cos \phi_1$$

$$\eta = \frac{V_1 I_1 \cos \phi_1 - \text{losses}}{V_1 I_1 \cos \phi_1}$$

$$= \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi_1} + \frac{W_i}{V_1 I_1^2 \cos \phi_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\frac{R_{01}}{V_1 \cos \phi_1} = \frac{W_i}{V_1 I_1^2 \cos \phi_1} \quad \text{or} \quad W_i = I_1^2 R_{01} \text{ or } = I_2^2 R_{02}$$

or **Cu loss = Iron loss**

The output current corresponding to maximum efficiency is

$$I_2 = \sqrt{\frac{W_i}{R_{02}}}$$

It is this value of the output current which will make the Cu loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

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In Fig. 21-32, Cu and iron losses are plotted as a percentage of power input and the efficiency curve as deduced from these is also shown. It is obvious that the point of intersection of the Cu and iron loss curves gives the point of maximum efficiency. It would be seen that the efficiency is high and is practically constant from 25% load to 25% overload.

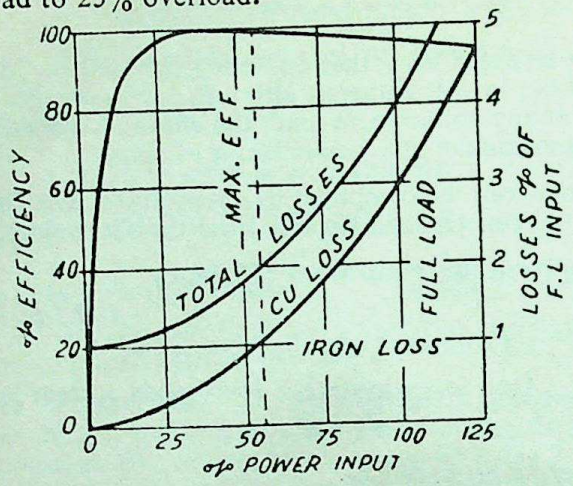

Fig. 21-32

21-21. Load Corresponding to Maximum Efficiency

If full-load Cu loss and iron loss of a transformer are known, then the load at which it will operate at maximum efficiency can be easily found.

Let W_i be the iron loss and W_{cu} the full-load Cu loss. Efficiency will be maximum at that load for which Cu loss becomes equal to W_i. If x is that load, then Cu loss when the transformer delivers this output, will be W_i *i.e.* equal to iron loss. So W_i becomes the Cu loss for x kVA output. Remembering that Cu loss is proportional to kVA^2, we have

$$W_{cu} \propto (\text{Full-load kVA})^2$$

$$W_i \propto x^2$$

$$\therefore \quad \left(\frac{x}{\text{F.L. kVA}}\right)^2 = \frac{W_i}{W_{cu}}$$

or

$$x = \text{F.L. kVA} \times \sqrt{\frac{W}{W_{cu}}}$$

$$\therefore \quad x = \text{F.L. kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{Full-load Cu loss}}}$$

Example 21-11. *A 200/400-V, 10-kVA, 50-Hz single-phase transformer has, at full-load, a copper loss of 120 W. If it has an efficiency of 98% at full-load unity power factor, determine the iron losses.*

What would be the efficiency of the transformer at half load 0·8 power factor lagging ? *(A.M.I.E. Summer 1985)*

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Solution. Output at full-load u.p.f. = 10 kW

Input = 10/0·98 = 10·204 kW

Total full-load losses = 10·204 − 10 = 0·204 kW = 204 W

Iron losses = 204 − 120 = 84 W

Half-load 0·8 p.f.

Cu loss = $(\frac{1}{2})^2 \times 120 = 30$ W, Iron loss = **84 W**

Total loss = 30 + 84 = 114 W = 0·114 kW

Output = (10/2) × 0·8 = 4 kW

$$\therefore \quad \eta = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{4}{4 \cdot 114} = 0 \cdot 97 \text{ or } 97\%$$

Example 21-12. *A 50-kVA 5000/500-V, 50-Hz, 1-phase transformer has the high-voltage winding with a resistance of 8 Ω and low-voltage winding with a resistance of 0·06 Ω. The no-load losses of the transformer amount to 1,000 W. Calculate the efficiency of the transformer when delivering its full-rated output at a power factor of 0·8. With the same power factor, what will be the efficiency if the output drops down to 80% of the rated value.*

Solution. As stated in Art. 21-15, the no-load loss of a transformer is practically equal to the iron loss. Hence, iron loss = 1000 W.

Full load Cu loss $= I_2^2 R_{02}$

$K = 500/5{,}000 = 1/10$

$R_{02} = R_2 + K^2 R_1$

$= 0 \cdot 06 + (1/10)^2 \times 8 = 0 \cdot 14\ \Omega$

Full-load $I_2 = 50{,}000/500 = 100$ A

$\therefore$ F.L. Cu loss $= 100^2 \times 0 \cdot 14 = 1400$ W

Total loss $= 1000 + 1400 = 2 \cdot 4$ kW

F.L. output $= 50 \times 0 \cdot 8 = 40$ kW

$\therefore \quad \eta = 40/42 \cdot 4 = 0 \cdot 9434$ or **94·34%**

When Output Drops

Cu loss $= 0 \cdot 8^2 \times 1400 = 896$ W

Iron loss $= 1000$ W as before

Total losses $= 1896$ W $= 1 \cdot 896$ kW

Output $= 80\%$ of F.L. output

$= 0 \cdot 8 \times (50 \times 0 \cdot 8) = 32$ kW

$\therefore \quad \eta = 32/33 \cdot 896 = 0 \cdot 9439$ or **94·39%**

Example 21-13. *In a 25-kVA, 3300/230-V, 1-phase transformer the iron and full-load copper losses are respectively 350 and 400 W. Calculate (i) the efficiency at half-load 0·8 p.f. and (ii) the load at which the efficiency is maximum.* ***(A.M.I.E. Winter 1985)***

Solution. (*i*) **Half-load, 0·8 p.f.**

Cu loss at half-load = $(1/2)^2 \times$ F.L. loss

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$400/4 = 100$ W

Output $= (25/2) \times 0{\cdot}8 = 10$ kW

Input $= 10 + 0{\cdot}1 = 10{\cdot}1$ kW

$$\therefore \quad \eta = \frac{10 \times 100}{10{\cdot}1} = 99{\cdot}02\%$$

(*ii*) Load for maximum efficiency

$$= \text{Full-load kVA} \times \sqrt{\frac{\text{iron loss}}{\text{F.L. Cu loss}}}$$

$$= 25 \times \sqrt{350/400} = 23{\cdot}4 \text{ kVA}$$

Tutorial Problems—35

1. Iron loss of a 100-kVA single-phase transformer is 1·5 kW and full-load copper loss is 1 kW. Calculate transformer efficiency at :

(*a*) full-load unity p.f. and

(*b*) half full-load 0·8 p.f. [97·56% ; 95·81%]

2. The primary resistance of a 440/110-V transformer is 0·5 Ω and the secondary resistance is 0·04 Ω. When 440 V is applied to the primary and the secondary is left open-circuited, 200 W is drawn from the supply. Find the secondary current which will give maximum efficiency and calculate this efficiency for a load having unity p.f. [53 A ; 93·58%]

3. A 200-kVA transformer has an efficiency of 98% at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate (*a*) iron loss (*b*) Cu loss at F.L. (*c*) efficiency at half-load. Ignore magnetising current and assume a p.f. of 0·8 at all loads.

[(*a*) 1·777 kW (*b*) 2·09 kW (*c*) 97·92 %]

4. At full-load the Cu and iron losses in a 100-kVA transformer are each equal to 2·5 kW. Find the efficiency at a load of 65 kVA power factor 0·8. [93·58 %]

5. A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1·5 kW at full-load current. Calculate the efficiency at (*a*) 100 kVA output at 0·8 p.f. lagging (*b*) 50 kVA output at unity power factor.

[(*a*) 97·44% (*b*) 98·09%]

6. A 150-kVA single-phase transformer has a core loss of 1·5 kW and a full load Cu loss of 2 kW. Calculate the efficiency of the transformer (*a*) at full load 0·8 p.f. lagging (*b*) at one-half full-load unity p.f. Determine also the secondary current at which the efficiency is maximum if the secondary voltage is maintained at its rated value of 240 V. [(*b*) 97·71% (*b*) 97·4% ; 541 A]

7. A 500-kVA, single-phase transformer has an iron loss of 2·5 kW and the maximum efficiency at 0·8 p.f. occurs when the load is 268 kW.

Calculate (*a*) the maximum efficiency at unity power factor and (*b*) the efficiency on full-load at 0·71 power factor. [(*a*) 98·53% at 335 kW (*b*) 97·78%]

8. A 500/250—V, 1-phase transformer has constant losses of 150 W. When delivering an output of 8 kW to a load, the transformer takes 21 A at a p.f. of 0·8 from 500 V supply mains. Calculate the variable losses of the transformer at this load. [250 W]

9. Calculate the efficiency at full-load of a 500 kVA transformer from the following results when the p.f. of the load is 0·8 (lag).

Primary volts = 4000

Secondary volts = 500

Primary resistance = 0·2 Ω

Secondary resistance = 0·002 Ω

Iron-loss (by no-load test) = 2,500 W [98·2 %]

10. Find the efficiency of a 100—kVA transformer at (*i*) half load (*ii*) full-load both at 0·8 p.f. lagging if the copper loss at full-load is 2 kW and iron loss is 1·2 kW. [95·94% ; 96·16%]

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11. A 400/230-V, 150-kVA, 1-phase transformer has a core loss of 1 kW and full-load copper loss of 2 kW. Calculate (*a*) the kVA load for maximum efficiency and (*b*) the efficiency at 50 percent full-load at 0·8 p.f. lagging. **[106·05 kVA ; 97·57%]**

21-22. All-day Efficiency

The ordinary or commercial efficiency of a transformer is given by the ratio

$$\eta = \frac{\text{Output in watts}}{\text{Input in watts}}.$$

But there are certain types of transformers whose performance cannot be judged by this efficiency. Transformers used for supplying lighting and general network *i.e.* distribution transformers have their primaries energised all the twenty-four hours, although their secondaries supply little or no-load much of the time during the day except during the house-lighting period. It means that whereas core loss occurs throughout the day, the Cu loss occurs only when the transformer is loaded. Hence, it is considered a good practice to design such transformers so that core losses are very low. The Cu losses are relatively less important, because they depend on the load. The performance of such a transformer should be judged by all-day efficiency also known as 'operational efficiency' which is computed on the basis of energy consumed during certain time period, usually a day of 24 hours.

$$\therefore \qquad \eta_{all-day} = \frac{\text{output in kWh}}{\text{input in kWh}} \text{ (for 24 hours)}$$

To find this all-day efficiency or (as it is also called) energy efficiency, we have to know the load cycle on the transformer *i.e.* how much and how long the transformer is loaded during 24 hours. Practical calculations are facilitated by making use of a *load factor.*

Example 21-14. *A 100-kVA lighting transformer has a full-load loss of 3 kW, the losses being equally divided between iron and copper. During a day, the transformer operates on full-load for 3 hours, one half-load for 4 hours, the output being negligible for the remainder of the day. Calculate the all-day efficiency.*

Solution. It is usually assumed that lighting transformers have a load p.f. of unity.

Iron loss for 24 hours $= 1{\cdot}5 \times 24 = 36$ kWh

Full-load Cu loss $= 1{\cdot}5$ kW

Cu loss for 3 hours on full-load $= 3 \times 1{\cdot}5 = 4{\cdot}5$ kWh

Cu loss at half full-load $= 1{\cdot}5/4$ kW

Cu loss for 4 hours at half F.L. $= 4 \times (1{\cdot}5/4) = 1{\cdot}5$ kWh

Total losses in 24 hours $= 36 + 4{\cdot}5 + 1{\cdot}5 = 42$ kWh

Total output in 24 hours $= (100 \times 3) + (50 \times 4) = 500$ kW

$$\eta_{all-day} = \frac{500 \times 100}{(500 + 42)} = 92{\cdot}92\%$$

Note. It may be noted that the ordinary or commercial or power efficiency of this transformer is $= 100/103 = 97{\cdot}1\%$.

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Example 21-15. *A 5-kVA distribution transformer has a full-*

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load efficiency of 95 per cent at which the copper loss is equal to iron loss. The transformer is loaded in a 24 hour period as follows :

No-load for 10 hours, $\frac{1}{4}$ full load for 7 hours ; $\frac{1}{2}$ full-load for 5 hours and full-load for 2 hours. Calculate the all-day efficiency of the transformer.

Solution. Being lighting transformer it has a load p.f. of unity *i.e.* $\cos \phi = 1$.

Full-load output $= 5$ kVA $\times 1 = 5$ kW

F.L. loss $=$ input $-$ output $= (5/0{\cdot}95 - 5) = 264$ W

F.L. Cu loss $=$ Iron loss $= 132$ W $= 0{\cdot}132$ kW

Iron loss for 24 hr $= 24 \times 0{\cdot}132 = 3{\cdot}168$ kWh

Cu loss during 7 hr $= (0{\cdot}132/16) \times 7 = 0{\cdot}058$ kWh

Cu loss during 5 hr $= (0{\cdot}132/4) \times 5 = 0{\cdot}165$ kWh

Cu loss during 2 hr $= 0{\cdot}132 \times 2 = 0{\cdot}264$ kWh

Total loss in 24 hr $= 3{\cdot}168 + (0{\cdot}058 + 0{\cdot}165 + 0{\cdot}264) = 3{\cdot}655$ kWh

Output in 24 hr $= (5 \times 2) + (5 \times 5/2) + (7 \times 5/4) = 31{\cdot}25$ kWh

All-day $\eta = 31{\cdot}25 \times 100/(31{\cdot}25 + 3{\cdot}655) = \mathbf{89{\cdot}54\%}$

21-23. Three-phase Transformers

Large scale generation of electric power is usually 3-phase at generated voltages of 132-kV or somewhat higher. Transmission is generally accomplished at higher voltages of 66, 110, 132, 220 and 275 kV for which purpose 3-phase transformers are necessary to step up the generated voltage to that of the transmission line. Next at load centres, the transmission voltages are reduced to distribution voltages of 6600, 4600 and 2300 volts. Further, at most of the consumers, the distribution voltages are still reduced to utilization voltages of 420, 220 or 110 volts. Years ago, it was a common practice to use suitably interconnected three single-phase transformers instead of a

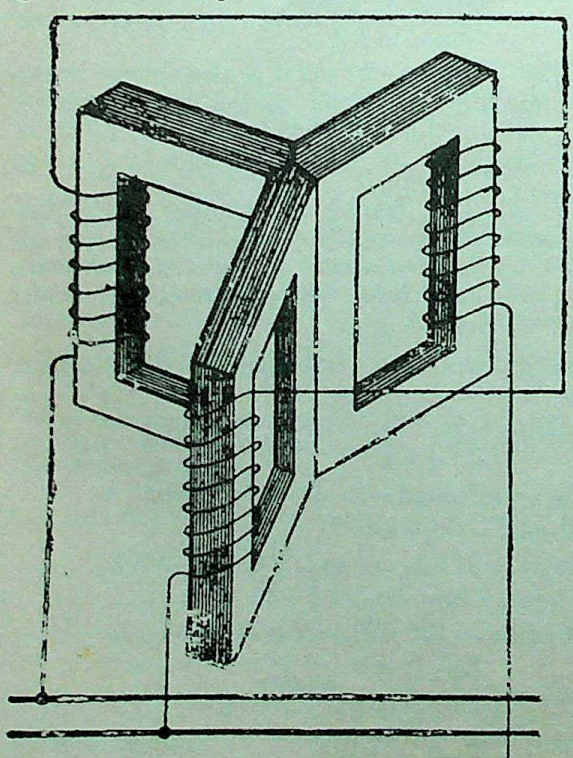

Fig. 21-33

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single 3-phase transformer. But these days, the latter is gaining popularity partly because of improvement in design and manufacture but principally because of better acquaintance of operating men with the three-phase type. As compared to a bank of single-phase transformers, the main advantages of a 3-phase transformer are that it occupies less floor space for equal rating, weighs less, costs about 15% less and further, that only one unit is to be handled and connected.

Like single-phase transformers, the three-phase transformers are also of the core type or shell type. The basic principle of a 3-phase transformer is illustrated in Fig. 21-33 in which only primary windings have been shown interconnected in star and put across 3-phase supply. The three cores are 120° apart and their empty legs are shown in contact with each other. The centre-leg, formed by these three, carries the flux produced by the three-phase currents I_R, I_Y and I_B. As, at any instant, $I_R+I_Y+I_B=0$, hence the sum of three fluxes is also zero. Hence, it will make no difference if the common leg is removed. In that case, any two legs will act as the return for the third just as in a 3-phase system any two conductors act as the return for the current in the third conductor. This improved design is shown in Fig. 21-34 where dotted rectangles indicate the three windings and numbers in the cores and yokes represent the directions and magnitudes of fluxes at a particular instant. It will be seen that at any instant, the amount of 'up' flux in any leg is equal to the sum of 'down' fluxes in the other two legs. The core-type transformers are usually wound with circular cylindrical coils.

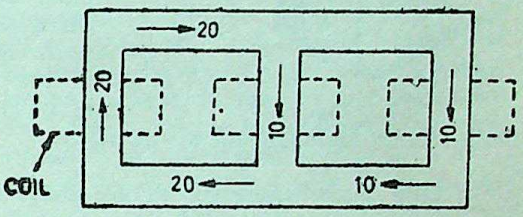

Fig. 21-34

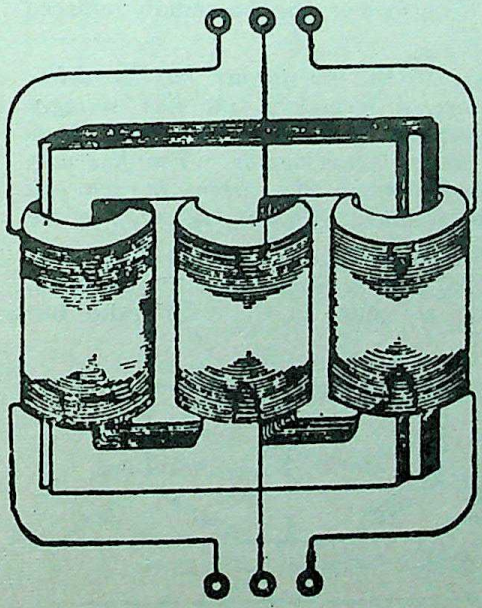
Fig. 21-35

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In a similar way, three single-phase shell-type transformers can be combined together to form a 3-phase shell-type unit. But some saving in iron can be achieved in constructing a single 3-phase transformer as shown in Fig. 21-36. It does not differ from three single-phase transformers put side by side. Saving in iron is due to the joint use of the magnetic paths between the coils. The three phases in this case are more independent than they are in the core-type transformer, because each phase has a magnetic circuit independent of the other.

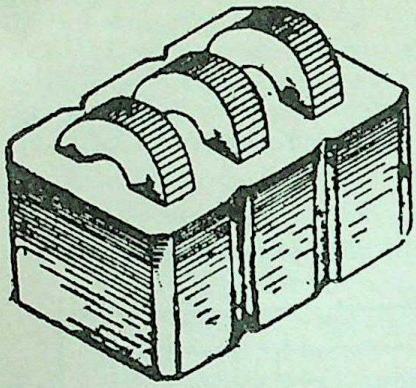

Fig. 21-36

One main drawback in a 3-phase transformer is that if any one phase becomes disabled, then the whole transformer has to be ordinarily removed from service for repairs (the shell-type may be operated open-Δ or Vee but this is not always feasible). However, in the case of a 3-phase bank of single-phase transformers, if one transformer goes out of order, the system can still be run open-Δ at reduced capacity or the faulty transformer can be rapidly replaced by a single spare.

Example 21-16. *A 3-phase, 50-Hz transformer has a delta-connected primary and s ar-connected secondary, the line voltages being 22,000 V and 400 V respectively. The secondary has a star-connecteu balanced load at 0·8 power factor lagging. The line current on the primary side is 5 A. Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in kW?*

Solution. It should be noted that in three-phase transformers, the *phase* transformation ratio is equal to the turn ratio but

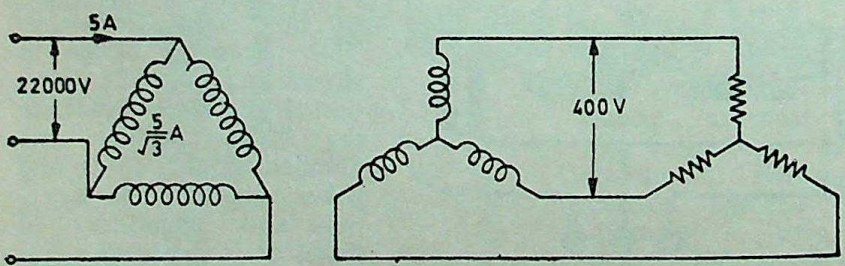

Fig. 21-37

the terminal or line voltages depend upon the method of connection employed. The Δ/Y connection is shown in Fig. 21-37.

Phase voltage on primary side = 22,000 V

Phase voltage on secondary side = $400/\sqrt{3}$

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$\therefore$ $K = 400/22{,}000 \times \sqrt{3} = 1/55\sqrt{3}$

Primary phase current $= 5/\sqrt{3}$ A

Secondary phase current $= \dfrac{5}{\sqrt{3}} \div \dfrac{1}{55\sqrt{3}} = 275$ A

Secondary line current $= 275$ A

Output $= \sqrt{3}\, V_L\, I_L \cos\phi$

$= \sqrt{3} \times 400 \times 275 \times 0{\cdot}8$

$= 15{\cdot}24$ kW

Example 21-17. *A 3-phase, 3,300/400-V transformer has its high-voltage winding connected in delta and the low-voltage winding connected in star. If a load consisting of three impedances each of 6+j8 ohm are joined in delta across the low-voltage side, calculate (a) the kW delivered to the load (b) currents in the low and high-voltage windings and the current drawn by the transformer from line. Neglect losses and no-load current of the transformer.*

Solution. The transformer connection diagram is shown in Fig. 21-38.

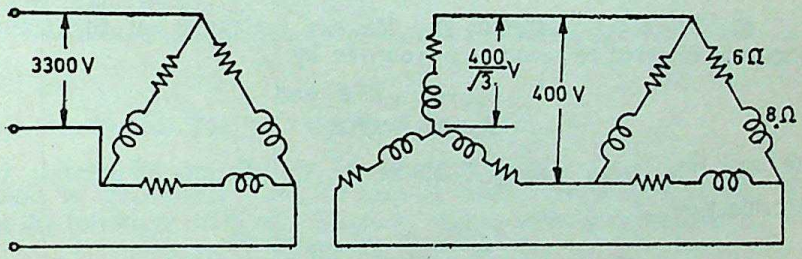

Fig. 21-38

(*a*) Power delivered to the load $= \sqrt{3} V_L\, I_L \cos\phi$

Now, consider the $\triangle$ -connected load

$V_{ph} = V_L = 400$ V $\qquad Z_{ph} = \sqrt{6^2 + 8^2} = 10\ \Omega$

$I_{ph} = 400/10 = 40$ A $\qquad I_L = \sqrt{3} \times 20 = 69{\cdot}3$ A

$\cos\phi = 6/10 = 0{\cdot}6$

$P = \sqrt{3} \times 400 \times 69{\cdot}3 \times 0{\cdot}6 = 28{,}800$ W

(*b*) Primary phase voltage $= 3{,}300$ V

Secondary phase voltage $= 400/\sqrt{3}$

$\therefore$ $K = \dfrac{400/\sqrt{3}}{3300} = \dfrac{4}{33 \times \sqrt{3}}$

Current in low-voltage winding *i.e.* secondary is

$I_2 = 40 \times \sqrt{3} = 69{\cdot}3$ A

Current in high-voltage winding *i.e.* primary is

$I_1{}^* = KI_2 = \dfrac{4}{33 \times \sqrt{3}} = 40 \times \sqrt{3}$

$= 4{\cdot}85$ A

Line current on the primary side $= \sqrt{3} \times 4{\cdot}85 = 9{\cdot}66$ A

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*Because, as given, the no-load current I_0 is neglected.

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HIGHLIGHTS

1. The e.m.f. equation of a transformer is

$$E_1 = 4{\cdot}44\, f\, N_1 \Phi_m \text{ volt}$$
$$= 4{\cdot}44\, f\, N_1 B_m A \text{ volt}$$

2. The voltage transformation ratio (K) is given by

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

3. The no-load primary current (I_0) of a transformer has two components :

(i) working component, $I_W = I_0 \cos \phi_0$

(ii) magnetising component $I_\mu = I_0 \sin \phi_0$

4. The full-load primary current (I_1) also has the following two components :

(i) no-load component I_0 which is very small but constant

(ii) load component I_2' which is large but variable

$$I_2' = KI_2$$

5. The total resistance and leakage reactance of the transformer as referred to secondary are given by

$$R_{02} = R_2 + K^2 R_1 \text{ and}$$
$$X_{02} = X_2 + K^2 X_1$$

6. The total voltage drop of the transformer as referred to secondary is

$$= I_2(R_{02} \cos \phi \pm X_{02} \sin \phi)$$

The $+$ve sign is for lagging power factor and the $-$ve sign for a leading power factor.

7. The efficiency and voltage regulation of a transformer can be calculated with tue help of the data obtained from the following tests :—

(i) Open-circuit ($O.C.$) Test. It gives the iron losses in the transformer.

(ii) Short-circuit ($S.C.$) Test. It gives the full-load Cu loss in the transformer.

8. The variation in the secondary terminal voltage from no-load to full-load is called the (voltage) regulation of the transformer.

9. There are only two types of losses in a transformer (i) Cu loss which is variable and (ii) iron loss which is constant *i.e.* independent of load.

10. The power or ordinary efficiency is maximum when Cu loss equals iron loss.

11. The all-day (or energy) efficiency of a transformer is given by

$$\eta_{all\text{-}day} = \frac{\text{output in kWh}}{\text{input in kWh}} \text{ —for 24 hours}$$

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INDUCTION MOTOR 22

1. Induction Motor : General Principle. 2. Construction 3. Production of Rotating Field 4. Principle of Operation 5. Slip 6. Frequency of Rotor Current 7. Relation Between Torque and Rotor p.f. 8. Starting Torque 9. Starting Torque of a Squirrel Cage Motor 10. Starting Torque of a Slip-ring Motor 11. Condition for Maximum Starting Torque 12. Effect of Change in Supply Voltage 13. Rotor E.M.F. and Reactance Under Running Conditions 14. Torque Under Running Conditions 15. Condition for Maximum Torque 16. Relation Between Torque and Slip 17. Effect of Changes in Supply Voltage on Torque and Speed 18. Full-load Torque and Maximum Torque 19. Starting Torque and Maximum Torque 20. Power Stages in an Induction Motor 21. Rotor Output 22. Highlights.

22-1. Induction Motor : General Principle

Of all the a.c. motors, the 3-phase induction motor is the one which is extensively used for various kinds of industrial drives. It has the following main advantages as well as some disadvantages.

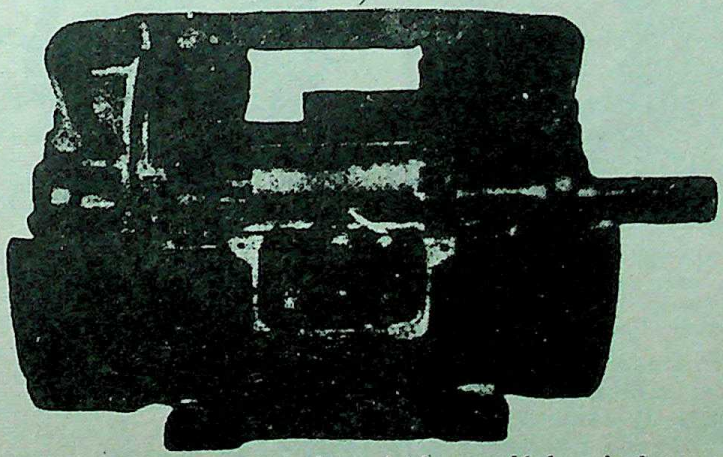

Fig. 22-1. Cut-away view of completely-assembled squirrel-cage Induction Motor (*Courtesy* : *Kirloskar Electric Company*).

Advantages :

(*i*) It has very simple and extremely rugged, almost unbreakable construction (especially squirrel cage type)

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(*ii*) Its cost is low and is very reliable.

(*iii*) It has sufficiently high efficiency. It has a reasonable good power factor.

(*iv*) It requires minimum of maintenance.

(*v*) It starts up from rest and needs no extra starting motor and has not to be synchronized. Its starting arrangement is simple, especially for squirrel-cage rotor type motor.

Disadvantages :

(*i*) Its speed can't be varied without sacrificing some of its efficiency.

(*ii*) Just like d.c. shunt motor, its speed decreases somewhat with increase in load.

(*iii*) Its starting torque is somewhat inferior to that of a d.c. shunt motor.

22-2. Construction (*A.M.I.E. May 1974*)

An induction motor consists of two main parts :

(*a*) a stator and (*b*) a rotor.

(*a*) Stator

The stator of an induction motor is, in principle, the same as that of an alternator. It is made up of a number of stampings

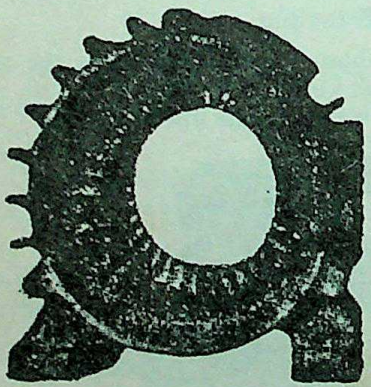

Fig. 22-2 (*a*). Unwound stator with semi-closed slots. Laminations are of high-quality low-loss silicon steel. (*Courtesy : Gautam Electric Motors*)

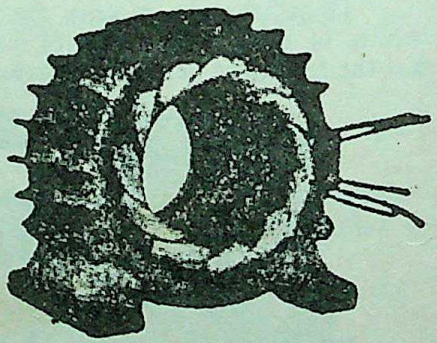

Fig. 22-2 (*b*). Completely wound stator for an induction motor. (*Courtesy : Gautam Electric Motors*)

which are slotted to receive the windings [Fig. 22-2 (*a*)]. The stator carries a 3-phase winding [Fig. 22-2 (*b*)] and is fed from a 3-phase supply. It is wound for a definite number of poles, the exact number of poles being determined by the requirements of speed. Greater the number of poles, lesser the speed and *vice versa*. It will be seen in Art. 22-3 that the stator windings, when supplied with 3-phase current, produce a magnetic field or flux which is of constant value but which revolves or rotates at synchronous speed (given by $N_S = 120f/P$). This revolving magnetic flux induces an e.m.f. in the rotor by mutual induction.

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(*b*) Rotor

(*i*) ***Squirrel-cage rotor.*** Motors employing this type of rotor are known as squirrel-cage induction motors.

(*ii*) ***Phase-wound or Wound rotor.*** Motors employing this type of rotor are variously known as 'phase-wound' motors or 'wound' motors or as 'slip-ring' motors.

(1) **Squirrel-cage Rotor**

Almost 90 per cent of induction motors are squirrel-cage type, because this type of rotor has the simplest and most rugged construction imaginable and is almost indestructible. The rotor consists of a cylindrical laminated core with parallel slots for carrying the rotor conductors which, it should be noted clearly, are not wires but consist of heavy bars of copper, aluminium or alloys. One bar is placed in each slot, rather the bars are inserted from the end when semi-closed slots are used. The rotor bars are brazed or electrically welded to two heavy and stout short-circuiting end-rings, thus giving us, what is so picturesquely called a squirrel-cage construction [Fig. 22-3 and 22-4].

Fig. 22-3. Squirrel-cage rotor with copper bars and alloy-brazed end-rings. (*Courtesy : Gautam Electric Motors*)

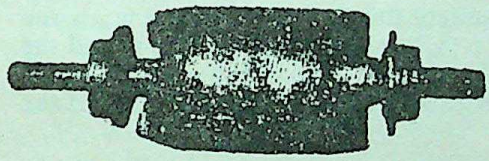

Fig. 22-4. Rotor with shaft and bearings. (*Courtesy : Gautam Electric Motors*)

It should be noted that the rotor bars are permanently short-circuited on themselves, hence it is not possible to add any external resistance in series with the rotor circuit for starting purposes.

(2) **Phase wound rotor**

This type of rotor is provided with 3-phase, double-layer, distributed winding consisting of coils as used in alternators. The rotor is wound for as many poles as the number of stator poles and is always wound 3-phase even *when the stator is wound two phase.*

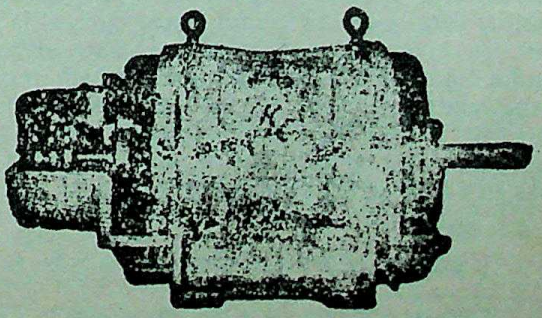

Fig. 22-5. Slip-ring motor with sliprings, brushes and short-circuiting device. (*Courtesy : Kirloskar Electric Company*)

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The three phases are starred internally. The other three winding terminals are brought out and connected to three insulated slip-rings mounted on the shaft with brushes resting on them. These three brushes are further externally connected to a 3-phase star-connected rheostat. This makes possible the introduction of additional resistance in the rotor circuit during the starting period for increasing the starting torque of the motor and for changing its speed/torque/current characteristics. When running under normal conditions, the slip-rings are automatically short-circuited by means of a metal collar which is pushed along the shaft and connects all the rings together. Next, the brushes are automatically lifted from the slip-rings to reduce the frictional losses and the wear and tear. Hence, it is seen that under normal running conditions, the wound rotor is short-circuited on itself just like the squirrel-cage rotor.

22-3. Production of a Rotating Field *(A.M.I.E. Summer 1978)*

It can be shown that when a 3-phase winding is energised by a 3-phase supply, then the resultant flux* produced rotates in space around the stator as though actual magnetic poles were being rotated.

The magnitude of the resultant flux is 1·5 Φ_m where Φ_m is the maximum value of the flux due to any phase. It rotates round the stator synchronously *i.e.* with synchronous speed $N_S = 120 f/P$ where P is the number of motor poles and f is the frequency of the a.c. supply.

22-4. Principle of Operation *(A.M.I.E. Summer 1985)*

The reason why the rotor of an induction motor is set into rotation is as follows : when the 3-phase stator windings are fed by a 3-phase supply, then, as said above, a magnetic flux of constant magnitude but rotating at 'synchronous speed,. is set up. The flux passes through the air-gap, sweeps past the rotor surface and so cuts the rotor conductors which, as yet, are stationary. Due to the relative speed between the rotating flux and the stationary conductors, an e.m.f. is induced in the latter according to Faraday's Law of Electro-magnetic Induction. The *frequency of the induced e.m. f. is the same as the supply frequency*. Its magnitude is proportional to the relative velocity between the flux and the conductors and its direction is as given by Fleming's Right-hand rule. Since the rotor bars or conductors form a closed circuit, rotor current is produced, whose direction, as given by Lenz's law, is such as to oppose the very cause producing it. In this case, the cause which produces the rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductors. Hence to reduce the relative speed, the rotor starts running in the *same* direction as that of the flux and tries to catch up with the rotating flux.

22-5. Slip

In practice, the rotor never succeeds in '*catching up*' with the

*Strictly speaking, it is the seat of the resultant flux which keeps on shifting around synchronously.

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stator flux: If it really did so, then there would be no relative speed between the two, hence no e.m.f., no current, and so, no torque to maintain rotation. That is why the rotor runs at a speed which is always less than the speed of the stator field. The difference in speed depends upon the load on the motor.

The difference between the synchronous speed N_S and the actual speed N of the rotor is known as **slip**. Though it may be expressed in so many rev/s yet it is usual to express it as a percentage of the synchronous speed. Actually, the term '*slip*' is descriptive of the way in which the rotor '*slips back*' from synchronism.

$$\therefore \qquad \% \text{ slip } s = \frac{N_S - N}{N_S} \times 100$$

Sometimes, $N_S - N$ is called the *slip speed*.

22-6. Frequency of Rotor Current

When the motor is stationary, the frequency of rotor current *is the same as the supply frequency*. But when the rotor starts revolving, then the frequency depends upon the relative speed or on slip-speed. Let at any slip-speed the frequency of the rotor current be f'. Then

$$N_S - N = \frac{120 f'}{P}$$

Also $$N_S = \frac{120 f}{P}$$

Dividing one by the other, we get

$$\frac{f'}{f} = \frac{N_S - N}{N_S} = s \qquad \therefore \quad f' = sf$$

Example 22-1. *A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor at full-load is 3%, calculate the full-load speed of the motor.*

Solution. Let N = actual motor speed

Supply frequency $f = 12 \times 500/120 = 50$ Hz

Synchronous speed $N_S = 120 \times 50/8 = 750$ r.p.m.

$$\% \text{ slip } s = \frac{N_S - N}{N_S} \times 100$$

$$\therefore \qquad 3 = \frac{750 - N}{750} \times 100 \qquad \therefore \quad N = 727{\cdot}5 \text{ r.p.m.}$$

Example 22-2. *If a 6-pole motor running from a 50-Hz supply has an e.m. f. in the rotor of frequency 2·5 Hz, determine (a) the slip (b) the speed of the motor.*

Solution. We have seen that

$$f' = sf$$

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(*a*) $\therefore \quad 2{\cdot}5 = s \times 50 \quad \therefore \quad s = 2{\cdot}5/50 = 0{\cdot}05$ or **5%**

$$N_S = 120 \times 50/6 = 1000 \text{ r.p.m.}$$

(*b*) $\therefore \quad 5 = \dfrac{1000 - N}{1000} \times 100 \; ; \quad \mathbf{N = 950 \text{ r.p.m.}}$

Example 22-3. *A 3-ϕ induction motor is wound for 4-poles and is supplied from a 50-Hz system. Calculate (i) synchronous speed (ii) speed of rotor when slip is 4% and (iii) rotor current frequency when rotor runs at 1200 r.p.m.* (*A.M.I.E. Winter 1981*)

Solution. (*i*) $N_s = 120\, f/P = 120 \times 5/4 = \mathbf{1500 \text{ r.p.m.}}$

(*ii*) $$s = \frac{N_s - N}{N_s} \times 100$$

$$4 = \frac{1500 - N}{1500} \times 100 \; ; \; N = \mathbf{1440 \text{ r.p.m.}}$$

(*iii*) When $N = 1200$ r.p.m.

$$s = \frac{1500 - 1200}{1500} = 0{\cdot}2$$

$\therefore$ frequency of rotor current

$$f' = sf = 0{\cdot}2 \times 50 = \mathbf{10 \text{ r.p.m.}}$$

Example 22-4. *An 8-pole alternator runs at 750 r.p.m. and supplies power to a 6-pole induction motor which has at full-load a slip of 3%. Find the full-load speed of the induction motor and the frequency of its rotor e.m.f.*

Solution. The supply frequency is found from the speed of the alternator and its number of poles.

Supply frequency $f = PN/120 = 8 \times 750/120 = 50$ Hz

Synchronous speed of motor

$$N_S = 120 \times 50/6 = 1000 \text{ r.p.m.}$$

$$\therefore \quad 3 = \frac{1000 - N}{1000} \times 100$$

$\therefore$ Actual full-load motor speed $N = \mathbf{970 \text{ r.p.m.}}$

$$f' = s \times f = 3 \times 50/100 = \mathbf{1{\cdot}5 \text{ Hz}}$$

22-7. Relation Between Torque and Rotor P.F.

In Art. 12-7 it has been shown that in the case of a d.c. motor, the torque T_a is proportional to the product of armature current and flux per pole *i.e.* $T_a \propto \Phi I_a$. Similarly, in the case of an induction motor, the torque is also proportional to the product of flux per stator pole and the rotor current. However, there is another factor to be taken into account *i.e.* the power factor of the rotor.

$$\therefore \quad T \propto \Phi \, I_2 \cos \phi_2$$

or $$T = k \, \Phi \, I_2 \cos \phi_2$$

where I_2 = rotor current at *standstill*

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ϕ_2 = angle between rotor e.m.f. and rotor current

k = a constant

Denoting rotor e.m.f. at *standstill* by E_2, we have that $E_2 \propto \Phi$

$\therefore \quad T \propto E_2 I_2 \cos \phi_2$

or $\quad T = k_1 E_2 I_2 \cos \phi_2$ where k_1 is another constant

22-8. Starting Torque

Let E_2 = rotor e.m.f. per phase *at standstill*

R_2 = rotor resistance/phase

X_2 = rotor reactance/phase

$\therefore \quad Z_2 = \sqrt{(R_2^2 + X_2^2)}$

= rotor impedance/phase *at standstill*

then $I_2 = E_2/Z_2 = E_2/\sqrt{(R_2^2 + X_2^2)}$

$$\cos \phi_2 = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} \qquad \text{(Fig. 22-6)}$$

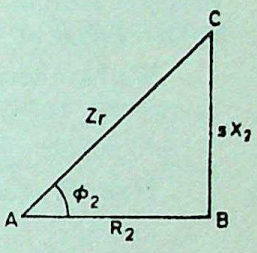

Fig. 22-6

Standstill or starting torque $T_S = k_1 E_2 I_2 \cos \phi_2$

$$= k_1 E_2 \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

$$= \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

If supply voltage V is constant, then flux Φ and hence E_2 both are constant.

$$\therefore \quad T_S = k_2 \frac{R_2}{R_2^2 + X_2^2} = k_2 \frac{R_2}{Z_2^2} \qquad \ldots(i)$$

where k_2 is some other constant.

22-9. Starting Torque of a Squirrel-cage Motor

The resistance of a squirrel-cage rotor is fixed and small as compared to its reactance which is very large especially at the start because at standstill, the frequency of the rotor current equals the supply frequency. Hence, the starting current I_2 of the rotor, though very large in magnitude, lags by a very large angle behind E_2 with the result that the starting torque per ampere is very poor. It is roughly 1·5 times the full-load torque although the starting current is 5 to 7 times the full-load current. Hence, such motors are not useful where the motor has to start against heavy loads.

22-10. Starting Torque of a Slip-ring Motor

The starting torque of such a motor is increased by improving its power factor by adding external resistance in the rotor circuit from the star-connected rheostat, the rheostat resistance being progressively cut out as the motor catches speed. Addition of externa' resistance, however, increases the rotor impedance and so reduces the rotor current. At first, the effect of improved power factor predominates the current-decreasing effect of impedance. Hence, starting torque is increased. But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

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22-11. Condition for Maximum Starting Torque

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It can be proved that starting torque is maximum when rotor resistance equals rotor reactance.

Now $$T_S = \frac{k_2 R_2}{R_2^2 + X_2^2}$$

$$\therefore \quad \frac{dT_S}{dR_2} = k_2 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

or $$R_2^2 + X_2^2 = 2R_2^2 \quad \text{or} \quad R_2 = X_2$$

22-12. Effect of Change in Supply Voltage

We have seen that

$$T = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \qquad \text{...from Art. 22-8}$$

Now, $E_2 \propto$ supply voltage V

$$T = \frac{k_3 V^2 R_2}{R_2^2 + X_2^2}$$

where k_3 is yet another constant.

$$\therefore \quad T \propto V^2$$

Clearly, the torque is very sensitive to any changes in the supply voltage. A change of 5 per cent in supply voltage, for example, will produce a change of approximately 10% in the rotor torque. This fact is of importance in star-delta and auto-transformer starters.

Example 22-5. *A 3-ϕ induction motor having a star-connected rotor has an induced e.m. f. of 80 volts between slip-rings at standstill on open-circuit. The rotor has a resistance and reactance per phase of 1 Ω and 4 Ω respectively. Calculate current/phase and power factor when (a) slip-rings are short-circuited (b) slip-rings are connected to a star-connected rheostat of 3 Ω per phase.*

Solution. Standstill e.m.f./rotor phase $= 80/\sqrt{3} = 46{\cdot}2$ V

(*a*) Rotor impedance/phase $= \sqrt{(1^2 + 4^2)} = 4{\cdot}12\ \Omega$

Rotor current/phase $= 46{\cdot}2/4{\cdot}12 = \mathbf{11{\cdot}2}$ **A**

Power factor $= \cos\phi = 1/4{\cdot}12 = \mathbf{0{\cdot}243}$

As p.f. is low, the starting torque is also small.

(*b*) Rotor resistance/phase $= 3 + 1 = 4\ \Omega$

Rotor impedance/phase $= \sqrt{(4^2 + 4^2)} = 5{\cdot}66\ \Omega$

$\therefore$ Rotor current/phase $= 46{\cdot}2/5{\cdot}66 = \mathbf{8{\cdot}16}$ **A**

$\cos\phi = 4/5{\cdot}66 = \mathbf{0{\cdot}707}$

Hence, the starting torque is increased due to improvement in the power factor. It will also be noted that improvement in p.f. is much more than the decrease in current due to increased impedance.

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Tutorial Problems—36

1. A 4-pole 50-Hz induction motor on full-load runs at 1,440 r.p.m. Calculate

(*i*) the synchronous speed

(*ii*) the speed of slip

(*iii*) the frequency of the rotor currents.

[(*i*) 1500 r.p.m. (*ii*) 60 r.p.m. (*iii*) 2 Hz]

2. Calculate the number of poles in the cases of the undermentioned 3-phase, 50-Hz induction motors, of which the full-load shaft speeds are given.

(*i*) A 482 r.p.m. winder motor

(*ii*) A 725 r.p.m. coal-cutter motor

(*iii*) A 1,445 r.p.m. centrifugal pump motor **[(*i*) 12 (*ii*) 8 (*iii*) 4]**

3. A 3-phase, 50-Hz, 4-pole, wound-rotor induction motor has a rotor standstill e.m.f. per phase of 115 V. If the motor is running at 1,440 r.p.m., calculate for this speed :

(*a*) the slip

(*b*) the frequency of the rotor induced e.m.f.

(*c*) the value of the rotor induced e.m.f. per phase.

[(*a*) 4% (*b*) 2 Hz (*c*) 4·6 V]

22-13. Rotor E.M.F. and Reactance under Running Conditions.

Let E_2 = *standstill* rotor induced e.m.f./phase

X_2 = „ „ „ reactance/phase

f_2 = rotor current frequency *at standstill*

When rotor is stationary *i.e.* $s=1$, then frequency of rotor e.m.f. is the same as that of the stator supply frequency. The value of e.m.f. induced in the rotor at standstill is maximum because the relative speed between the rotor and the revolving stator flux is maximum. In fact, the motor is equivalent to a 3-phase transformer with a short-circuited rotating secondary.

When rotor starts running, the relative speed between it and the rotating stator flux is decreased. Hence, the rotor induced e.m.f. which is directly proportional to this relative speed, is also decreased (and may disappear altogether if rotor speed were to become equal to the speed of stator flux). Hence, for a slip s, the rotor induced e.m.f. will be s times the induced e.m.f. at standstill.

Therefore, under *running* conditions,

$$E_r = sE_2$$

The frequency of the induced e.m.f. will likewise become

$$f_r = sf_2$$

Due to decrease in frequency of the rotor e.m.f., the rotor reactance will also decrease,

$$\therefore \quad X_r = sX_2$$

where E_r and X_r are rotor e.m.f. and reactance under *running* conditions.

22-14. Torque Under Running Conditions

$$T \propto E_r I_r \cos \phi_2$$

or

$$T \propto \Phi I_r \cos \phi_2 \qquad (\because \ E_r \propto \Phi)$$

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where E_r = rotor e.m.f./phase under *running conditions*
I_r = ,, current/phase ,, ,, ,,

Now $E_r = sE_2$

$$\therefore \quad I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{[R_2^2 + (sX_2)^2]}}$$

$$\cos \phi_2 = \frac{R_2}{\sqrt{[R_2^2 + (sX_2)^2]}} \quad \text{(Fig. 22-7)}$$

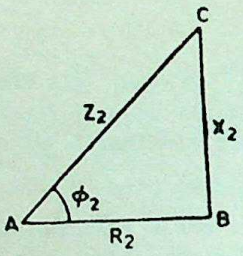

Fig. 22-7

$$\therefore \quad T \propto \frac{\Phi\, s\, E_2 R_2}{R_2^2 + (sX_2)^2} = \frac{k\,\Phi . s . E_2 R_2}{R_2^2 + (sX_2)^2}$$

Also $$T = \frac{k_1\, s\, E_2^2\, R_2}{R_2^2 + (sX_2)^2} \qquad (\because \; E_2 \propto \Phi)$$

where k_1 is another constant.

At standstill when $s = 1$, obviously

$$T = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} \quad \text{...the same as in Art. 22-8}$$

22-15. Condition for Maximum Torque

The torque of a rotor under running conditions is

$$T = \frac{k\,\Phi\, s\, E_2\, R_2}{R_2^2 + (sX_2)^2}$$

The condition for maximum torque may be obtained by differentiating the above expression with respect to slip s and then by putting it equal to zero. However, it is simpler to put $Y = \frac{1}{T}$ and then differentiate it.

$$\therefore \quad Y = \frac{R_2^2 + (sX_2)^2}{k\,\Phi\, s\, E_2\, R_2} = \frac{R_2}{k\,\Phi\, s\, E_2} + \frac{sX_2^2}{k\,\Phi E_2 R_2}$$

$$\frac{dY}{ds} = \frac{-R_2}{k\Phi E_2\, s^2} + \frac{X_2^2}{k\,\Phi\, E_2\, R_2} = 0$$

$$\therefore \quad \frac{R_2}{k\,\Phi E_2\, s^2} = \frac{X_2^2}{k\,\Phi\, E_2\, R_2}$$

or $$R_2^2 = s^2 X_2^2 \quad \text{or} \quad R_2 = sX_2$$

Hence, torque *under running conditions* is maximum at that value of slip s which makes rotor reactance/phase equal to rotor resistance per phase.

Slip corresponding to maximum torque is

$$s_{max} = R_2/X_2$$

Putting $R_2 = sX_2$ in the above equation for the torque, we get

$$T_{max} = \frac{k\,\Phi\, s^2\, E_2\, X_2}{2s^2 X_2^2} \quad \left(\text{or} \quad \frac{k\,\Phi\, s\, E_2\, R_2}{2R_2^2}\right)$$

$$= \frac{k\,\Phi\, E_2}{2X_2} \quad \left(\text{or} \quad \frac{k\,\Phi\, sE_2}{2R_2}\right)$$

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(It can be seen by putting the value of s in the expression in bracket that it is the same as the other expression).

From above, it is found

(*i*) *That the maximum torque is independent of rotor resistance as such.*

(*ii*) *However, the speed or slip at which maximum torque occurs is determined by the rotor resistance. As seen above, torque becomes maximum when rotor reactance equals its resistance. Hence, by varying rotor resistance (possible only with slip-ring motors) maximum torque can be made to occur at any desired slip (or motor speed).*

(*iii*) *Maximum torque varies inversely as standstill reactance. Hence, it should be kept as small as possible.*

22-16. Relation Between Torque and Slip

A family of torque/slip curves is shown in Fig. 22-8 for a range of $s=0$ to $s=1$ with R_2 as the parameter. We have seen above that

$$T=\frac{k\,\Phi\,s\,E_2\,R_2}{R_2^2+(sX_2)^2}$$

It is clear that when $s=0$, $T=0$, hence the curve starts from point O.

At normal speeds, close to synchronism, the term (sX_2) is small and hence negligible with respect to R_2

$$\therefore \quad T \propto \frac{s}{R_2}$$

or $T \propto s$ if R_2 is constant.

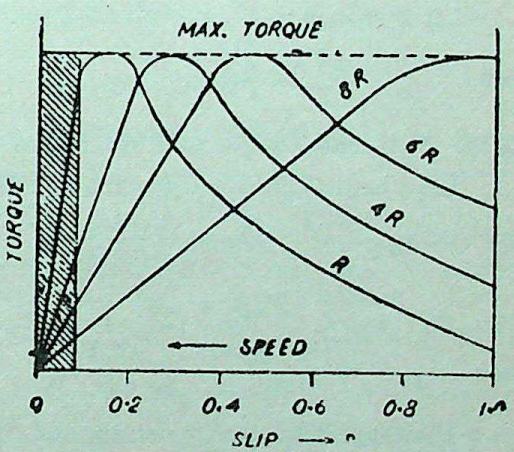

Fig. 22-8

Hence, for low values of slip, the torque/slip curve is approximately a straight line. As slip increases (for increasing load on the motor) the torque also increases and becomes maximum when $s=R_2/X_2$. This torque is known as 'pull out' or 'break down' torque. As the slip further increases (*i.e.* motor speed falls) with further increase in motor load, R_2 becomes negligible as compared to (sX_2). Therefore,

$$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$$

Hence, the torque/slip curve is a rectangular hyperbola. So we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has

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been so protected. In fact, the stable operation of motor lies between the values of $s=0$ and that corresponding to maximum torque. The operating range is shown shaded in Fig. 22-8.

It is seen that although maximum torque does not depend on R_2, yet *the exact location of T_{max} is dependent on it*. Greater the R_2, greater is the value of slip at which the maximum torque occurs.

22-17. Effect of Change in Supply Voltage on Torque and Speed

$$T = \frac{k \Phi s E_2 R_2}{R_2^2 + (sX_2)^2}$$

As $E_2 \propto \Phi \propto V$ where V is supply voltage

$\therefore$ $T \propto V^2$

Change in supply voltage not only affect the starting torque T_S but torque under running conditions also. If V decreases, then T also decreases. Hence, for maintaining the same torque, slip increases *i.e.* speed falls.

Let V change to V', s to s' and T to T'

then $$\frac{T}{T'} = \frac{sV^2}{s'V'^2}$$

22-18. Full-load Torque and Maximum Torque

Let s be the slip corresponding to full-load torque, then

$$T_f \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

$$T_{max} \propto \frac{1}{2 \times X_2} \quad \text{...(Art. 22-15)}$$

then $$\frac{T_f}{T_{max}} = \frac{2sR_2X_2}{R_2^2 + (sX_2)^2}$$

Dividing both the numerator and the denominator by X_2^2, we get

$$\frac{T_f}{T_{max}} = \frac{2s\ R_2/X_2}{(R_2/X_2)^2 + s^2} = \frac{2\,a\,s}{a^2 + s^2}$$

where $a = R_2/X_2$ = resistance/standstill reactance.

22-19. Starting Torque and Maximum Torque

$$T_S \propto \frac{R_2}{R_2^2 + X_2^2} \quad \text{...(Art. 22-8)}$$

$$T_{max} \propto \frac{1}{2X_2} \quad \text{...(Art. 22-15)}$$

$\therefore$ $$\frac{T_S}{T_{max}} = \frac{2R_2\ X_2}{R_2^2 + X_2^2} = \frac{2R_2/X_2}{1 + (R_2/X_2)^2} = \frac{2\ a}{1 + a^2}$$

where $a = \frac{R_2}{X_2}$ = $\frac{\text{rotor resistance per phase}}{\text{standstill reactance}}$

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Example 22 6. *A 3,300-V, 24-pole, 50-Hz, 3-ϕ star-connected induction motor has a slip-ring rotor resistance of 0·016 Ω and stand-still reactance of 0·265 Ω per phase. Calculate (a) the speed at maximum torque (b) ratio of full-load torque to maximum torque if full load torque is obtained at 247 r.p.m.*

Solution. We have seen that for maximum torque

$$R_2 = sX_2$$

or $$s = R_2/X_2 = 0{\cdot}016/0{\cdot}265 = 0{\cdot}0604$$

or $$s = 6{\cdot}04\%$$

Rotor speed $$N = (1-s)\,N_S$$

Now $$N_S = 120 \times 50/24 = 250 \text{ r.p.m.}$$

(*a*) $$\therefore \quad N = (1 - 0{\cdot}0604) \times 250 = \mathbf{235 \text{ r.p.m.}}$$

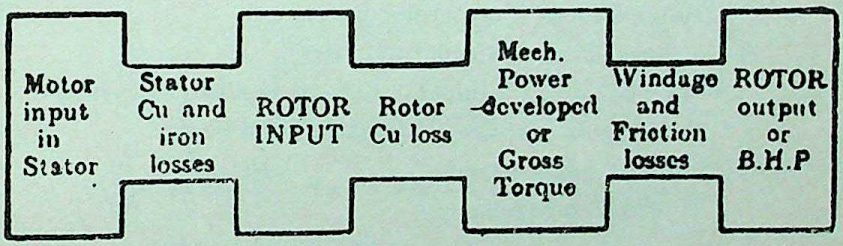
(*b*) Full-load slip $$= \frac{250-247}{250} = \frac{3}{250} = 0{\cdot}012 \text{ or } 1{\cdot}2\%$$

Now $$\frac{T_f}{T_{max}} = \frac{2\,a\,s}{a^2+s^2}$$

where $$a = X_2/R_2 = 0{\cdot}0604$$

$$\therefore \quad \frac{T_f}{T_{max}} = \frac{2 \times 0{\cdot}0604 \times 0{\cdot}012}{0{\cdot}012^2 + 0{\cdot}0604^2} = \mathbf{0{\cdot}382}$$

22-20. Power Stages in an Induction Motor



Stator iron loss (consisting of eddy and hysteresis losses) depends on the supply frequency and the flux density in the iron core. It is practically constant. The iron loss of the rotor is, however, negligible because frequency of rotor currents under normal running conditions is always small.

Total rotor Cu loss $= 3I_2^2\,R_2$

22-21. Rotor Output

Stator input = stator output + stator losses

The stator output is transferred entirely inductively to the rotor circuit.

Obviously, rotor input = stator output

Rotor output = rotor input − rotor Cu losses

This rotor output is converted into mechanical energy and gives rise to gross torque T_g Out of this gross torque developed, some is

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lost due to windage and friction losses in the rotor and the rest appears as the useful or shaft torque T_{sh}.

Let N r.p.s. be the actual speed of the rotor and T_g the gross torque in N-m, then

$$T_g \times 2\pi N = \text{rotor output in watts}$$

$$\therefore \quad T_g = \frac{\text{rotor output in watts}}{2\pi N} \qquad \ldots(1)$$

If there were no Cu losses in the rotor, then rotor output will equal rotor input and the rotor will run at synchronous speed.

$$\therefore \quad T_g = \frac{\text{rotor input}}{2\pi N_S} \qquad \ldots(2)$$

From (1) and (2), we get

$$\text{Rotor output} = T_g \times 2\pi N$$

$$\text{Rotor input} = T_g \times 2\pi N_S \qquad \ldots(3)$$

The difference between the two equals rotor Cu losses.

$$\therefore \quad \text{Rotor Cu loss} = T_g \times 2\pi (N_S - N) \qquad \ldots(4)$$

From (3) and (4)

$$\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_S - N}{N_S} = s$$

$$\therefore \quad \text{Rotor Cu loss} = s \times \text{rotor input} \qquad \ldots(5)$$

$$\text{Rotor gross output} = \text{input} - \text{Cu loss}$$

$$= \text{input} - s \times \text{rotor input} = (1 - s) \text{ input}$$

$$\therefore \quad \text{Rotor gross output} = (1 - s) \text{ rotor input} \qquad \ldots(6)$$

$$\therefore \quad \frac{\text{Rotor gross output}}{\text{Rotor input}} = 1 - s = 1 - \frac{N_S - N}{N_S} = \frac{N}{N_S}$$

$$= \frac{\text{actual speed of rotor}}{\text{synchronous speed}}$$

$$\therefore \quad \text{Rotor efficiency} = N/N_S$$

$$\text{Also} \quad \frac{\text{Rotor Cu loss}}{\text{Rotor gross output}} = \frac{s}{1 - s}$$

Important Conclusion

If some power P_2 is delivered to a rotor, then a part sP_2 is lost in the rotor itself as copper loss (and appears as heat) and the remaining $(1-s)$ P_2 appears as gross mechanical power P_m (including friction and windage losses).

$$\therefore \quad P_2 : P_m : I^2R \;::\; 1 : 1 - s : s$$

The rotor input power will always divide itself in this ratio, hence it is advantageous to run the motor with as small a slip as possible.

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Example 22-7. *A 7·46-kW, 230-V, 3-phase, 50-Hz, 6-pole squirred-cage induction motor operates at a full load slip of 4% when rated voltage and rated frequency are applied. Determine*

(i) full load speed (ii) full load torque in newton-metres (iii) frequency of rotor current under this condition (iv) speed of rotation of the stator m.m.f. *(A.M.I.E. Winter 1978)*

Solution. *(iv)* Stator m.m.f. revolves with synchronous speed given by $N_s = 120\,f/P = 120 \times 50/6 = \mathbf{1000\ r.p.m.}$

(i) $N = N_s\,(1-s) = 1000\ (1-0{\cdot}04) = \mathbf{960\ r.p.m.}$

(iii) $f' = sf = 0{\cdot}04 \times 50 = \mathbf{2\ Hz}$

(ii) If we neglect the windage and friction losses of the rotor, then 7·46 kW also represents the gross mechanical power developed in rotor (Art. 22-21)

$$\therefore \quad T_g = \frac{\text{rotor output}}{2\pi N} = \frac{7{\cdot}46 \times 10^3}{2\pi \times (960/60)} = \mathbf{74{\cdot}2\ N\text{-}m}$$

Example 22-8. *A 3-phase, 6-pole induction motor delivers 3·73 kW at 950 r.p.m. What is the stator input if the stator loss is 250 W ?*

Solution. Rotor output = 3,730 W

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m.}$$

$$N = 950 \text{ r.p.m.}$$

$$\text{Now} \quad \frac{\text{Rotor output}}{\text{Rotor input}} = \frac{N}{N_s}$$

$$\therefore \quad \text{Rotor input (or stator output)} = \frac{3{,}730 \times 1000}{950} = 3{,}930 \text{ W}$$

$$\text{Stator input} = \text{stator output} + \text{stator losses}$$

$$= 3930 + 250 = \mathbf{4180\ W}$$

Example 22-9. *A 6-pole, 3-phase induction motor rnns at a speed of 960 r.p.m. and the shaft torque is 135·7 N-m. Calculate the rotor Cu loss if the friction and windage losses amount to 150 watts. The frequency of supply is 50 Hz.*

Solution. Output $= 2\pi \times N \times T_{sh}$

$$= 2\pi \times (960/60) \times 135{\cdot}7 = 13{,}630 \text{ W}$$

$$\text{Gross rotor output} = 13{,}630 + 150 = 13{,}780 \text{ W}$$

$$N_s = 120 \times 50/6 = 1000 \text{ r.p.m.}$$

$$s = (1000 - 960)/1000 = 0{\cdot}04$$

$$\text{Now} \quad \frac{\text{Rotor Cu loss}}{\text{Rotor gross output}} = \frac{s}{1-s}$$

$$\therefore \quad \text{Rotor Cu loss} = \frac{13{,}780 \times 0{\cdot}04}{1 - 0{\cdot}04} = \mathbf{574\ W}$$

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HIGHLIGHTS

1. An induction motor essentially consists of (*a*) a stator and (*b*) a rotor.

Rotors are of two types (*i*) squirrel-cage rotor and (*ii*) phase-wound or wound rotor.

2. When stationary coils, wound for three phases, are fed by three-phase supply, then a uniformly revolving flux is produced.

(*i*) whose value is (3/2) $\Phi_m = 1{\cdot}5$ times the maximum flux due to either phase and

(*ii*) which rotates or revolves at synchronous speed.

3. Slip is defined as the difference between the synchronous speed N_s and the actual speed N of the rotor.

4. The synchronous speed is given by

$$N_s = \frac{120\,f}{P}$$

5. Torque developed by the rotor is

$$T = k_1 E_2 I_2 \cos \phi_2$$

where E_2 = *standstill* e.m.f. induced in the rotor/phase

6. Starting torque of an induction motor is

$$T_s = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2} = k_2 \cdot \frac{R_2}{R_2^2 + X_2^2}$$

where R_2 = rotor resistance/phase

X_2 = rotor reactance/phase *at standstill.*

7. Condition for maximum starting torque is

$$R_2 = X_2$$

8. The torque of an induction motor under running conditions is

$$T = \frac{k_1 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

9. Condition for maximum torque under running condition is

$$R_2 = sX_2$$

The value of maximum torque is

$$T_{max} = \frac{k \Phi E_2}{2X_2} \quad \text{or} \quad \frac{k \Phi s E_2}{2R_2}$$

10. The relation between torque and slip is given by

$T \propto s$ —for light loads

$T \propto 1/s$ —for heavy loads

11. The relation between maximum torque (T_{max}) and full-load torque (T_f) is given by

$$\frac{T_f}{T_{max}} = \frac{2as}{a^2 + s^2}$$

where $a = R_2/X_2$

= resistance/standstill reactance per phase

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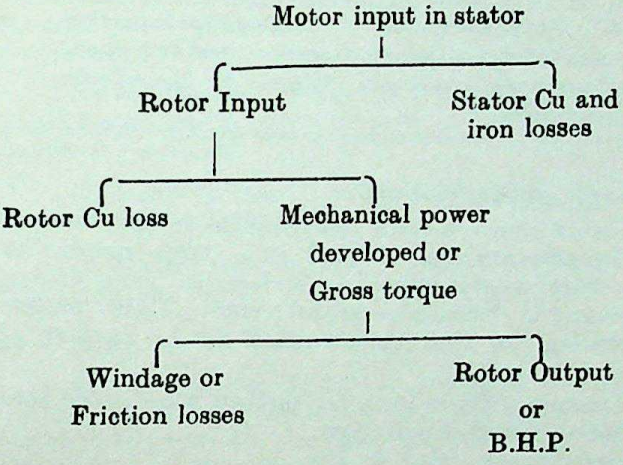
12. The relation between standstill torque (T_s) and maximum torque is given by

$$\frac{T_s}{T_{max}} = \frac{2a}{1+a^2}$$

13. Power stages in an induction motor are as follows :—



14. (*i*) Gross torque $T_g = \dfrac{\text{rotor output in watts}}{2\pi \times N}$

(*ii*) Rotor Cu loss $= T_g \times 2\pi\,(N_S - N)$

(*iii*) $\dfrac{\text{Rotor Cu loss}}{\text{Rotor input}} = s$

(*iv*) Rotor Cu loss $= s \times$ rotor input

(*v*) $\dfrac{\text{Rotor gross output}}{\text{Rotor input}} = 1 - s = \dfrac{N}{N_S}$

(*vi*) Rotor efficiency $= N/N_S$

(*vii*) $\dfrac{\text{Rotor Cu loss}}{\text{Rotor gross output}} = \dfrac{s}{1-s}$

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SINGLE-PHASE MOTORS 23

1. Types of Single-phase Motors 2. Single-phase Induction Motor 3. Split-phase Induction Motor 4. Capacitor-start Induction-run Motors 5. Capacitor-start-and-run Motors 6. Shaded-pole Motor. 7. Motor Troubles.

23.1 Types of Single-phase Motors

As the name shows, such motors are designed to operate from a single-phase supply and are manufactured in a large number of types to perform a wide variety of useful services in home, offices, factories, workshops and in business establishments. These motors are classified according to their construction and method of starting :—

(*i*) Induction motors. These may be further sub-divided into split-phase, capacitor and shaded-pole type.

(*ii*) Repulsion Motors

(*iii*) A.C. Series Motors

(*iv*) Unexcited Synchronous Motors (like Reluctance Motor and Hysteresis Motor).

23.2 Single-phase Induction Motor

Like 3-phase motor, a single-phase motor also consists of

(*i*) a stator which carries single-phase winding and

(*ii*) a squirrel-cage type of rotor.

However, there is one very fundamental difference between the two : whereas a 3-phase motor is self-starting, a single-phase motor is non-self-starting. This is due to the fact that whereas the stator winding of a 3-phase motor produces a rotating (or revolving) flux, the single-phase winding produces merely a pulsating or alternating flux. A synchronously rotating flux can be produced only by either a 2-phase or 3 phase stator winding when energised from a 2-phase or 3-phase supply respectively

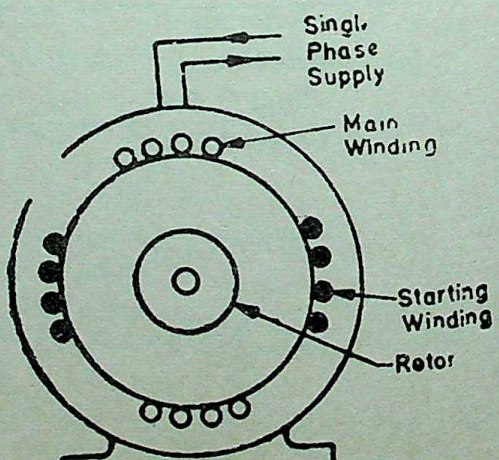

Fig. 23-1

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An alternating flux acting on a *stationary* squirrel-cage rotor cannot produce rotation (only a rotating flux can).

However, if the rotor of a single-phase motor is given an initial start by hand or otherwise in *either* direction, then immediately a torque arises and the motor accelerates to its final speed provided it is not heavily loaded.

There are many methods to make a single-phase motor self-starting and single-phase induction motors are classified and named according to the method used as detailed below :

(*a*) Split-phase motors : These motors are started by two-phase motor action with the help of an additional winding known as starting winding.

(*b*) Capacitor motors : These are also started by the two-phase motor action by using capacitor.

(*c*) Shaded-pole motors : These are started by using shaded-pole principle.

23-3 Split-phase Induction Motor

As shown in Fig. 23-2 (*a*), a single-phase motor is temporarily converted into a 2-phase motor by providing an extra winding on the stator in addition to the main or running winding. The circuit connections are shown in Fig. 23-2 (*b*). By making starting winding highly resistive and main winding highly reactive, the phase difference

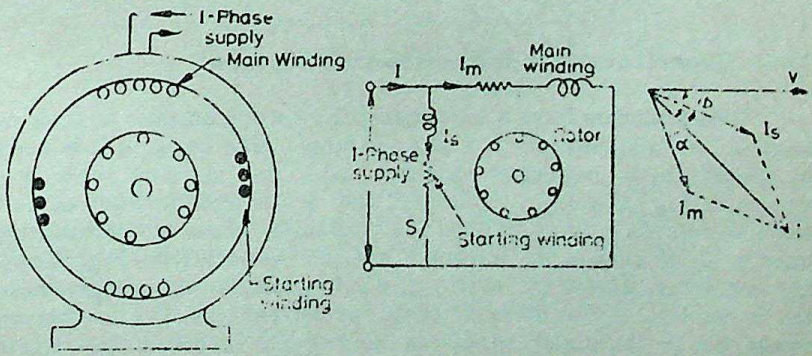

Fig. 23-2

between the currents drawn by them can be made sufficiently large (the ideal value being 90°). The motor behaves like a two-phase motor for starting purposes. The two currents produce a revolving flux and hence make the motor self-starting. The starting torque $T_S = k\, I_s\, I_m \sin \alpha$ where k is a constant governed by motor design parameters [Fig. 23-2 (*c*)].

A centrifugal switch S is connected in series with the starting winding and is located outside the motor. Its function is to auto-

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matically disconnect the starting winding from the supply when the motor reaches 70 to 80 per cent of its full-load speed.

Typical torque/speed characteristic for such a motor is shown in Fig. 23-3. As seen, the starting torque is 150 to 200 per cent of the full-load torque with a starting current of 6 to 8 times the full-load current.

This motor is also known as resistance-start split-phase motor.

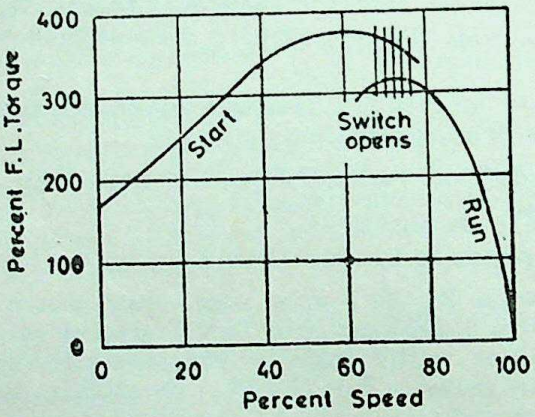

Fig. 23-3

23-4 Capacitor-start Induction-run Motors

These motors have a higher starting torque because in their case angle α between currents I_s and I_m is large. The angle α is increased by connecting a capacitor in series with the starting winding as shown in Fig. 23-4 (*a*). As before, the centrifugal switch cuts off both the starting winding and the capacitor when motor runs up to nearly 75 per cent of its full-load speed. As seen from Fig. 23-4 (*b*), the current I_s drawn by starting winding leads the voltage whereas the current I_m in the main winding as before lags V. In this way, value of α is increased to about 80° which increases the starting torque to twice that developed by a standard split-phase induction motor. Such motors have starting torques as high as 450 per cent of the full-load value.

Typical performance curve of such a motor is shown in Fig. 23-4 (*c*).

23-5 Capacitor start-and-run Motors

These are similar to the capacitor-start induction-run motors except that the starting winding and capacitor are connected in the circuit at *all times*. In other words, the capacitor remains in the

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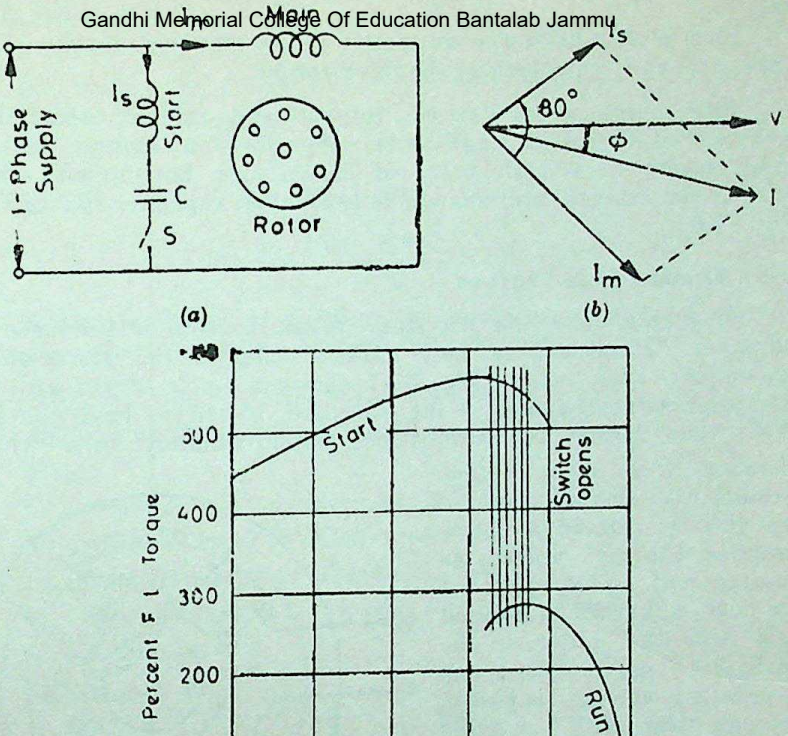

Fig. 23-4

circuit permanently as shown in Fig. 23-5. That is why such motors are also known as permanent-split capacitor motors. Obviously, there is no need for a centrifugal switch. The advantages of leaving the capacitor permanently in circuit are (*i*) improvement of overload capacity of the motor (*ii*) a higher p.f. (*iii*) higher efficiency and (*iv*) quieter running of the motor.

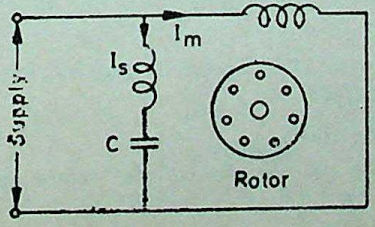

Fig. 23-5

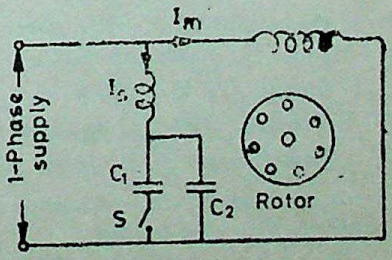

Fig. 23-6

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Such motors have a comparatively low starting torque which is about 50 to 100 per cent of the rated torque.

Such motors which start and run with one value of capacitance in the circuit are called single-value capacitor-run motors. Others which start with a high value of capacitance but run with a low value of capacitance are known as two-value capacitor-run motors (Fig. 23-6).

23-6 Shaded-pole Motors

In such motors, the necessary phase-splitting is produced by induction. These motors have salient poles on the stator and a squirrel cage type rotor. Fig. 23-7 shows a 4-pole motor with the field poles connected in series for alternate polarity. Each pole has a slot cut across its laminations approximately one third distance from one edge. Around the smaller part of the pole is placed a short-circuited Cu coil known as *shading coil*. This part of the pole is known as shaded pole and the other as the unshaded pole. When an alternating current is passed through the field winding surrounding the whole pole, the magnetic axis of the pole shifts from the unshaded part to the shaded part. This shifting of the magnetic axis is, in effect, equivalent to the actual physical movement of the pole. Hence, the rotor starts rotating in the direction of this shift *i.e.* from the unshaded part to the shaded part.

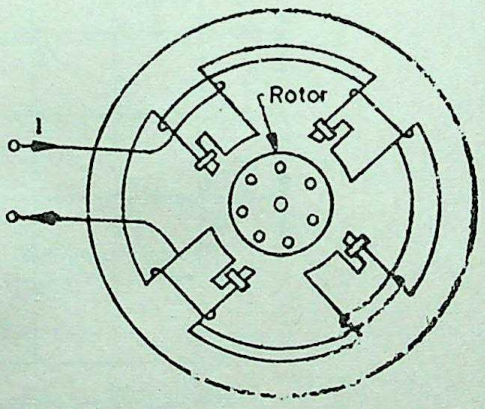

Fig. 23-7

Such motors are built in very small sizes and although simple in construction and cheap, suffer from the disadvantages of (*i*) low starting torque (*ii*) very little overload capacity and (*iii*) very low efficiency ranging from 5% (for tiny sizes) to 35% (for higher ratings).

Example 23 1. *The name plate of a 1-phase, 4-pole induction motor gives the following data :*

Output 373 W ; 230 V

Frequency 50 Hz ; input current 2·9 A

Power factor 0·71 ; speed 1410 r.p.m.

Calculate (a) the efficiency and (b) the slip of the motor when delivering the rated output.

Solution. Input $= VI \cos \phi = 230 \times 2{\cdot}9 \times 0{\cdot}71 = 474$ W

Output $= 373$ W

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(*a*) $\eta = 373/474 = 0{\cdot}785$ or **78·5 %**

(*b*) $\text{Slip} = \dfrac{N_S - N}{N_S} \times 100$

Now $N_S = 1200 \times 50/4 = 1500$ r.p.m.

$\therefore$ $s = \dfrac{1500 - 1410}{1500} \times 100 = 6\%$

Example 23-2. *At starting, the windings of a 220-V, 50-Hz split-phase induction motor have the following values :*

Main winding : $R = 10\ \Omega$, $X_L = 25\ \Omega$

Starting winding : $R = 25\ \Omega$, $X_L = 15\ \Omega$

Find

(*a*) *current in main winding and* ϕ_m

(*b*) *current in starting winding and* ϕ_s

(*c*) *phase displacement between* I_m *and* I_s

(*d*) *value of constant k if initial starting torque is 20 N-m.*

(*e*) *line current*

Solution

(*a*) $I_m = \dfrac{V}{Z_m} = \dfrac{220}{\sqrt{10^2 + 25^2}} = \dfrac{220}{26{\cdot}9} = \mathbf{8{\cdot}17\ A}$

$\cos \phi_m = R/Z_m = 10/26{\cdot}9 = 0{\cdot}37$; $\phi_m = \mathbf{68{\cdot}2°\ (lag)}$

(*b*) $I_s = \dfrac{V}{Z_s} = \dfrac{220}{\sqrt{15^2 + 25^2}} = \dfrac{220}{29{\cdot}1} = \mathbf{7{\cdot}56\ A}$

$\cos \phi_s = R/Z_s = 25/29{\cdot}1 = 0{\cdot}86$, $\phi_s = \mathbf{30{\cdot}5°}$

(*c*) $\alpha = \phi_m - \phi_s = 68\ 2° - 30{\cdot}5° = \mathbf{37{\cdot}7°}$

(*d*) $T_s = kI_mI_s \sin \alpha$

$\therefore$ $k = \dfrac{T_s}{I_mI_s \sin \alpha}$

$= \dfrac{20}{8{\cdot}17 \times 7{\cdot}56 \times \sin 37{\cdot}7°} = \mathbf{0{\cdot}53}$

(*e*) As shown in Fig. 23.8, line current I is equal to the vector sum of I_m and I_s.

Total X-component $= I_m \cos \phi_m + I_s \cos \phi_s$

$= 8{\cdot}17 \cos 68{\cdot}2° + 7{\cdot}56 \cos 30{\cdot}5°$

$= 9{\cdot}55$ A

Total Y-component $= I_m \sin \phi_m + I_s \sin \phi_s$

$= 8\ 17 \sin 68{\cdot}2° + 7{\cdot}56 \sin 30{\cdot}5°$

$= 11{\cdot}42$ A

$\therefore$ $I = \sqrt{9{\cdot}55^2 + 11{\cdot}42^2} = \mathbf{14{\cdot}9\ A}$

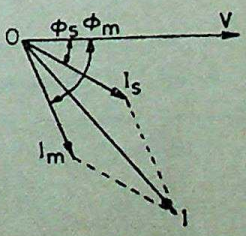

Fig. 23.8

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Example 23-3. *If a 160-μF capacitor is connected in series with the starting winding of the motor discussed in Example 23-2, what will be the value of its starting torque ?*

Solution. $$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 160} = 19{\cdot}9\ \Omega$$

$$I_s = \sqrt{25^2 + (15 - 19{\cdot}9)^2} = 25{\cdot}5\ \Omega$$

$$I_s = 220/25{\cdot}5 = 8{\cdot}62\ \text{A}$$

$$\cos \phi_s = R/Z_s = 25/25{\cdot}5 = 0{\cdot}98 \text{ (lead)}$$

$$\phi_s = 11°$$

$$\alpha = \phi_m + \phi_s$$

$$= 68{\cdot}2° + 11° = 79{\cdot}2°$$

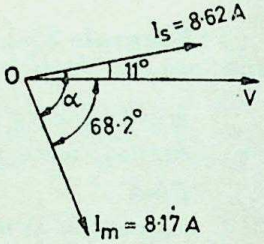

Fig. 23-9

The phasor diagram is shown in Fig. 23-9.

$$T_s = kI_mI_s \sin \alpha$$

$$= 0{\cdot}53 \times 8{\cdot}17 \times 8{\cdot}62 \times \sin 79{\cdot}2° = \mathbf{36{\cdot}7\ N\text{-}m}$$

The increase in the value of starting torque is considerable.

23-7. Motor Troubles

Common single-phase motor troubles as diagnosed by different human senses are summarized in Table 23-1 given below :

Table 23-1 Motor Troubles

Sense	*Symptom*	*Possible Causes*	
		Mechanical	*Electrical*
Sight	Motor will not start smoking	seized bearing, over-heated bearing	supply faulty, burnt out winding
Hearing	bearing rumble, vibration	faulty bearing, installation fault or out-of-balance rotor	—
Touch	overheating	insufficiently or incorrectly lubricated bearing	continued overload or faulty winding
Smell	burning smell	overheated bearing	burning winding

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ALTERNATORS 24

1. Basic Principle and Construction 2. Principle of Operation 3. Speed and Frequency 4. Equation of Induced E.M.F. 5. Alternator on Load 6. Phasor Diagram of a Loaded Alternator 7. Voltage Regulation 8. Highlights.

24-1. Basic Principle and Construction *(A.M.I.E. Nov. 1972)*

A.C. generators or alternators (as they are usually called) operate on the same fundamental principle of electromagnetic induction as d.c. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in d.c. generators, the *armature rotates* and the field system is stationary, the arrangement in alternators is just the reverse. In their case, standard construction consists of armature windings mounted on a stationary element called *'stator'* and field windings on a rotating element called *'rotor'*. The details of construction are shown in Fig. 24-1.

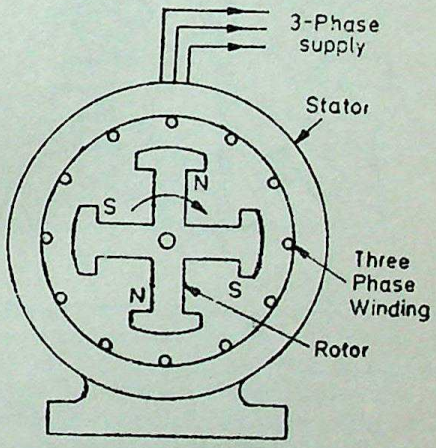

Fig. 24-1

(*a*) Stator

It consists of a cast-iron frame which supports the laminated armature core having slots on its inner periphery for housing the 3-phase winding.

(*b*) Rotor

These are of two types :

(*i*) *Salient (or projecting) pole type*

It is like a flywheel which has a large number of alternate North and South poles bolted to it as shown in Fig. 24-1. The magnetic wheel is made of cast iron or steel of good magnetic quality.

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The magnetic poles are excited by a small d.c. generator mounted on the shaft of the alternator itself.

Such rotors are used in low and medium-speed alternators which are characterised by their large diameters and short axial lengths.

(*ii*) *Smooth Cylindrical Type*

It consists of a smooth solid forged-steel cylinder having a number of slots milled out at intervals along the outer periphery for accommodating field coils as shown in Fig. 24-2. Two or four regions corresponding to the central polar areas are left unslotted. The central polar areas are surrounded by the field windings placed in slots. Obviously, in this case, the poles are non-salient *i.e.* they do not project out from the surface of the rotor.

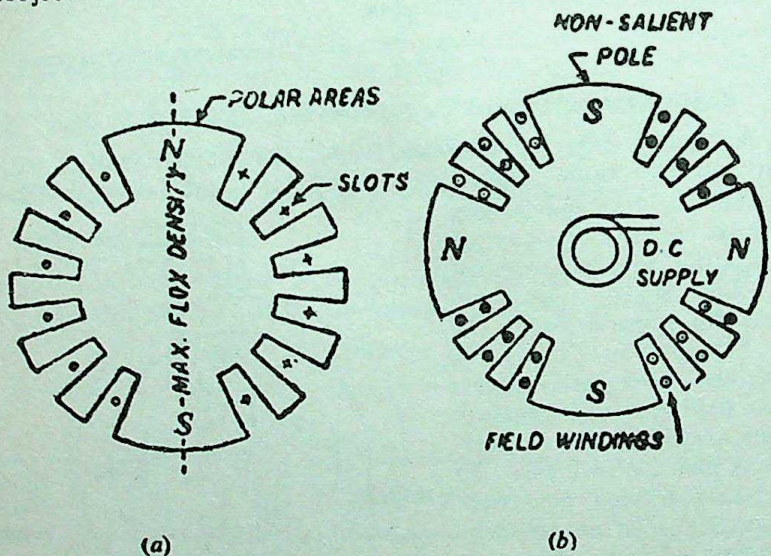

Fig. 24-2

Such rotors are used in steam-turbine driven alternators *i.e.* turbo-alternators or turbo-generators which run at very high speeds and are characterised by their small diameters and very long axial lengths.

24-2. Principle of Operation

(*A.M.I.E. Winter 1983*)

When the rotor is rotated by the prime-mover, the stator winding or conductors are cut by the magnetic flux of the rotor poles. Hence, an e.m.f. is induced in the stator conductors. Because the rotor poles are alternately N and S, they induce an alternating e.m.f. in the stator conductors. The frequency of this induced e.m.f. is given by $f = PN/120$ (Art. 24-3) and its direetion can be found by applying Fleming's Right-hand rule.

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The e.m.f. generated in the stator conductors is taken out from the three leads connected to the stator winding as shown in Fig. 24-1.

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24-3. Speed and Frequency

In an alternator, there exists a definite relationship between the rotational speed (N) of the rotor, the frequency (f) of the generated e.m.f. and the uumber of poles P.

Consider an armature conductor marked X in Fig. 24-3 situated at the centre of a N-pole rotating in clockwise direction. The conductor being situated at the place of maximum flux density, will have maximum e.m f. induced in it.

The direction of the induced e.m.f. is given by the Fleming's Right-hand rule. But while applying this rule, one should be careful to note that the thumb indicates the direction of the motion of the *conductor* relative to the *field*. To an observer stationed on the clockwise revolving poles, the conductor would seem to be rotating anticlockwise. Hence, thumb should point to the left. The direction of the induced e.m.f. is downwards in a direction at right angles to the plane of the paper.

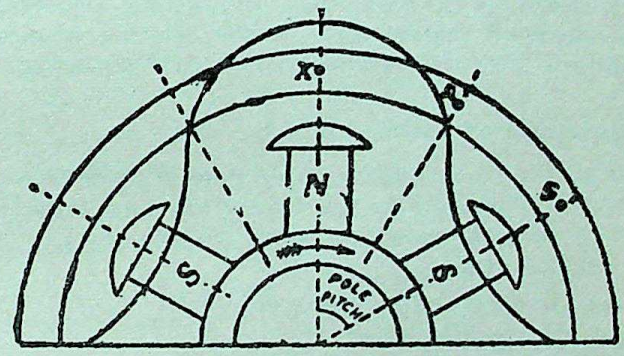

Fig. 24-3

When the conductor is in interpolar gap as at A in Fig. 24-3, it has minimum e.m.f. induced in it because flux is minimum there. Again when it is at the centre of a S-pole, it has maximum e.m.f. induced in it, because flux density at B is maximum. But the direction of the e.m f. when conductor is over a N-pole is opposite to that when it is over a S-pole.

Obviously, one cycle of e.m.f. is induced in a conductor when one pair of poles passes over it. In other words, the e.m.f. in an armature conductor goes through one cycle in an angular distance equal to twice the pole pitch as shown in Fig. 24-3.

Let P = total number of magnetic poles

N = rotative speed of the rotor in r.p.m.

f = frequency of generated e.m.f. in Hz

Then, in one revolution of the rotor, a conductor is cut by $P/2$ North poles and $P/2$ South poles.

No. of cycles/revolution $= P/2$

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No. of revolutions/second $= N/60$

$$\therefore \quad \text{frequency} = \frac{P}{2} \times \frac{N}{60} = \frac{PN}{120} \text{ Hz}$$

or $$f = PN/120 \text{ Hz}$$

N is known as the synchronous speed, because it is the speed at which an alternator must run in order to generate an e.m.f. of the required frequency. In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, the alternator will have to run at the following speeds.

No. of poles	2	4	6	12	24	36
Speed (r.p.m.)	3600	1800	1200	600	300	200

Referring to the above equation, we get $P = 120\,f/N$

It is clear from the above that because of slow rotative speeds of engine-driven alternators, their number of poles is much greater as compared to that of the turbo-generators which run at very high speeds.

24-4. Equation of Induced E.M.F.

Let Z = No. of conductors or coil sides in series/phase
$= 2T$ where T is the No. of coils or turns per phase (remember one turn or coil has two sides)
P = No. of poles
f = frequency of induced e.m.f. in Hz
Φ = flux/pole in webers
N = rotative speed of the rotor in r.p.m.

In one revolution of the rotor (*i.e.* in $60/N$ second), each stator conductor is cut by a flux of $P\Phi$ webers.

$$\therefore \quad d\Phi = P\Phi \text{ and } dt = 60/N$$

$\therefore$ average e.m.f. induced per conductor

$$= \frac{d\Phi}{dt} = \frac{P\Phi}{60/N} = \frac{PN\Phi}{60} \text{ volt} \qquad \ldots(i)$$

Now, we know that $f = PN/120$ or $N = 120\,f/P$

Eliminating N from Eq. (i), we have

average e.m.f. per conductor

$$= \frac{\Phi P}{60} \times \frac{120\,f}{P} = 2\,f\Phi \text{ volt}$$

If there are Z conductors in series/phase, then

average e.m.f./phase $= 2\,f\Phi Z = 4\,f\Phi T$ volt

R.M.S. value of e.m.f./phase $= 1{\cdot}11 \times 2\,f\Phi Z = 2{\cdot}22\,fZ\Phi$ volt
$= 4{\cdot}44\,f\Phi T$ volt*

*It is exactly the same as e.m.f. equation of a transformer (Art. 21-6).

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If the alternator is star-connected (as is usually the case) then the line voltage is $\sqrt{3}$ times the phase voltage (as found from the above formula).

Example 24-1. *Calculate the synchronous speed of a four-pole 50-Hz alternator.*

Solution. $f = \frac{PN}{120}$ or $N = \frac{120f}{P}$

$\therefore$ $N = 120 \times 50/4 = \mathbf{1500\ r.p.m.}$

Example 24-2. *What is the frequency of voltage generated by an alternator having 10 poles and rotating at 720 r.p.m.?*

Solution. $f = PN/120 = 10 \times 720/120 = \mathbf{60\ Hz}$

Example 24-3. *A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 30 mWb sinusoidally distributed and the speed is 375 r.p.m. Find the frequency, the phase and the line e.m.f.*

Solution. Formula used

$E = 2{\cdot}22\, f\, Z\Phi$ volt —per phase

$f = PN/120 = 16 \times 375/120 = \mathbf{50\ Hz}$

No. of slots per phase $= 144/3 = 48$

No. of conductors/slot $= 10$

$\therefore$ No. of conductors/phase $= Z = 48 \times 10 = 480$

$\therefore$ e.m.f. per phase $= 2{\cdot}22 \times 50 \times 30 \times 10^{-3} \times 480$

$= \mathbf{1,600\ V}$

For a star-connection, $V_L = \sqrt{3} V_{ph}$

$\therefore$ Line voltage $= \sqrt{3} \times 1600 = \mathbf{2770\ V}$

Tutorial Problems—37

1. A 3-phase, star-connected, 2-pole alternator runs at 3600 r.p.m. If there are 500 conductors per phase in series on the armature winding and the sinusoidal flux per pole is 0·1 Wb, calculate the magnitude and frequency of the generated voltage. **[60 Hz ; 11·5 kV]**

2. A 4-pole, 3-phase, star-connected alternator has 24 slots with 12 conductors per slot and the flux per pole is 0·1 Wb. Calculate the line e.m.f. generated when the alternator is driven at 1,500 r.p.m. **[1,785 V]**

24-5. Alternator on Load

As load on an alternator is varied, its terminal voltage V is also found to vary as in dc generators. This variation in V is due to the following reasons :

(*i*) voltage drop due to armature resistance R_a

(*ii*) voltage drop due to armature leakage reactance X_L

(*iii*) voltage drop due to armature reaction

The drop due to armature reaction is theoretically accounted for by assuming a fictitious reactance X_a in armature winding. The vector sum of X_L and X_a gives *synchronous reactance* X_s.

Hence, it can now be said that an alternator possesses (*i*) resistance R_a and (*ii*) synchronous reactance X_s. Their *vector* sum

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gives synchronous impedance Z_s.

$$Z_s = \sqrt{R_a^2 + X_s^2}$$

When an alternator is on load, there is voltage drop due to R_a and X_s or due to Z_s.

If V is the terminal voltage/phase on load and E the generated emf/phase on no-load, then

$$\mathbf{E} = \mathbf{V} + \mathbf{I}(\mathbf{R}_a + \mathbf{jX}_s) = \mathbf{V} + \mathbf{I}_a\mathbf{Z}_s$$

24-6. Phasor Diagram of a Loaded Alternator

Let E = no-load emf/phase
V = terminal voltage/phase
I_a = armature current/phase
ϕ = load p.f. angle
Z_s = synchronous impedance/phase

(a) Unity Load p.f.

In Fig. 24-4, V is taken as the reference vector. Current vector I_a is in phase with V. The voltage drop I_aR_a is in phase with I_a whereas drop I_aX_s is at right angles to it, their vector sum giving I_aZ_s. When I_aZ_s is combined with V, we get E.

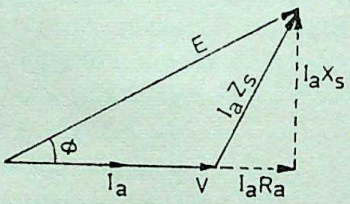

Fig. 24-4

(b) Lagging Load p.f.

In this case, I_a lags behind V by an angle ϕ. As usual, I_aR_a is in phase with I_a whereas I_aX_s is at right angles to it. When I_aZ_s is combined with V, we get E as shown in Fig. 24-5 (*a*). Fig. 24-5 (*b*) shows the diagram when instead of V, I_a is taken as the reference vector.

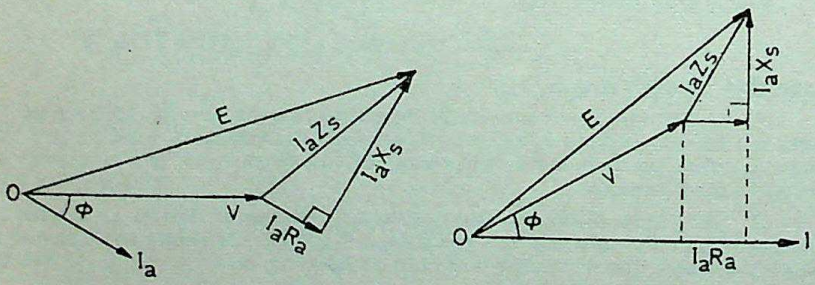

Fig. 24-5

(c) Leading Load p.f.

Such a case is shown in Fig. 24-6. Here, I_a leads V by ϕ. As usual, vector sum of V and I_aZ_s gives E.

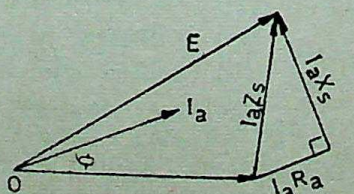

Fig. 24-6

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24-7. Voltage Regulation

The terminal voltage V of an alternator is found to vary with load and its power factor. Voltage regulation of an alternator is defined as the *rise* in voltage when full-load is removed divided by the rated terminal voltage

$$\%\text{regulation 'up'} = \frac{E-V}{V} \times 100$$

Example 24-4. *A 500-kVA, 1100-V, 50-Hz, Y-connected 3-ϕ alternator has armature resistance/phase of 1·0 Ω and synchronous reactance/phase of 1·5 Ω. Find its voltage regulation for (a) unity p.f. (b) 0·9 lagging p f. and (c) 0·8 leading p f. Also calculate voltage regulation in each case.*

Solution. $\sqrt{3}V_L I_L = 500{,}000$

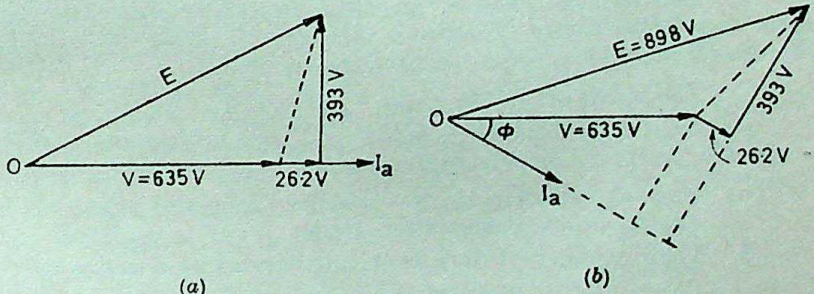
$$I_L = 500{,}000/\sqrt{3} \times 1100 = 262 \text{ A}$$

armature current, $I_a = 262$ A

$$I_a R_a = 262 \times 0{\cdot}1 = 26{\cdot}2 \text{ V}$$

$$I_a X_s = 262 \times 1{\cdot}5 = 393 \text{ V}$$

$$V = V_L/\sqrt{3} = 1100/\sqrt{3} = 635 \text{ V}$$



(*a*) (*b*)

Fig. 24-7

(*a*) Unity p.f.

As shown in Fig. 24-7 (*a*)

$$E = \sqrt{(V + I_a R_a)^2 + (I_a X_s)^2}$$

$$= \sqrt{(635 + 26{\cdot}2)^2 + (393)^2} = 769{\cdot}2 \text{V}$$

$$\text{regn} = \frac{769{\cdot}2 - 635}{635} \times 100 = 21{\cdot}1\%$$

(*b*) Lagging p.f.

This case is shown in Fig. 24-7 (*b*)

$$\cos\phi = 0{\cdot}2, \quad \phi = 25{\cdot}83°$$

$$\sin\phi = 25{\cdot}83° = 0{\cdot}436$$

As seen from Fig. 24-7 (*b*)

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$$E = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi + I_aX_s)^2}$$

$$= \sqrt{(635\times 0{\cdot}9 + 26{\cdot}2)^2 + (635\times 0{\cdot}436 + 393)^2}$$

$$= \mathbf{898\ V}$$

$$\%\text{regn} = \frac{898-635}{635}\times 100 = \mathbf{41{\cdot}4\%}$$

(*c*) **Leading p.f.**

As seen from Fig. 24-8

$$E = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi - I_aX_s)^2}$$

$$= \sqrt{(635\times 0{\cdot}8 + 26{\cdot}2)^2 + (635\times 0{\cdot}6 - 393)^2}$$

$$= \mathbf{534\ V}$$

$$\%\ \text{regn} = \frac{534-635}{635}\times 100$$

$$= \mathbf{-15{\cdot}9\%}$$

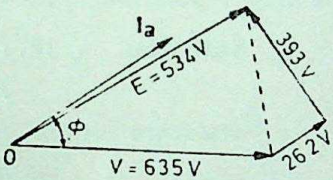

Fig. 24-8

HIGHLIGHTS

1. Like induction motor, an alternator also consists of (*i*) a stator and (*ii*) a rotor. The stator provides the armature windings whereas rotor provides the rotating magnetic field.

2. The rotors are of two types :

(*i*) Salient (or projecting) pole type—used for low and medium-speed (engine-driven) alternators.

(*ii*) Smooth cylindrical type—used for turbo-alternators *i.e.* for steam-turbine driven alternators.

3. The frequency of the alternating current produced is

$$f = PN/120\ \text{Hz}$$

4. The e.m.f. equation of an alternator is

$$E = 2{\cdot}22 f Z\Phi\ \text{volt/phase}$$

$$= 4{\cdot}44 f\,\Phi T\ \text{volt/phase}$$

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SYNCHRONOUS MOTORS 25

1. Synchronous Motor 2. Construction 3. Principle of Operation 4.Making Synchronous Motor Self-starting 5. Characteristics of Synchronous Motor 6. Motor on Load 7. Motor Phasor Diagram 8. Power Stages in a Synchronous Motor 9 Values of E_b and E_R 10. Mechanical Power Developed by Motor 11. Synchronous Capacitor 12. Highlights.

25-1. Synchronous Motor—Construction

A synchronous motor is electrically identical with an alternator. A synchronous machine may be used, at least theoretically, as an alternator when driven mechanically or as a motor when driven electrically, just as in the case of d.c. machines (Art. 12-2).

As shown in Fig. 25-1, essential mechanical elements of such a motor are the same as those of an alternator *viz*

(*i*) a stator—which houses 3-phase windings in the slots of the stator core.

(*ii*) a rotor—which has a number of alternately North and South poles.

The rotor can be either salient pole type or smooth cylindrical type (Art. 24-2). The rotor poles are excited by a small d.c. shunt generator mounted on the rotor shaft.

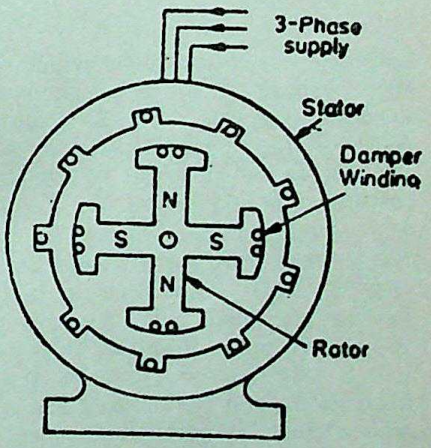

Fig. 25-1

25-2. Principle of Operation *(A.M.I.E. Winter 1983)*

One of the basic limitations of a synchronous motor is that it is not inherently self-starting as may be understood from the discussion to follow.

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It was shown in Art. 22-3 that when a 3-phase winding is fed from a 3-phase supply, a resultant magnetic flux is produced whose

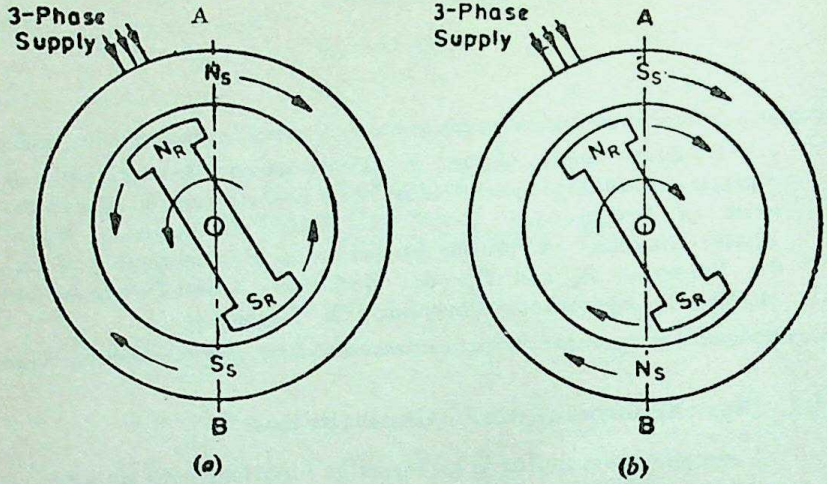

Fig. 25-2

magnitude is constant but which revolves synchronously. For all practical purposes it can be assumed as if two actual magnetic poles rotate around the stator. Fig. 25-2 shows two *stator* poles marked N_S and S_S revolving synchronously in clockwise direction. With the rotor position as shown in the figure, suppose the stator poles are at that instant at points A and B. The two similar poles, N_R (of rotor) and N_S (of stator) as well as S_R and S_S will repel each other, with the result that the rotor tends to rotate in anticlockwise direction [Fig. 25-2 (*a*)]. But half a period later [Fig. 25-2 (*b*)], stator poles, having travelled half way around, interchange their positions *i.e.* N_S goes to point B and S_S to point A. During this period, the rotor has hardly moved from its position. Under these conditions, N_S attracts S_R and S_S attracts N_R. Hence, rotor tends to rotate clockwise (which is just the opposite of the condition half a period earlier). So, we find that due to continuous and rapid rotation of stator poles, rotor is subjected to a rapidly reversing torque. In other words, rotor is subjected to a torque which tends to move it first in one direction and then in the opposite. Owing to its large moment of inertia, rotor cannot respond to such quickly-reversing torques with the result that it remains stationary. Now, consider the condition shown in Fig. 25-3 (*a*). The stator and rotor poles attract each other. Within half a period, stator poles will interchange their position as shown in Fig. 25 3 (*b*). Suppose that, by some means, the rotor poles are also made to rotate through half a revolution during the same period as shown in Fig. 25-3 (*b*). Then again the stator and rotor poles will attract each other. Obviously, if the rotor poles

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also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque.

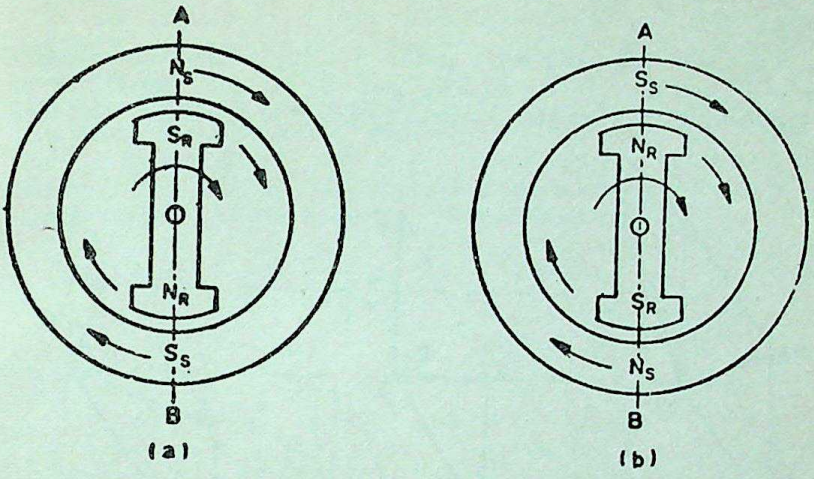

Fig. 25-3

One method of starting the synchronous motor is as under :

The rotor is speeded up by another motor (called pony motor) to its synchronous or near synchronous speed. Then, the exciting current of the rotor poles is switched on. The moment rotor poles are excited, they get magnetically locked into position with the stator poles of opposite polarity *i.e.* the rotor poles are engaged with the stator poles and both run synchronously in the same direction. It is because of this interlocking of rotor and stator poles that the motor has either to run at synchronous speed or not at all.

25-3. Making Synchronous Motor Self-starting

The motor can be made self-starting by providing squirrel-cage winding (also called damper winding) consisting of Cu bars embedded in the rotor poles-shoes and short-circuited at both ends (Fig. 25-1). Obviously, such a motor will start readily, acting as a 3-phase induction motor during the starting period.

The supply voltage is switched on to the stator winding while the rotor pole field circuit is left open. The synchronously rotating stator flux will produce e.m.f. in the squirrel cage rotor bars as discussed in Art. 22-4. The motor starts up as induction motor but when it reaches nearly 90 per cent of its synchronous speed, the rotor field circuit is closed. The rotor poles are magnetised and get immediately interlocked with the stator poles thus pulling the motor into synchronism.

25-4. Characteristics of a Synchronous Motor

One of the most important property of this motor is that it can be made to operate at a power factor varying from lagging to a leading one. This peculiar behaviour of the motor can be understood by noting that, as in the armature of a d.c. motor, there are two opposing voltages in the stator (or armature) winding of a synchronous motor. The two voltages are :

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(*a*) the supply voltage V

(*b*) the back e.m.f. E_b induced in the stator winding by the rotor pole flux.

The net voltage across the stator winding is the vector difference of V and E_b. If $V - E_b = E_R$ and Z_S is the impedance of the stator winding, then stator winding current is $I_a = E_R/Z_S$ and lags

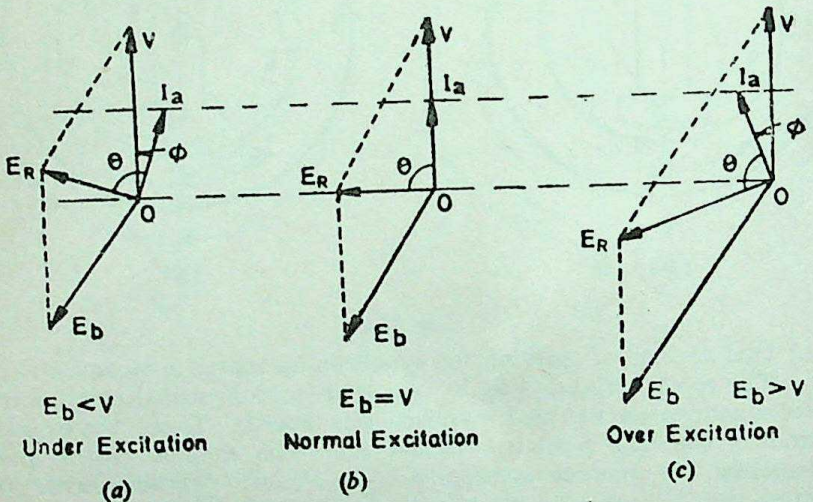

Fig. 25-4

behind E_R by a fixed angle $\theta = \tan^{-1}(X_a/R_a)$. If ϕ is the phase angle between V and I_a, then motor power factor is $\cos\phi$. As shown in Fig 25-4, the power factor can be varied by varying rotor field excitation and hence E_b.

(*i*) if E_b is lesser in magnitude than V *i.e.* when rotor field excitation is weak or motor is underexcited, then I_a (which is $\theta°$ behind E_R) lags behind V by an angle. ϕ. Hence, motor has a lagging power factor [Fig. 25-4 (*a*)].

(*ii*) when rotor field excitation is now increased, so that E_b equals V in magnitude, I_a is in phase with V and has minimum value for the given load. The motor has a power factor of unity.

(*iii*) when rotor field excitation is further increased *i.e.* when motor is overexcited, then E_b becomes greater than V in magnitude and I_a leads V so that motor power factor becomes leading as shown in Fig. 25-4 (*c*).

The fact that synchronous motor can run with a leading power factor makes it extremely useful for phase advancing purposes in the case of industrial loads driven by induction motors and lighting and heating loads supplied through transformers (Art. 20-21).

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25-5. Motor on Load

As explained in Art. 25-2, the stator and rotor poles of a running synchronous motor are magnetically interlooked and hence both run with same synchronous speed. But the two poles are not in phase with each other even when motor runs on no-load. There is a small phase difference between the two poles, with rotor pole trailing behind stator pole even though running synchronously as shown in Fig. 25-5 (*a*). This phase angle is essential for developing motor torque sufficient enough to meet no-load losses.

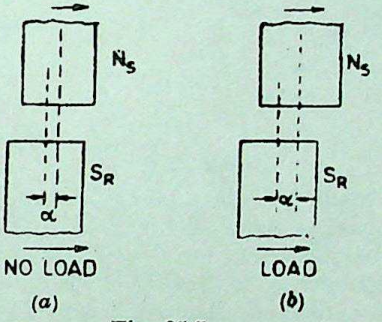

Fig. 25-5

As the load on the motor is increased, the rotor falls back in phase (but not in speed) by a greater angle α as shown in Fig. 25 5 (*b*) but *it still continues to run synchronously*. The value of this load angle or coupling angle α depends on the load to be met by the motor. In other words, torque developed by the motor depends on this angle.

25-6. Motor Phasor Diagram

As explained in Art. 25-4, there are *two* opposing voltages in the armature (stator) winding of a synchronous motor :—

(*a*) the supply voltage V

(*b*) back emf E_b induced in the stator winding by the synchronously-revolving pole flux.

This E_b depends on the rotor speed and flux. Since rotor runs at a constant speed, its value depends on flux produced by rotor poles.

The net voltage/phase in stator winding is

$$E_R = V - E_b \qquad \text{—vector difference}$$

If R_a is stator resistance/phase, X_S stator synchronous reactance/phase, Z_S, stator synchronous impedance/phase, then

$$Z_S = \sqrt{R_a^2 \times X_S^2}$$

Stator armature current/phase is $I_a = E_R/Z_S$.

This current I_a lags behind E_R by a fixed angle $\theta = \tan^{-1}(X_S/R_a)$ in the phasor diagram of Fig. 25-6 (*a*) which is similar to Fig. 25-4 (*a*) though turned 90° clockwise. If $R_a = 0$, then $\tan\theta = X_S/R_a = X_S/0 = \infty$, hence $\theta = 90°$

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In practice, howerver, $R_a \neq 0$ though negligible as compared to X_S and so $\theta \cong 85°$ or so.

Fig. 25-6 (*a*), in fact, represents the no-load condition of an under-excited motor *i.e.* one in which $E_b < V$. If ϕ is the angle between V and I_a, motor power factor is cos ϕ. Obviously, motor power input per phase $= V\ I_a \cos \phi$.

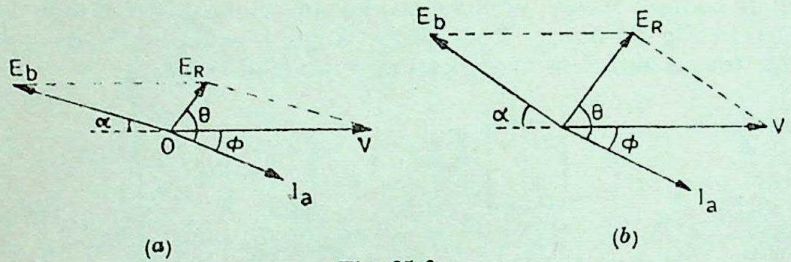

Fig. 25-6

When load is applied to the motor, its rotor further falls behind the stator poles in phase as shown in Fig. 25-6 (*b*). Consequently, vector for E_b further falls behind vector for V (you should imagine the vectors in Fig. 25-6 to be revolving in anti-clockwise direction) though its magnitude remains unaffected. The angle α in Fig. 25-6 (*b*) is greater than its value in Fig. 25-6 (*a*). It is seen that magnitude of E_R and hence I_a is increased with the result that now motor draws more power in order to supply the extra load put on it.

The mechanical power developed by the motor is

P_{mech} = back emf × stator current × cosine of the angle between the two

$= E_b\ I_a \cos(\alpha - \phi)$ per phase.

25-7. Power Stages in a Synchronous Motor

We will consider the phase values.

Power input/phase $= V\ I_a \cos \phi$

Out of this, some goes to meet armature Cu loss $I_a^2 R_a$ and the rest is converted into gross mechanical power in armature *i.e.* $P_{mech} = E_b I_a \cos(\alpha - \phi)$. Out of it, some power is used for meeting iron, friction and excitation losses and the balance is available as output power as shown below :

Power input/phase
$V\ I_a \cos \phi$

- armature Cu loss $I_a^2\ R_a$
- Mech. power in armature $P_{mech} = E_b\ I_a \cos(\alpha - \phi)$
  - iron, friction and excitation losses
  - output power, P_{out}

* If $E_b > V$, then motor is said to be overexcited and draws a leading current. If $E_b = V$, motor draws in-phase current.

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25-8. Values of E_b and E_R

Fig. 25-7 shows the phasor diagram of an under-excited motor running loaded.

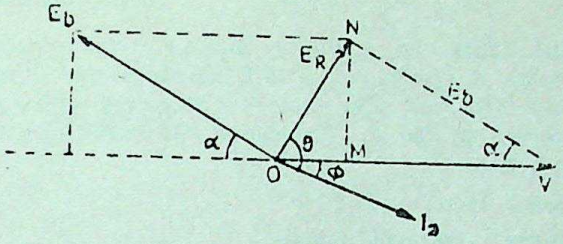

Fig. 25-7

(*a*) Value of E_b

As seen from right-angled $\triangle VMN$

$$VN^2 = MN^2 + VM^2$$
$$= [E_R \sin(\theta - \phi)]^2 + [V - E_R \cos(\theta - \phi)]^2$$
$$= E_R^2 \sin^2(\theta - \phi) + V^2 + E_R \cos^2(\theta - \phi) - 2VE_R \cos(\theta - \phi)$$

$$\therefore \quad E_b^2 = V^2 + E_R^2 - 2VE_R \cos(\theta - \phi)$$

or $$E_b = \sqrt{V^2 + E_R^2 - 2V\,E_R \cos(\theta - \phi)}$$

(*b*) Value of E_R

Again with reference to Fig. 25-7,

$$E_R = \sqrt{OM^2 + MN^2}$$
$$= \sqrt{(V - E_b \cos\alpha)^2 + (E_b \sin\alpha)^2}$$
$$= \sqrt{V^2 + E_b^2 - 2V\,E_b \cos\alpha}$$

The angle between V and E_R is $\angle NOM = (\theta - \phi)$

$$\tan(\theta - \phi) = \tan\delta = NM/OM$$

$$\therefore \quad (\theta - \phi) = \tan^{-1}\left(\frac{E_b \sin\alpha}{V - E_b \cos\alpha}\right) \qquad \text{—Fig. 25-7}$$

25-9. Mechanical Power Developed by Motor

With reference to Fig. 25-8, it can be proved that mechanical power/phase developed by a synchronous motor is given by

$$P_{mech} = \frac{E_b V}{Z_S} \cos(\theta - \alpha) - \frac{E_b^2}{Z_S} \cos\theta$$

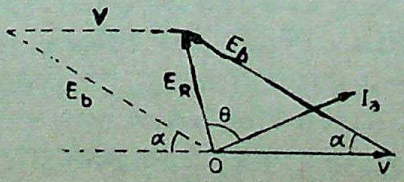

Fig. 25-8

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Now, $\theta \cong 90°$, hence $\cos\theta = 0$ and $\cos(\theta - \alpha) = \cos(90 - \alpha) = \sin\alpha$

$$\therefore \quad P_{mech} = \frac{E_b V}{Z_s} \sin\alpha \qquad \text{—per phase}$$

$$= 3\,\frac{E_b\,V}{Z_s} \sin\alpha \qquad \text{—for 3-phases}$$

Example 25·1. *A 2,300-V, 3-phase, star-connected synchronous motor has a resistance of 0·2 Ω/phase and a synchronous reactance of 2·2 Ω/phase. The motor is operating at 0·5 p.f. leading with a line current of 200 A. Determine the value of generated (or back) e.m.f. per phase.*

Solution. Here,

$$\phi = \cos^{-1}(0{\cdot}5) = 60° \text{ (lead)}$$

$$\tan\theta = X_s/R_a = 2{\cdot}2/0{\cdot}2 = 11$$

$$\therefore \quad \theta = \tan^{-1}(11) = 84{\cdot}4°$$

$$\therefore \quad \theta + \phi = 84{\cdot}4° + 60° = 144{\cdot}8°$$

$$\cos 144{\cdot}8° = -\cos 35{\cdot}2°$$

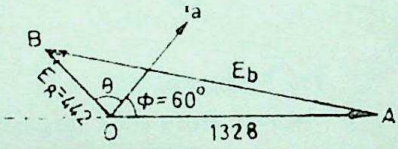

Fig. 25-9

$$\therefore \quad V/\text{phase} = 2300/\sqrt{3} = 1328 \text{ V}$$

$$Z_s = \sqrt{R_a^2 + X_s^2} = \sqrt{0{\cdot}2^2 + 2{\cdot}2^2} = 2{\cdot}21\ \Omega$$

Now, for Y-connection, phase current equals line current.

$$E_R = I_a\, Z_s = 200 \times 2{\cdot}21 = 442 \text{ V}$$

As seen from vector (or phasor) diagram of Fig. 25-9

$$E_b^2 = V^2 + E_R^2 - 2\,V\,E_b \cos(\theta + \phi)$$

$$= 1328^2 + 442^2 - 2 \times 1328 \times 442 \times \cos 144{\cdot}8°$$

or $$E_b = \sqrt{1328^2 + 442^2 - 2 \times 1328 \times 442 \times (-\cos 35{\cdot}2°)}$$

$$= \mathbf{1708 \text{ volts/phase}}$$

Example 25·2. *A 18·65 kW, 220-V 50-Hz, 4-pole, Y-connected synchronous motor is running with a light load. The load angle is 4° and back emf generated per phase is 110 V. If armature resistance/phase is 0·1 Ω and synchronous reactance/phase is 1·5 Ω, find*

(a) resultant armature voltage/phase

(b) armature current/phase

(c) internal angle θ

(d) p f. angle ϕ and p.f.

(e) power input to motor

(f) gross torque developed by motor.

Solution. Here, $\alpha = 4°$, $V = 220/\sqrt{3} = 127$ V. $E_b = 110$ V

(a) $$E_R = \sqrt{V^2 + E_b^2 - 2\,V\,E_b \cos\alpha}$$

$$= \sqrt{127^2 + 110^2 - 2 \times 127 \times 110 \times \cos 4°}$$

$$= \mathbf{18{\cdot}9 \text{ V per phase}}$$

(b) $$Z_s = \sqrt{R_a^2 + X_s^2} = \sqrt{0{\cdot}1^2 + 1{\cdot}5^2} \cong 1{\cdot}5\ \Omega$$

$$I_a = E_R/Z_s = 18{\cdot}9/1{\cdot}5 = \mathbf{12{\cdot}6\ A}$$

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(c) $\tan\theta = X_s/R_a$, $\theta = \tan^{-1}(1{\cdot}5/0{\cdot}1) = \mathbf{86{\cdot}2°}$

The phasor diagram is shown in Fig. 25-10.

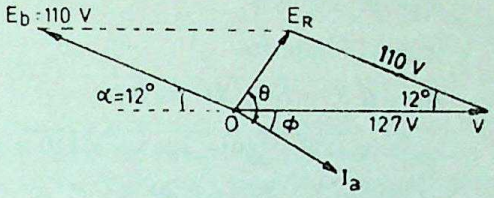

Fig. 25-10

(*d*) As seen from Art. 25-8,

$$(\theta - \phi) = \tan^{-1}\frac{E_b \sin\alpha}{V - E_b\cos\alpha}$$

$$= \tan^{-1}\left(\frac{110 \times \sin 4°}{127 - 110\cos 4°}\right)$$

$$= \tan^{-1}(0{\cdot}4435) = 23{\cdot}9°$$

$$\therefore \quad \phi = \theta - 23{\cdot}9° = 86{\cdot}2° - 23{\cdot}9° = \mathbf{62{\cdot}3°}$$

power factor $= \cos 62{\cdot}3° = \mathbf{0{\cdot}46\ (lag)}$

(*e*) $P_{in} = \sqrt{3}\, V_L\, I_L \cos\phi$

$$= \sqrt{3} \times 220 \times 12{\cdot}6 \times \cos 62{\cdot}3° = \mathbf{2210\ W}$$

(*f*) Total armature Cu loss

$$= 3\, I_a^2\, R_a = 3 \times 12{\cdot}6^2 \times 0{\cdot}1 = 48\ \text{W}$$

Internal power developed

$$= 2210 - 48 = 2162\ \text{W}$$

$$N_s = 120\, f/P = 120 \times 50/4 = 1500\ \text{rpm} = 25\ \text{rps}$$

If T_g is the gross torque developed in newton-metre (N-m), then

$$T_g \times 2\pi\, N_s = 2162$$

or $T_g = 2162/2\pi \times 25 = \mathbf{13{\cdot}8\ N\text{-}m.}$

Example 25-3. *The load on the motor of Example 25-2 is increased so that torque angle becomes 12°. If excitation remains constant, calculate*

(*a*) *armature current*

(*b*) *power factor*

(*c*) *total power input to the motor*

(*d*) *gross torque developed by the motor.*

Solution. Now, $\alpha = 12°$, $\cos\alpha = 0{\cdot}978$, $\sin\alpha = 0{\cdot}208$

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From example 25-2, we have

$$Z_s = 1{\cdot}5\ \Omega\ ;\ \theta = 86{\cdot}2°$$

$$V = 127\ \text{V/phase}$$

$$E_b = 110\ \text{V/phase}.$$

(*a*) $$E_R = \sqrt{V^2 + E_b^2 - 2V\,E_b \cos\alpha}$$

$$= \sqrt{127^2 + 110^2 - 2 \times 127 \times 110 \times 0{\cdot}978} = 30\ \text{V}$$

$$I_a = E_R/Z_s = 30/1{\cdot}5 = \mathbf{20\ A}$$

(*b*) $$\tan(\theta - \phi) = \frac{E_b \sin\alpha}{V - E_b \cos\alpha}$$

$$= \frac{110 \times 0{\cdot}208}{127 - 110 \times 0{\cdot}978} = 1{\cdot}167$$

$$(\theta - \phi) = \tan^{-1}(1{\cdot}167) = 49{\cdot}4°$$

$$\therefore \quad \phi = 86{\cdot}2° - 49{\cdot}4° = 36{\cdot}8°$$

$\therefore$ power factor $= \cos\phi = \cos 36{\cdot}8° = \mathbf{0{\cdot}8\ (lag)}$

Phasor diagram is shown in Fig. 25-11

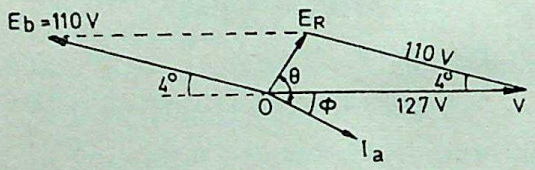

Fig. 25-11

(*c*) $$P_{in} = \sqrt{3}\,V_L\,I_L \cos\phi$$

$$= \sqrt{3} \times 220 \times 20 \times 0{\cdot}8 = \mathbf{6097\ W}$$

(*d*) Total armature Cu loss

$$= 3\,I_a^2\,R_a = 3 \times 20^2 \times 0{\cdot}1 = 120\ \text{W}$$

$\therefore$ internal power developed

$$= 6097 - 120 = 5977\ \text{W}$$

$$N_s = 120\,f/P = 120 \times 50/4 = 1500\ \text{rpm} = 25\ \text{rps}$$

$$\therefore \quad T_g = 5{,}977/2\pi \times 25 = \mathbf{38\ N\text{-}m}$$

From the above examples, it is clear that

(*i*) increase in load angle α is almost directly proportional to increase in motor input

(*ii*) increase in gross torque is almost directly proportional to increase in load angle α

(*iii*) as load is increased, lagging power factor tends to approach unity.

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25.10. Synchronous Capacitor

When a synchronous motor is over-excited, its generated or back emf E_b is much greater than supply voltage V. When such a motor operates without load, α is small and E_R is almost 180° ahead of V. As shown in Fig. 25-12, I_a lags E_R by nearly 90° and hence *leads* V by about 90°.

Due to this leading power factor, a synchronous motor acts as a capacitor and can be used to correct the p.f. of a lagging load. Such a motor, designed to carry no load, is called a *synchronous capacitor*. It is used where load is so large that construction of a static capacitor is impractical.

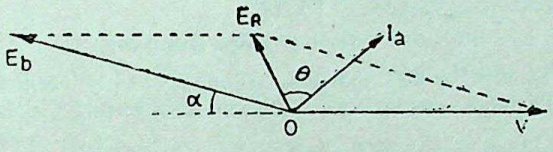

Fig. 25-12

In factories which use induction motors, the p.f. rarely exceeds 85 per cent. Taking into account the fact that starting power factor of such motors is poor and also that not all the motors operate at full-load, the overall p.f. is as low as 60 per cent.

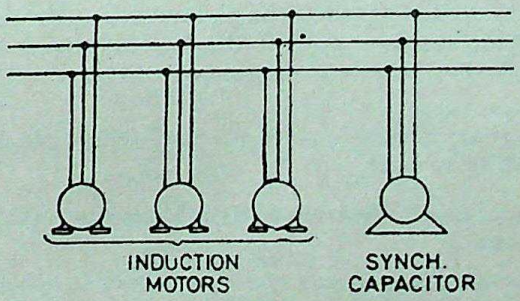

Fig. 25-13

As shown in Fig. 25-13, installation of a synchronous capacitor in parallel with the induction motors improves the p.f. of the line and hence of supply generators and transformers. It is obvious that in order to raise the p.f. to unity, current taken by the capacitor must equal the total reactive current drawn by the induction motors.

Example 25-4. *A load connected to a 3-ϕ, 2300-V supply line draws 1500 kW at a lagging p.f. of 0·6. Find (a) kVA rating of a synchronous capacitor required to be installed in parallel with the load in order to raise the line p.f. to unity and (b) kVA rating of an alternator required to supply the new load.*

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Solution.

(*a*) Load kW = 1500

$\cos \phi = 0{\cdot}6$, $\sin \phi = 0{\cdot}8$

kVA = kW/cos ϕ

= 1500/0·6 = 2500

kVAR = kVA sin ϕ

= 2500 × 0·8

= 2000 (lag)

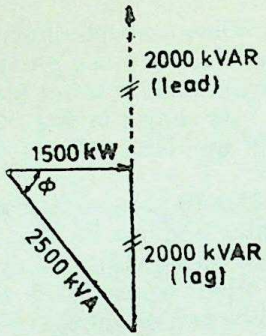

Fig. 25-14

For bringing the power factor to unity, the synchronous capacitor must supply an equal number of lagging kVAR. Hence, kVAR supplied by synchronous motor is **2000 (leading)**

(*b*) The alternator now has to supply only 1500 kW as shown in Fig. 25-14 which represents the alternator rating *i.e.* **1500 kVA**.

25-11. Applications of Synchronous Motors

1. In view of their high efficiency and constant speed, these motors can be advantageously employed for loads where constant speeds are desirable.

2. Since these motors have high efficiency and can be built in low speeds, they are will-suited for direct connection to reciprocating compressors.

3. Over-excited synchronous motors are most commonly used for power factor improvement of lagging industrial loads. When employed in such a role, they are referred to as synchronous capacitors.

4. They are generally used for improving voltage regulation of long transmission lines.

25-12. Comparison Between Synchronous and Induction Motors

1. Whereas synchronous motor runs only at synchronous speed, an induction motor never runs with synchronous speed.

2. Synchronous motor can be operated under a wide range of power factors both lagging and leading. But induction motors always run with lagging power factor.

3. A synchronous motor is not inherently self-starting whereas an induction motor is.

4. The torque of a synchronous motor is much less affected by changes in applied voltage than that of an induction motor

5. A d.c. excitation is required by a synchronous motor but not by an induction motor.

6. Synchronous motors are usually more costly and complicated than induction motors.

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25-13. Motor Classification By Speed

Such a list is given below :

Speed	*Motor*	*Applications*
Constant	(*i*) synchronous (*ii*) reluctance and hysteresis	useful for p.f. correction, small power applications, clocks, turntables, conveyors
Almost constant	(*i*) induction (*ii*) D.C. shunt	used when small speed change can be tolerated-fans, blowers, machine tools pumps, compressors, crushers
Varying speed	(*i*) series (*ii*) universal, repulsion	cranes, hoists, traction
Marginal speed control	(*i*) wound rotor induction (*ii*) d.c. compound with field control	cranes, hoists
Variable speed	(*i*) d.c. motor with armature voltage control (*ii*) induction motor with variable frequency supply.	used for any drive requiring a wide speed variation—expensive.
Multispeed	(*i*) cascaded induction motor (*ii*) pole changihg induction motors (*iii*) single-phase induction motor, FHP universal motors	used when two discrete speeds are required. Two speed fans. all domestic applications : cleaners, washers, tumble dryers, central heating, pumps.

HIGHLIGHTS

1. Synchronous motor is nothing else but an alternator operated in the reverse direction.

2. A synchronous motor runs at one speed only *i.e.* synchronous speed and can work at both lagging and leading power factors.

3. Mechanical power developed by synchronous motor is

$$P_{mech} = E_b I_a \cos(\alpha - \phi) \cong \frac{E_b V}{Z_s} \sin \alpha$$

4. Back e.m.f. generated in stator/phase is

$$E_b^2 = V^2 + E_R^2 - 2VE_R \cos(\theta - \phi)$$

5. The resultant voltage in stator/phase is

$$E_R^2 = V^2 + E_b^2 - 2VE_b \cos \alpha$$

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Q AND A ON ELECTRIC MACHINERY 26

1. Direct Current Generators 2. Direct Current Motors 3. Three-phase Induction Motors 4. Single-phase Motors 5. Alternators 6. Synchronous Motors.

A. Direct Current Generators

26-1. *What type of voltage is generated in a standard d.c. generator ?*

Ans. Generated voltage is alternating *i.e.* a.c. voltage.

26-2. *How can the output voltage of a d.c. generator be regulated ?*

Ans. Either by changing excitation with the help of field rheostat or by controlling generator speed.

26-3. *Which parts, if any, of a d.c. generator need to be laminated ? And why ?*

Ans. Armature has to be laminated in order to reduce eddy current loss. Field poles need not be laminated because they have constant field and hence there is no eddy current loss.

26-4. *What is meant by the term voltage 'build up' of a d.c. generator ?*

Ans. It means the gradual increase in the generator voltage to its maximum value after the generator is started from rest.

26-5. *How should a generator be started ?*

Ans. It is usually brought up to speed with the help of the driving engine called prime mover.

26-6. *How should a shunt or compound generator be started ?*

Ans. Such machines excite best when all switches controlling the external circuits are open.

26-7. *How about a series generator ?*

Ans. In this case, the external circuit must be closed otherwise the generator will not build up.

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26-8. *What is the procedure for shutting down a generator ?*

Ans. First, the load should be gradually reduced, if possible by easing down the driving engine, then when the generator is supplying little or no current, the main switch should be opened. When the voltmeter reads almost zero, the brushes should be raised from the commutator.

26-9. *What are the indications and causes of an overloaded generator ?*

Ans. A generator is said to be overloaded if a greater output is taken from it than it can safely carry. Overloading is indicated by (*i*) excessive sparking at brushes and (*ii*) overheating of the armature and other parts of the generator.

Most likely causes of overloading are :

(*a*) excessive voltage—as indicated by the voltmeter or the increased brilliancy of the pilot lamp. This could be due to over-excitation of field magnets or too high speed of the driving engine.

(*b*) excessive current—which could be due to bad feeding of the load.

(*c*) reversal of polarity—this happens occasionally when the series or compound-wound generators are running in parallel. Polarity reversal occurs during stopping by the current from the machines at work.

(*d*) short-circuit or ground in the generator itself or in the external circuit.

26-10. *Mention and explain the various causes for the failure of the generator to build up.*

Ans. Principal causes due to which a generator may fail to excite are :

(*i*) *brushes not properly adjusted*—if brushes are not in their proper positions, then whole of the armature voltage will not be utilized and would be insufficient to excite the machine.

(*ii*) *defective contacts*—unclean contacts may interpose large resistance in the path of the exciting current and reduce it to such a small value that it fails to excite the machine.

(*iii*) *incorrect adjustment of regulators*—in the case of shunt- and compound-generators, it is possible that the resistance of field regulator may be too great to permit the passage of sufficient current through the field windings.

(*iv*) *speed too low*—in the case of shunt and compound-wound generators, there is certain critical armature speed below which they will not excite (Art. 11-7).

(*v*) *open-circuit*—in the case of series machines.

(*vi*) *short-circuit*—in the generator or external circuit.

(*vii*) *wrong connections*—particularly when connecting the field coils to the armature.

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(*viii*) *reversed field polarity*—usually caused by the reversed connections of the field coils.

(*ix*) *insufficient residual magnetism*—this trouble normally occurs when the generator is new. It can be remedied by passing a strong direct current through the field coils.

26-11. *Is there any major difference between a d.c. motor and a d.c. generator ?*

Ans. Fundamentally, there is none. A d.c. motor can run as a d.c. generator when driven by a prime mover. Similarly, a d.c. generator will run as a d.c. motor when connected across suitable d.c. supply.

B. Direct Current Motors

26-12. *Which end of the motor is called 'front end' ?*

Ans. The end opposite the shaft.

26-13. *What is the standard direction of rotation of a motor ?*

Ans. Counterclockwise when looking at the front of the motor.

26-14. *How may the direction of rotation of a d.c. motor be reversed ?*

Ans. By reversing either the field current or current through the armature. Usually, reversal of current through the armature is adopted.

26-15. *What will happen if both currents are reversed ?*

Ans. The motor will run in the original direction.

26-16. *What will happen if the direction of current at the terminals of a series motor is reversed ?*

Ans. It will not reverse the direction of rotation of the motor.

26-17. *What will happen if the field of a d.c. motor is opened ?*

Ans. The motor will achieve dangerously high speed and may destroy itself.

26-18. *Explain what happens when a d.c. motor is connected across an a.c. supply ?*

Ans. (*i*) since on a.c. supply, reactance will come into the picture, the a.c. supply will be offered impedance (not resistance) by the armature winding. Consequently, with a.c. supply, current will be much less. The motor will run but it would not carry the same load as it would on d.c. supply.

(*ii*) there would be more sparking at the brushes.

(*iii*) though armature is laminated, the field poles are not. Consequently, the eddy currents will cause the motor to heat up and eventually burn on a.c. supply.

26-19. *What will happen if a shunt motor is directly connected to the supply line ?*

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Ans. Small motors up to 1 kW rating may be line-started without any adverse results being produced. High rating motors must be started through a suitable starter in order to avoid the huge starting current which will

(*i*) damage the motor itself and

(*ii*) badly affect the voltage regulation of the supply line.

26-20. *What is the function of interpoles and how are the interpole windings connected ?*

Ans. Interpoles are small poles placed it between the main poles. Their function is to assist commutation by producing the auxiliary or commutating flux. Consequently, brush sparking is practically eliminated. Interpole windings are connected in series with the armature winding.

26-21. *How can we control the speed of a d.c. motor ?*

Ans. By changing either the pole flux or the voltage across armature.

C. Three-phase Induction Motors

26-22. *What is, in brief, the basis of operation of a 3-phase induction motor.*

Ans. The revolving magnetic field which is produced when the 3-phase stator winding is fed from a 3-phase supply.

26-23. *What factors determine the direction of rotation of the motor ?*

Ans. The phase sequence of the supply lines and the order in which these lines are connected to the stator winding.

26-24. *How can the direction of rotation of the motor be reversed ?*

Ans. By transposing or changing over any two line leads as shown in Fig. 26-1.

26-25. *Why are induction motors calle 'asynchronous' ?*

Ans. Because their rotors can never run with the synchronous speed.

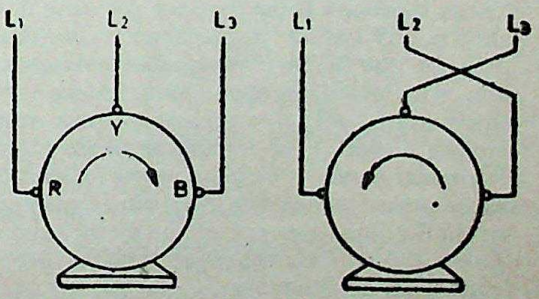

Fig. 26-1

26-26. *How does the slip vary with load ?*

Ans. The greater the load, greater is the slip or slower is the rotor speed.

26-27. *What modifications would be necessary if a motor is required*

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to operate on a voltage different from that for which it was designed ?

Ans. The number of conductors per slot will have to be changed in the same ratio as the change in voltage. If the voltage is doubled, the number of conductors per slot will have to be doubled.

26-28. *Enumerate the possible reasons if a 3-phase motor fails to start.*

Ans. Any one of the following reasons could be responsible :

(*i*) one or more fuses may be blown

(*ii*) voltage may be too low

(*iii*) the starting load may be too heavy

(*iv*) worn bearings due to which the armature may be touching field laminae thus introducing excessive friction.

26-29. *A motor stops after starting i.e. it fails to carry load. What could be the causes* ?

Ans. Any one of the following :

(*i*) hot bearing which increase the load by excessive friction.

(*ii*) excessive tension on belt which causes the bearings to heat

(*iii*) failure of short cut-out switch

(*vs*) single-phasing on the running position of the starter.

26-30. *Which is the usual cause of blow outs in induction motors* ?

Ans. The commonest cause is single-phasing.

26-31. *What is meant by 'single-phasing' and what are its causes* ?

Ans. By single-phasing is meant the opening of one wire (or leg) of a three-phase circuit whereupon the remaining leg at once becomes single-phase. When a three-phase circuit functions normally, there are three distinct currents flowing in the circuit. As is known, any two of these currents use the third wire as the return path *i.e.* one of the three phases acts as a return path for the other two. Obviously, an open circuit in one leg kills two of the phases and there will be only one current or phase working even though two wires are left intact. The remaining phase attempts to carry all the load. The usual cause of single-phasing is, what is generally referred to as, *running fuse* which is a fuse whose current-carrying capacity is equal to the full-load current of the motor connected in the circuit. This fuse will blow out whenever there is overload (either momentary or sustained) on the motor.

26-32. *What happens if single-phasing occurs when the motor is running* ? *And when it is stationary* ?

Ans. (*i*) If already running and carrying half load or less, the motor will continue running as a single-phase motor on the remaining single-phase supply without damage because half loads do not blow normal fuses.

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(*ii*) If it be running on full load or overload when single phasing occurs, it will try to keep on running throwing all the load on the intact winding which sooner or later burns out.

(*iii*) If motor is very heavily loaded, then it stops under single-phasing and since it can neither restart nor blow out the remaining fuses, the burn out is very prompt.

A stationary motor will not start with one line broken. In fact, due to heavy standstill current, it is likely to burn out quickly unless immediately disconnected.

26-33. *Which phase is likely to burn out in a single-phasing delta-connected motor.*

Ans. The Y-phase connected across the live or operative lines carries nearly three times its normal current and is the one most likely to burn out (Fig. 26-2).

The other two phases R and B which are in series across L_2 and L_3 carry more than their full-load current.

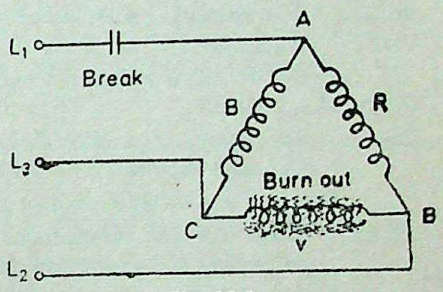

Fig. 26-2

26-34. *What currents flow in single-phasing star-connected motor ?*

Ans. With L_1 disabled, the currents flowing in L_2 and L_3 and through phases Y and B in series will be of the order of 250 per cent of the normal full-load current, 160 per cent on 3/4 load and 100 per cent on 1/2 load.

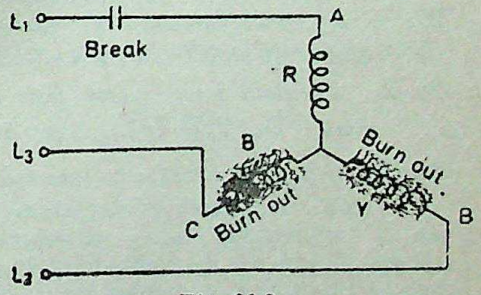

Fig. 26-3

26-35. *How can the motors be protected against single-phasing ?*

Ans. (*i*) By incorporating a combined overload and single-phasing relay in the control gear.

(*ii*) By incorporating a phase-failure relay in the control gear.

The relay may be either voltage or current-operated.

26-36. *Can a 3-phase motor be run on a single-phase line ?*

Ans. Yes, it can be. But a phase-splitter is essential.

26-37. *What is meant by a phase-splitter ?*

Ans. It is a device consisting of a number of capacitors so connected in the motor circuit that it produces, from a single input wave, three output waves which differ in phase from each other.

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D. Single-phase Motors

26-38. *Can a single-phase motor be run on a 3-phase supply* ?

Ans. Yes, by connecting it across two phase wires.

36-39. *How would you reverse the direction of rotation of a capacitor-start induction-run motor* ?

Ans. By reversing either the running or starting-winding leads where they are connected to the lines. Both must not be reversed.

26-40. *Does a resistance-start split-phase motor have a high starting torque* ?

Ans. No. It has low starting torque which is 150-200 percent of full-load torque.

26-41. *How about the starting torque of a capacitor-start induction-run motor* ?

Ans. It has a starting torque as high as 350 to 450 percent of its full-load torque.

26-42. *Are capacitor-start-and-run motors single-value or two-value capacitor motors* ?

Ans. Two-value capacitor motors.

26-43. *What are the advantages of a capacitor-start-capacitor-run motor over the capacitor-start induction-run motors* ?

Ans. (*i*) smoother running
(*ii*) higher p.f.
(*iii*) less input current

26-44. *In which direction does a shaded-pole motor turn* ?

Ans. It runs from the unshaded to the shaded pole.

26-45. *Do shaded-pole motors have much starting torque* ?

Ans. No. That is why they are built in very small sizes.

26-46. *Can such a motor be reversed* ?

Ans. Normally, such motors are not reversible because that would involve mechanical dismantling and re-assembly. However, special motors are made having two rotors on a common shaft, each having one stator assembled for rotation in opposite direction.

26-47. *What is a Universal motor* ?

Ans. It is built like a series d.c. motor with the difference that both its stator and armature are laminated. They can be used either on d.c. or a.c. although the speed and power is greater on direct current. They cannot be satisfactarily made to run at less than about 2000 r.p.m.

26-48. *How can a Universal motor be reversed* ?

Ans. By reversing either the field leads or armature leads but not both.

26-49. *What could be the possible faults if a capacitor-start induction-run motor fails to start when switched on to its proper supply* ?

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Ans. Anyone of the following could be responsible :

1. open in connection to line
2. open circuit in motor main or running winding
3. centrifugal switch contacts open
4. defective capacitor
5. open in starting winding

26-50. *A permanent-split capacitor motor does not always start even on no-load but runs in either direction when started manually. What could be the reasons ?*

Ans. Any one of the following :

(*i*) defective capacitor
(*ii*) one or both windings open

26-51. *A capacitor-start-and-run motor does not start when switched on but runs in either direction when started manually, though it overheats. List the causes responsible for this behaviour.*

Ans. The possible causes are as under :

1. defective capacitor
2. either winding short-circuited or grounded
3. one or more winding open

26-52. *A shaded-pole motor is found to develop less power but gets too hot. Reason ?*

Ans. Either of the following :

(*i*) winding short-circuited or grounded
(*ii*) sticky or tight bearings
(*iii*) some interference between stator and rotor

26-53. *Mention the causes if a capacitor-start induction-run motor blows fuse or does not stop when switch is turned to OFF position.*

Ans. (*i*) winding short-circuited or grounded
(*ii*) ground near switch end of the winding

E. Alternators

26-54. *Basically, what does an alternator consist of ?*

Ans. (*a*) stator—which has 3-ϕ armature winding
(*b*) rotor—which has salient or smooth magnetic poles

26-55. *How is the field of an alternator usually excited ?*

Ans From a small dc generator mounted on the shaft of the alternator itself.

26-56. *What kind of rotor is most suitable for turboalternators which run at very high speed ?*

Ans. Non-salient pole type *i.e.* smooth cylindrical type.

26-57. *What is the formula for the frequency of the ac voltage produced by an alternator ?*

Ans. $f = PN/120$ Hz

where P = No. of magnetic poles on the rotor
N = r.p.m. of the rotor

26-58. *When will a 50-Hz alternator run at the greatest possible speed ?*

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Ans. When its rotor has two poles.

26-59. *How will the generated e.m.f./phase of an alternator change when its speed is doubled ?*

Ans. It will be doubled.

26-60. *How can the generated voltage of an alternator be controlled ?*

Ans. By varying field excitation.

26-61. *Which factors cause variations in the terminal voltage of an alternator when loaded ?*

Ans. (*a*) drop across armature resistance *i.e.* $I_a R_a$
(*b*) drop due to armature leakage reactance *i.e.* $I_a X_L$
(*c*) armature reaction

26-62. *Is voltage regulation of an alternator always negative ?*

Ans. No. It depends on the load power factor. It is positive for lagging p.f. and negative for leading p.f.

26-63. *What is the unit for expressing rating of an alternator ?*

Ans. kVA

F. Synchronous Motors

26-64. *What happens when the field of a synchronous motor is* (*i*) *under-excited* (*ii*) *over-excited ?*

Ans. (*i*) It has a lagging p.f. and (*ii*) it has a leading p.f.

26-65. *When is a synchronous motor said to be over-excited ?*

Ans. When its excitation is such that generated or back emf/phase E_b becomes greater than applied voltage/phase, V.

26-66. *What is a synchronous capacitor ?*

Ans. An overexcited unloaded synchronous motor is called synchronous capacitor because, like a capacitor, it takes a leading current.

26-67. *What happens when load on a synchronous motor increases ?*

Ans. (*a*) first, torque angle α increases as rotor falls back in phase
(*b*) then, E_R increases
(*c*) next, I_a increases
(*d*) hence, ultimately motor input increases to meet the extra load.

26-68. *What are the causes of faulty starting of a synchronous motor ?*

Ans. It could be due to the following causes :

(*i*) voltage may be too low—at least half voltage is required for starting.

(*ii*) there may be open-circuit in any phase—due to which motor may heat up.

(*iii*) static friction may be large—either due to great belt tension or too tight bearings.

(*iv*) stator windings may be incorrectly connected.

(*v*) field excitation may be too much.

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1. General 2. Electrons 3. Methods of Producing Electronic Emission 4. Thermionic Emission 5. Cathodes 6. Filamentary Cathodes 7. Heater Cathode 8. Electron Tubes 9. Physical Characteristics of High Vacuum Diode 10. How Does a Diode Operate 11. Space Charge 12. Diode Characteristics 13. Rectification of A.C. 14. Half-wave Rectification of 1-phase A.C. 15. Full-wave Rectification of Single-phase A.C. 16. Filter Circuit 17. Half-wave Rectification of Three-phase Current 18. High-vacuum Triode 19. Physical Characteristics of a Triode 20. Electrical Characteristics of a Triode 21. Triode—Static Characteristics 22. Tube Coefficients 23. Amplification Factor 24. Plate Resistance 25. Transconductance 56. Inter-relation of Coefficients 27. Values of Tube Coefficients from Characteristics 28. Dynamic Characteristics 29. Load Line 30. Triode as an Amplifier 31. The A.C. Equivalent Circuit of an Amplifier. 32. Self-biasing of a Triode 33. Classification of Amplifiers 34. Simple Amplifier 35. Amplifier Coupling Methods. 36. Interelectrode Capacitances of a Triode 37. Tetrode 38. Tetrode Characteristics 39. Pentode 40. Oscillators. 41. Oscillations Due to the Discharge of a Capacitor 42. Essentials of a Feedback Oscillator 43. Tickler Oscillator 44. Tuned-plate Oscillator 45 Tuned-plate Tuned-grid Oscillator 46. Hartley Oscillator 47. Colpitts Oscillator 48. Photoelectric Emission 49. Highlights.

27-1. General

The term Electronics includes, in a broad sense, all the electric phenomena because all electric conduction is due to flow of electrons in a circuit. However, at present, the common interpretation of the term includes only the conduction of electricity through vacuum by electrons alone or through gases by electrons and ions. Electronics has been defined as *that branch of science and technology which relates to the conduction of electricity through gases or in vacu.*

In this chapter, some of the fundamental characteristics of a (thermionic) electron tube or vacuum tube will be discussed in brief. It consists of an evacuated vessel enclosing two or more electrodes and in which there can be an electric current only when electrons pass from one electrode to another. One of the electrodes, which is then called cathode, is made capable of emitting electrons. The enclosing vessel is either of glass or metal or some insulating material with the electrodes supported by wires which are sealed through the glass or metal covers to make connections to the outside.

27-2. Electrons

Some elementary ideas about the atomic structure of matter have already been given in Chapter 1. As explained there, electricity

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is composed of extremely small negative charges called *electrons*. The charge of each electron is found to be $1{\cdot}602 \times 10^{-19}$ coulomb and its mass $9{\cdot}1 \times 10^{-31}$ kg or 1/1845 of the mass of a hydrogen atom. An electrically neutral atom of matter consists of a positively charged nucleus surrounded by orbital electrons whose number is such as to give an equal negative charge, so that the *resultant charge on the entire atom is zero*. In bad conductors of electricity, these electrons are comparatively more rigidly attached to their nuclei, hence it is very difficult to detach them away from the nuclei of atoms of such materials. But in the so-called good conductors of electricity like metals, a small portion of electrons is found free in the sense that they can easily pass from one atom to another. In other words, these electrons are not firmly attached to their parent nuclei. These free electrons form a kind of electron *gas* within a conductor and, in fact, it is their circulation around a closed circuit that constitutes the ordinary electric current.

The electrons constituting electron gas within metallic conductors are in a state of constant agitation and keep on colliding with each other and with other atoms, thereby executing very erratic and chaotic motion.

When such a free electron is darting around well within the interior of the metal, the resultant attractive force acting on it due to various positive ions lying all round it, is zero because it is pulled equally from all sides. Now, let us suppose that the electron is moved towards the metal surface. As it approaches the surface, the number of positive ions lying behind it increases and of those lying in front of it decreases till when it finally reaches the boundary, all ions lie behind it and none in front of it. Hence, during this journey of the electron towards the boundary, the backward pull on it goes on increasing till it is maximum at the surface beyond which it tapers off.

It is clear that as the electron is moved towards the surface of the metal, work has to be done because the electron is being moved against the united backward pulling force of all ions lying behind the electron in motion.

As soon as electron tries to pass out of the interior of the metal through the boundary surface, it is immediately pulled back by the united force of all the ions which it is leaving behind, whereas before this, it was immersed in their force fields. Therefore, the outgoing electron experiences a potential energy barrier to its escape, this barrier being known as work function (ϕ) of the material.

27-3. Methods of Producing Electronic Emission

(A.M.I.E. Winter 1985)

In the light of above discussion, it may be guessed that if an electron in the electron gas of a metal be given sufficient energy and if this speeded-up electron approaches metal surface perpendicularly, then it will escape from the metal surface.

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Ordinarily, none of the electrons in the electronic gas has

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kinetic energy equal to or greater than the amount necessary to overcome the "potential-energy barrier" Hence, they cannot leave the surface and are pulled back every time they try to approach the surface.

There are many ways in which electronic emission can be brought about. In all these methods, energy, in one form or the other, is imparted to the free electrons, in a quantity which gives them kinetic energy sufficient enough to overcome potential energy barrier represented by the work function of the metal. Some of these ways are discussed below :

(*i*) By heating the metals sufficiently high, thermal energy may be imparted to electrons to enable them to escape from the metal surface. This is known as *thermionic emission* and is employed in vacuum tubes.

(*ii*) Electrons can also be pulled out by the application of a strong electrostatic field on the surface of the metal. Such an emission is known as *field emission.*

(*iii*) A beam of high-velocity electrons may strike a metal surface and so eject electrons out of it. This process is known as *secondary emission.*

(*iv*) When the surfaces of certain metals are illuminated by a beam of light, electrons are ejected out of metals by the photons incident on the metal surface. Such an electronic emission is known as *photo-electric emission* and is used in photo-electric cells.

27-4. Thermionic Emission - (*A.M.I.E. Summer 1986*)

Many formulae for the thermionic emission of electrons have been developed. But the formula employed today is the Richardson-Dushman equation

$$J_S = AT^2 e^{-b/T} \text{ A/m}^2$$

where J_S = emission current/m² of the emitter

T = temperature in °K

A = a constant

e = natural logarithmic base

b = a constant for the given metal.

As explained earlier, normally the free electrons in a metal do not have enough energies to escape the confines of their metal boundaries or to surmount the potential energy barrier. For doing so, they must be given additional energy. In a thermionic cathode, the additional boost of energy is given to the electrons through the medium of heat. As the metal is heated more and more, the average kinetic energy of the electrons increases. If the temperature becomes high enough, then some of the electrons may get sufficient energy so as to overcome the potential-energy barrier and may escape. At high

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temperatures, the electrons are literally being 'boiled' off from the metal surface. This is analogous to the boiling of water.

From practical point of view, the work function of the metal used in a thermionic emitter, should be as low as possible to permit operation at the lowest possible temperature. This is very much desirable for increasing the efficiency of emission *i.e.* to increase the milliamperes of emitted current per watt of heating power supplied to the emitter.

27-5. Cathodes

A cathode is an essential part of an electron tube because it emits electrons which are essential for tube operation. When heat energy is given to the material of the cathode, electrons are released. Different forms of cathodes can be distinguished from each other by the manner in which they are heated. In general, cathodes may be classified into two categories :

(1) *Directly-heated cathodes or filamentary cathodes*

(2) *Indirectly-heated cathodes or heater cathodes.*

27-6. Filamentary Cathodes

Present-day directly-heated cathodes in wide use, can be further classified according to the active material on the cathode into (*a*) tungsten filaments (*b*) thoriated-tungsten filaments and (*c*) oxide-coated filaments.

(*a*) Tungsten Filamentary Cathode

Tungsten cathodes are made from pure metal because it is very well suited for thermionic emission. Tungsten is exceedingly refractory and has the highest melting temperature of all metals (3655°K). It can be safely operated at 2500°K for wires of small diameters and at much higher temperatures for wires of large diameter. Despite its higher work function (4·54 eV) tungsten is particularly suited for high power transmitting tubes where the cathode has to be rugged in order to withstand all the mechanical and chemical requirements particularly the continuous bombardment by fast moving positive ions.

(*b*) Thoriated-tungsten Filamentary Cathode

The heating power required by filamentary cathodes of *pure* tungsten is rather high, because its work function is high. Hence, other suitable electron emitters have been sought—the thoriated tungsten filament being one of these. These filaments are made from tungsten impregnated with thorium oxide. The presence of a monoatomic layer of thorium on the tungsten surface lowers its work function, hence copious electronic emission can be obtained at a more moderate temperature of 1700°C (a bright yellow). Therefore, thoriated-tungsten filamentary cathodes are much more economical of filament heating power than are the filamentary cathodes made of *pure* tungsten.

(*c*) Oxide-coated Filamentary Cathode

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Oxide-coated cathodes are prepared by coating a wire of a

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ribbon of a base metal like nickel having a few percents of cobalt, silicon or konel metal (an alloy of nickel, cobalt, iron and titanium) with a 50-50 mixture of carbonates of barium and strontium. The latter are applied to the filaments by dragging the wire through suspension of carbonates in water and then successive layers are baked.

Oxide-coated filamentary cathodes produce a copious supply of electrons at relatively low temperature of about 700°C to 750°C (a dull red). Hence, they require relatively small heating power and operate with higher efficiency.

27-7. Indirectly-heated Cathode or Heater Cathode

The construction of such a cathode is shown in Fig. 27-1. It consists of a thin metal sleeve (of nickel or konel) which is coated on the outside with electron-emitting oxides. Within the sleeve is a heater wire of tungsten coated with a layer, about 0·5 mm thick of the oxides of aluminium and beryllium and electrically insulated from the sleeve. The purpose of the heater wire is merely to heat the sleeve and the sleeve coating by radiation and conduction to an electron-emitting temperature. It is to be noted that useful electronic emission does not take place from the heater wire. The heater wire usually operates at a temperature of about 1000°K and maintains the sleeve at about 800°K which is sufficient for copious emission from the oxide coating on the metal sleeve.

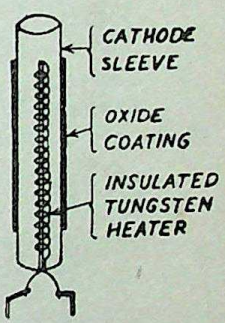

Fig. 27-1

Because of (*i*) separate parts for emission and heating and (*ii*) the insulation of emitter from heater, it is possible to minimize the introduction of hum from the A.C. heater supply and also to minimize electrical interference which might enter the tube circuit via the heater supply line. Because of these advantages and many others, such types of heater cathodes are used in almost all present-day receiving tubes designed for a.c. operation.

27-8. Electron Tube

So far we have discussed primarily the physical phenomena involved in electronic conduction. Next, we will discuss how these phenomena combine to determine the characteristics and limitations of some important electron tubes. As explained at the beginning of this chapter, electron tube is an electronic device in which electronic conduction takes place in an evacuated vessel or enclosure.

27-9. High Vacuum Diode—Physical Characteristics

A typical diode is shown in Fig. 27-2. It consists of two electrodes *i.e.* an anode or plate *P* surrounding a filamentary cathode. Different shapes of filamentary cathodes are shown in Fig. 27.4. In some tubes, an indirectly-heated cathode of the type shown in Fig. 27-1 is also used. Both the elements are enclosed

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in an evacuated glass envelop having the necessary connections brought out through air-tight seals. The standard symbol for a diode is shown in Fig. 27-3 in which the straight line represents the anode or plate and the inverted *V* represents the filamentary type cathode.

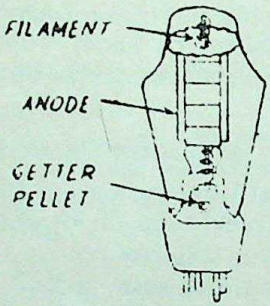

Fig. 27-2

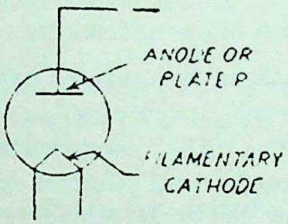

Fig. 27-3

A diode having an indirectly-heated cathode is symbolically shown in Fig. 27-5. In their case, the heater element is represented

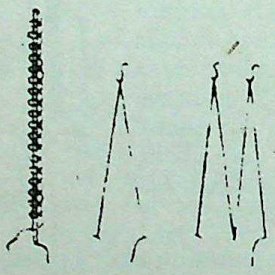

Fig. 27-4

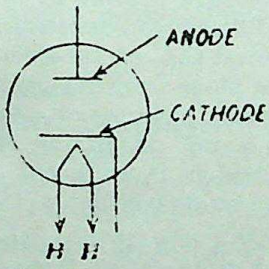

Fig 27-5

by the inverted *V* and the electron-emitting cathode by straight line with sides folded down.

Sometimes, two diodes are located in one bulb (Fig. 27-6). Such a diode is known as double-diode or duo-diode. Examples are : $5Y3-GT$ and $5V4-G$ (filamentary cathodes) and 5*Z*4 (indirectly heated cathode).

It should be noted that description of a tube by a name like 5 *Z* 4 gives a significant information *i.e.* first number 5 stands for the heater voltage and the last number 4 indicates the number of connections used in connecting the tube in a circuit.

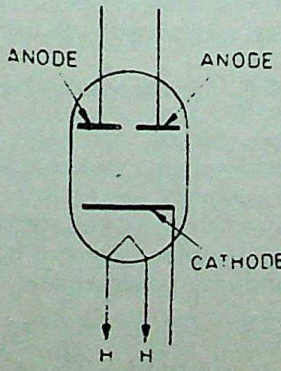

Fig. 27-6

27-10. How Does a Diode Operate ?

(A.M.I.E. Summer 1975)

The working of a high-vacuum thermionic diode can be descri-

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bed roughly as follows. When the cathode is heated either by a battery or an a.c. snpply, it produces a substantial supply of electrons. The battery used for heating the filament is known as *A*-battery. If, now, the plate (or anode) is positive with respect to the cathode, then electrons are attracted towards the plate and hence plate or anode current is set up and is registercd by the ammeter included in the plate circuit. In Fig. 27-7 is shown a diode whose plate is connected to the plate battery *B* and whose heater is heated by battery *A*. Fig. 27-8 shows the connections for a diode having an indirectly-heated cathode. It makes no difference even if points *'a'* and *'b'*

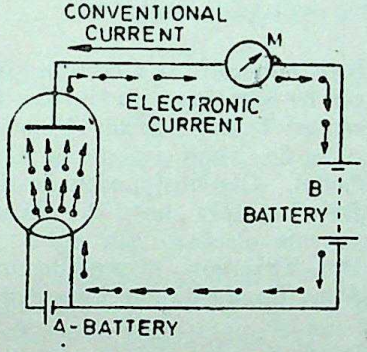

Fig. 27-7

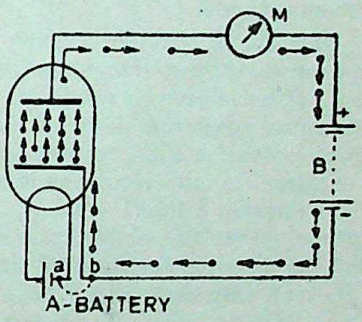

Fig. 27-8

are joined together. Now, if the anode is given a negative potential with respect to the cathode (Fig. 27-9), then the electrons are repelled back by the anode, hence there is pratically no plate current.

So, we find that a diode is such a device that it conducts current in one direction only. That is why it is sometimes referred to as *electron valve* because of its unidirectional conductivity. If an alternating voltage is applied across the plate and cathode, then current conduction takes place only in one direction and current is zero during alternate half-cycles. Under these conditions, the tube (or diode) makes the alternating voltage to send unidirectional but pulsating current through the load resistance. The diode is then said to act as a rectifier.

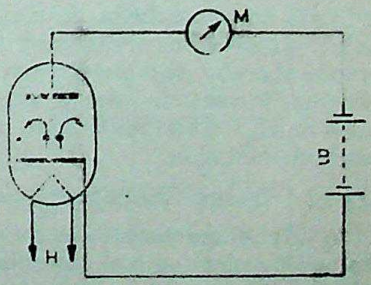

Fig. 27-9

27-11. Space Charge *(A.M.I.E. Summer 1983)*

It is the negative charge of the electron cloud which is formed in the inter-electrode space between the cathode and anode of a diode. Exactly how it is formed and how it affects the anode current is discussed below :

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For a given operating temperature, the total number of electrons emitted by the cathode of a diode is the same and is determined by Richardson's emission equation (Art. 27-5). These electrons are emitted by the cathode in random directions with very small initial velocities. These electrons tend to bunch up in the form of an electron cloud immediately above the cathode and do not flow towards the anode (which for the purpose of present discussion may be assumed not connected to any source of voltage). The cloud of negative electrons constitutes *space charge*. It exerts a repelling force on the electrons being emitted from the cathode.

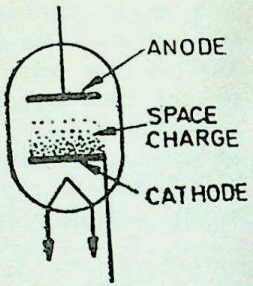

Fig. 27-10

It is found that for a given cathode material at a certain given temperature, only so many electrons can be contained in the electron cloud. Other electrons would be prevented from entering the cloud unless some electrons leave it (as some do when anode is given positive potential with respect to cathode). Obviously, cathode does not continue to emit electrons indefinitely but only until the electron cloud achieves *critical density*. Then one electron can enter the cloud only when one electron leaves it. This sort of equilibrium is known as *emission* (*or cathode*) *saturation*. Each cathode temperature has its own emission saturation.

The anode current is determined jointly by the space charge and the anode voltage—the two acting in opposite directions. The electric field of the positive anode reaches through the space charge to attract the electrons thereby partially neutralizing its repelling effect. At low anode voltages, only electrons nearest to the anode are attracted to it so that anode or plate current is small. At this stage, space charge has strong effect on anode current. As anode voltage is increased, a greater number of electrons is now attracted towards the anode and correspondingly fewer electrons are repelled back to the cathode. A certain value of anode voltage is reached when all electrons emitted by the cathode are attracted towards the anode thereby completely overcoming the effect of space charge. Further increase in anode voltage does not change the anode current much. This maximum value of anode current is limited by cathode emission.

27-12 Diode Characteristics *(A.M.I.E. Summer 1975)*

By characteristic is meant a graph or a curve which is obtained by plotting two variables of the tube in order to determine the third. The two characteristics of a vacuum diode are as under :

(*i*) V_p/I_p Characteristic. It is also known as volt-ampere characteristic. It gives the relation between plate current when cathode temperature is kept constant at different values.

(*ii*) I_p/T Characteristic. It gives relation between plate current and cathode temperature for different fixed values of plate voltage. Since it is not of much significance, it would not be discussed any further.

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Volt-ampere Characteristic

The circuit connections for obtaining this characteristic are shown in Fig. 27-11. The voltage applied to the diode plate can be varied with the help of movable tap over the potentiometer *CD*. Similarly, cathode temperature can be varied by changing heater current with the help of a variable resistance *R* connected in the cathode circuit.

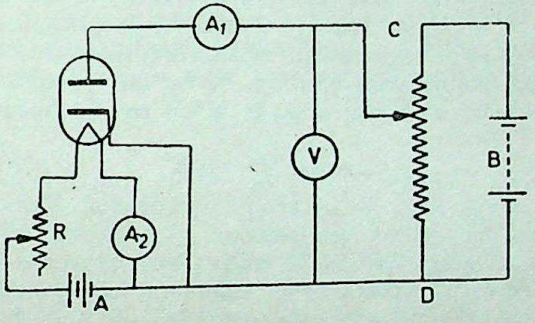

Fig. 27-11

It is seen from Fig. 27-12 that all curves are the same *i.e.* coincident at low anode voltage where anode current is limited by space charge. The region *oa* is known as space charge limited region. Here, the anode or plate current increases as the voltage to the three-halves power. This relation is known as Child's Three-halves Power law which can be written as

$$I_p = K V_p^{3/2}$$

where K is a constant depending on the shape of the electrodes and the geometry of the diode.

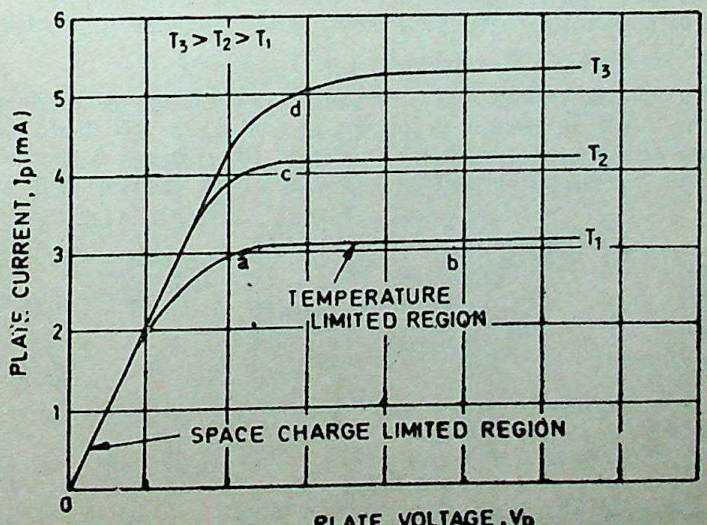

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Fig. 27-12

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Obviously, in the space charge-limited region, I_p is completely controlled by V_p and is independent of cathode temperature.

But ultimately the plate voltage reaches a value (for a given cathode temperature) at which its attractive force on the cathode becomes greater than the mutual repulsive force due to the negative space charge, hence all the electrons are drawn to the plate as fast as they are emitted from the cathode. The current becomes constant and is practically independent of the plate voltage. Hence, the curve *ab* is practically a straight line. This is known as *cathode saturation* or temperature saturation because the current is limited by thermionic emission only. Over the portion *ab*, the value of saturation current density J_s is given by Richardson-Dushman equation

$$J_s = AT^2 e^{-b/T} \text{ A/m}^2$$

It is worth noting that at higher temperatures, both space charge saturation limit as well as temperature saturation limit are higher. As discussed earlier in Art. 27-10 when plate voltage is reversed *i.e.* made negative with respect to the cathode, there would be no plate current. Hence, there is no curve to the left of point *O* in Fig. 27-12.

27-13. Rectification of Alternating Current

By a rectifier is meant a device *which converts alternating current into unidirectional current by virtue of a characteristic permitting appreciable flow of current in one direction only.*

Rectification of an alternating current *i.e.* its conversion into a unidirectional current is carried out with the help of a diode which is strictly a unidirectional conductor. As discussed before, when the plate is given a positive potential with respect to the cathode, a current flows in the plate circuit whereas no current flows when plate (or anode) is negative with respect to the cathode.

In subsequent figures, indirectly-heated cathodes have been taken, although directly-heated cathodes are also used in practice.

27-14. Half-wave Rectification of 1-phase A.C.

In the half-wave rectifier, current flows through the diode every other half-cycle of the a.c. supply voltage whenever the diode plate is positive with respect to cathode. Fig. 27-13 (*a*) shows a simple half-wave rectifier circuit containing a diode, a load resistor and an a.c.

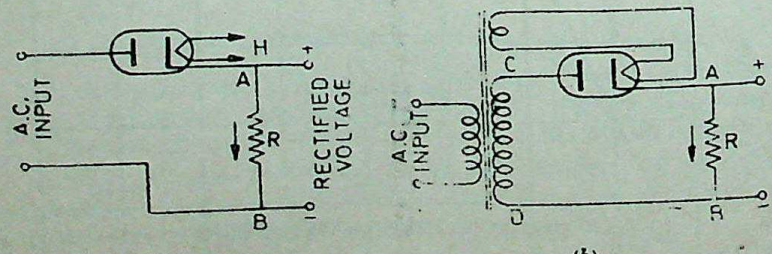

Fig. 27-13

source. Fig. 27-13 (*b*) shows the rectifier circuit which uses a 3-winding transformer whose function is to increase the supply voltage

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before rectification. The heater element of the cathode is heated by the low-voltage a.c. power obtained from an additional low-voltage winding on the main transformer. The diode plate is joined to one end of the transformer secondary. In each of the two circuits shown above, the diode plate becomes alternately positive and negative with respect to the cathode.

Let us consider one cycle of the a.c. input. When the secondary lead C (and hence the plate) becomes positive with respect to the cathode in one half cycle, conventional current flows from plate to cathode and then *via* load resistance R to the lead D. In the

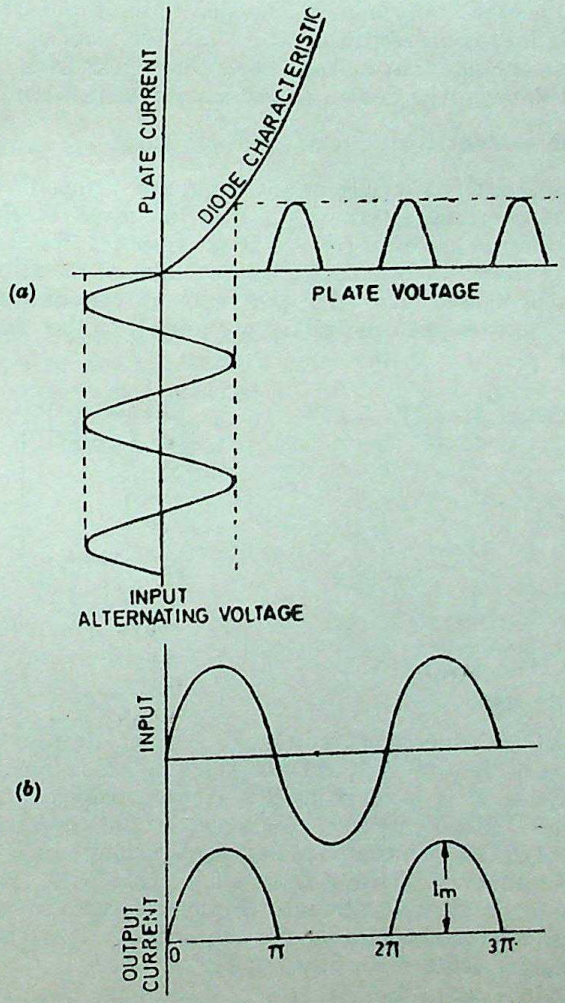

Fig. 27-14

next half-cycle, when lead C (and hence the plate) becomes negative relative to the cathode, no current flows through R. So, we find that

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in one cycle of a.c. input, current flows through load resistance R from A to B only for one half-cycle and not for the other half-cycle.

Again, in the next cycle, current will flow through R for the first positive half-cycle but none during the negative half cycle. Hence, the current through R is unidirectional (*i.e* always from A to B) but discontinuous or intermittent as shown in Fig. 27-14 (*b*). (It should be particularly noted by the reader that for output currents, the cathode end is the *positive* end because output current flows out of it). In Fig. 27-14 (*a*) is shown graphically the rectification of single-phase alternating current.

The above type of rectification is known as half-wave rectification because in this case, only one half-cycle of the input from the transformer secondary is utilized, the other half-cycle being suppressed. But if we so arrange that current flows through R from A to B in both half-cycles, then it is known as full-wave rectification.

27-15. Full-wave Rectification of 1-phase A.C.

In this case, both half-cycles of a.c. input are utilized with the help of two diodes working alternately. *For full-wave rectification, use of a transformer is essential (though it is optional for half-wave rectification).* As shown in Fig. 27-15, the cathodes of the two diodes are joined together at F to give the +ve end of the output. The two ends of the centre-tapped secondary are joined to the two plates

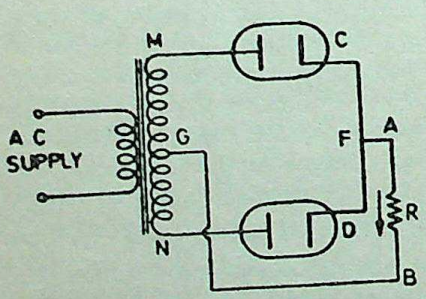

Fig. 27-15

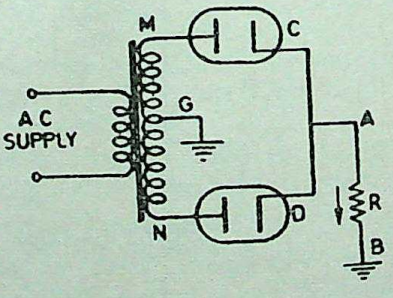

Fig. 27-16

of C and D. The centre point G is joined to point B and may be earthed. The ends M and N of the secondary become +ve and −ve alternately. When M is +ve, point G is at zero potential and N is at −ve potential. Hence, valve C conducts (but not (D) and current flows along $CFABGM$. In the next half-cycle, when end N becomes +ve, then D conducts and current flows along $DFABGN$. So, we find that current keeps on flowing through R in the same direction (*i.e.* from A to B) in both half cycles of the a.c. input. In other words, both half-cycles are utilized as shown in Fig. 27-17 (*b*).

Usually, the two separate diodes are combined into one valve known as *double diode*. The circuit for such a valve is exactly similar to that for the two separate valves and is shown in Fig. Fig. 27-17 (*a*).

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We have seen that with the help of a diode valve, which is fundamentally a rectifier (or non-return) valve, we are able to get

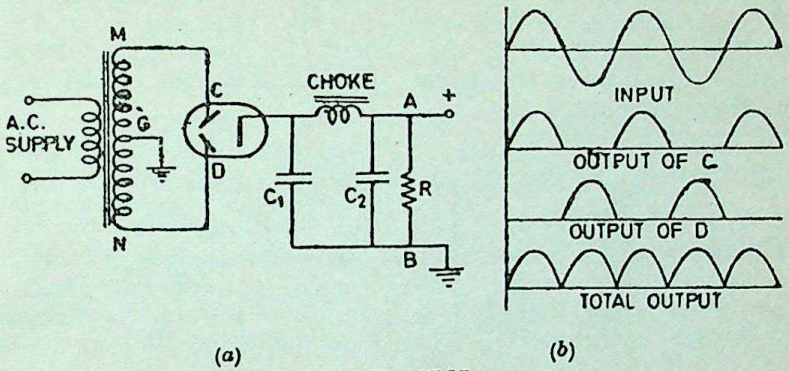

Fig. 27.17

unidirectional but intermittent current output (as distinguished from a pure direct current). But for most purposes, this pulsating or fluctuating output is unsatisfactory. Hence, it is further smoothened up by the use of filters or by using polyphase circuits or both as discussed below.

27-16. Smoothening or Filter Circuit

We have seen above that a rectifier with a resistive load produces a current which, although *unidirectional*, is not *smooth*. The output current rather consists of intermittent pulses as shown by curves in Fig. 27-17. In some electro-chemical applications such as charging of storage batteries and electroplating, such a unidirectional pulsating current may be satisfactory but in many other applications such as in radio receivers and radio-transmitter power supplies, a substantially steady current is necessary. Hence, special filter circuits are employed for smoothening out the pulsations in the output current.

In Fig. 27-18 are shown the complete connections for a full-wave rectifier generally used to supply d.c. power in radio apparatus. The functional diagram represents symbolically the divisions which can be made in the rectifier circuit. As seen, the filter circuit is inserted between the output terminal of the double-diode and the load. It consists of two capacitors (of electrolytic type) connected across the output terminals on either side of a series iron-choke. Capacitor C_1 is known as the input or reservoir capacitor.

When current is increasing [portion *ab* in Fig. 27-18 (*c*) (*iii*)] then C_1 is charged. But by discharging itself during those portions of the cycle when the rectifier output is falling [portion *bc* in Fig. 27-18 (*c*) (*iii*)] it tries to maintain the current in the load circuit at a uniform mean value. During the portion '*cd*', it is again charged and then it becomes ready to supply back the current during the falling portion '*de*'. Hence, it tries to fill up the gaps or 'valleys' shown shaded in Fig. 27-18 (*c*) (*iii*) between two halves of the cycle. The series choke, being highly inductive, tries to choke down or oppose

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out any pulsations in the output. It opposes the increase of current during the portion '*ab*' but tries to maintain it at its original value during the falling portion '*bc*' thereby toning down the fluctuations. The function of capacitor C_2 is similar to that of C_1 and it tries to absorb any ripple still left in the output. By choosing a filter of proper design and value, the output voltage and current can be kept substantially constant.

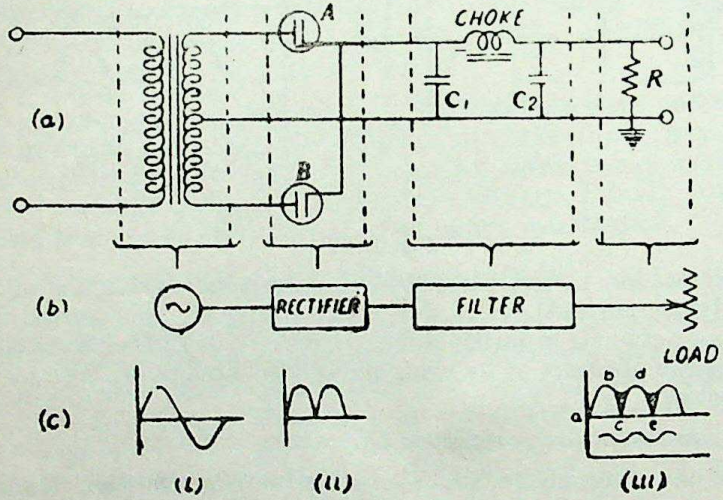

Fig. 27-18

27-17. Half-wave Rectification of 3-phase Currents

Half-wave rectification of a 3-phase supply with the help of three diodes is shown in Fig. 27-19 along with a smoothening circuit.

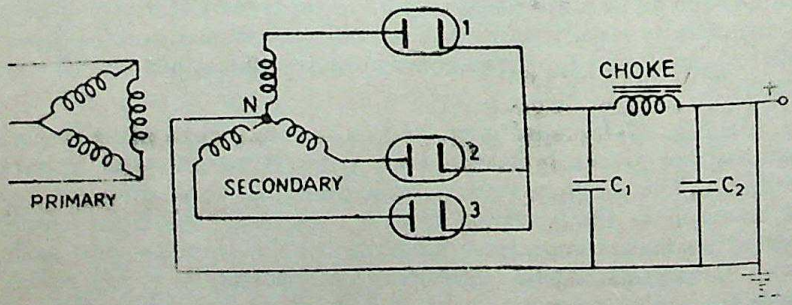

Fig. 27-19

The three diodes are connected to the three phases of a star-connected secondary of a 3-phase transformer. Neutral-point N of the secondary is the negative terminal for the rectified output and is earthed as shown.

The shape of the output is shown in Fig. 27-20. The horizontal line AB represents the potential of the negative d.c. terminal output and the sine waves 1, 2 and 3 each represent the plate potentials of the three valves.

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During one-third of the cycle i.e. during time t_1, only valve No. 1 will conduct. It will cease conducting at c and then valve

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No. 2 will conduct next after which valve No. 3 will take over and will supply current till *e*. When one valve conducts, the other two remain inactive because then their plates are more negative than their cathodes. This process repeats itself during each ensuing cycle, with the conducting period of each tube being as indicated. The output is given by the vertica distance '*ab*' between the upper envelop and the line *AB* Obviously, the output fluctuates between the maximum and minimum values thrice in each cycle. But it should be noted that the magnitude of these fluctuations or pulsations is less than for a single-phase full-wave rectified output since the current ever touches zero value. It is further smoothened up by a filter circuit (Fig. 27-19).

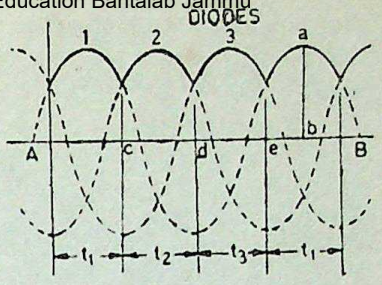

Fig. 27-20

27-18. High-vacuum Triode

Historically speaking, a high vacuum diode was the first electron tube to be produced. However, Electronics would never have assumed such tremendous importance as it enjoys today had not the high-vacuum triode been invented. The introduction of a third electrode between the anode (or plate) and cathode of a diode by De Forest in 1907 opened up practically unlimited areas of experimentation and research in electronics. Whereas the applications of diodes are comparatively limited to rectification only, the applications of triodes are practically without any count. Triodes are used for amplification of weak a.c. signals of frequencies ranging from zero to over 1,000 Hz. as oscillators, for detection, modulation and for a large number of applications of such a diversified nature that it is almost impossible to classify them. It has been aptly remarked that *diodes were the doorways to Electronics and triodes were the key that unlocked the door.*

27-19. Triode—Physical Characteristics

A typical cut-away sketch of a high-vacuum triode has been shown in Fig. 27-21. As its name implies, a triode contains three active electrodes. The cathode is located at the centre of the tube and is surrounded by a grid which is, in turn, surrounded by an anode. Its symbol is shown in Fig. 27-22 and it is the same as for a diode except for the addition of a dotted line between the anode and the cathode representing the grid. The grid may be in the form of a helix, a squirrel cage or it has apertures or perforations through which electrons can freely pass from cathode to anode. The shape of the anode is influenced by the shape of cathode and grid and is usually round, elliptical or even rectangular.

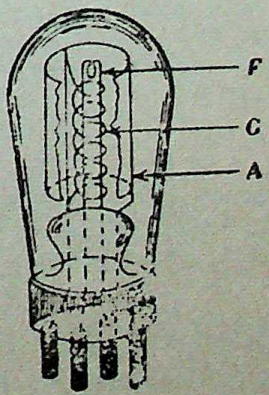

Fig. 27-21

The primary function of the addition

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al third electrode *i.e.* grid is to serve as an electrostatic screen and thus partially shield the cathode from the electrostatic field of the anode. The operation of the grid can be explained thus :

First, let it be pointed out that grid is directly situated in the path of the electronic beam and secondly, it is nearer to cathode which is the source of electrons. Because of this advantageous position, is exerts a very sensitive and powerful control on the flow of electrons which constitute plate current. When the grid is given some negative potential with respect to the cathode, it repels back or retards the flow of electrons towards the plate, thereby reducing the plate current. Because of its advantageous position as pointed out, even with a small negative potential, it is capable of exerting greater controlling effect on plate current than anode itself even with a much large positive potential. Because of this current-controlling property of the grid, it is often referred to as *control* grid. As the negative potential of the grid (called *grid bias*) is increased, I_p decreases continuously till ultimately it is reduced to zero. Grid-bias corresponding to zero plate current is known as *cut-off* potential of the grid. Its value can be found by putting $I_p = 0$ in the triode equation given below. If on the other hand, it is made positive relative to cathode, it helps the anode in pulling the electrons up by partially neutralizing the negative space charge. Hence, I_p is increased. It is found that small changes in grid voltage produce such changes in plate current which can be produced by comparatively large changes in plate voltage. It is because of this that a triode possesses that valuable property of amplification. If a change of 2 volts on the grid produces the same change in plate current as produced by a change of 50 volts in plate voltage, then the amplification factor (μ) is $50/2 = 25$. It means that the grid (because of its advantageous positioning) is 25 times more effective than anode in controlling the plate current.

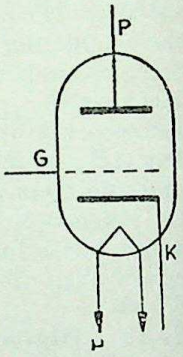

Fig. 27-22

From the above discussion it is obvious that the plate current depends not only on cathode temperature and plate voltage V_p but on grid voltage V_g also. In the region where plate current is principally limited by space-charge, the expression for plate current is given by

$$I_p = K\left(V_g + \frac{V_p}{\mu}\right)^{3/2}$$

where K is a constant depending upon the geometry of the tube and μ is the amplification factor. The above equation can also be put in an alternate form

$$I_p = A(V_p + \mu V_g)^{3/2}$$

where A is another constant equal to $K/\mu^{3/2}$.

It would be noticed that the above equation is similar to

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Child's law except for the addition of a new term μV_g. This new term is explained by the fact that a grid voltage of V_g is equivalent to μV_g volt on the plate.

27-20. Triode—Electrical Characteristics

The term '*characteristic*' is used for identifying the distinguishing electrical features and values of an electron tube. These values may be represented by means of a family of curves or may be tabulated. When the characteristic values are given in the form of curves, then these curves may be used for determining the tube performance and for calculating additional tube factors.

Various tube characteristics are obtained from electrical measurements made on a tube in various circuits under certain definite conditions of voltages. Another way of describing the characteristics is by denoting the conditions under which the measurements are made. For example, *static or steady characteristics* represent the values obtained with different d.c. potentials applied to the tube electrodes, whereas the *dynamic* characteristics are the values obtained with a.c. voltages applied across the control grid under various conditions of d.c. potentials on the other tube electrodes. Hence, *dynamic characteristics are indicative of the performance capabilities of a tube under actual working conditions.*

There are three static characteristics of a triode

(*i*) plate (or anode) characteristic (*i.e.* V_p/I_p characteristic).

(*ii*) transfer (or mutual) characteristic (*i.e.* V_g/I_p characteristic) and

(*iii*) V_g/V_p characteristic (or constant-current characteristic).

These three characteristics give the same information but in three different ways to enhance its usefulness. The plate current *Vs* plate voltage characteristic or simply the plate characteristic is obtained by varying plate voltage and measuring the plate current corresponding to different grid-bias voltages whereas transfer characteristic is obtained by varying grid bias voltage and measuring plate current for different anode voltages. The plate-voltage *Vs.* grid-voltage characteristic is obtained by changing grid voltage and then by changing plate voltage so as to keep plate current constant.

27-21. Static Characteristics of a Triode

In Fig. 27-23 is shown the experimental arrangement for determining the static characteristics of a triode.

(*a*) Plate Characteristic

The plate of the triode is connected *via* a sensitive milliammeter *A* to a potentiometer R_1 with the help of a slider. With this arrangement, the anode or plate can be maintained at any desired constant potential above the cathode. Meanwhile, the grid can be given any positive or negative potential with the help of a reversing

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switch RS from a potentiometer R_2 and its value noted on the voltmeter V_2 connected for the purpose. Keeping grid voltage constant

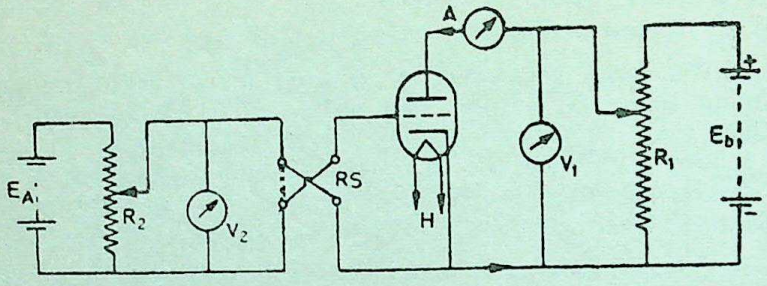

Fig. 27-23

at a given value, the plate voltage is changed and the corresponding values of I_p are noted on the milliammeter connected in the plate circuit.

The test is repeated with $V_g = -2V$ and $V_g = -4V$ etc. The results so obtained are plotted in Fig. 27-24 which are typical for a triode. Curves have been drawn for only zero and negative values of the grid potentials.

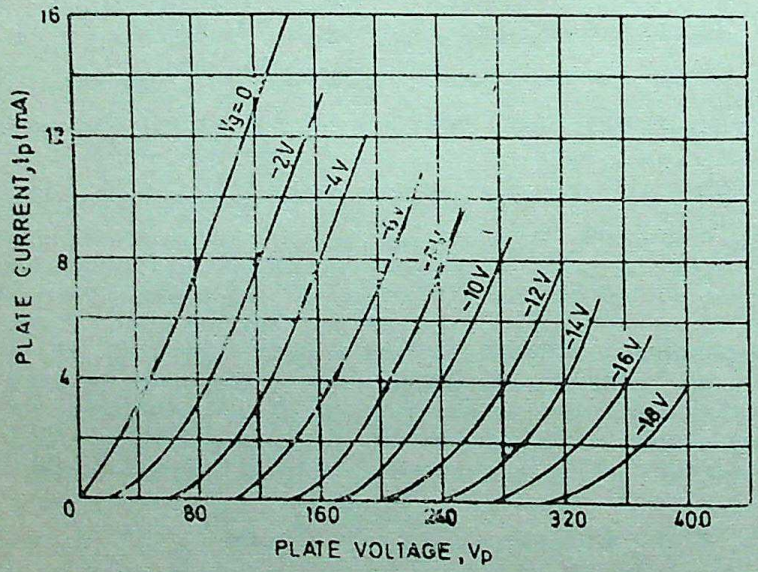

Fig. 27-24

(*b*) Transfer (or mutual) Characteristic

The curves showing the variation of I_p with V_g for fixed values of V_p are known as *transfer characteristics*.

For obtaining data for these curves, V_p is kept constant at a certain value and changes in I_p due to changes in V_g are noted from the plate milliammeter. When the grid is positive, it helps the anode to pull the electrons up, hence I_p is increased. If the grid is given

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a negative potential with respect to cathode by reversing the connections with the help of reversing switch RS, then it tends to retard the plate-ward journey of the electrons. Hence, I_p is reduced. If V_g is made too much negative, it may totally stop the passage of electrons through it (by repulsion) and so reduce I_p to zero. The approximate value of this 'cut off' grid bias can be found by putting $I_p=0$ in the equation

$$I_p=K(V_g+V_p/\mu)^{3/2}$$

$$\therefore \qquad 0=K(V_g+V_p/\mu)^{3/2} \quad \text{or} \quad V_g=-V_p/\mu$$

It is obvious that value of 'cut off' potential is directly proportional to the plate potential. The test is repeated by keeping V_p constant at different fixed values and by changing V_g from positive values to negative values. The data so obtained is plotted in the form of curves as shown in Fig. 27-25. It is seen from the figure that

(*i*) The transfer characteristic is practically straight over an appreciable portion of its length.

(*ii*) Curves for equal differences of plate voltages are equally-spaced.

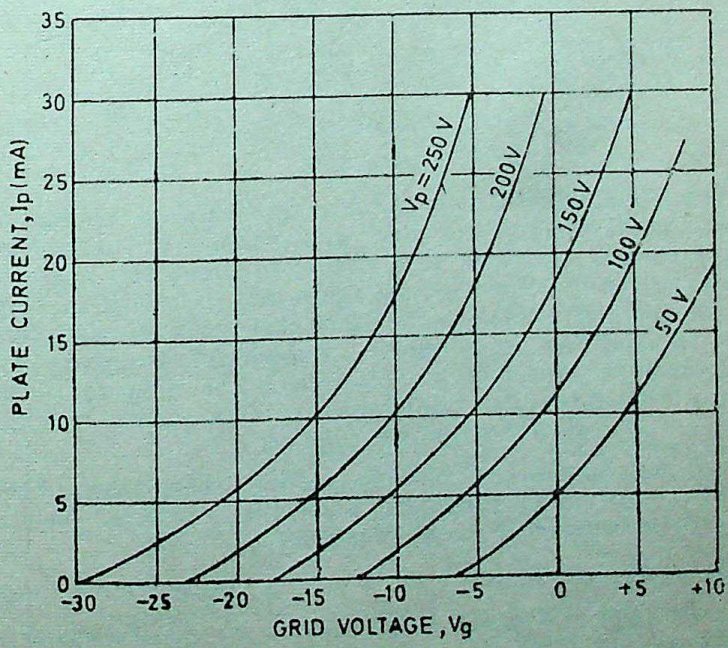

Fig. 27-25

(*iii*) Cut-off grid potentials for different curves (corresponding to different values of V_p) are different.

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(c) Constant-current Characteristic

For obtaining data for this characteristic, the same experimental arrangement is used as for obtaining the other two charac-

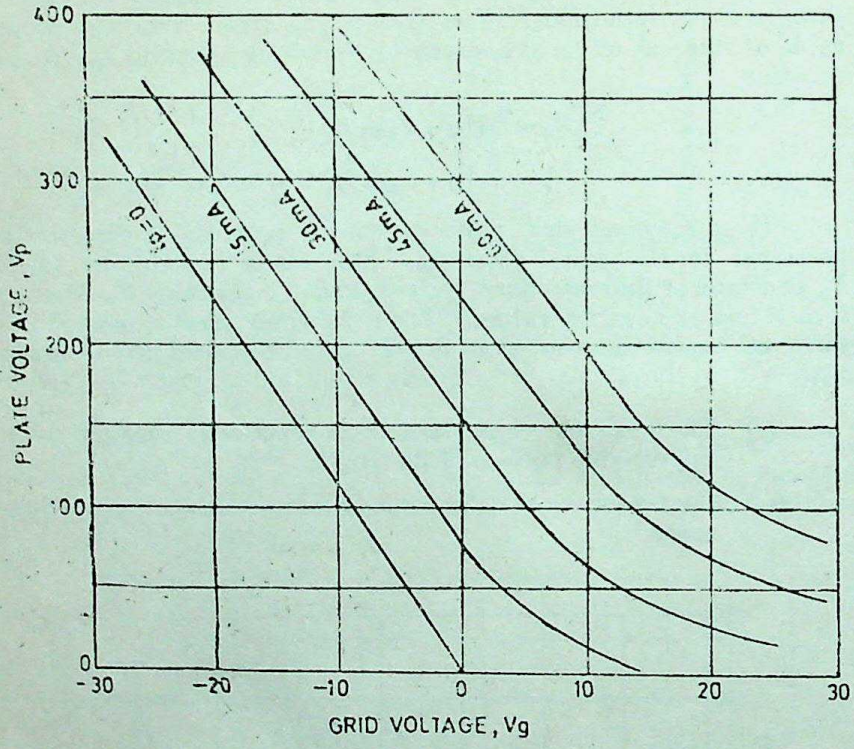

Fig. 27-26

teristics. The experiment is begun by setting plate voltage at some suitable positive value and then grid voltage is changed until the highest current for which it is desired to obtain the characteristic, passes through it. The plate voltage is then decreased in steps and at each step grid voltage readjusted so that the same plate current is obtained as at the original point. Experiment is repeated for some other constant values of plate current. The data so obtained is plotted in the form of a family of curves as shown in Fig. 27-26 which are typical of the results so obtained.

It may be remarked here, in passing, that if any of the three sets of characteristics (discussed above) is available, the other two characteristics can be deduced therefrom.

27-22. Tube Coefficients

(A.M.I.E. Summer 1975)

The slope of a static characteristic at its usual working point is known as a tube coefficient or tube factor or tube constant.

The three coefficients are :

(*i*) **Amplification factor** denoted by the Greek letter μ. It equals the slope of constant-current characteristic.

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(*ii*) **Plate resistance.** The other alternative names are dynamic or variational plate resistance or a.c. plate resistance, anode-slope resistance or differential resistance. It is denoted by R_p. It is equal to the reciprocal of the slope of plate characteristic curve.

(*iii*) **Mutual conductance** *or* **transconductance.** It is the slope of transfer characteristic.

We will discuss their practical significance and definitions in the following sections.

27-23. Amplification Factor (*A.M.I.E. Summer 1983*)

In a qualitative sense, the amplification factor of a triode is the measure of the comparative effectiveness of the grid and plate as controllers of plate current. In Fig. 27-27 are shown the constant-current characteristics. Let us assume that the operating point is B where $V_p = 200$ V, $V_g = -4$ V and $I_p = 30$ mA. Suppose, now that the grid is given a negative increment of $\triangle V_g = 8$V so that it becomes -12 V. It is found that plate current is decreased from its value of 30 mA to 15 mA *i.e.* the operating point shifts to 15 mA plate-current line. To keep the new operating point on the same plate-current line or in other words, in order to restore the plate current

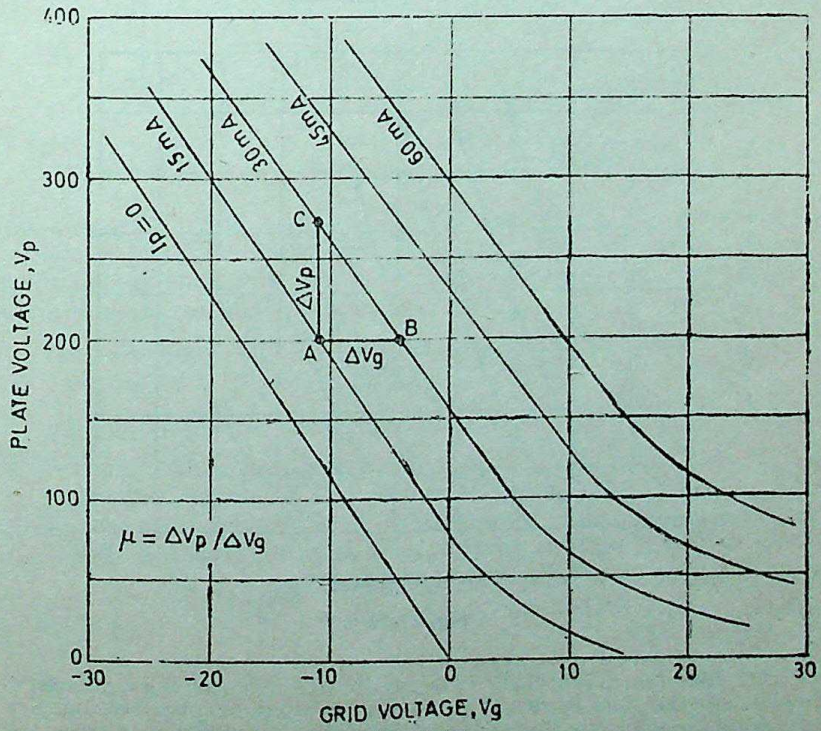

Fig. 27-27

to its original value of 30 mA, the plate voltage must be increased by an amount $\triangle V_p = 80$ V ($=AC$). It means that a change of 8 volts on

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grid produces the same change in plate current as 80 volts on the anode or plate. The ratio $\triangle V_p/\triangle V_g$ for constant plate current is called the amplification factor and being a ratio, is dimensionless number.

$$\therefore \qquad \mu = \frac{\triangle V_p}{\triangle V_g} \Big| I_p \text{ constant}$$

Its value in the present case is equal to $80/8 = 10$

Sometimes, a negative $(-)$ sign is put before the right-hand expression simply to indicate that the changes in the two voltages are in opposite directions. It is clear that amplification factor represents the slope of the constant-current characteristic.

27-24. Plate Resistance *(A.M.I.E. Summer 1983)*

As said above, it is equal to the reciprocal of the slope of the plate or anode characteristic which is shown in Fig. 27-28. Considering point A, it is seen that an increment in plate voltage of $\triangle V_p$ $(AB = 42$ V) demands a corresponding increment $\triangle I_p (= BC = 6$ mA) in order that the tube may operate on the same grid-voltage line (*i.e.* -6 volts in the figure). This ratio is known as dynamic plate resistance* (the subscript p standing for plate).

$$\therefore \qquad R_p = \frac{\triangle V_p}{\triangle I_p} \Big| V_g \text{ constant.}$$

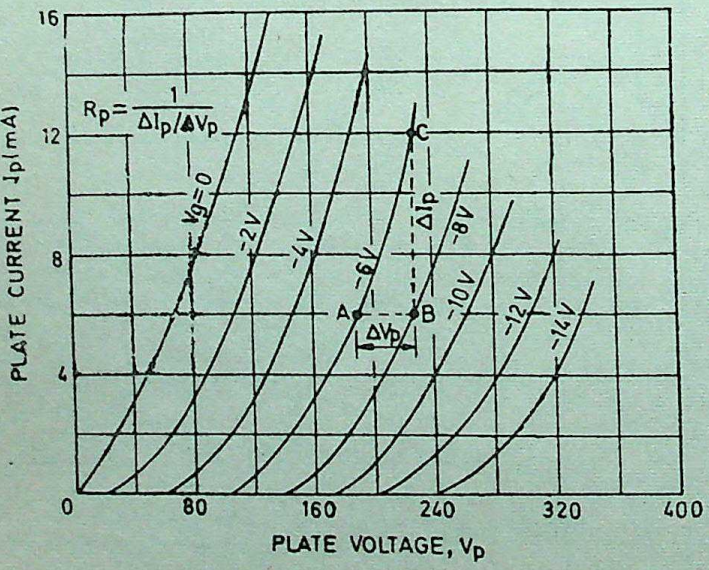

Fig. 27-28

*It should be noted that R_p is not the ratio of the d.c. potential of the plate and the plate current. This ratio is known as d.c. plate resistance or static plate resistance to distinguish it from the variational or incremental plate resistance (also known as a.c. plate resistance in contrast with d.c. plate resistance or dynamic plate resistance in contrast with the static plate resistance). Static plate resistance is seldom employed in analytical methods of solution.

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In other words, this plate resistance is the ratio of an incremental change in plate voltage to the corresponding change in plate current, grid voltage remaining constant. Its value, in the present case, is $42/6 \times 10^{-3} = 7000\ \Omega$.

27-25. Transconductance *(A.M.I.E. Summer 1983)*

It is equal to the slope of the transfer characteristic (Fig. 27-29). The ratio of change in plate current $\triangle I_p$ ($=BC=8$ mA) to

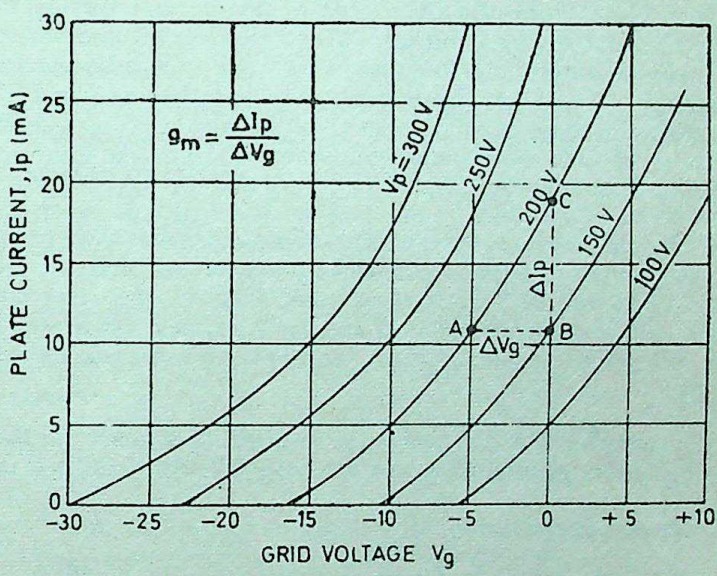

Fig. 27-29

the change in grid voltage $\triangle V_g (=AB=5$ V) for constant plate voltage of 200V in Fig. 27-29 is called the transconductance or mutual conductance of the tube and its symbol is g_m (the subscript m standing for mutual).

$$g_m = \frac{\triangle I_p}{\triangle V_g}\Bigg|_{V_p \text{ constant}}$$

Its value in the present case is $8 \times 10^{-3}/5 = 1{\cdot}6 \times 10^{-3}$ mho or 1600 micro-mhos.

The designation of transconductance is used because this coefficient involves quantities in both the grid and plate circuit. It is also sometimes referred to as grid-plate transconductance.

In the end, it may be pointed out that the above three coefficients are in fact the partial derivatives of equation given in Art. 27-19.

27-26. Inter-relation of Three Coefficients

We have seen above that,

$$\mu = \frac{\triangle V_p}{\triangle V_g} = \frac{\triangle V_p}{\triangle I_p} \times \frac{\triangle I_p}{\triangle V_g} = R_p \times g_m$$

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or $\mu = R_p \times g_m$.

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27-27. Values of Tube Coefficients from Characteristics

The three tube coefficients discussed and defined above can be found graphically from *any one* of the three static characteristics.

In Fig. 27-28 is shown the method of computing the three coefficients from plate characteristics. It is assumed that the operating point lies on one of the curves. First, the operating point A is located by the intersection of grid-voltage curve with either a horizontal line from the given plate current or vertical line from the given value of plate voltage. Next, horizontal and vertical lines are drawn through A till they intersect with the adjacent grid voltage curve. These lines have been shown dotted in Fig 27-28. As indicated, the three quantities $\triangle V_p$, $\triangle I_p$ and $\triangle V_g$ are determined. The required tube coefficients can be found from the following three equations $R_p = \triangle V_p / \triangle I_p$; $\mu = \triangle V_p / \triangle V_g$ and $g_m = \triangle I_p / \triangle V_g$.

The methods for determining these coefficients from transfer characteristic or constant-current characteristic respectively are shown in Fig. 27-29 and 27-27. In each case, first the operating point A is determined and then horizontal and vertical lines are drawn through A to find $\triangle V_p$, $\triangle I_p$ and $\triangle V_g$. Then the coefficients can be calculated as before.

Example 26-1. *Estimate (a) amplification factor (b) the anode-slope resistance of a triode which gives the following readings on test :*

Anode voltage (volts)	*100*	*80*
Anode current in mA with grid at 0 V	*15*	*10*
Anode current in mA with grid at −2 V	*8*	*3*

Solution. An inspection of the above data shows that the values refer to the linear portion of the characteristic.

When V_p is held constant at 100 V, then a change of −2V in grid potential changes the plate current by 15−8=7 mA. When V_g is kept constant at 0 V, then a change of 100−80=20 V in plate voltage produces a change of (15−10)=5 mA in plate current. Since we are working on the linear portion of the characteristic, we can calculate the change in plate voltage which would be necessary to produce a change of 7 mA in plate current.

Necessary change in plate voltage $= 20 \times 7/5 = 28$ V. Now, we can say that a change of 7 mA in plate current can be produced either by a change of 28 volts in plate voltage or by a change of −2 volts in grid voltage.

(*a*) $\therefore \quad \mu = -28/2 = -14$ (*b*) $R_p = \triangle V_p / \triangle I_p = 20/50 \times 10^{-3}$

$= 1000\ \Omega$

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Example 26-2. *The following readings were taken on a 3-electrode valve :*

Grid voltage	*0*	*—2*	*—4*	*—6*	*—8*	*—12*	*—14*	*—16*	*—18*
Anode current (mA) ...									
(a) with 130 V H.T. ...	*15*	*13*	*11*	*6*	*7*	*3*	*1·5*	*0·7*	*0·2*
(b) with 100 V H.T. ...	*10*	*8*	*6*	*4*	*2*	*0·4*	*0·1*	—	—

Plot the characteristic grid-voltage/anode-voltage curves and determine the internal impedance at zero grid voltage, the amplification factor and the mutual conductance.

Solution. The characteristics as drawn from the given data are shown in Fig. 27-30. For calculations, linear portions of the curves are utilized.

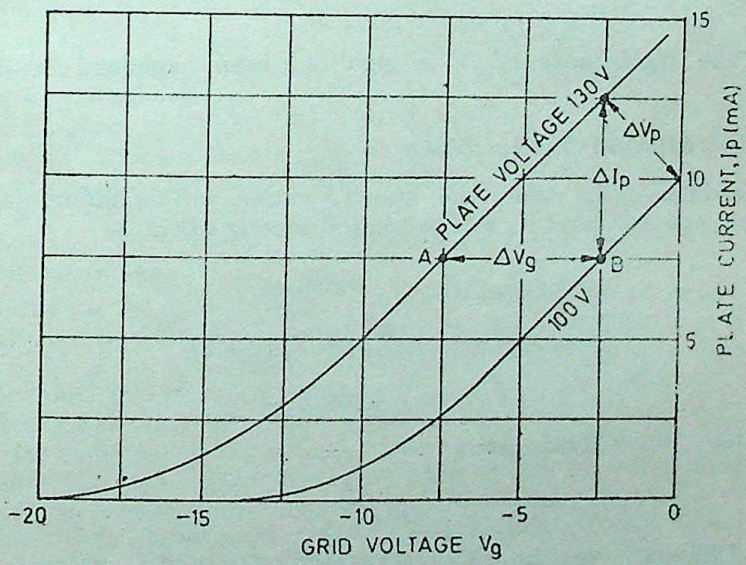

Fig. 27-30

As seen, $\triangle V_p = 130 - 100 = 30$ V

$$\triangle V_g = -5 \text{ V} \quad \text{and} \quad \triangle I_p = 5 \text{ mA}$$

$$g_m = \triangle I_p / \triangle V_g = 5 \times 10^{-3}/5 = 1{\cdot}0 \times 10^{-3} \text{ mho}$$

$$R_p = \triangle V_p / \triangle I_p = 30/5 \times 10^{-3} = 6000\ \Omega$$

$$\mu = \triangle V_p / \triangle V_g = 30/5 = 6$$

(or $\mu = R_p \times g_m = 6000 \times 10^{-3} = 6$)

Example 26-3. *The anode current characteristic of a triode is represented by the following expression*

$$I_a = 2 \times 10^{-6} (V_a + 10 V_g)^2$$

(*A.M.I.E., U.P.E.E.E. Winter 2003*)

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where I_a is the anode current in amperes, V_a and V_g are the anode and control grid voltages respectively, Determine mathematically, the values of (i) the amplification factor (ii) the mutual conductance (g_m) and (iii) the anode resistance (R_a) for the triode at the point where $V_a = 200$ V and $V_g = -10$ V. (*A.M.I.E. Winter 1978*)

Solution. As stated in Art. 27-25, the three tube coefficients are partial derivatives of current equation of a triode.

(i) Amplification factor

$$\mu = \triangle V_a / \triangle V_g \mid I_a \text{ constant}$$

Hence, differentiating the equation $I_a = 2 \times 10^{-6} (V_a + 10 V_g)^2$ with respect to V_g (and keeping I_a constant) we get

$$0 = 2 \times 10^{-6} \times 2 (V_a + 10\ V_g) \left(\frac{\triangle V_a}{\triangle V_g} + 10 \right)$$

Now, putting $V_a = 200$ V and $V_g = -10$ V, we get

$$0 = 4 \times 10^{-6} (200 - 100)(\mu + 10)$$

$$\therefore \quad \mu + 10 = 0 \quad \text{or} \quad \mu = -10$$

(The significance of $-$ve sign has been explained in Art. 27-23).

(ii) Mutual Conductance

For finding its value, the above equation will be differentiated with respect to V_g and V_a will be treated constant because

$$g_m = \triangle I_a / \triangle V_g \mid V_a \text{ constant}$$

$$\therefore \quad \frac{\triangle I_a}{\triangle V_g} = 2 \times 10^{-6} \times 2 (V_a + 10 V_g)(0 + 10)$$

or $$g_m = 10^{-6} \times 4000 = \mathbf{4{,}000\ \mu\text{-}\mho}$$

(iii) Plate Resistance

$$R_a = \frac{\triangle V_a}{\triangle I_a} \Big| V_g \text{ constant}$$

Differentiating the the equation with respect to I_a, we get

$$1 = 2 \times 10^{-6} \times 2 (V_a + 10 V_g) \left(\frac{\triangle V_a}{\triangle I_a} + 0 \right)$$

$$\therefore \quad 1 = 4 \times 10^{-6} \times 100\ R_a$$

$$\therefore \quad R_a = 10^4 / 4 = \mathbf{2{,}500\ \Omega}$$

Tutorial Problems—38

1. A triode valve has a characteristic given by

$$I_a = 0{\cdot}002 (V_a + 10 V_g)^2$$

where I_a is in milliamperes and V_a and V_g are in volts. Plot the characteristic curves for an anode voltage of 160 between values of V_g of -2 V and

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-16 V. What is the mutual conductance of the valve at zero grid potential with the above anode voltage ? Find also the a.c. resistance.

[6·4 mA/V ; 40,000 Ω]

2. The characteristic curves of a vacuum triode can be represented by the equation $i_a = 0{\cdot}003\ (V_a + 30V_g)^{1.5}$ where i_a is in milliamperes and V_a and V_g are in volts. If $V_a = 250$ V and $V_g = -3$ V, determine (*i*) the mutual conductance (*ii*) the amplification factor and (*iii*) the anode-slope resistance of the valve.

[1·74 mA/V ; 30 ; 16,950 Ω]

3. The transconductance of a triode is 800 μΩ. The plate current at $V_g = -4$ V is 2 mA. What would be the plate current at $V_g = -2$V ?

[3·6 mA]

4. With a 15 V bias, the plate current of a triode is 45 mA and when the bias is increased to 25V, the plate current is reduced to 20 mA. If, however, at this value of grid bias (−25 V) the plate voltage is increased from 150 to 200 volts, the plate current comes back to its original value of 45 mA. What is the value of amplification factor of the tube ? **(5)**

27-28. Dynamic Characteristics

The three static characteristics discussed above show the relationship between V_p, I_p and V_g and each characteristic gives the relationship between two quantities *when the third is kept constant*. For example, the transfer (or mutual) characteristic represents variations of plate current with grid voltage, when plate voltage is held constant. Now, keeping of plate voltage constant *when the plate current changes*, is possible only when there is no load resistance in the plate circuit (Fig. 27-23). Because, in that case, there is no voltage drop in the plate circuit, hence plate voltage would be the same as battery e.m.f. whatever the value of plate current. Hence, the matter is simple. But under actual operating conditions, there is always some load resistance (or impedance) in the plate circuit. As shown in Fig. 27-31, let there be a load resistance R_L in the plate circuit. The voltage applied across plate is now equal to $(E_b - I_p R_L)$.

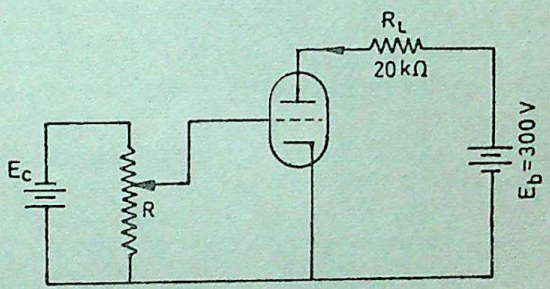

Fig. 27-31

Hence, in practice, as I_p changes due to changes in V_g, V_p also changes in inverse manner with the plate current (and this change, in turn, changes the variation of the plate current).

To sum up, we can say, that the plot of variation of I_p with V_g *when plate load resistance (or impedance) is zero* is called *static characteristic* because V_p remains static or constant. But the plot of the variation of I_p with V_g *when the plate load resistance (or impedance) is not zero* is known as dynamic characteristic because V_p is variable (*i.e.* changing or dynamic).

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27-29. Load Line

It is a straight line drawn on the V_p/I_p characteristic of a triode valve and represents the load resistance *i.e.* resistance connected in the plate circuit. It can be drawn as follows :

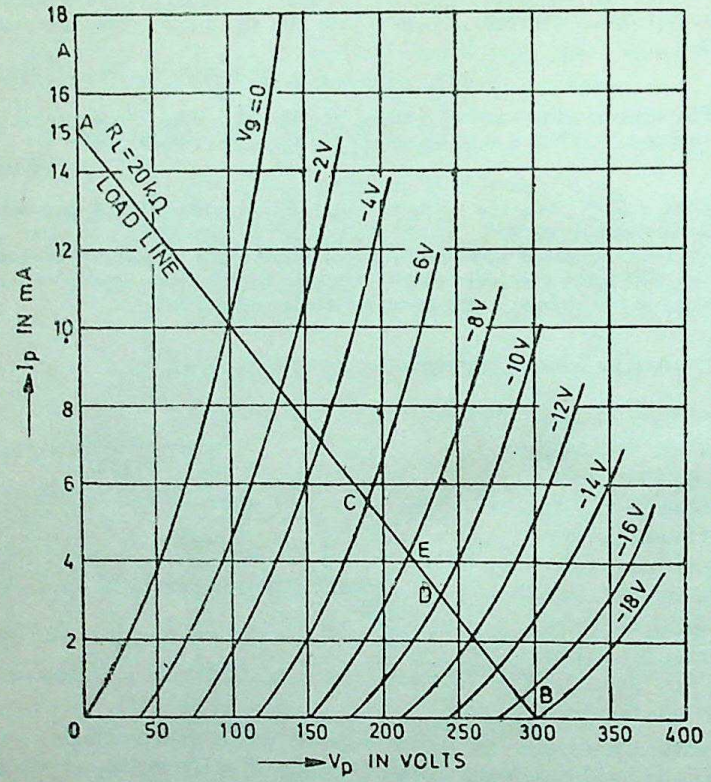

Fig. 27-32

When V_g is sufficiently negative, $I_p=0$, hence $V_p=E_b=300$ V. This helps to locate the point B (300,0) on the load line. The other point A can be located from the consideration that when valve is short-circuited, plate current is maximum and is given by $I_{max}=300/20{,}000=15$ mA. This gives point A (0, 15). Line AB represents the load line for a load resistance of 20 kΩ.

The load-line is a very valuable dynamic indicator. It helps in finding the following :

(*i*) plate potential for a given grid bias. For example, with $V_g=-2$V, $V_p=140$ V.

(*ii*) voltage drop across load resistance for a given grid bias provided plate supply voltage is known.

In the present case, $V_p=140$ V, hence

Voltage across load resistor $=300-140=160$ V

(*iii*) change in voltage drop across load resistor for a given change in grid voltage.

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Such a situation arises when an a.c. signal is applied to the grid of a triode amplifier. Suppose $V_g = -8$ V so that $V_p = 220$ V (point E) and drop across $R_L = 300 - 220 = 80$ V. Let an a.c. signal of peak value ± 2V be now applied. Obviously, grid-bias will vary between -6V and -10V which will give points of intersection C and D on the load line (Fig. 27-32). The corresponding plate voltages are 195 V and 245 V. Subtracting these from the plate supply voltage of 300 V, we get 105 V and 55 V respectively. This gives variation of voltage drop of ± 25 V across load resistor.

Hence, we find that an incoming signal of peak value ± 2V is amplified to a value of ± 25 V.

27-30. Triode as an Amplifier *(A.M.I.E. Winter 1983)*

The ability of a triode to amplify weak electrical impulses or signals is its most important property and the one that is most commonly employed. It is used extensively in long-distance telephony, talking pictures, radio reception, television and lots of other less familiar and less spectacular applications. The amplifying property is due to the fact that any small change produced in grid potential produces a big change in I_p and hence large potential drop across load in the plate circuit. In order to use this property of the triode, it is necessary to introduce some form of resistive or inductive load in the plate circuit.

Fig. 27-33 shows a simple triode amplifier circuit having a load resistor of 20 kΩ and a plate supply of 300 V. Grid bias is supplied by the battery E_e which fixes the operating point A on the V_g/I_p or transfer characteristic shown in Fig. 27-34. This point is so chosen that under quiescent conditions, it lies approximately at the centre of the linear portion of the transfer curve. It is seen from Fig. 27-34 that with no signal applied, $V_g = -3$ V, $I_p = 7$ mA, drop across load resistor $= 7 \times 20 = 140$ V and $V_p = 300 - 140 = 160$ V. Hence, all values are fixed for the operating point A on the 160 V curve of Fig. 27-34.

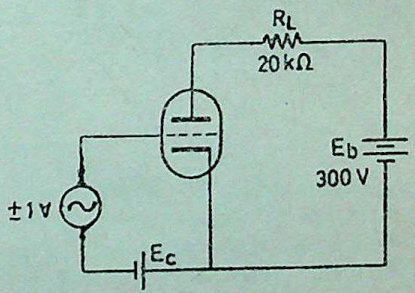

Fig. 27-33

Suppose, that an a.c. signal of peak value ± 1 V is now introduced into the grid-cathode circuit. This signal voltage will be impressed on the existing d.c. grid bias of -3 V. Grid bias will, therefore, vary between values of -4 V and -2 V. It is so because when positive half-cycle of the signal is applied, V_g becomes $(-3+1) = -2$ V and when negative half-cycle is applied, it becomes $(-3-1) = -4$ V. As mentioned earlier, point A (and hence V_g) is so chosen that grid becomes less negative or more negative but *never positive during the whole signal cycle*. Accordingly, plate current varies between limits of 9·5 mA and 4·5 mA corresponding to points C and B respectively. The variations in I_p can be obtained by projecting points A, B and C to the right side as shown in Fig. 27-34.

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Let us calculate the corresponding voltage drop across R_L and hence value of V_p.

(a) When $V_g = -2$ V, $I_p = 9{\cdot}5$ mA

Drop across $R_L = 9{\cdot}5 \times 20 = 190$ V

$\therefore \quad V_p = 300 - 190 = 110$ V

(b) When $V_g = -4$ V, $I_p = 4{\cdot}5$ mA

Drop across $R_L = 4{\cdot}5 \times 20 = 90$ V

$\therefore \quad V_p = 300 - 90 = 210$ V

Under quiescent conditions (*i.e.* before the a.c. signal was applied) I_p was steady at 7 mA and V_p was 160 V. Hence, we find

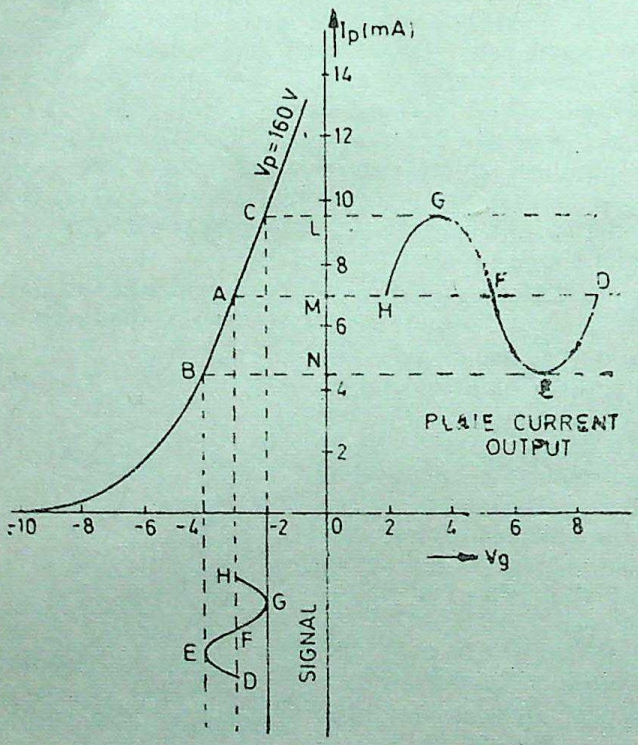

Fig. 27-34

that a variation of $\pm$ 1 V in grid-cathode circuit produces a variation of $\pm$ 50 V in plate voltage.

Similarly, under quiescent conditions, drop across load resistor was 140 V. Now, the drop varies between 190 V and 90 V giving a variation of $\pm$ 50 V around the steady value. It shows that an input signal variation of $\pm$ 1 V in the grid-cathode circuit appears as a variation of $\pm$ 50 V in the output plate-cathode circuit. Hence, the signal is amplified $50/1 = 50$ times.

Another point worth noting is that variations in V_p are 180° out of phase with the signal voltage as shown in Fig. 27-35. With

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positive half-cycle of signal voltage impressed, V_g becomes less negative, hence I_p increases thereby producing greater drop across

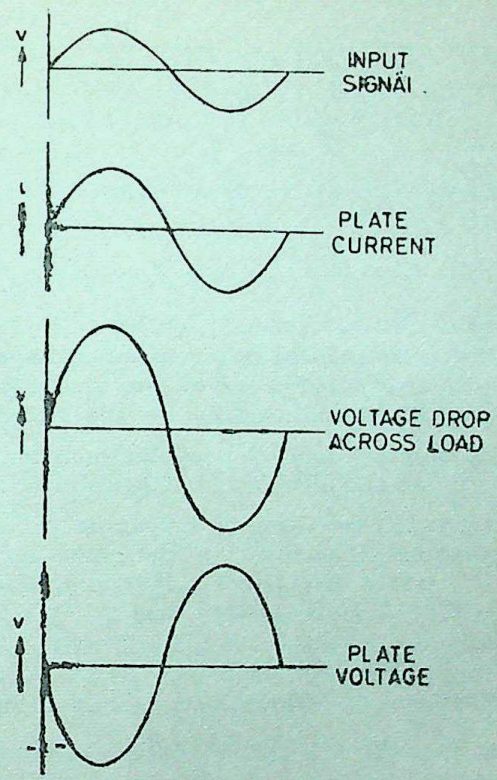

Fig. 27-35

R_L. Accordingly, V_p is reduced. When negative half-cycle of signal voltage is applied, V_g becomes more negative, hence I_p decreases, thus producing smaller drop across R_L. Consequently V_p is reduced. However, variations in output voltage across R_L are in step with the signal voltage.

27-31. The A.C. Equivalent Circuit of an Amplifier

In Fig. 27-36 is shown the actual circuit of a triode amplifier. E_g is the r.m.s value of the grid-signal voltage. The plate current consists of (*i*) one steady component I_0 and (*ii*) an alternating component of r m.s. value I_p. For example, as discussed in Art. 27-30, the steady value I_0 of plate current under quiescent conditions was 7 mA but under operating conditions it had an a.c. component of peak value $\pm$ 2 5 mA imposed on it.

The plate battery has an e.m.f of E_b and grid battery provides necessary grid bias voltage E_c. Since we are interested in the behaviour of the tube for alternating voltages and currents only,*

*The d.c. voltages and currents merely serve to fix the operating point and the particular values of r_p, R_L and μ, and to supply the energy required.

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the equivalent circuit is reduced to the simple form shown in Fig. 27-37. This is known as a.c. *equivalent circuit* of the amplifier.

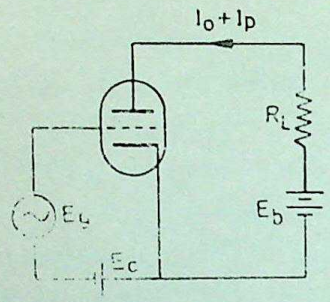

Fig. 27-36

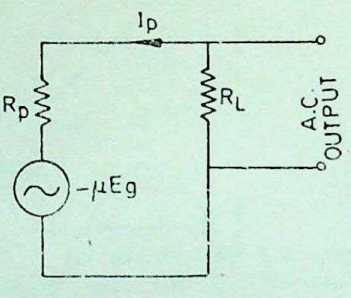

Fig. 27-37

The triode has been replaced by an equivalent alternator generating an r.m.s voltage of $(-\mu E_g)$ and having a resistance of R_p in series (this R_p being the internal resistance of the triode).

The negative sign in $(-\mu E_g)$ is a helpful reminder of the phase reversal produced by an amplifier in the plate voltage (Art. 27-30).

The plate current I_p represents r.m.s. value of the *alternating* plate current flowing in the circuit. In the following calculations, it is assumed that the tube parameters μ, R_p and g_m remain constant. With reference to Fig. 27-37, it is found that

$$I=(-\mu E_g)/(R_L+R_p)$$

The r.m.s. value of a.c. voltage drop across load resistance R_L is

$$V_0=I_pR_L=(-\mu E_g\times R_L)/(R_L+R_p)$$

Voltage amplification $A=V_0/E_g$

or
$$A=\frac{-\mu E_g\times R_L}{(R_L+R_p)E_g}=\frac{-\mu R_L}{R_L+R_p}$$

or
$$A=\frac{-\mu}{1+(R_p/R_L)}$$

The negative sign in the above expression is usually omitted for simplicity.

It is seen from the above expression that greater the value of plate load resistance R_L, greater is the voltage amplification obtained *but it never exceeds the value of amplification factor* μ.

If the load consists of a coil having a resistance of R_L, inductance of X_L and hence impedance of Z_L, then

$$A=\frac{-\mu Z_L}{\sqrt{(R_p+R_L)^2+X_L^2}} \qquad \text{(Ex. 27-6)}$$

Another point is that a larger value of R_L not only produces greater gain but additionally tends to produce a straight line dynamic characteristic and distortionless operation.

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The power output to the load resistance is,

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$$P_0 = I_p^2 R_L = \left(\frac{\mu E_g}{R_L + R_p}\right)^2 \times R_L = \frac{\mu^2 E_g^2 R_L}{(R_L + R_p)^2}$$

If the load is not purely resistive, then power factor is also taken into account.

The value of R_L needed to make this power output maximum is found by differentiating the above expression with respect to R_L and equating the result to zero. It will be found that required condition is that $R_L = R_p$.

In that case, the maximum power output is given by $\mu^2 E_g^2/4R_p$.

The value of a.c. voltage drop across R_L is

$$V_0 = \mu E_g R_L/(R_L + R_p) = AE_g$$

Example 27-4. *A certain triode has an amplification factor of 27 and an anode a.c. resistance of 18 kΩ. Calculate the magnitude of the voltage amplification when the load is a non-reactive resistor of 25 kΩ.* (*A.M.I.E. Summer 1980*)

Solution. $A = \dfrac{\mu R_L}{R_L + R_p}$ Now, $\mu = 27$, $R_p = 18$ kΩ, $R_L = 25$ kΩ

$$\therefore \quad A = \frac{27 \times 25}{(25 + 18)} = \mathbf{15{\cdot}7}$$

Example 27-5. *A load resistance of 50 k Ω is used in a triode amplifying stage. The valve has an amplification factor of 16 and anode resistance of 30 k Ω. Calculate the r.m.s. value of the output voltage when 2·5 volt r.m.s. is applied between grid and cathode.*

Solution. With reference to the a.c. equivalent circuit of a triode in Fig. 27-37, it is clear that

$$I_p = \frac{\mu E_g}{R_L + R_p}$$

where E_g = r.m.s. value of input voltage.

∴ r.m.s. value of output voltage across load

$$= I_p R_L = \frac{\mu E_g R_L}{R_L + R_p}$$

$$= \frac{16 \times 2{\cdot}5 \times 50\,000}{30{,}000 + 50{,}000}$$

$$= \mathbf{25\ volts}$$

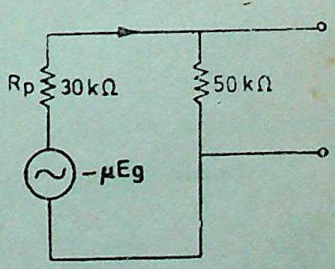

Fig. 27-38

Example 27-6. *A triode has an anode-slope resistance of 12 kΩ and an amplification factor of 15. Determine the stage gain if it is used as an amplifier with an anode load consisting of (i) a non-inductive resistance of 20 kΩ and (ii) a choke having a resistance of 20 kΩ and an inductance of 7500 μH when the input voltage has a frequency of 600 kHz.* (*A.M.I.E. Winter 1974*)

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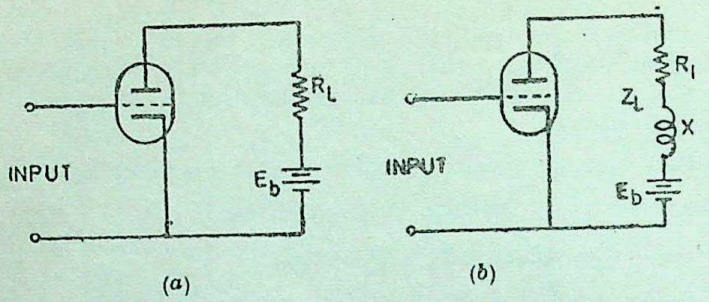
Solution. (*i*) This case is shown in Fig. 27-39 (*a*).

$$A=\frac{-\mu R_L}{R_p+R_L}=\frac{-15\times 20}{(12+20)}=-9{\cdot}4$$



(*a*) (*b*)

Fig. 27-39

(*ii*) This case is shown in Fig. 27-39 (*b*)

$$X_L=2\pi f L=2\pi\times 600\times 10^3\times 7500\times 10^{-6}=28{\cdot}3\ \text{k}\Omega$$

$$Z_L=\sqrt{R_L^2+X_L^2}=\sqrt{20^2+28{\cdot}3^2}=34{\cdot}64\ \text{k}\Omega$$

$$A=\frac{-\mu Z_L}{\sqrt{(R_p+R_L)^2+X_L^2}}=\frac{-15\times 34{\cdot}64}{\sqrt{32^2+28{\cdot}3^2}}=-12{\cdot}17$$

Example 27-7. *A triode valve, used as a voltage amplifier with a resistance load, has an input of 0·2 V and gives an output voltage of 1·6 V when the load is 10,000 Ω and 1·2 V when the load is 5000 Ω. Find the anode-slope resistance and mutual conductance of the triode.*

Solution. First Case

Voltage amplification $=1{\cdot}6/0{\cdot}2=8$; $R_L=10{,}000\ \Omega$

Now, $$A=\frac{\mu R_L}{R_p+R_L}$$

$\therefore$ $$8=\frac{\mu\times 10{,}000}{R_p+10{,}000} \qquad \ldots(i)$$

Second Case

$$A=1{\cdot}2/0{\cdot}2=6$$

$\therefore$ $$6=\frac{\mu\times 5000}{R_p+5000} \qquad \ldots(ii)$$

Dividing Eq. (*i*) by (*ii*), we get

$$\frac{8}{6}=\frac{2(R_p+5000)}{(R_p+10{,}000)}$$

$\therefore$ $$R_p=5{,}000\ \Omega$$

Substituting this value of R_p in Eq. (*i*) we get

$$8=\mu\times 10{,}000/(5000+1000)$$

$$\mu=12$$

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Now, $\mu = R_p \times g_m$

$\therefore$ $g_m = 12/5000 = 2{\cdot}5$ mA/V

Tutorial Problems — 39

1. The slopes of the mutual and anode characteristics of a triode valve are respectively 1·2 and 0·2 mA/V. Calculate the value of the voltage amplification factor obtainable for the tube when the anode load is a resistance of 20,000 Ω. [4·8]

2. A certain triode has an amplification factor under operating condition of 100 and mutual conductance of.2 mA/V. It has connected in the anode circuit an external ohmic resistance of 50,000Ω. Calculate the voltage gain. [50]

3. A non-inductive resistance of 20,000Ω is connected in the anode circuit of a triode having an amplification factor of 15 and an anode slope resistance of 12,000Ω. If an a.c. voltage of effective value 0·5 V is applied to the grid, what is the effective value of alternating voltage across the load resistance, assuming the anode voltage and grid bias to be such as to give distortionless amplification ? [4·69 V]

27.32. Self-biasing of a Triode *(A.M.I.E. Summer 1975)*

By grid *bias* is meant the difference of d.c. potential that exists between the control grid and cathode of a triode, with the grid being negative with respect to the cathode. The importance of grid bias lies in the fact that it determines the operating point of the triode valve when used as an amplifier. In the circuits drawn so far, grid bias was provided by the C-battery. However, such a battery is costly as well as troublesome and needs periodic replacements. Following two methods are commonly employed for obtaining grid bias by using currents flowing in valve circuit itself rather than by using a separate C-battery. In that case the triode is said to be *self-biased.*

(*i*) Cathode Bias

It is the most common form of self-biasing used in amplifier circuits. As shown in Fig. 27-40 (*a*), it is accomplished by placing a resis-

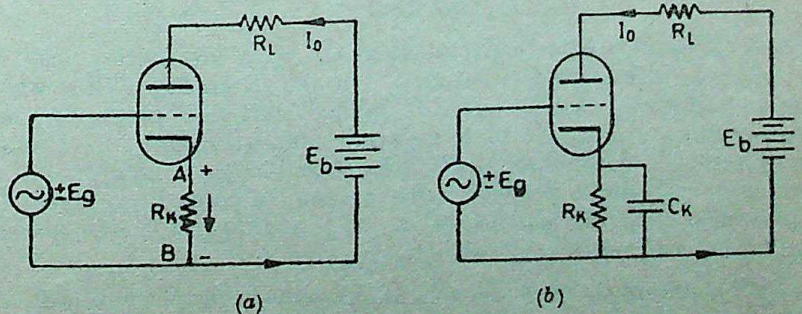

Fig. 27-40

tor (R_k) in series with the cathode circuit so that plate current passes through it. The voltage drop across $R_k = I_0 R_k$ is such as to make grid *negative* with respect to cathode. It is so because *conventional* current

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through R_k flows from point A to B so that point A is at a higher potential than B. In other words, potential of point B (and hence grid) is lower than that of point A (*i.e.* cathode) by an amount $=I_0R_k$ where I_0 is the steady value of plate current. With zero signal input, this steady plate current while passing through R_k establishes the fixed grid bias. When, however, signal is applied, variations are produced in plate current which lead to variations in the drop across R_k. Hence, grid-bias varies. To prevent these variations and thus maintain a steady or constant grid-bias, a by-pass capacitor C_k is connected in parallel with R_k [Fig. 27-40 (*b*)]. This capacitor allows an easy passage to the alternating component of plate current whereas steady current passes through R_k thereby producing a steady grid-bias.

(*ii*) Grid-leak Bias

This form of grid-bias is used in transmitters, oscillators and other types of class-*C* amplifiers [Art. 27-33 (*a*)]. It is obtained with the help of a high resistance R_g connected in the grid-cathode circuit. This resistance is known as grid-leak resistance because the small grid current leaks through it.

With zero signal, there is no grid bias because there is no grid current and hence no voltage drop across R_g. When a.c. signal is

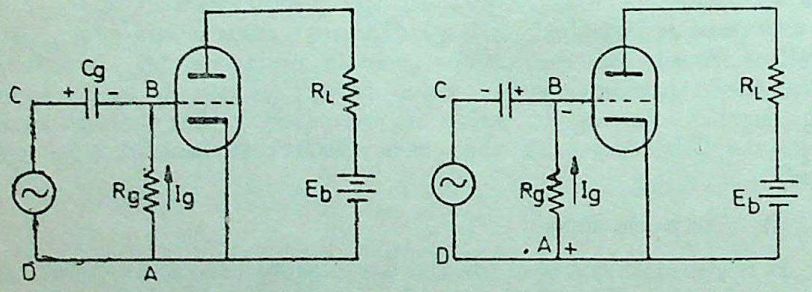

Fig. 27-41

applied, it appears across R_g, making the grid alternately positive or negative with respect to cathode. As shown in Fig. 27-41 when positive half-cycle of the signal is applied, left-side plate of C_g becomes positive, right-side plate negative and grid positive. Hence, grid current I_g flows through R_g from A to B. Obviously, point A (and hence cathode) is positive with respect to point B (*i.e.* grid) by an amount$=I_gR_g$. It should be noted that this grid current passes from grid to cathode *through* the triode valve.

When the negative half-cycle of the signal is applied [Fig. 27-41 (*b*)], it makes end A positive and B negative. There is no current through the triode circuit but only around circuit $ABCD$. This current after passing through R_g acts as a discharging current for C_g.

Hence, it is seen that in this case of self-biasing, grid-bias is obtained by the grid current initiated by the input signal through C_g

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27.33. Classification of Amplifiers (*A.M.I.E. Winter 1974*)

Different amplifiers may be classified on the basis of their grid-bias values. This classification leads to following types of amplifiers :

(*a*) Class-A Amplifiers

These are the amplifiers in which the grid-bias is so chosen that the operating or quiescent point lies on the linear portion of the dynamic transfer characteristic as shown in Fig. 27-42. Class-*A* operation has the following properties :

(*i*) the grid never goes positive

(*ii*) the plate current flows all the time (360°)

(*iii*) the tube operates only on the linear portion of the curve (Fig. 27-42).

(*iv*) the output signal has least distortion *i.e.* the output signal is almost of exactly same shape as the input signal although greatly magnified. Class-*A* operation is employed where it is essential to amplify a signal without distorting its shape such as in radio receivers and television.

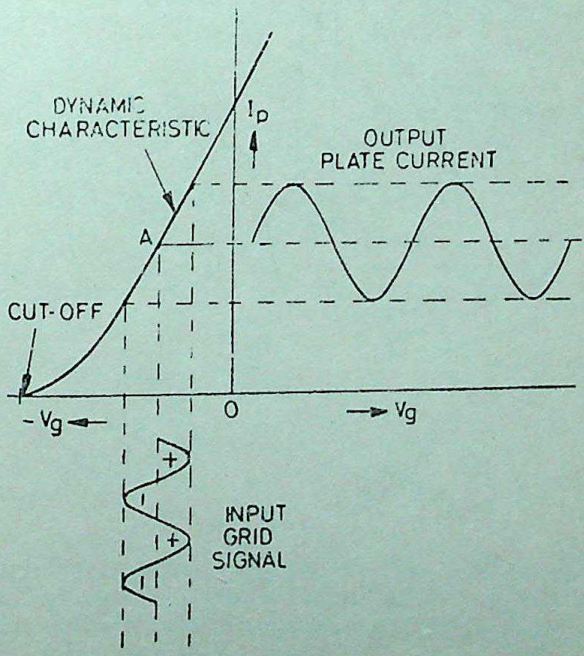

Fig. 27-42

However, the two disadvantages of class-*A* amplifiers are as under :

(*i*) they cannot supply large power outputs

(*ii*) they have low (about 20%) plate efficiency which is defined as the ratio of a.c. power output developed across R_L to the d.c. power supplied to the plate.

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(*b*) Class-B Amplifiers

In these amplifiers, the grid is biased to cut-off value so that no plate current flows when no signal is applied. As shown in Fig. 27-43 the tube conducts only during the positive half-cycle of the input signal. Following are the important points about class-*B* operation :

(*i*) control grid is biased at cut-off

(*ii*) no plate current flows during conditions of no-signal input *i.e.* during quiescent conditions.

(*iii*) plate current flows only during the positive half cycle of the input signal (180°).

(*iv*) the output consists of positive half-cycles as shown in Fig. 27-43.

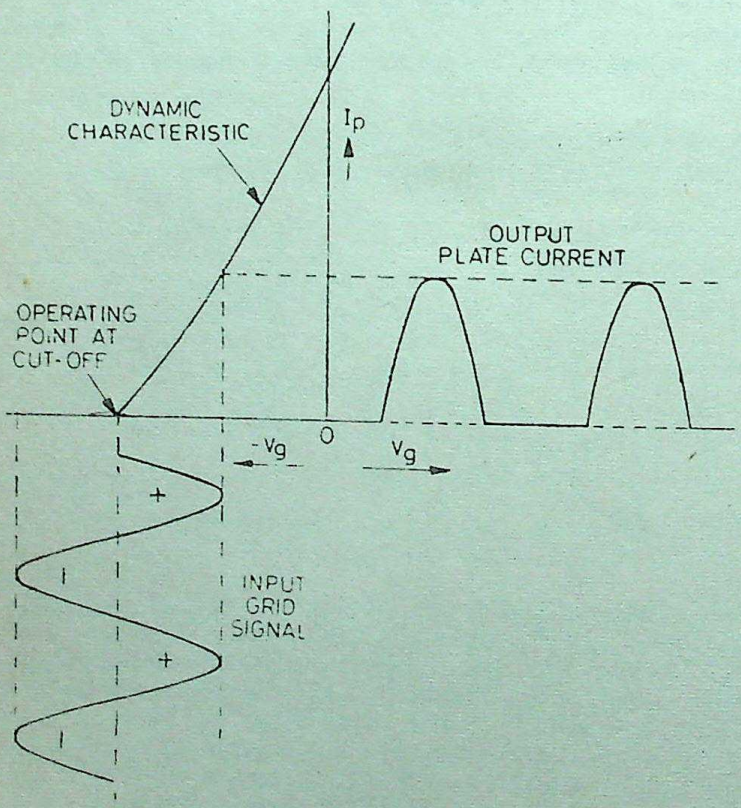

Fig. 27-43

However, class-*B* amplifiers furnish large output power (sometimes as much as 5 times that supplied by class-*A* amplifiers) and have a comparatively high efficiency of about 50 percent. Because they produce large distortion of the output signal, such amplifiers are not much used except for radio-frequency amplifiers using push-pull arrangement.

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(c) Class-C Amplifiers

In these amplifiers, grid is highly biased much below the cut-off point (Fig. 27-44) and is driven with a very large grid signal. Here, the plate current flows only for two-third of the positive half-cycle of the input signal (120°). The output plate current does not resemble the incoming signal. Instead, it appears to consist of pulses which occur at the rate of the incoming signal. Obviously, the negative half-cycles of the input signal are completely suppressed. Class-*C* operation may be defined by the following conditions :

(*i*) control grid is biased much below the cut-off point.

(*ii*) plate current flows for about one-third of the input cycle (120°).

(*iii*) output consists of incomplete half-cycles *i.e.* pulses of short duration.

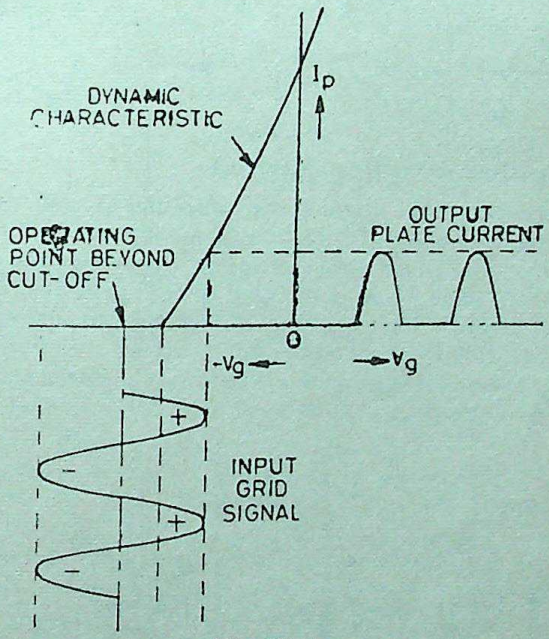

Fig. 27-44

Such amplifiers have very high distortion although their plate efficiency is an high as 85 percent. Main use of such amplifiers is in oscillators for radio-frequency amplification where high efficiency is very essential regardless of distortion.

27-34 Simple Amplifier *(A.M.I.E. Winter 1974)*

The circuit of a basic class-*A* triode amplifier is shown in Fig. 27-45. The triode has been self-biased by means of a cathode bias resistor R_k which has a by-pass capacitor C_k connected across it. An alternating input signal voltage E_g is applied to the grid-cathode circuit. An amplified alternating output voltage appears across the load resistance R_L.

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Working

As grid bias varies, I_p also varies in step with it. This varying plate current produces a varying voltage drop across R_L which is an amplified version of E_g. This alternating drop V_0 is taken out through the output coupling capacitor C. If necessary, this output voltage V_0 can be further amplified by another amplifier. In that case, the arrangement would be known as two-stage amplifier.

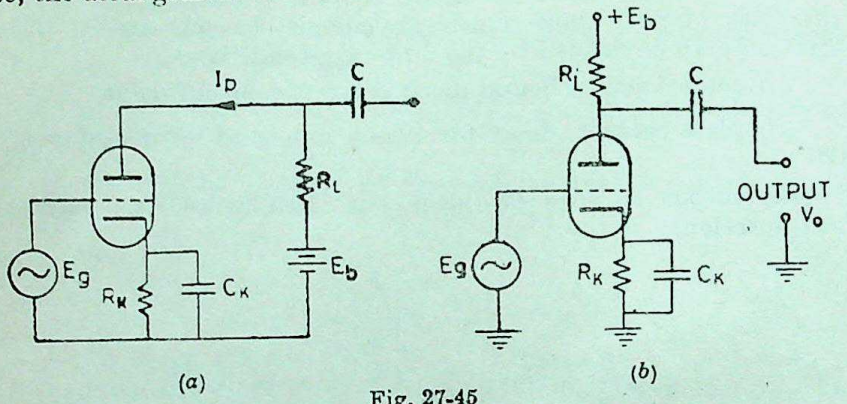

Fig. 27-45

27-35. Amplifier Coupling Methods

The output from a single-stage amplifier is generally not large enough to drive an output device, say, a loudspeaker. It needs further amplification over two or three stages. For this purpose, the a.c. output voltage of each stage is coupled to the grid of the next amplifier valve and so on. The manner in which output of one stage is fed for amplification to the succeeding stage is called *coupling*. The stages that are coupled together are said to be *cascaded*. Following three coupling methods are in general use :

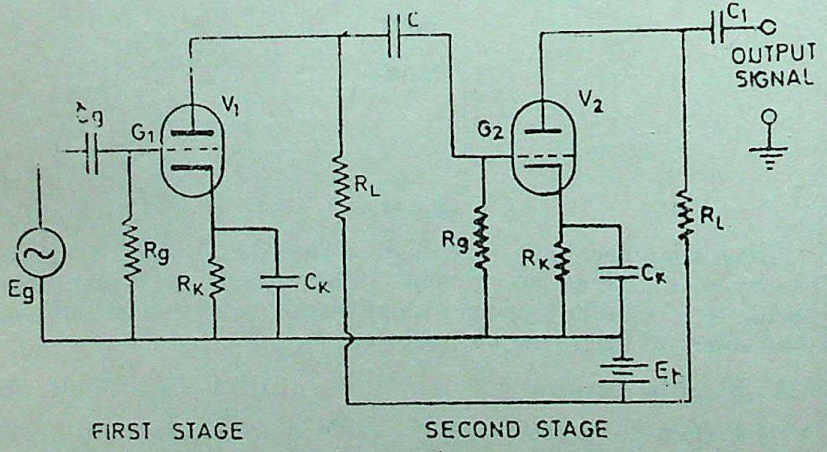

Fig. 27-46

(a) Resistance-capacitance or R-C coupling. It is also known as resistance coupling and is shown in Fig. 27-46. It is the most popular coupling method because it is cheap and provides undis-

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torted audio signals over a wide band of frequency. When the input signal of r.m.s. value E_g is applid to G_1, it appears across R_L in amplified form. This amplified signal passes through C to G_2 where it undergoes further amplification by valve V_2. Since coupling capacitor C blocks the application of d.c. voltage to G_2, it is sometimes known as *blocking* capacitor. If necessary, the varying voltage developed across load resistor of valve V_2 can be applied through C_1 to the grid of the succeeding valve for still further amplification. A simplified circuit diagram is shown in Fig. 27-47.

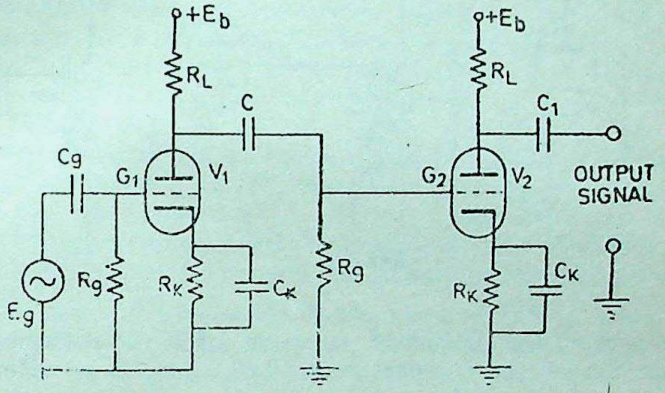

Fig. 27-47

(*b*) Impedance Coupling

In this case, the plate load is in the form of an inductor having an impedance of Z_L. The a.c. voltage drop across Z_L is passed on

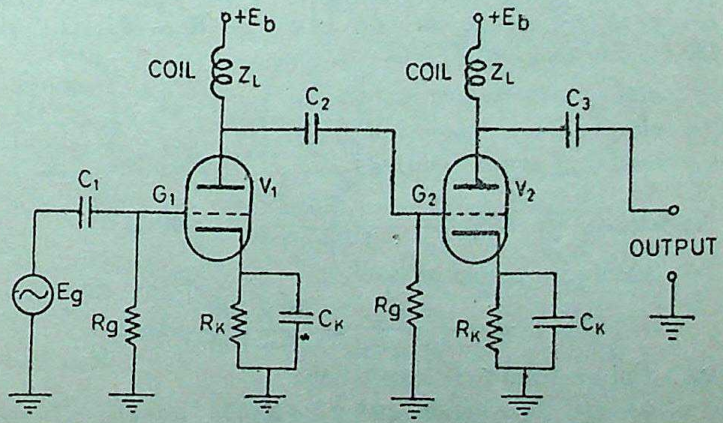

Fig. 27-48

to G_2 through coupling capacitor C_2 for further amplification (Fig. 27-48). Such coupling is generally used in amplifiers where only a narrow range of frequencies is to be amplified.

(*c*) Transformer Coupling

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As shown in Fig. 27-49, in this case, an interstage transformer is used to couple V_1 and V_2 with primary connected in the plate

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circuit of V_1 and secondary winding to the grid circuit of V_2. This coupling has following advantages over other types of couplings.

(*i*) The output voltage in the first stage itself can be stepped up by using a step-up transformer.

(*ii*) The grid of V_2 is completely isolated from the d.c. plate supply of V_1.

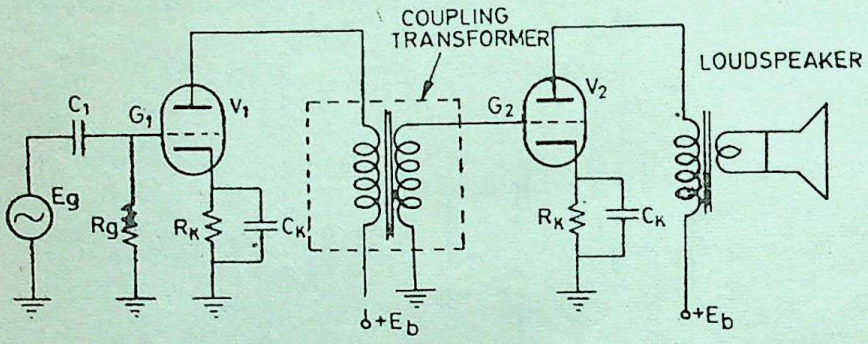

Fig. 27-49

However, the frequency response of a transformer-coupled amplifier is poor as compared to *R-C* amplifier. Hence, this coupling is not commonly used because the additional voltage gain available with step-up turn ratio is not sufficient compensation for its poor frequency response.

Example 27-8. *A single-stage voltage amplifier uses 6 C 5 triode to amplify a 0·25 V audio signal from a phono pick up to at least 50 V for driving a succeeding power amplifier. With $V_g = -7·5$ V, steady plate current of the triode is 1·5 mA. If $R_p = 10$ kΩ, $\mu = 20$, $R_L = 100$ kΩ and $E_b = 300$ V, find*

(*i*) *value of cathode bias resistor R_k*

(*ii*) *voltage gain per stage and*

(*iii*) *number of stages required.*

Solution. (*i*) $R_k = \dfrac{V_g}{I_0} = \dfrac{7·5}{1·5 \times 10^{-3}} = 5{,}000\ \Omega$

(*ii*) Stage gain per stage is

$$A = \frac{\mu R_L}{R_L + R_p} = \frac{20 \times 100}{10 + 100} = 18·2$$

(*iii*) Output from first stage

$$= 0·25 \times 18·2 = 4·55 \text{ V}$$

Output from second stage

$$= 4·55 \times 18·2 = 82·8 \text{ V}$$

Obviously, two amplifier stages are required. Of course, volume control would be necessary to prevent overloading of the amplifier.

27-36. Inter-electrode Capacitances of a Triode

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(A.M.I.E. Winter 1974)

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The nearness of elements in a triode causes small capacitances to exist between (*i*) grid and cathode (*ii*) grid and plate and (*iii*) plate and cathode. These capacitances (shown in Fig. 27-50) are called the interelectrode capacitances of a triode valve. Although they are quite small (from 2 to 10 pF) they can be very significant especially when a triode is used as amplifier at very high frequencies *i.e.* as a radio frequency amplifier.

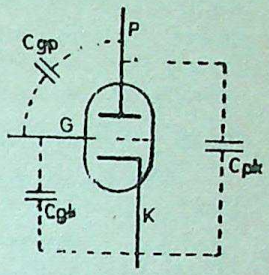

Fig. 27-50

Out of the three interelectrode capacitances shown in Fig. 27-50, C_{gp} is the most important one. At higher radio frequencies, its reactance ($=1/2\pi f C_{gp}$) becomes very small. Hence, some energy is fed back from the plate circuit (output) to the grid circuit (input) which leads to instability and oscillations in the amplifier. Since at lower audio frequencies, reactance offered by C_{gp} is very large, feedback between the plate and grid circuits is small That is why interelectrode capacitances of a triode are of not much significance at lower frequencies.

Inter-electrode capacitance C_{gp} can be reduced by inserting an additional grid between the plate and the control grid. This grid is, therefore, known as *screen* grid. Such a tube is called a tetrode.

27-37. Tetrode

It is a four-electrode valve. It contains a plate, cathode, control grid G_1 and another grid G_2 known as screen grid as shown in Fig.

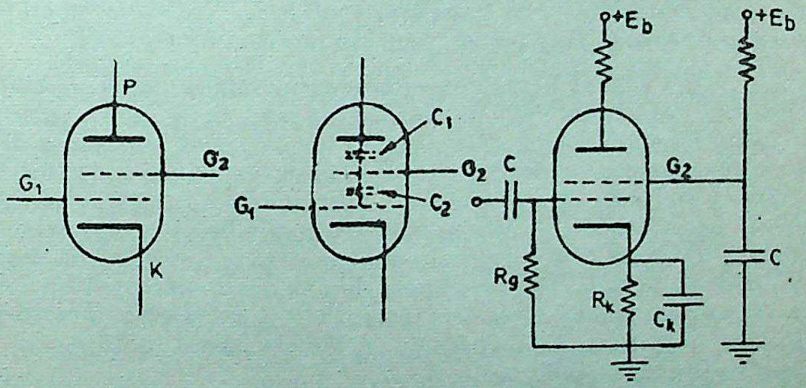

Fig. 27-51

27-51 (*a*). It is called screen grid because its main purpose is to screen or shield the plate from the control grid in order to reduce grid-plate inter-electrode capacitance C_{gp}. As shown in Fig. 27-51 (*c*), screen grid is grounded so far as a.c. potentials are concerned with the help of the by-pass capacitor C which by offering very low reactance to high frequency currents acts as a short-circuit for them. G_2 has to be an a.c. ground if it is to act as an effective shield. The bypass capacitor keeps G_2 and cathode at the same a.c. potential. However, G_2 is given large d.c. potential which is only slightly less then V_p.

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As seen from Fig. 27-51 (*b*), screen grid breaks up the control grid-plate capacitance into two capacitances marked C_1 and C_2 which are connected in series. Hence, their combined capacitance is reduced. In fact, its value is reduced to about 0·01 pF. Another advantage of adding the screen grid is that it makes I_p almost independent of V_p thereby increasing the amplification factor to about 800 or so.

27-38. Tetrode Characteristics *(A.M.I.E. May 1974)*

The V_p/I_p characteristic of a triode has been shown in Fig. 27-52. For plotting this curve, screen grid G_2 is given a large but fixed d.c. potential from *B*-battery [Fig 27-51 (*c*)] and control grid G_1 has a fixed bias. Then V_p is gradually increased in steps and corresponding values of I_p noted.

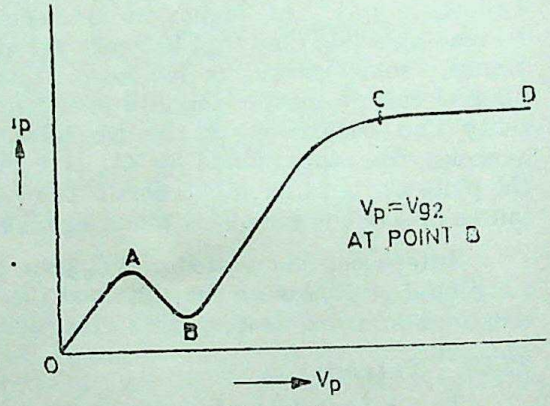

Fig. 27-52

(*i*) It is seen from Fig. 27-52 that starting from zero, as V_p is increased I_p keeps on increasing up to point *A*. It is so because screen grid voltage helps this small V_p to attract more electrons most of which pass straight through the comparatively large spaces between the screen grid wires. Very few of them are retained by G_2 itself.

(*ii*) As V_p is further increased between points *A* and *B*, I_p starts *decreasing*. Over this region, tetrode is said to possess *negative* resistance because, contrary to Ohm's law, current decreases with increase in applied voltage. This has been explained in terms of *secondary emission* from the plate. In the region *AB*, electrons starting from cathode, gather up large speed and hence energy as they approach the plate. As they strike the plate, they dislodge electrons from it. These 'splashed' secondary electrons are attracted towards the screen grid whose fixed d c. potential V_{g2} is *as yet* higher than the plate potential. Consequently, I_p is decreased upto point *B* where V_p becomes equal to V_{g2}.

(*iii*) As V_p is further increased (portion *BC*) and begins to exceed the value of screen-grid voltage, the force of attraction exerted by the plate on the secondary electrons becomes greater than that exerted by screen-grid with the result that the secondary electrons return to it. Consequently, I_p rises sharply. Since now V_p exceeds the value of screen-grid voltage, plate collects all the electrons coming from the cathode.

(*iv*) From point *C* onwards, I_p remains practically constant. Over the region *CD*, V_p hardly affects I_p due to the screening effect of the screen grid G_2.

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27-39. Pentode

(A.M.I.E. Summer 1979)

When one more grid is added to a tetrode, it becomes pentode. This additional grid is known as *suppressor grid*. Its main job is to

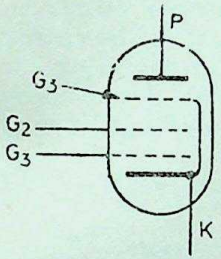

Fig. 27-53

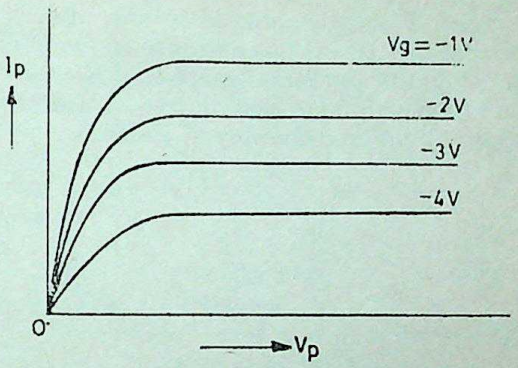

Fig. 27-54

suppress the bad effects of secondary emission which takes place in a tetrode. For this purpose, suppressor grid G_3 is joined (internally) to the cathode so that it is at negative potential with respect to the plate. Hence, it repels the secondary electrons emitted by the plate and drives them back to the plate itself. Thus, while not eliminating secondary emission from the plate. the suppressor grid does eliminate the bad effects of secondary emission, Consequently, I_p increases smoothly with increasing V_p from zero upto maximum value for each control grid voltage V_{g1}.

Another point worth noting is that suppressor grid provides further shielding between the plate and control grid and thus further reduces the grid-to-plate capacitance C_{gp}. This makes I_p even more independent of V_p than in the tetrode as demonstrated by the further flattening of the V_p/I_p characteristics (Fig. 27-54). This results in making the amplification factor of a pentode as high as 1500 as compared to about 800 for tetordes and about 50 for triodes. Consequently, pentode is capable of being used at still higher frequencies.

27-40. Oscillators

It is a device capable of converting a direct current into a high-frequency alternating current. The high frequency electro-magnetic waves produced by this oscillatory current are used for many purposes including radio transmission.

27-41. Oscillations due to the Discharge of a Capacitor

It can be shown that when a charged capacitor is discharged through an inductive coil, it sets up an oscillatory current in the LC circuit whose frequency is given by $f = 1/2\pi\sqrt{LC}$. Fig 27-55 (a) shows a fully-charged capacitor connected across a coil of self-inductance L through a switch S. Now let us see what happens when S is closed.

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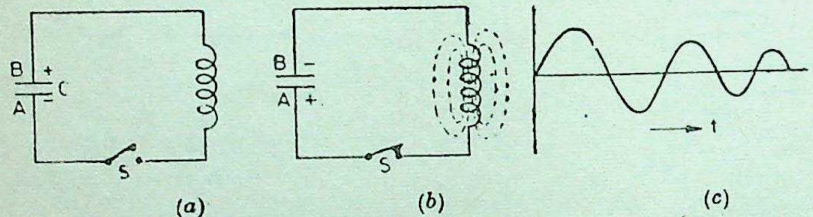
(*i*) The moment *S* is closed, electrons start rushing from plate *A* to plate *B* through the coil.

(*ii*) The rush of electrons is heavy at the beginning but then slows down as more of them go from *A* to *B*.

(*iii*) Finally, electronic current dies down when the excess of electrons on plate *A* has been driven to plate *B* so that both plates are reduced to the same potential. However, due to the self-induction of the coil, more electrons are transferred to *B* than are necessary to fill up its deficiency of electrons.



(*a*) (*b*) (*c*)

Fig. 27-55

(*iv*) It means that now plate *B* has more electrons than plate *A*. Hence, the capacitor becomes charged again although this time in opposite direction as shown in Fig. 27-55 (*b*).

(*v*) After this, the capacitor starts discharging in the opposite direction *i.e.* now electrons move from plate *B* to *A*.

(*vi*) In doing so, these electrons again overshoot and an excess of electrons flows to plate *A*, thereby charging the capacitor once more. By now one cycle of the alternating current has been completed.

(*vii*) This sequence of charging and discharging continues till circuit energy is spent up. This to and fro motion of electrons between the two plates of the capacitor constitutes an oscillatory current.

It is seen that discharge of a capacitor through a coil is oscillatory in nature. This oscillatory nature is due to the self-induction possessed by the coil. Another point worth noting is that during this process of charging and discharging, the electrostatic energy of the capacitor is converted into electromagnetic energy of the coil and *vice versa*.

One serious drawback of these oscillations is that they are damped *i.e.* of diminishing amplitude as shown in Fig. 27-55 (*c*). It is so because there is continuous loss of energy from the circuit due to the following two reasons :

(*i*) some energy is dissipated in the form of heat developed in the resistance of the coil and the connecting wires.

(*ii*) and some energy is lost in the form of electromagnetic waves radiating out from the coil.

Both these losses subtract energy from the circuit with the result that current decreases gradually till it ceases altogether.

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It may, however, be pointed out here that damped oscillations so produced are no good for radio transmission purposes because of their limited range and excessive distortion. For this purpose, we need undamped oscillations *i.e.* oscillations whose amplitude remains constant (Fig. 27-56). These undamped oscillations can be produced by supplping some additional energy in correct phase and in correct direction to the *LC* circuit for making good the I^2R losses continually occuring in the circuit.

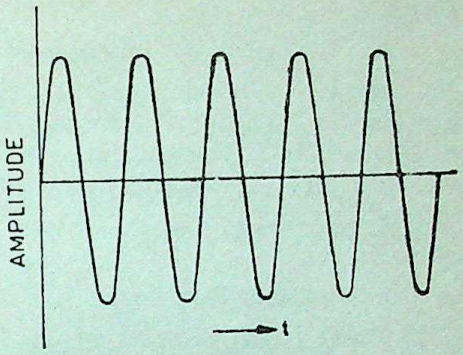

Fig. 27-56

27-42. Essentials of a Feedback Oscillator

(A.M.I.E. Winter 1979)

The essential components of a feedback type oscillator are :—

(*i*) a resonator which consists of an *LC* circuit. It is also known as frequency-determining network (FDN) or tank circuit.

(*ii*) an amplifier whose function is to amplify the oscillations produced by the resonator.

(*iii*) a positive feedback network (PFN) whose function is to transfer part of the output energy to the resonant *LC* circuit in proper phase. The amount of energy fed back is sufficient to meet I^2R losses in the *LC* circuit.

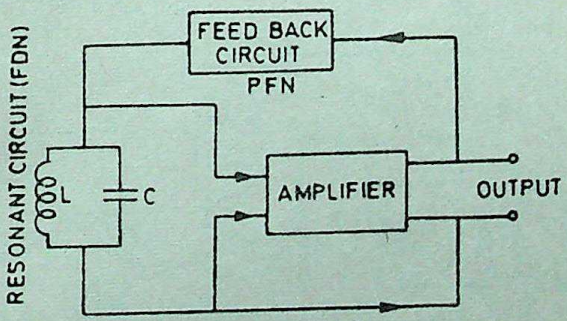

Fig. 27-57

These fundamental components of an oscillator have been shown in the block diagram of Fig. 27-57. The majority of oscillators used in radio receivers and transmitters are of the feedback type. Variations in circuits are due only to the different ways in which this feedback is supplied. A few of the more common type oscillators are as under :

(*i*) Tickler or Tuned-grid Oscillator

Here, the *LC* circuit is placed in the grid-cathode circuit and feedback is accomplished by mutual inductance between the plate coil and the grid coil.

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(*ii*) Tuned-plate Oscillator

Here, the resonant LC circuit is included in the plate-cathode circuit.

(*iii*) Tuned-plate Tuned-grid Oscillator

In this case, there are two LC circuits placed in the plate-cathode and grid-cathode circuits. The feedback between the two circuits is through the interelectrode capacitance C_{gp}.

(*iv*) Hartley Oscillator

It uses part of the coil of the resonant circuit for feedback purposes.

(*v*) Colpitts Oscillator

It uses capacitive part of the resonant circuit to feedback energy from the plate to the grid circuit.

27-43. Tickler Oscillator *(A.M.I.E. Winter 1983)*

Historically, it is one of the earliest electronic oscillator circuits and is shown in Fig. 27-58. The oscillatory tank circuit is made up of L and C placed in the grid-cathode circuit. The feedback circuit consists of a 'tickler' coil L_1 which is magnetically coupled to L. The capacitor C is made variable for adjusting the frequency of oscillations. The combination of C_g and R_g provides self grid bias so that triode works as a class-C amplifier. The capacitor C_1 placed across the B-battery provides a path of low reactance to the a.c. component of plate current.

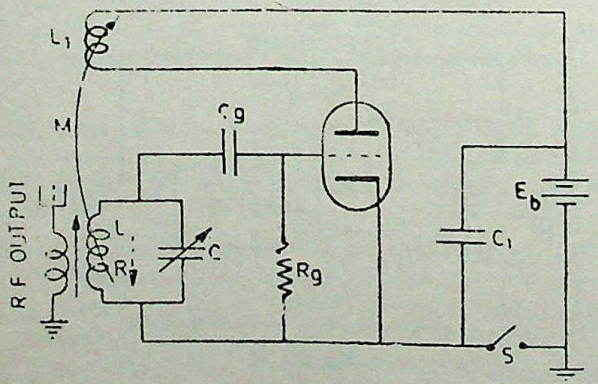

Fig.27-58

Working

When S is closed, plate current is established. This sudden establishment of current through L_1 produces an induced e.m.f. in L by mutual induction. The induced current (shown by solid arrow) charges C and also makes grid more negative which tries to reduce I_p. As I_p is reduced through L_1, it again induces current in L, this time in opposite direction (shown by dotted arrow). The capacitor starts discharging through L and the grid becomes less negative thereby allowing the plate current flowing through L_1 to increase.

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The charging and discharging of C through L sets up oscillations whose frequency is given by $f_0 = 1/2\pi \sqrt{LC}$ Hz. The maintenance equation of the oscillator is $M \geqslant CR/g_m$ where R is resistance of tank coil and g_m is the dynamic transconductance of the triode and M is the mutual inductance between L and L_1. Substituting the value of g_m, the above condition becomes $M \geqslant CRR_p/\mu$. Once oscillations are set up in the LC tank circuit, they appear in amplified form in the plate c ircuit. Part of the plate energy is fed back to the grid circuit by transformer action between L and L_1. This energy is sufficient to overcome losses and sustain oscillations in the resonant LC circuit. The phase of the feedback is correct because the triode amplifier itself produces a phase difference of 180° between grid and plate voltages (Art. 27-30). The combination of L and L_1 (which act as the primary and secondary of a transformer) produces a further phase difference of 180°, thereby making the total phase difference between the plate and grid circuits equal to 360°. Hence, the voltage fed back is in phase with the voltage in the grid circuit and, hence, positive or regenerative feedback takes place. The high-frequency output can be taken out of the tank coil by another coil connected to an antenna as shown.

27-44. Tuned-plate Oscillator

As shown in Fig. 27-59, in this case, resonator circuit has been placed in the plate circuit. As before, the feedback is by mutual inductance between L_1 and L. The value of mutual inductance for maintaining oscillations is the same as given in Art. 27-43. The frequency of oscillation is, as before, given by $f_0 = 1/2\pi \sqrt{LC}$.

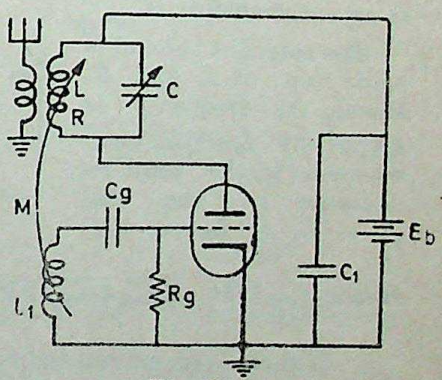

Fig. 27-59

27-45. Tuned-plate Tuned-grid Oscillator

The oscillator uses two tuned tank circuits ; one in the grid circuit and the other in the plate circuit as shown in Fig. 27-60. The two coils L_1 and L_2 are not inductively coupled. The feedback is via the interelectrode capacitance C_{gp}. The grid tank circuit L_1C_1 is tuned to the desired frequency. These oscillations are amplified by the valve and appear in the L_2C_2 circuit. The L_2C_2 circuit is tuned to a fre-

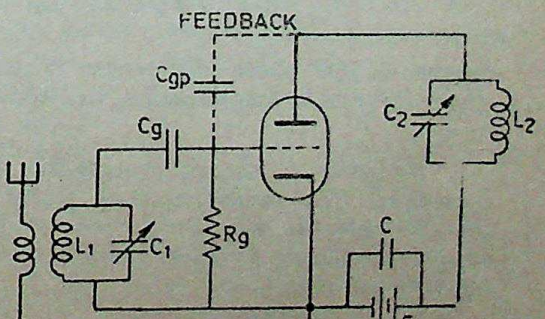

Fig. 27-60

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quency slightly higher than the resonant frequency of L_1C_1 circuit. The reason for this is that if L_2C_2 is tuned exactly to the grid circuit, the phase of the feedback is degenerative because of 180° phase difference inherent in a triode. By this slight detuning, a phase shift is brought about.

27.46. Hartley Oscillator

It is a modified version of tickler feedback circuit discussed in Art. 27-43. The main difference is that instead of using two separate coils, it uses a single tapped coil which is common to both plate and grid circuits. It is sometimes referred to as split tank inductance circuit.

As shown in Fig. 27-61 the lower portion of the tank coil (which is in plate circuit) is inductively coupled to the upper portion L_2 (which is included in the grid circuit), the two coils together functioning as an autotransformer.

The resonant circuit is made up of L and C. Hence, its frequency is given by $f_0 = 1/2\pi\sqrt{LC}$ where L is the total inductance of the coil ($L = L_1 + L_2 + 2M$)

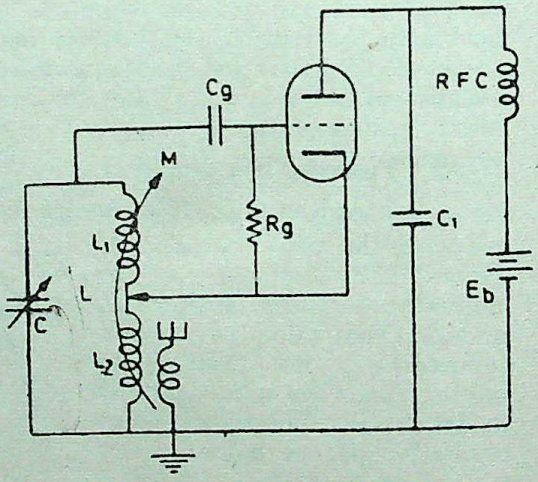

Fig 27-61

Hence, $f_0 = 1/2\pi\sqrt{C(L_1+L_2+2M)}$

If there is no mutual coupling between L_1 and L_2, then

$$f_0 = 1/2\pi\sqrt{C(L_1+L_2)}$$

Because of 180° phase difference due to transformer action and another 180° due to amplification, the feedback is positive.

As before, C_g and R_g provide the necessary grid bias for class-C operation. The radio-frequency choke (RFC) and capacitor C_1 together keep the a.c. component of plate current out of the B-supply, the former by offering high reactance and the latter by offering very low reactance.

Hartley oscillator is particularly suitable for producing low audio-frequency oscillations where good waveform is essential.

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27-47. Colpitts Oscillator

It is also known as split-tank capacitor oscillator. It is similar to Hartley oscillator except that it uses two variable capacitors C_1 and C_2 in the tank circuit instead of the tapped coil (Fig. 27-62). C_2 provides the necessary feedback through C_b from the plate to the grid circuit. The voltage at the ends of the tank circuit are 180° out of phase with each other. It is so because whenever top of C_1 is positive, the plate end of C_2 is negative. By adjusting C_1 and C_2, it is possible to vary the oscillator frequency as well as the amount of feedback necessary to sustain oscillations.

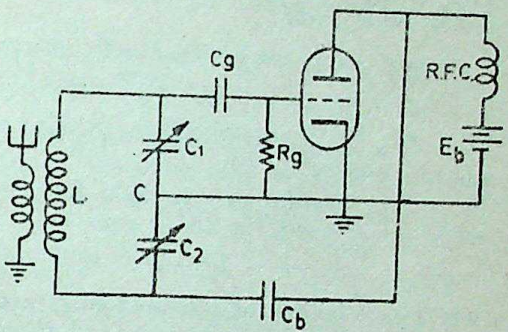

Fig. 27-62

The blocking capacitor C_b prevents the d.c. component of plate current from reaching the tank circuit while the *RFC* keeps radio frequency currents out of the *B*-supply.

The frequency of oscillations is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where C is the total capacitance of C_1 and C_2 connected in series.

Now, $$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\therefore \quad f_0 = \frac{1}{2\pi}\sqrt{\frac{C_1 + C_2}{LC_1C_2}}$$

From the standpoint of stability, Colpitts oscillator has advantage over Hartley oscillator.

27-48. Photo-electric Emission *(A.M.I.E. May 1974)*

Emission of electrons from a metal plate when exposed to radiations of suitable frequency (or wavelength) is called *photoelectric emission*. The ejected electrons are known as photoelectrons. Photoelectric emission can be explained with the help of Planck's Quantum theory. According to this theory, every radiation consists of *photons* which have an energy content of '*hf*' and travel with the velocity of light. Einstein proposed in 1905 that when a photon falls on a metal surface, (Fig. 27-63) it is completely absorb-

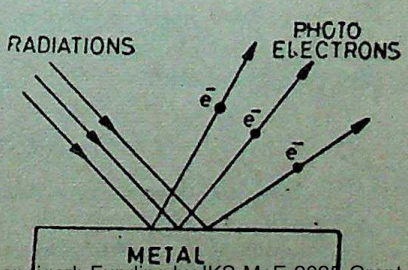

Fig. 27-63

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bed and in the process, gives its energy to a single electron of the metal. This energy is utilized for two purposes :

(*i*) partly for dislodging the electron from its orbit in the atom. This energy is known as photoelectric *work function* and is represented by W_0 and

(*ii*) partly for giving kinetic energy of $\frac{1}{2} mv^2$ to the electron

$$\therefore \quad hf = W_0 + \tfrac{1}{2} mv^2$$

It is known as Einstein's photoelectric equation. In case, the photon energy is just sufficient to liberate the electron only, then no energy would be available for imparting kinetic energy to the electron. Hence, the above equation would reduce to

$$hf_0 = W_0$$

where f_0 is called the *threshold frequency*. It is defined as the *minimum frequency of the incident radiation which can cause photoelectric emission.*

For frequencies lower than f_0, there would be no emission of electrons whereas for frequencies greater than f_0, electrons would be ejected with a certain definite velocity and hence kinetic energy.

HIGHLIGHTS

1. Thermionic emission, as given by Richardson-Dushman equation is

$$J_S = AT^2 e^{-b/T} \text{ A/m}^2$$

2. A diode valve consists of two electrodes—one positive called anode or plate and the other negative called cathode.

3. A diode is mostly used as a rectifier *i.e.* for converting alternating currents into unidirectional currents.

4. The plate current for a diode is given by Child's Three-halves Law

$$I_p = K V_p^{3/2}$$

5. A triode consists of a plate (or anode), cathode and a grid. Its plate current is given by

$$I_p = A(V_p + \mu V_g)^{3/2}$$

6. The plate resistance of a triode is defined as the ratio of change in plate voltage to change in plate current when grid potential is held constant

$$\therefore \quad R_p = \frac{\Delta V_p}{\Delta I_p} \Bigg|_{V_g \text{ constant}}$$

It is also known as dynamic or variational plate resistance or a.c. plate resistance or anode-slope resistance or differential resistance. It is equal to the reciprocal of the slope of the plate characteristic.

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7. Amplification factor of a triode is defined as the ratio of change in plate voltage to the change in grid voltage for keeping

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plate current constant.

$$\therefore \quad \mu = -\left.\frac{\Delta V_p}{\Delta V_g}\right| I_p \text{ constant}$$

8. Transconductance or mutual conductance is defined as the ratio of the change in plate current to the change in grid voltage when plate voltage is held constant

$$g_m = \left.\frac{\Delta I_p}{\Delta V_g}\right| V_p \text{ constant}$$

9. The relation between these three constants is

$$\mu = R_p \times g_m$$

10. The voltage amplification of a triode is given by

$$A = \frac{-\mu}{1+(R_p/R_L)}$$

11. The power output to the load resistance is

$$P_0 = \frac{\mu^2 E_g^2 R_L}{(R_L+R_p)^2}$$

where R_L = load resistance

R_p = plate resistance

E_g = grid input voltage

12. Different stages of a multi-stage amplifier can be coupled by the following three methods :

(*i*) *R-C* coupling (*ii*) impedance coupling and (*iii*) transformer coupling.

13. Essentials of a feedback oscillator are

(*i*) a frequency-determining network (*FDN*)

(*ii*) an amplifier

(*iii*) a positive feedback network (*PFN*)

14. The maintenance equation of a tickler oscillator is

$$M \geqslant CR/g_m \geqslant CRR_p/\mu$$

15. The oscillating frequency of a Hartley oscillator is

$$f_0 = \frac{1}{2\pi\sqrt{C(L_1+L_2+2M)}}$$

$$= \frac{1}{2\pi\sqrt{C(L_1+L_2)}} \quad \text{—neglecting } M$$

16. The oscillating frequency of Colpitts oscillator is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{C_1+C_2}{LC_1C_2}}$$

17. Einstein's photoelectric equation is

$$hf = W_0 + \tfrac{1}{2}mv^2$$

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GAS-FILLED VALVES

28

1. Gas-filled Valves 2. Mechanism of Gaseous Conduction
3. Cold-cathode Gas-filled Diode or Glow tube 4. Thyraton.

28.1 Gas-filled Valves

The thermionic diodes, triodes and tetrodes etc. used so far were almost completely evacuated with the result that their action took place in vacuum. However, there is a specialised group of valves in which the glass envelope after evacuation is purposely filled with a specified gas like nitrogen, argon, helium, neon or mercury vapours at a comparatively low pressure (ranging from 10 mm of Hg to 50 mm). Such valves are known as gas-filled valves. Their operation depends on the effects of electron-atom collisions. Usually, a gas-filled valve is called a 'soft' valve whereas an evacuated one is known as 'hard' valve.

The construction of gas-filled valves is almost similar to that of high-vacuum valves except that their cathodes, grids and anodes are usually larger in order to carry heavier currents.

Gas-filled valves are classified into two groups depending on the type of electron emission employed :

(*a*) *Cold-cathode type*—in which electron emission is obtained from a cold cathode.

(*i*) a two-element cold-cathode valve is called a *glow tube* because its action depends on the glow discharge through the gas.

(*ii*) a three-element cold-cathode valve is known as *grid-glow tube.*

(*b*) *Hot cathode* (*or thermionic*) *type*—in which electron emission is obtained from oxide-coated heated cathode just as in the conventional thermionic vacuum valves.

(*i*) a two-element hot-cathode valve is known as *phanotron* and its action depends on the arc discharge through the gas.

(*ii*) a three-element hot-cathode valve is known as *thyratron* and its operation also depends on the arc discharge.

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The schematic symbols for the different types of gas-filled valves are shown in Fig. 28-1. A small dot put anywhere in the envelope indicates that the valve is gas-filled.

Cold-cathode diodes (also called glow tubes) are used as

(*i*) indicating and testing lamps and

(*ii*) as voltage regulators (for which role they are called VR tubes).

These tubes are available in various sizes ranging from 1/25 to 3 W capacity.

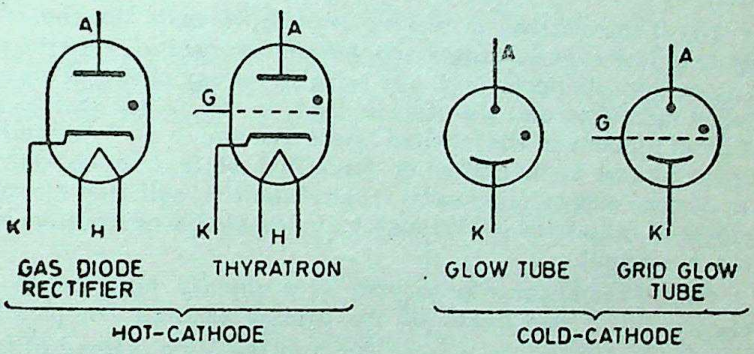

Fig. 28-1

Low-pressure hot-cathode diodes are usually designed for use as rectifiers in preference to high vacuum diodes because they have low tube drop and tube loss.

High-pressure hot-cathode diodes (with gas pressure of 50 mm of Hg) are known as Tunger or rectigon tubes and are extensively used in chargers for storage batteries.

Electrically speaking, a hot cathode triode or thyratron acts as a switch because of its trigger characteristics. Hence, it is put to diverse uses such as :

(*i*) for arcless switching and relay applications as in the control of d.c. motors and d.c. power regulating systems.

(*ii*) in relaxation type sweep-circuit generators or saw-tooth oscillators for TV and radar applications.

(*iii*) in controlled rectifiers used for electric welding and lighting control installations in theatres etc.

28-2 Mechanism of Gaseous Conduction

Consider the case of a gas-filled diode either of the cold-cathode or the hot-cathode type. In the former type of diode, there are no emitted electrons present in the gas although some electrons are usually produced by cosmic rays or by radiations from a radioactive material lying in the neighbourhood. In the hot-cathode type of diode, plenty of electrons are emitted by the cathode.

When a potential difference is applied between the anode and cathode of such a diode, an electric field is set up between the two.

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Under the influence of this field, the free electrons start moving towards the anode with increasing speed and hence increasing kinetic energy. During their flight, the electrons collide with the gas atoms (or molecules). Now, when an electron bumps into a gas atom, three things can happen depending on the energy of the electron at the time of its impact :

(*i*) if the electron is moving *slowly*, it will produce an elastic collision with conservation of total momentum. Since the electron is much lighter than the gas atom, it simply bounces off without producing any change in the gas atom except a slight change in its motion.

(*ii*) if the electron is moving *rapidly*, it may use some or all of its kinetic energy in raising the outermost orbital electron of the gas atom from its normal orbit to a higher-energy orbit. This action is called *excitation* and the atom is said to be in an excited state. The atom remains in the excited state for about 10^{-8} second after which it returns to its normal or unexcited state. In so doing, it gives out the energy previously taken from the colliding electron in the form of radiations (which may be in the visible or invisible region of the spectrum).

(*iii*) if the electron is moving *very rapidly* (and hence more energetically), it may knock off the orbital electron completely out of the atom instead of merely raising it to a higher energy orbit. In that case, the gas atom becomes a positive ion (because of having lost one electron) and the process is known as *ionisation by collision*. As seen, ionisation of a gas atom produces one positive ion and one free electron. Altogether, now there are three charged particles (one positive ion, one released electron and the original colliding electron) which are free to move in the electric field—positive ion moving towards the cathode and the electrons towards the anode. If the two electrons repeat the ionization process outlined above, the number of charged particles will multiply rapidly and the valve or diode will start conducting.

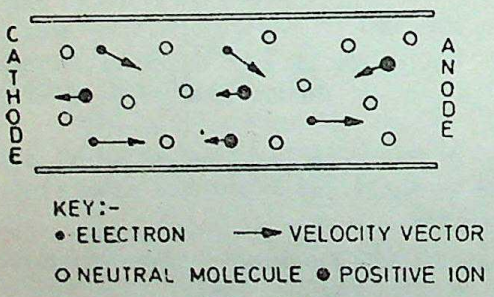

Fig. 28-2

Fig. 28-2 shows the condition which exists in a gas-filled tube after the ionization has started up. It is seen that electrons have much higher velocities than positive ions and that their general direction of movement is towards anode whereas the positive ions drift towards the cathode. The neutral gas molecules have random motion.

28.3 Cold-cathode Gas-filled Diode or Glow Tube

(A.M.I.E., Winter 1986)

It is a device which uses the self-maintained glow discharge between two cold electrodes in a gas at reduced pressure.

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(*a*) **Construction.** Fig. 28-3 shows the construction of one such simple tube used commercially. It consists of a central anode wire coaxial with a surrounding cathode. Both the anode wire and cathode are made of nickel, with the inner surface of the cathode being oxide-coated. The gases commonly used in such tubes are neon, argon and helium.

There is a 'starting probe' which consists of a metal piece attached to the cathode and directed towards and near to the anode. Its purpose is to reduce the breaking or firing potential of the tube.

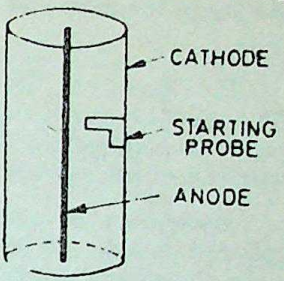
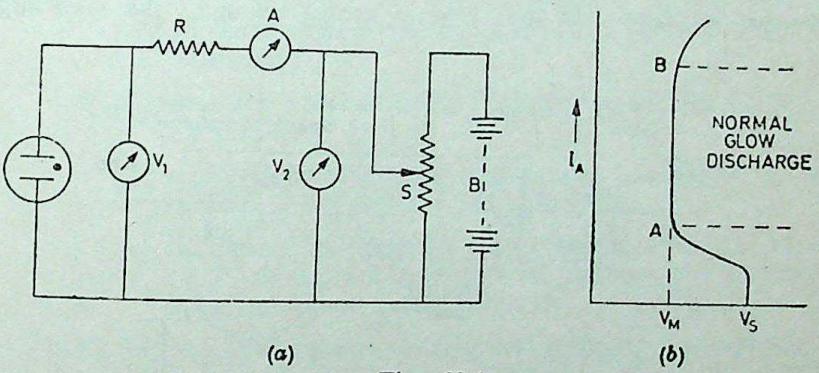


Fig. 28-3

(*b*) **Characteristic.** The volt-ampere (V_A/I_A) characteristic of such a tube may be found with the help of the circuit diagram shown in Fig. 28-4 (*a*).



(*a*) (*b*)

Fig. 28-4

As seen, the glow tube is connected to a high-voltage battery B through a current limiting resistance R and an indicating ammeter A. The voltmeter V_1 reads anode voltage V_A whereas V_2 measures the supply voltage applied across both R and the tube.

When anode voltage V_A is gradually increased from zero with the help of potentiometer slider S, no measurable current is found to flow through the tube at first because A reads zero. However, soon, a certain value of V_A is reached when there is a sudden flow of current. This value of the anode voltage is known as *breakdown* or *sparking voltage* V_S* (and is also known as striking or ionizing potential). After the breakdown occurs, the voltage across the tube (*i.e.* reading of V_1) falls to a lower value V_M and a glow appears in the tube. This voltage V_M is known as *maintenance* voltage or sometimes the *operating* or *burning* voltage. Like V_S, its value also depends on the type of gas used, its pressure and the

*Its value depends on many factors such as gas pressure and geometrical arrangement of electrodes. The function of the starting probe is to reduce the breakdown voltage. Sparking takes place first to the starting probe and then spreads to the rest of the cathode.

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nature of the electrodes. It usually varies from 80 V to several hundred volts for different diodes. The colour of the luminous glow depends on the nature of the gas, being pink for neon and blue for argon.

It is seen from Fig. 28-4 (*b*) that while working over the normal glow region *i.e.* over the portion AB of the V_A/I_A characteristic, the voltage drop across the tube remains practically constant and is independent of the current flowing through the tube.

Mechanism of operation of such a tube is exactly like that explained in Art. 28-2. The initial start of the operation depends on the presence of a free electron within the tube, such an electron may be released by ionization due to collision between a gas molecule and cosmic rays or by radiations from a neighbouring radioactive material or photoelectric emission. With the application of a potential difference between the anode and cathode of the tube, the electron drifts towards the anode and eventually ionizes the gas molecules by collision. As a consequence, the number of charged particles increases to three. If the field is strong enough, the resulting

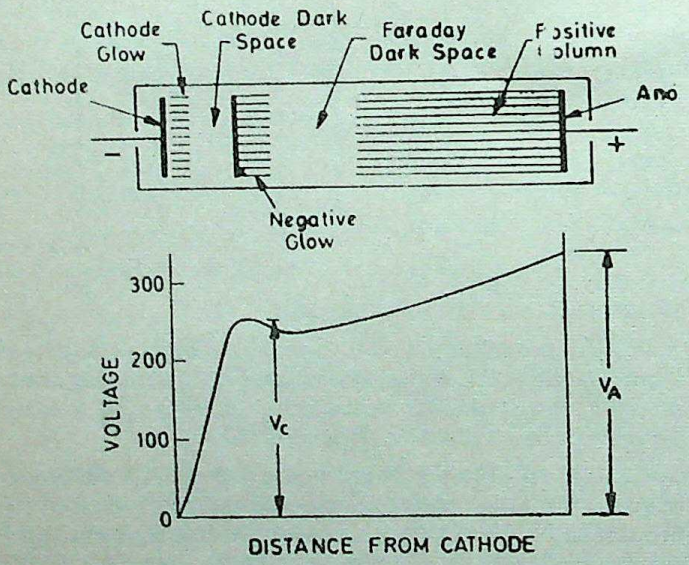

Fig. 28-5

cummulative ionization may continue till breakdown occurs. Once breakdown occurs, the potential distribution within the tube is completely modified and most of the region of the discharge (extending from negative glow to positive column) becomes virtually equipotential or force-free because it contains as many positive charges as the negative ones. As seen from Fig. 28-5, most of the potential drop occurs in the very narrow region near the cathode. Normal values of cathode-fall potential V_c range from 60 V(a potassium surface and helium gas) to about 350 V.

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(*c*) **Uses**

1. The principal use of glow tubes is as voltage regulators (or stabilisers) *i.e.* as VR tubes. For example, the voltage across the load R_L in the circuit of Fig. 28-6 will remain constant at 150 V over a range of current from 5 to 50 mA if a VR-150 glow tube is used. The difference between the supply voltage and the tube drop appears across the resistance *R*.

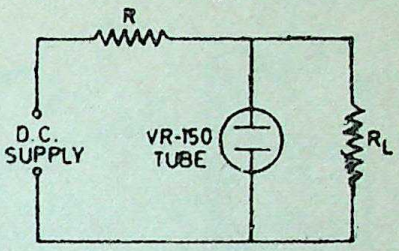

Fig. 28-6

2. as trigger relays
3. as rectifiers at currents lower than those associated with hot-cathode discharge tubes (usually for currents ranging from 1 μA to 100 mA.)
4. smaller glow tubes are used as test lamps
   - (*i*) to check circuit continuity
   - (*ii*) to check voltage whether it is 110 V or 220 V etc. by brilliancy.
   - (*iii*) to check whether supply frequency is 50 Hz or higher by flicker.
   - (*iv*) to check polarity of a d.c. source because it is the cathode which gets covered with glow.
   - (*v*) to detect the presence of a strong radio-frequency (*R.F.*) field which is capable of ionizing the gas without direct connection to the tube.
   - (*vi*) as pilot lamps to indicate that a circuit is energised.
   - (*vii*) as dim-light sources for exit markers, for fire-station markers and for location markers in general.

(*A.M.I.E. Winter 1984*)

28-4 Thyratron

It is a low-voltage non-self-maintainig arc tube having three electrodes *i.e.* anode, an indirectly-heated cathode and a grid and is filled with an inert gas like neon and argon etc. or mercury vapours at low pressure.

(*a*) **Construction.** The electrode structure of one type of a gas tube or thyratron* is as shown in Fig. 28-7. It consists of an anode and cathode which are approximately planar. The grid is a cylinder which *surrounds* both the anode and the cathode, a baffle or a series of baffles having small holes being inserted between the anode and the cathode. Obviously, the grid acts as a complete electrostatic shield between anode and cathode except for the holes in the grid baffles. Hence, even a small negative grid potential is sufficient to neutralize the effect of a large anode potential and

*Which in Greek means a 'door'. Originally, it was the trade name of G.E.C. (U.S.A.).

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thereby prevent the arc being initiated. However, once the arc has been struck between the anode and the cathode, the grid has no

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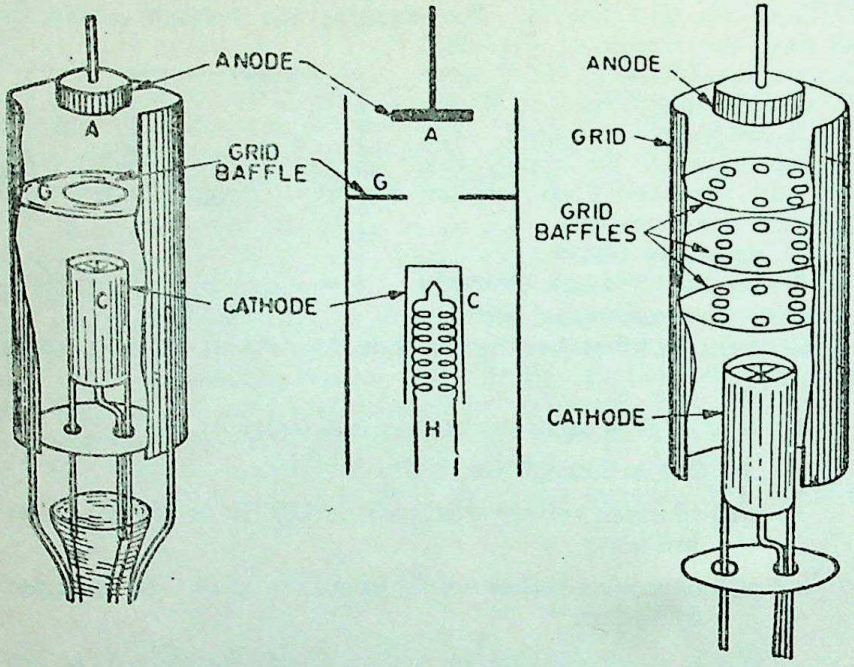

Fig. 28-7

control over the anode current. In other words, grid is used only to *start* conduction of anode current by ionization; it cannot be employed to control anode current or to stop it.

(*b*) **Working.** When grid voltage is sufficiently negative, the electrons emitted from the cathode are prohibited by the grid to go to the anode even when it has large positive potential. As grid potential is made less negative, an occasional electron from the cathode passes through the hole in the grid baffle but still does not have enough energy to produce ionization. As the negative grid voltage is gradually reduced further, the electrons acquire more speed and hence energy and a point called *critical grid voltage* is reached, when ionization occurs and the anode current changes rapidly from zero to a large value. At the same time, an arc discharge appears inside the valve and the anode-to-cathode voltage falls to about 15 to 20 V.

Once 'firing' has taken place, grid has no control over anode current (because during conduction, its negative charge is neutralized by the positive ions that collect on it). Obviously, once fired, the thyratron behaves like a closed switch. To stop anode current and to re-establish grid control, it is necessary to reduce the anode voltage below the maintaining voltage or to open the anode circuit. When discharge ceases, de-ionization is generally complete in about 0·1 to 1 millisecond.

(*c*) **Grid-control Characteristic.** It is the curve which shows relation between the anode voltage and the critical grid voltage at

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which the valve strikes. It is also known as *starting characteristic* or *breakdown characteristic* and may be found with the help of the circuit shown in Fig. 28-8.

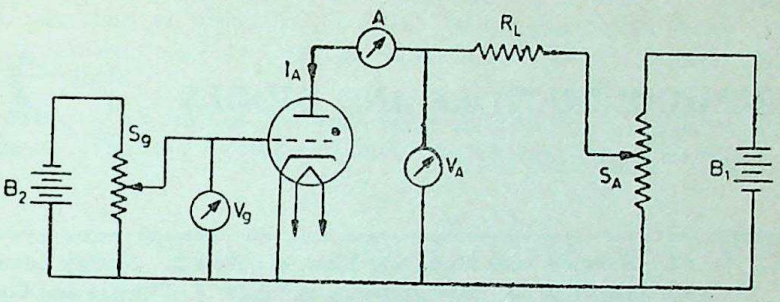

Fig. 28-8

The grid can be given any negative voltage with the help of the slider S_g and its value read on the voltmeter V_g. Similarly, anode voltage can be varied with the help of S_A and its value read on V_A. Ammeter A reads anode current I_A.

Suppose we adjust $V_A = 100$ V when grid is at -10 V. The anode current is found to be zero because this grid bias does not permit the emitted electrons to go to the anode. When grid voltage is gradually reduced to about -2 V, it is found that all of a sudden I_A achieves large value due to ionization by the high-velocity electrons. The value of this critical grid voltage is noted.

If the test is repeated for higher anode voltages, somewhat larger critical grid voltages would be obtained. Fig. 28-9 shows a typical grid-control characteristic of a thyratron. The slope of this characteristic gives (what is known as) control ratio which is defined as $= \triangle V_A / \triangle V_g$.

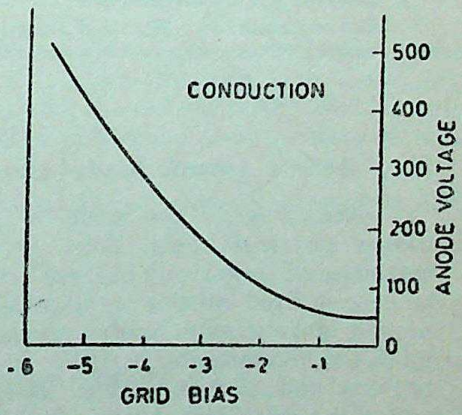

Fig. 28-9

(*d*) Uses

(*i*) Thyratrons are used for arcless switching and relay applications as in the control of d.c. motors etc.

(*ii*) The small argon-filled thyratron is generally used in saw-tooth sweep generators for TV and radar equipment.

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SEMICONDUCTORS AND DIODES 29

1. Bohr's Atomic Model and Electron Orbits 2. Energy Levels in a Single Atom 3. Energy Bands in Solids 4. Valence and Conduction Bands 5. Conductors, Semiconductors and Insulators 6. Atomic Binding in Semiconductors 7. Intrinsic Semiconductors 8. Extrinsic Semiconductors 9. Majority and Minority Charge Carriers 10. Mobile Charge Carriers and Immobile Ions 11. Zener Diode 12. P-N Junction 13. Formation of Depletion Layer 14. Junction or Barrier Voltage 15. Forward-biased P-N Junction 16. Reverse-biased Junction 17. Combined Forward and Reverse V/I Characteristics 18. Junction Breakdown 19. Junction Capacitance. 20. P-N Junction Diode. 21. Highlights.

29-1 Bohr's Atomic Model and Electron Orbits

According to the model of atom proposed by Nobel prize-winner physicist Niels Bohr, an atom is composed of negatively-charged electrons moving in fixed circular or elliptical orbits around a relatively heavy nucleus made up of two fundamental particles known as protons and neutrons (Fig. 29-1). The proton is a relatively heavy particle (1840 times heavier than an electron) while the neutron has no charge *i.e.* it is electrically neutral. The positive charge of the proton is equal in magnitude to the negative charge of the electron and has a value of $1{\cdot}6 \times 10^{-19}$ coulomb. The diameter of an atom is about 10^{-10} m and that of the nucleus is approximately 10^{-14} m*. By applying Quantum Theory, Bohr was able to determine the radii of the electron orbits as well as the energy associated with them.

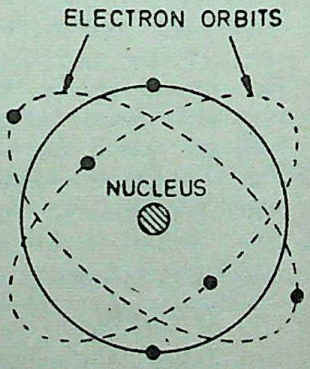

Fig. 29-1

*The relative distance of the electron from the nucleus is about the same as the relative distance of the earth from Pluto.

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The electron orbit nearest to the nucleus is designated by the letter K and is given the principal quantum number $n=1$. The succeeding orbits are referred to alphabetically as L, M, N,.....etc. and have principal quantum numbers of $n=2$, 3, 4 etc. The maximum number of electrons that can be accommodated in any orbit $=2n^2$ where n is the principal quantum number of the electron orbit or shell.

Fig. 29-2 shows the schematic distribution of electrons in different orbits for hydrogen, boron, phosphorus and germanium.

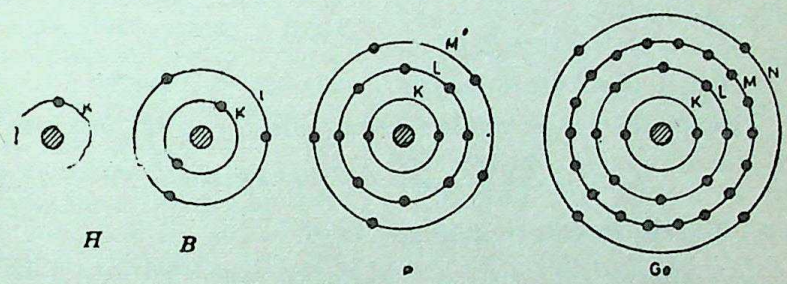

Fig. 29-2

The outermost electrons *i.e.* those farthest from the nucleus are known as *valence* electrons. These electrons have the highest energy* and may fully or partially fill the valence orbit. For example, boron has 3 valence electrons whereas it could accommodate 8 electrons in L shell.

The radii of the various permitted electron orbits (assumed circular) are given by

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$$

where n = principal quantum number

h = Planck's constant $= 6{\cdot}62 \times 10^{-34}$ J-s

m = electron mass $= 9{\cdot}1 \times 10^{-31}$ kg

e = electron charge $= 1{\cdot}6 \times 10^{-19}$ C

ϵ_0 = absolute permittivity of free space

$= 8{\cdot}854 \times 10^{-12}$ F/m

Z = atomic number

Substituting values of different constants, we get

$$r_n = 5{\cdot}3 \times 10^{-11}\, n^2/Z \text{ metre} \qquad \ldots(i)$$

When an electron moves in any permitted orbit, it has a definite energy which is the sum of

*Algebraically but not in magnitude.

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(*i*) kinetic energy due to motion and

(*ii*) potential energy due to the attraction of the nucleus.

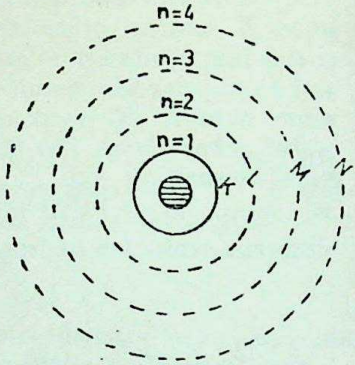

Fig. 29-3

Total energy

$$E_n = -\frac{m\,e^4\,Z^2}{8\,\epsilon_0^2\,n^2\,h^2} \text{ joule}^*$$

Again, substituting different values, we have

$E_n = -21{\cdot}76 \times 10^{-19}\, Z^2/n^2$ joule

Now, 1 eV $= 1{\cdot}6 \times 10^{-19}$ joule

Hence, when expressed in electron-volts, the energy becomes

$$E_n = \left(\frac{-21{\cdot}76 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}}\right)\frac{Z^2}{n^2} = -13{\cdot}6\,\frac{Z^2}{n^2}\text{ eV} \qquad \ldots(ii)$$

29-2. Energy Levels in a Single Atom

Let us find out the value of energy associated with different electron orbits for a *single isolated* hydrogen atom which has one

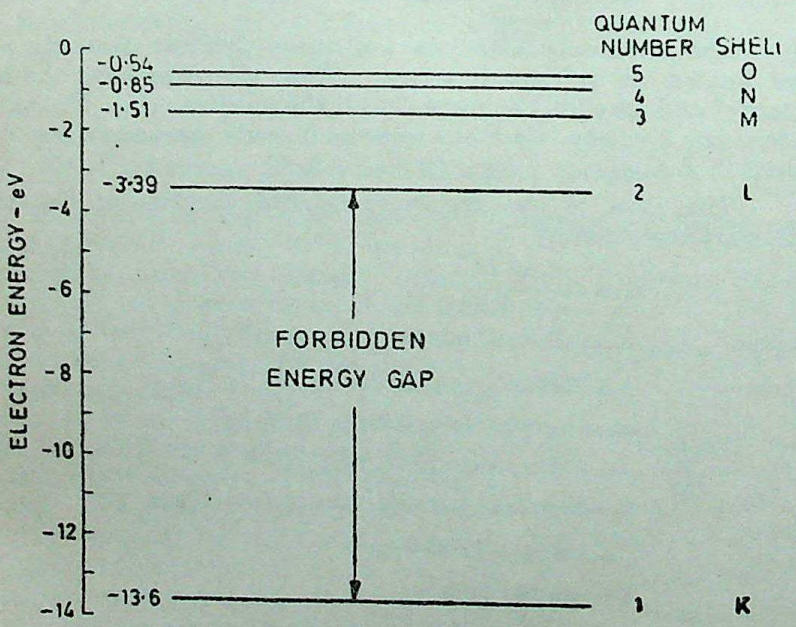

Fig. 29-4

proton in the nucleus surrounded by one orbital electron. Obviously, for hydrogen $Z = 1$. Substituting this value in Eq. (*ii*) of Art. 29-1, we have,

*The negative sign appears because the energy of the electron when at infinity is taken as zero.

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$E_n = -13{\cdot}6/n^2$ eV.

Energies of various electron orbits are :

$$E_1 = -13{\cdot}6 \text{ eV}$$
$$E_2 = -13{\cdot}6/2^2 = -3{\cdot}39 \text{ eV}$$
$$E_3 = -13{\cdot}6/3^2 = -1{\cdot}51 \text{ eV}$$
$$E_4 = -13{\cdot}6/4^2 = -0{\cdot}85 \text{ eV}$$

Different permitted (or possible) energy levels are shown in Fig. 29-4. Each level is represented by a horizontal line of the same length although the length of the lines has no significance. Moreover, there is no indication as to the actual occupancy of these levels. These levels are separated by forbidden energy gaps. Which orbit an electron occupies at a given time depends on the total energy possessed by it. At 0°K or 273°C, the electron in a hydrogen atom occupies the first orbit or first quantum level in the atom (which is associated with the least energy). If the electron acquires correct amount of additional energy from an external source (as by heating or by impact with a fast-moving particle), it can be excited to a higher energy level or state. For hydrogen atom, an extra amount of energy $=(-13{\cdot}6) -(-3{\cdot}4) = -10{\cdot}2$ eV is needed to excite the electron from K shell to the L shell. In that case, the atom is said to be *excited*. Similarly, minimum energy of $(-13{\cdot}6)-(-1{\cdot}51) = -12\ 5$ eV is required to raise

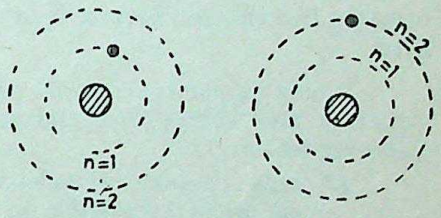

Fig. 29-5

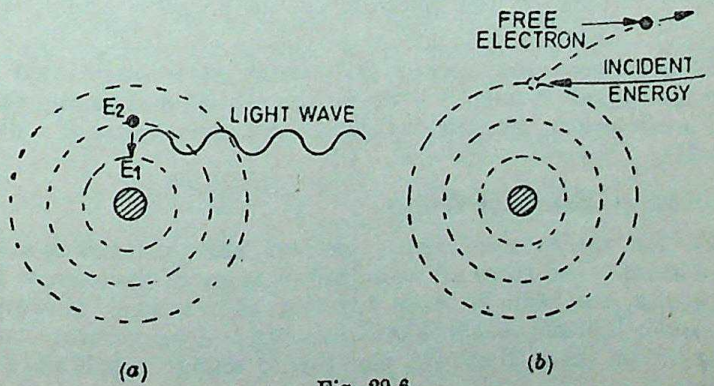

Fig. 29-6

the electron to M shell. However, the electron does not remain in the excited state or level longer than about 10^{-8} second* and eventually, returns to the ground state (or unexcited or stable state) in one or more 'jumps'. During each jump, a photon of energy or

*Some atoms like those of mercury have excitation levels which are occupied for quite long time of 10^{-2} second. Such atoms are said to be in a metastable state.

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light waves are emitted (or radiated) in accordance with the relation [Fig. 29-6 (*a*)].

$$E_2 - E_1 = hf$$

where f is the frequency of the light wave emitted. It is worth noting that when revolving in its normal orbit, the electron does not radiate out any energy.

When an electron is removed altogether [Fig. 29-6 (*b*)] the atom is said to be ionized and is left with a positive charge. The energy required to just ionize a normal atom is called *ionization* energy. For hydrogen atom, its value is 13·6 eV and hydrogen is said to have an ionization potential of 13·6 V. The electron thus freed from the atom becomes free electron because it can drift freely away from its parent atom.

It may be pointed out in passing that the account of atomic structure and energy levels given above is oversimplified for the following reasons :

(*i*) only circular orbits have been considered although elliptical orbits are possible for all values of n except for $n = 1$.

(*ii*) in addition to the principal quantum number n, there are three other quantum numbers which slightly affect the electron energy. These are :

(*a*) subsidiary quantum number which is a measure of the eccentricity of the electron orbits.

(*b*) magnetic quantum number which is associated with the rotation of the electron about the nucleus.

(*c*) spin quantum number which is associated with the spin of the electron *i.e.* its rotation about its own axis.

It so happens that there are $2n^2$ energy states associated with a particular quantum number n which are distributed among various energy levels—each grouping of states for a given value of n defines the 'shell'.

29-3. Energy Bands in Solids

So far we have considered electron energy levels in a *single isolated* atom. But there are significant changes in the energy levels when atoms are brought close together as in solids. However, in gases, under normal conditions of pressure and temperature, atomic spacing is so great that the changes in energy levels are almost negligible.

As is well known, a solid consists of an ordered array of many closely-packed atoms which is usually referred to as crystal lattice. Since atoms are brought very close to each other, there is some intermingling of electrons from adjacent atoms. The effect of intermixing is greatest with the outermost or valence electrons which have the *highest* energy. This fact results in the modification of energy levels in a solid. The number of permissible orbits is vastly increased, which, in other words, means that the number of permitted energy

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levels is increased. Consequently, the single energy level of *isolated* atom becomes a *band* of energies in a solid (Fig. 29-7) The indivi-

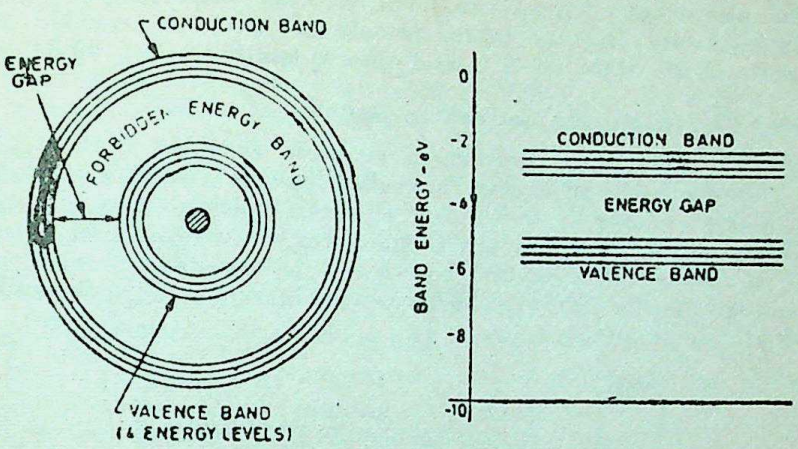

Fig. 29-7

dual energies within the band are so close together that the energy band may be considered to be continuous for many purposes.

For example, in an assembly of N electrons, the number of possible energy states is N. Since only two electrons of opposite spin can occupy the same state*, maximum number of electrons which these N states can accommodate is $2N$.

Consider the case of lithium metal—the simplest atom which forms a solid at ordinary temperature. Its atom has three electrons, two of which have the same energy and the third one has higher value of energy. In an isolated single atom, two electrons move round the electron orbit with $n=1$ whereas the third occupies the

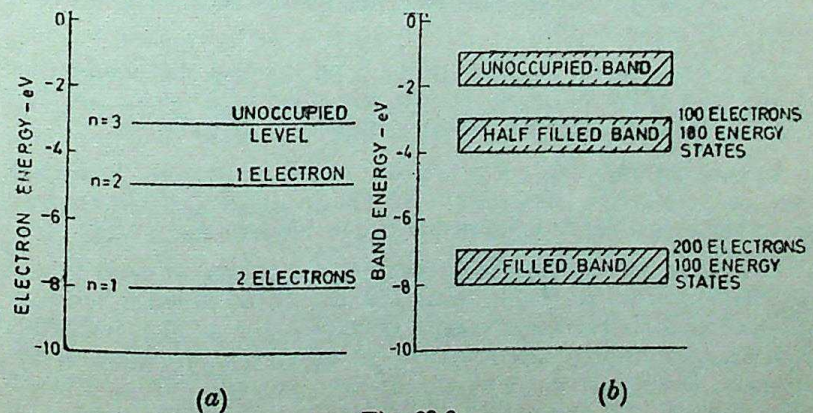

Fig. 29-8

orbit with $n=2$ as shown in Fig. 29-8 (*a*). Now, consider a piece of lithium metal containing 100 atoms. It will be found that the

*According to Pauli's Exclusion Principle which states that no two electrons may have the same four quantum numbers described in Art. 29-2.

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lower *level* (with $n=1$) forms a *band* of 200 electrons occupying 100 different energy states. The higher level (with $n=2$) forms a wider band also of 100 energy states which could, as before, accommodate 200 electrons. But as there are only 100 electrons available (one from each atom) this energy band remains half-filled [Fig. 29-8 (*b*)].

29-4. Valence and Conduction Bands

The outermost electrons of an atom *i.e.* those in the shell farthermost from the nucleus are called *valence* electrons and have the *highest* energy.* It is these electrons which are most affected when a number of atoms are brought very close together as during the formation of a solid. The states of lower-energy electrons orbiting in shells nearer to the nuclues are little, if at all, affected by this atomic proximity.

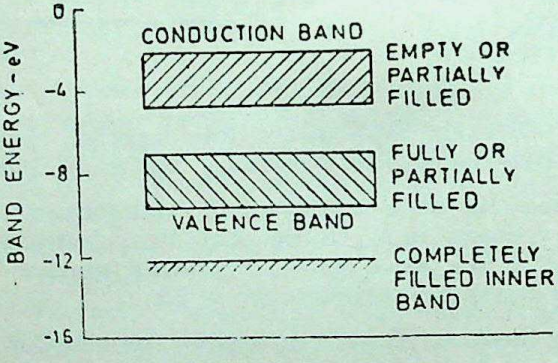

Fig. 29-9

The band of energy occupied by the valence electrons is called the *valence band* and is, obviously, the highest occupied band. It may be completely filled or partially filled with electrons, but can, obviously, be never empty.

The next higher permitted energy band is called the *conduction* band and may either be empty or partially filled with electrons. In fact, it may be defined as the lowest unfilled energy band.

In conduction band, electrons can move freely and hence are known as *conduction* electrons. As shown in Fig. 29-7, the gap between these two bands is known as the forbidden energy gap.

It may be noted that the covalent forces of the crystal lattice have their source in the valence band. If a valence electron happens to absorb enough energy, it jumps across the forbidden energy gap and enters the conduction band. An electron in the conduction band can jump to an adjacent conduction band more readily than it can jump back to the valence band from where it had come earlier. However, if a conduction electron happens to radiate too much energy, it will suddenly reapear in the valence band once again.

*Algebraically, not numerically.

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When an electron is ejected from the valence band, a covalent bond is broken and a positively-charged hole is left behind. This hole can travel to an adjacent atom by acquiring an electron from that atom which involves breaking an existing covalent bond and then re-establishing a covalent bond by filling up the hole. It is to be noted carefully that holes are filled by electrons which move from adjacent atoms without passing through the forbidden energy gap. It is simply another way of saying that conditions in the conduction band have nothing to do with the hole flow. It points to a very important distinction between the hole current and electron current—although holes flow with ease, they experience more opposition than electron flow in the conduction band.

To summarize the above, it may be repeated that :

1. conduction electrons are found in and freely flow in the conduction band.
2. holes exist in and flow in the valence band
3. conduction electrons move almost twice as fast as the holes.

29-5. Conductors, Semiconductors and Insulators

The electrical conduction properties of different elements and compounds can be explained in terms of the electrons having energies in the valence and conduction bands. The electrons lying in the lower energy bands, which are normally filled, play no part in the conduction process.

(*i*) **Insulators**. Stated simply, insulators are those materials in which valence electrons are bound very tightly to their parent atoms thus requiring very large electric field to remove them from the attraction of the nuclei.

In terms of energy bands, it means that insulators

(*a*) have a full valence band

(*b*) have an empty conduction band and

(*c*) have a large energy gap (of several eV) between them.

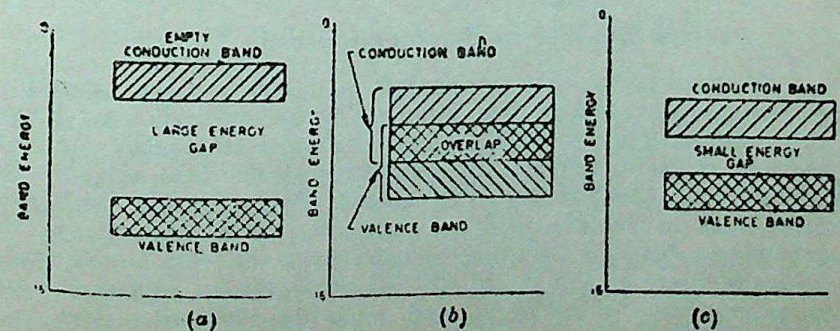

Fig. 29-10

This is shown in Fig. 29-10 (*a*). For conduction to take place, electrons must be given sufficient energy to jump from the valence

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band to the conduction band. Increase in temperature enables some electrons to go to the conduction band which fact accounts for the negative resistance temperature coefficient of insulators.

(*ii*) **Conductors.** Conducting materials are those in which plenty of free electrons are available for electric conduction.

In terms of energy bands, it means that electrical conductors are those which have overlapping valence and conduction bands as shown in Fig. 29-10 (*b*).

In fact, there is no physical distinction between the two bands. Hence, the availability of a large number of conduction electrons.

Another point worth noting is that in the absence of forbidden energy band in good conductors, there is no structure to establish holes. The total current in such conductors is simply a flow of electrons. It is exactly for this reason that the existence of holes was not discovered until semi-conductors were studied thoroughly.

(*iii*) **Semiconductors.** A semiconductor material is one whose electrical properties lie in between those of insulators and good conductors. Examples are germanium and silicon.

In terms of energy bands, semiconductors can be defined as those materials which have almost an empty conduction band and almost filled valence band with a very narrow energy gap (of the order of 1 eV) separating the two.

At 0°K, there are no electrons in the conduction band and the valence band is completely filled. However, with increase in temperature, width of the forbidden energy band is decreased so that some of the electrons are liberated into the conduction band. In other words, conductivity of semi-conductors increases with temperature.

Semiconductors are of two types :—

(*i*) pure or intrinsic semiconductors and

(*ii*) impure or extrinsic semiconductors.

These are further discussed in Art. 29-7 and 29-8.

29-6 Atomic Binding in Semiconductors

Semiconductors like germanium and silicon have crystalline structure. Their atoms are arranged in an ordered array known as crystal lattice. Both these materials are tetravalent *i.e.* each has four valence electrons in its outermost shell.* The neighbouring atoms form covalent bonds by sharing four electrons with each other so as to achieve inert gas structure (*i.e.* 8 electrons in the outermost orbit). A two-dimensional view of the germanium crystal lattice is shown in Fig. 29-11 (*b*) in which circles represent atom cores consisting of the nuclei and inner 28 electrons. Each pair of lines represents a covalent bond. The dots represent the valence

*A single germanium atom has 32 electrons out of which only four electrons take part in the electrical properties of germanium, the remaining 28 electrons being tightly bound to the nucleus. The four electrons revolve in the outermost shell and are called valence electrons.

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electrons. It is seen that each atom has 8 electrons under its influence.

A 3-dimensional view of germanium crystal lattice is shown in Fig. 29-11 (*a*) where each atom is surrounded symmetrically by four

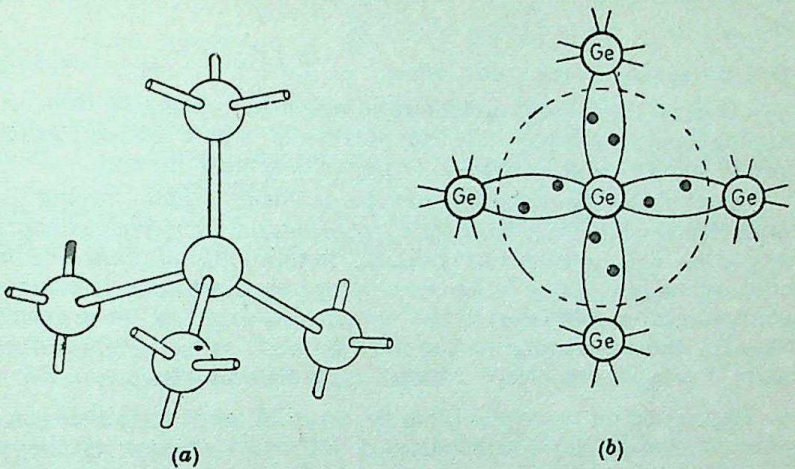

Fig. 29-11

other atoms forming a tetrahedral crystal. Each atom shares valence electrons with each of its four neighbours thereby forming a stable structure.

In the case of pure (intrinsic) germanium, the covalent bonds have to be broken to provide electrons for conduction.

29-7 Intrinsic Semiconductors *(A.M.I.E. Winter 1981)*

An intrinsic semiconductor is one which is made of the semiconductor material in its extremely pure form.

Common examples of such semiconductors are pure germanium and silicon which have forbidden energy gap of 0·72 eV and 1·1 eV respectively. The energy gap is so small that even at ordinary room temperature, there are many electrons which possess sufficient energy to jump across the small energy gap from the valence to the conduction band. However, it is worth noting that for each electron liberated into conduction band, a positively-charged hole is created in the valence band.

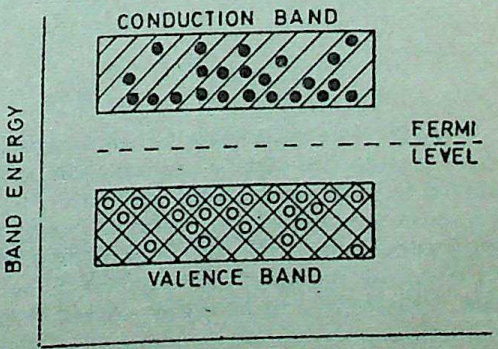

Fig. 29-12

When an electric field is applied to an intrinsic semiconductor at a temperature greater than 0°K, conduction electrons move to the anode and the holes in the valence band move to the cathode. Hence semiconductor current consists of movement of electrons and holes in opposite directions.

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Alternatively, an intrinsic semiconductor may be defined as one in which the number of conduction electrons is equal to the number of holes.

Schematic energy band diagram of an intrinsic semiconductor at room temperature is shown in Fig. 29-12. Here, Fermi level* lies in the middle of the forbidden energy gap.

29-8. Extrinsic Semiconductors *(A.M.I.E. Summer 1984)*

Those intrinsic semiconductors to which some suitable impurity or doping agent has been added in extremely small amount (about 1 part in 10^8) are called *extrinsic* or *impurity* semiconductors.

Usually, the doping agents are pentavalent atoms having five valence electrons (arsenic, antimony, phosphorus) or trivalent atoms having three valence electrons (gallium, indium, aluminium, boron). Pentavalent doping atom is known as *donor* atom because it donates or contributes one electron to the conduction band of pure germanium. The trivalent atom, on the other hand, is called *acceptor* atom because it accepts one electron from the germanium atom.

Depending on the type of doping material used, extrinsic semiconductors can be further subdivided into two classes (*i*) *N*-type semiconductors and (*ii*) *P*-type semiconductors.

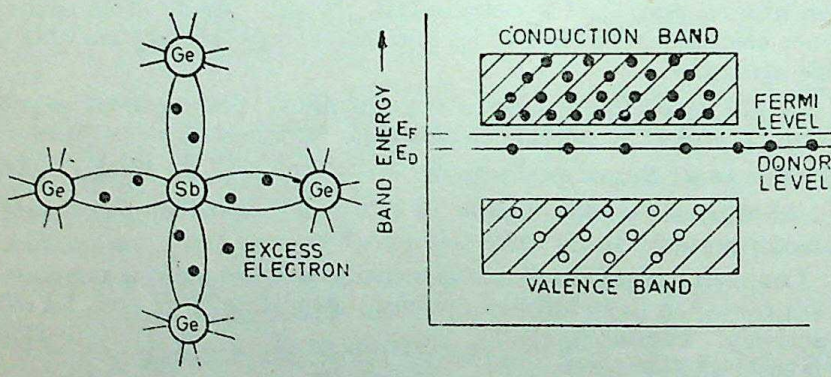

Fig. 29-13

(*i*) **N-type Extrinsic Semiconductor.** This type of semiconductor is obtained when a pentavalent material like antimony (Sb) is added to pure germanium crystal. As shown in Fig. 29-13 (*a*), each antimony atom forms covalent bonds with the surrounding four germanium atoms with the help of four of its five electrons. The fifth electron is superfluous and is loosely bound to the antimony atom. Hence, it can be easily excited from the valence band to the conduction band by the application of electric field or increase in its thermal energy. Thus, practically every antimony atom introduced into the germanium lattice contributes a conduction electron *without creating* a positive hole. Antimony is called donor impurity and

*For the present discussion, Fermi level may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes weighted according to their energies.

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makes the pure germanium an *N*-type (*N* for the negative) extrinsic semiconductor.

It may be noted that by giving away its one valence electron, the donor atom becomes a positively-charged ion. But it cannot take part in conduction because it is firmly fixed or tied into the crystal lattice. As seen, in addition to the electrons and holes intrinsically available in germanium, the addition of antimony greatly increases the number of conduction electrons. Hence, concentration of electrons in the conduction band is increased and exceeds the concentration of holes in the valence band. Because of this, Fermi level shifts upwards towards the bottom of the conduction band as shown in Fig. 29-13 (*b*)*.

In terms of energy levels, the fifth antimony electron has an energy level (called donor level) just below the conduction band. Usually, the donor level is 0·01 eV below conduction band for germanium and 0·054 eV for silicon.

It is seen from the above description that in an *N*-type semiconductor, electrons are the majority carriers while holes constitute the minority carriers.

(*ii*) **P-type Extrinsic Semiconductor.** This type of semiconductor is obtained when traces of a trivalent impurity like boron (B) are added to a pure germanium crystal.

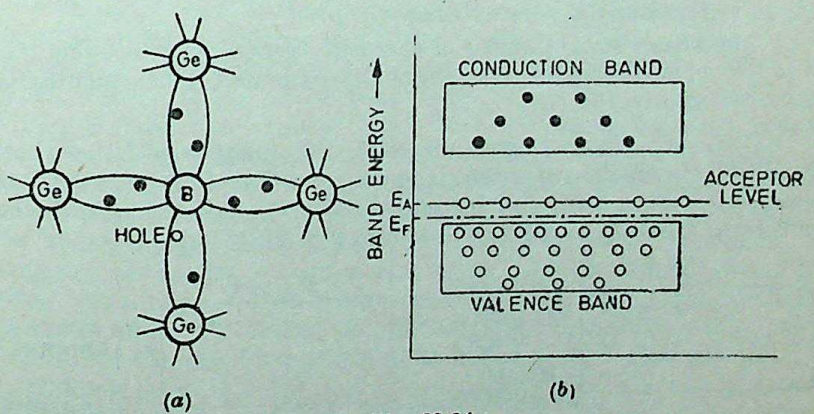

Fig. 29-14

In this case, the three valence electrons of boron atom form covalent bonds with four surrounding germanium atoms but one bond is left incomplete and gives rise to a hole as shown in Fig. 29-14 (*a*).

Thus, boron which is called an acceptor impurity causes as many positive holes in a germanium crystal as there are boron atoms thereby producing a *P*-type (*P* for the positive) extrinsic semiconductor.

*Since the number of electrons as compared to the number of holes increases with temperature, the position of Fermi level also changes considerably with temperature.

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In this type of semiconductor, conduction is by means of holes in the valence band. Accordingly, holes form the majority carriers whereas electrons constitute minority carriers.

Since concentration of holes in the valence band is more than the concentration of electrons in the conduction band, Fermi level shifts nearer to the valence band [Fig. 29-14 (*b*)]. The acceptor level lies immediately above the Fermi level. Conduction is by means of hole movement at the top of valence band, the acceptor level readily accepting electrons from the valence band.

29-9. Majority and Minority Charge Carriers

In a piece of pure germanium or silicon, no free charge carriers are available at 0 °K. However, as its temperature is raised to room temperature, some of the covalent bonds are broken by heat energy and as a result, electron-hole pairs are produced. These are called thermally-generated charge carriers. They are also known as intrinsically-available charge carriers. Ordinarily, their number is quite small.

An intrinsic or pure germanium can be converted into a *P*-type semiconductor by the addition of an acceptor impurity which adds a large number of holes to it. Hence, a *P*-type material contains following charge carriers :

(*a*) large number af positive holes—most of them being the added impurity holes with only a very small number of thermally-generated ones.

(*b*) a very small number of thermally-generated electrons (the companions of the thermally-generated holes mentioned above).

Obviously, in a *P*-type material, the number of holes (both added and thermally-generated) is much more than that of electrons. Hence, in such a material, holes constitute majority carriers and electrons form minority carriers as shown in Fig. 29-15 (*a*).

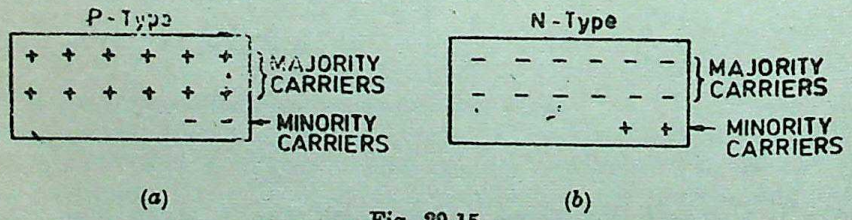

Fig. 29-15

Similarly, in an *N*-type material, the number of electrons (both added and thermally-generated) is much larger than the number of thermally-generated holes. Hence, in such a material, electrons are majority carriers whereas holes are minority carriers as shown in the Fig. 29-15 (*b*).

29-10. Mobile Charge Carriers and Immobile Ions

As discussed in Art. 29-8, *P*-type material is formed by the addition of acceptor impurity atoms like boron to the pure Ge or Si crystals. The number of holes added is equal to the number of

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boron atoms because each such atom contributes one hole. Now, when a hole moves *away* from its parent atom, the remaining atom becomes a negative ion. Unlike the mobile and free-moving hole, this ion cannot take part in conduction because it is fixed in the crystal lattice. In Fig. 29-16 (*a*) these immobile ions are shown by circled minus signs whereas free and mobile holes are shown by uncircled plus signs. Thermally-generated electrons (which form minority carriers) are shown by uncircled minus signs.

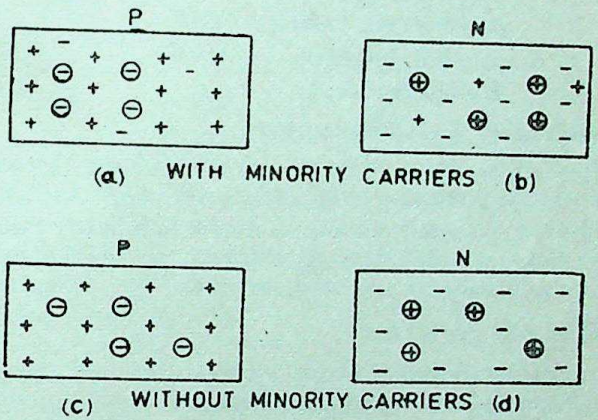

Fig. 29-16

Similarly, addition of pentavalent atoms like antimony to pure Ge or Si crystal produces an *N*-type material. The number of free and mobile electrons which are added equals the number of donor Sb atoms. Again, when an electron moves *away* from its parent atom, it leaves behind a positive ion. This ion, being fixed in crystal structure, cannot take part in conduction. As shown in Fig. 29-16 (*b*), these immobile ions are represented by *circled* plus signs whereas free and mobile electrons are represented by *uncircled* minus sigtns. The thermally-generated holes (which form minority carriers in his case) are shown by *uncircled* plus signs. In Fig. 29-16 (*c*) and (*d*), minority carriers of both types have been neglected. Hence, the figure does not show the small number of free electrons in the *P*-type material or the small number of holes in the *N*-type material :

29-11. Current Carriers in Semiconductors

(A.M.I.E. Winter 1979)

We will consider the following cases separately.

(*a*) **Intrinsic Semiconductors.** In their case, current flow is due to the movement of electrons and holes in opposite directions. Even though the number of electrons equals the number of holes, hole mobility μ_h is practically half of electron mobility μ_e.

The total current flow which is due to the sum of that for electrons and holes is given by

$$I = I_e + I_h = n_I \, e \, v_e A + p_I \, e \, v_h \, A$$

where v_e = drift velocity of electrons

v_h = drift velocity of holes

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n_I = number of free electrons in an intrinsic semiconductor

p_I = number of holes in an intrinsic semiconductor

e = electron charge

A = cross-section of the semiconductor

Since in an intrinsic conductor $n_I = p_I$.

$$\therefore \quad I = n_I e\,(v_e + v_h)\,A$$
$$= n_I e\,(\mu_e + \mu_h)\,EA$$

where μ_e = electron mobility $= v_e/E$

μ_h = hole mobility $= v_h/E$

E = electric field

(*b*) **Extrinsic Semiconductors**

(*i*) In *N*-type semiconductors, electrons form the majority carriers although holes are also available as minority carriers.

(*ii*) In *P*-type semiconductors, conduction is by means of holes in the valence band which form majority carriers in this case although electrons are available as minority carriers.

29-12. The P-N Junction

It is possible to manufacture a single piece of a semiconductor material (either Ge or Si) one half of which is doped by *P*-type impurity and the other half by *N*-type impurity as shown in Fig. 29-17. The plane dividing the two halves or zones is called a *P-N junction*.

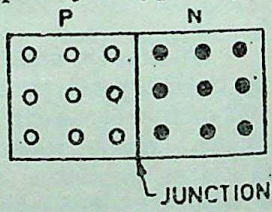

Fig. 29-17

During the formation of junction, following two phenomena take place :

(*i*) a thin depletion layer or region is set up on both sides of the junction and is so called because it is depleted or devoid of free charge carriers. Its width is about 1 μm (10^{-6} m).

(*ii*) a junction or barrier potential V_B is developed across the junction whose value is about 0·3 V for Ge and 0·7 V for Si.

When a *P-N* junction is packed as a semiconductor device, it is called a *P-N* junction diode.

29-13. Formation of Depletion Layer

Suppose that a *P-N* junction has just been formed. At that instant, holes are still in the *P*-region and electrons in the *N*-region. However, there is greater concentration of holes in *P*-region (where they form majority carriers) than in *N*-region (where they form minority carriers). Similarly, concentration of electrons is greater in *N*-region than in *P*-region (where they exist as minority carriers). This difference in concentration establishes a density gradient across the junction resulting in carrier diffusion. Holes diffuse from *P*-to *N*-region and electrons from *N*-to *P*-region and terminate their existence by recombination as shown in Fig. 29-18 (*a*).

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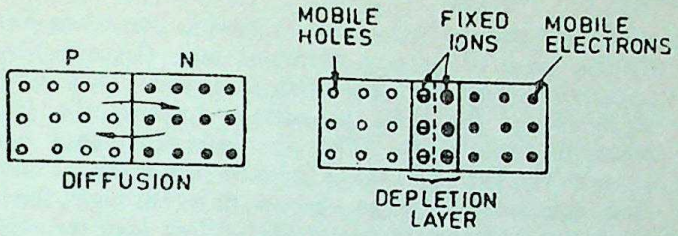
This recombination of free and mobile holes and electrons produces the narrow region at the junction called *depletion layer*. It is so called because this region is *devoid or depleted of free and mobile charge carriers* though it does contain fixed or immobile positive and negative ions.



(*a*) Fig. 29-18 (*b*)

This production of ions is due to the fact that the impurity atoms which provide migratory electrons and holes are themselves left behind in an ionised state bearing a charge which is opposite to that of the departed carrier. For example, when an electron migrates across the junction from *N*-region to *P*-region, it leaves behind an atom that is one electron short of its normal quota. This atom is now *ionised* and has a *positive* charge. These impurity ions so produced are fixed in their positions in the crystal lattice in the *P*- and *N*-regions. Hence, as shown in Fig. 29-18 (*b*) they form parallel rows or *plates* of opposite charges facing each other across the depletion layer. As seen, depletion layer contains *no free and mobile charge* carriers but only fixed and immobile ions. Hence, this layer or region behaves like an insulator (and due to the presence of rows of fixed charges, it possesses capacitance as explained in Art. 29-19).

29-14. Junction or Barrier Voltage (V_B)

As explained in Art. 29-13, depletion layer of a *P-N* junction diode has no free charge carriers but only fixed rows of oppositely-charged ions on its two sides. Because of this charge separation, an electric potential V_B is established across the junction even when *the junction is not connected to any external source of e.m.f.* (Fig. 29-19)*. It is known as *junction* or barrier potential. It stops further flow of carriers across the junction unless supplied by energy from an external source. At room temperature of 300 °K, V_B is about 0·3 V for Ge and 0·7 V for Si.

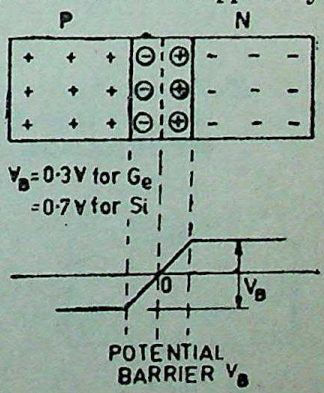


Fig. 29-19

29-15. Forward Biased P-N Junction

Suppose positive battery terminal

*In this figure, mobile holes have been shown by crosses and not circles as done hitherto. Similarly, electrons have been shown by negative signs rather than by black dots.

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is connected to *P*-region of a semiconductor and the negative battery terminal to the *N*-region as shown in Fig. 29-20. In that case, the junction is said to be biased in the forward direction because it permits easy flow of current across the junction. The current flow may be explained in the following two ways :

(*i*) As soon as battery connection is made, holes are *repelled* by the positive battery terminal and electrons by the negative battery terminal with the result that both the electrons and the holes are driven *towards* the junction where they recombine. This *en masse* movement of electrons to the left and that of holes to the right of the junction constitutes a large current flow through the semiconductor. Obviously, the crystal offers *low resistance in the forward direction.*

Incidentally, it may be noted that though there is movement of both electrons and holes *inside* the crystal, only free electrons move in the external circuit *i.e.* in the battery-connected wires.

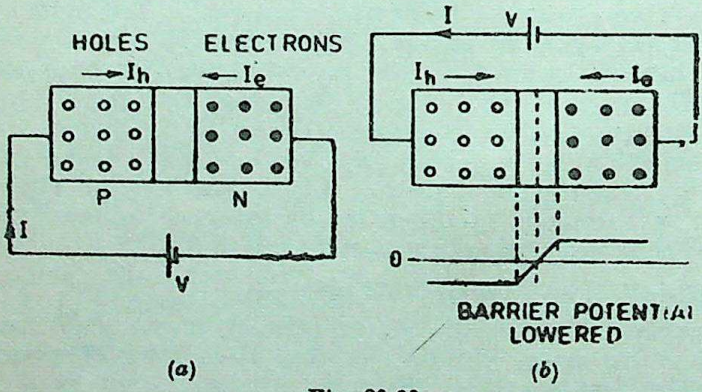

Fig. 29-20

(*ii*) Another way to explain current flow in forward direction is to say that due to the applied external voltage, the barrier potential is reduced which now allows more current

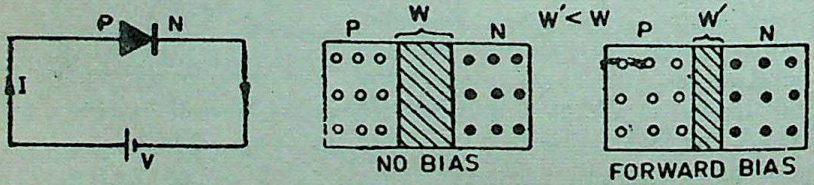

Fig. 29-21

to flow across the junction [Fig. 29-20 (*b*)]. Incidentally, it may be noted that forward bias reduces the thickness of the depletion layer as shown in Fig. 29-21.

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A typical V/I characteristic for a forward-biased P-N junction is shown in Fig. 29-22. It is seen that forward current rises exponentially with the applied forward voltage. However, at ordinary room temperature, a p.d. of about 0·3 V is required before a reasonable amount of forward current starts flowing in a germanium junction. This voltage is known as *threshold* voltage (V_{th}) or *cut-in* voltage. It is the same as barrier voltage V_B. Its value for silicon junction is about 0·7 volt. For $V < V_{th}$, current flow is negligible. But as applied voltage increases beyond the threshold value, the forward current increases sharply. If forward voltage is increased beyond a certain safe value, it will produce an extremely large current which may destroy the junction due to overheating.

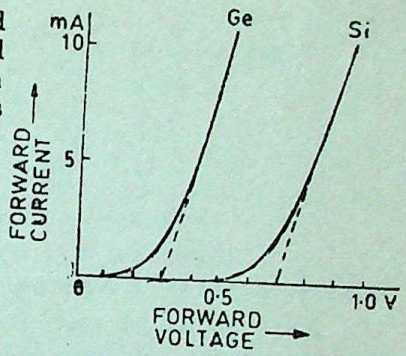

Fig. 29-22

Ge devices can stand junction temperatures around 100°C whereas Si units can function upto 175°C.

29-16. Reverse Biased P-N Junction

When battery connections to the semiconductor are made as shown in Fig. 29-23 (*a*), the junction is said to be reverse-biased. In this case, holes are attracted by the negative battery terminal and electrons by the positive terminal so that both holes and electrons move *away* from the junction and away from each other. Since there is no electron-hole combination, no current flows and the junction offers high resistance.

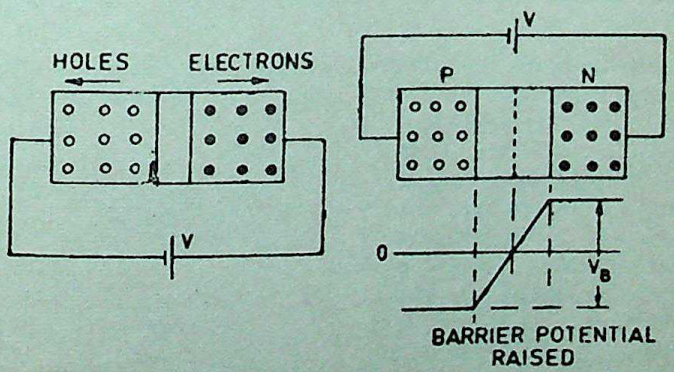

Fig. 29-23

Another way of looking at the above process is that, in this case, the applied voltage increases the barrier potential thereby blocking the flow of *majority* carriers as shown in Fig. 29-23 (*b*).

Incidentally, it may be noted that under reverse bias conditions

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width of depletion layer is increased because of increased barrier potential (Fig. 29-24).

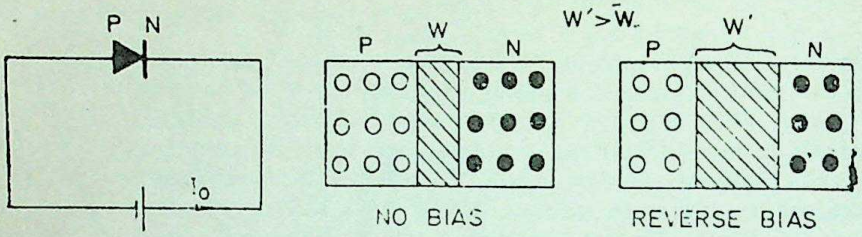

Fig. 29-24

Reverse Saturation Current

Although, in this case, there is practically no current due to majority carriers, yet there is a small amount of current (a few μA only) due to the flow of *minority carriers* across the junction. As explained earlier in Art. 29-9, due to thermal energy, there are always generated some holes in the *N*-type region and some electrons in the *P*-type region of the semiconductor as shown in Fig. 29-15. The applied voltage acts as a *forward bias for these minority carriers* though it is a reverse bias for the majority carriers. The battery drives these minority carriers across the junction thereby producing a small current called reverse current or *reverse saturation current* I_S or I_0. Since minority carriers are thermally-generated, I_0 is extremely temperature dependent. For the same reason, forward current is also temperature dependent but to a much less degree because minority current forms a very small percentage of the majority current.

I_0 is found to double for every 10°C rise for Ge and for every 6°C rise in the case of Si. Usually, it is of the order of μA for Ge and nA for Si.

It may be noted that reverse saturation current is also referred to as *leakage current* of the *P-N* junction. Fig. 29-25 shows *V/I* characteristics of a reverse-biased *P-N* junction. It is seen that as reverse voltage is increased from zero, the reverse current quickly rises to its maximum or saturation value. Keeping temperature constant, as the reverse voltage is increased, I_0 is also found to increase slightly. This slight increase is due to the impurities on the surface of the semiconductor which behaves as an effective resistance and hence obeys Ohm's law. This gives rise to a very small current called *surface leakage* current. Unlike the main leakage (or saturation) current, this surface leakage current is independent of temperature but depends on the magnitude of the reverse voltage.

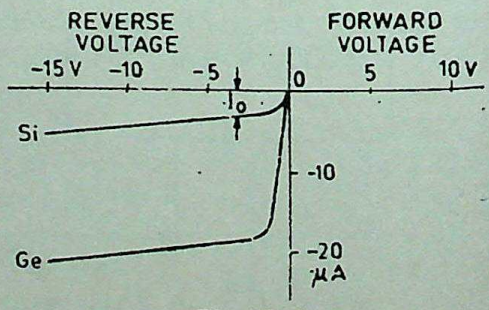

Fig. 29-25

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29-17. Combined Forward and Reverse V/I Characteristics

Combined forward and reverse voltage-current characteristics for both Ge and Si are shown in Fig. 29-26. It is seen that leakage

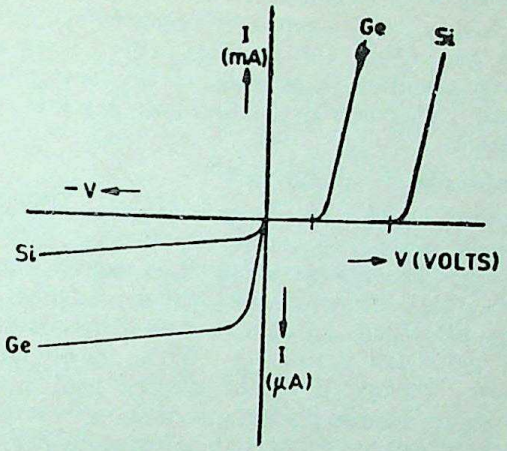

Fig. 29-26

current of Ge junction is much more than that of Si junction.

29-18. Junction Breakdown

If the reverse bias applied to a *P-N* junction is increased, a point is reached when the junction breaks down and reverse current rises sharply to a value limited only by the resistance connected in series with the junction (Fig. 29-27) This critical value of the voltage is known as *breakdown* voltage (V_{BR}). It is found that once breakdown has occurred, very little further increase in voltage is required to increase the current to relatively high values. The junction offers almost *zero* resistance at this point.

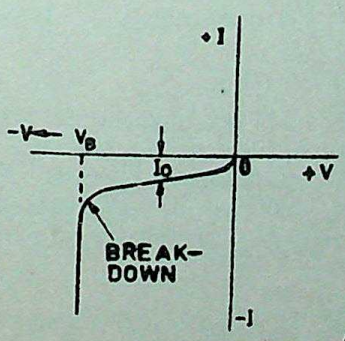

Fig. 29-27

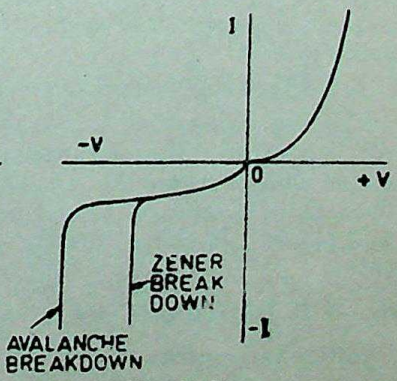

Fig. 29-28

The breakdown voltage depends on the width of the depletion region which, in turn, depends on the doping level.

The following two mechanisms are responsible for breakdown under increasing reverse voltage :

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(*a*) **Zener Breakdown**

This form of breakdown occurs in junctions which, being heavily doped, have *narrow* depletion layers. The breakdown voltage sets up a very strong electric field (about 10^8 V/m) across this narrow layer. This field is strong enough to break or *rupture* the covalent bonds thereby generating electron-hole pairs. Even a small further increase in reverse voltage is capable of producing large number of current carriers. That is why junction has very low resistance in the breakdown region.

(*b*) **Avalanche Breakdown**

This form of breakdown occurs in junctions which, being lightly-doped, have *wide* depletion layers where the electric field is not strong enough to produce Zener breakdown. Instead, the minority carriers (accelerated by this field) collide with the semiconductor atoms in the depletion region. Upon collision, covalent bonds are broken and electron-hole pairs are generated. These newly-generated charge carriers are also accelerated by the electric field resulting in more collisions and hence further production of charge carriers. This leads to an avalanche of charge carriers and, consequently, to a very low reverse resistance. The two breakdown phenomena are shown in Fig. 29-28.

29-19. Junction Capacitance

Capacitive effects are exhibited by *P-N* junctions when they are both forward and reverse-biased.

(*a*) *Transition Capacitance* (C_T) *or Space-charge Capacitance*

When a *P-N* junction is *reverse-biased*, the depletion region acts like an insulator or as a dielectric material essential for making a capacitor. The *P*-and *N*-type regions on either side have low resistance and act as the plates. We, therefore, have all the components necessary for making a parallel-plate capacitor. This junction capacitance is called *transition* or *space charge* capacitance (C_T). It is voltage-dependent.

(*b*) *Diffusion or Storage Capacitance* (C_D)

This capacitive effect is present when the junction is *forward-biased*. It is called diffusion capacitance to account for the time delay in moving charge across the junction by diffusion process. Due to this fact, this capacitance cannot be identified in terms of a dielectric and plates. It varies directly with the magnitude of forward current.

29-20. P-N Junction Diode

(*A.M.I.E. Summer 1985*)

(*a*) **Construction**

It is a two-terminal device consisting of a *P-N* junction formed either in Ge or Si crystal. Its circuit symbol is shown in Fig. 29-29 (*a*). The *P*-and *N*-type regions are referred to as *anode* and *cathode* respectively. In Fig. 29-29 (*b*), arrowhead indicates the *conventional* direction of current flow when forward-biased. It is the same direction in which hole flow takes place.

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Commercially available diodes usually have some means t indicate which lead is P and which lead is N. Standard notation consists of type numbers preceded by 'IN' such as IN 240 and IN 1250. Here, 240 and 1250 correspond to colour bands.

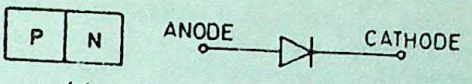

Fig. 29-29

(*b*) Working

A P-N junction diode is a one-way device offering low resistance when forward-biased and behaving almost as an insulator when reverse-biased. Hence, such diodes are mostly used as rectifiers *i.e.* for converting alternating current into direct current.

(*c*) V/I Characteristic

Fig. 29-30 shows the static voltage-current characteristics for a low-power P-N junction diode.

(*i*) Forward Characteristic

When the diode is forward-biased and the applied voltage is increased from zero, hardly any current flows through the device in the beginning. It is so because the external voltage is being opposed by the internal barrier voltage V_B whose value is 0·7 V for Si and 0·3 V for Ge. As soon as V_B is neutralised, current through the diode increases rapidly with increasing battery voltage. It is found that as little a voltage as 1·0 V produces forward current of about 50 mA .

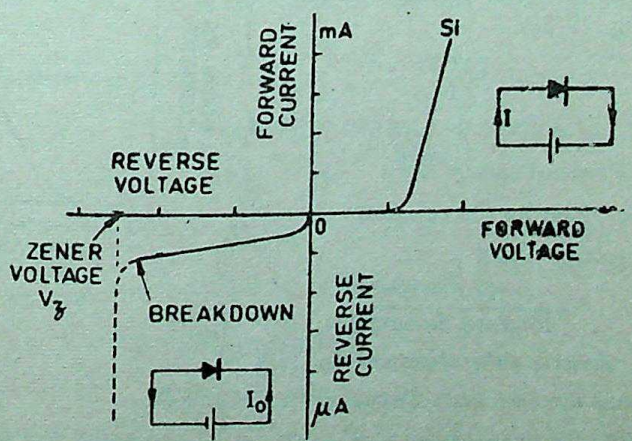

Fig. 29-30

(*ii*) Reverse Characteristic

When the diode is reverse-biased, majority carriers are blocked and only a small current (due to minority carriers) flows through the diode. As the reverse voltage is increased from zero, the reverse current very quickly reaches its maximum or saturation value I_0 which

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is also known as *leakage current*. It is of the order of nanoamperes (nA) for Si and microamperes (μA) for Ge.

As seen from Fig. 29-30, when reverse voltage exceeds a certain value called *Zener voltage* V_Z, the leakage current suddenly and sharply increases, the curve indicating zero resistance at this point. When *P-N* junction diodes are employed primarily because of this property as voltage regulators, they are called Zener diodes (Art. 29-21).

(*d*) Diode Parameters *(A.M.I.E. Summer 1984)*

The diode parameters of greatest interest are :

1. *bulk resistance* (r_B)

It is the sum of the resistance values of the *P*- and *N*-type semiconductor materials of which the diode is made of. Usually, it is very small.

2. *Junction resistance* (r_j)

Its value for forward-biased junction depends on the magnitude of forward dc current.

$$r_j \cong \frac{25 \text{ mV}}{I_F} \quad \text{...for Ge}$$

$$\cong \frac{50 \text{ mV}}{I_F} \quad \text{...for Si}$$

where I_F is the forward current in mA.

3. *Dynamic* or *ac resistance*

$$r_{ac} \quad \text{or} \quad r_d = r_B + r_j$$

As seen from forward characteristic of Fig. 29-31,

$$r_{ac} = \frac{\triangle V_F}{\triangle I_F}$$

...at a given dc forward current.

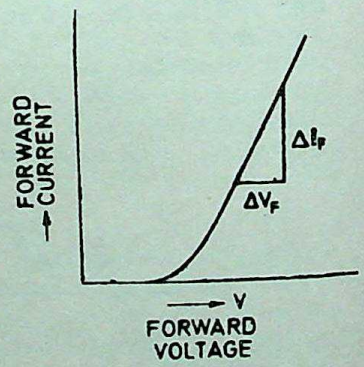

Fig. 29-31

4. *Forward voltage drop*

It is given by the relation .

$$\text{orward voltage drop} = \frac{\text{power dissipated}}{\text{forward dc current}}$$

5. *Reverse saturation current* (I_o)

It has already been discussed in Art. 29-16.

6. *Reverse breakdown voltage* (V_{BR})

It has already been discussed in Art. 29-18.

7. *Reverse dc resistance* R_R

$$R_R = \frac{\text{reverse voltage}}{\text{reverse current}}$$

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(e) Applications

The main applications of semiconductor diodes in modern electronic circuitry are as under :

1. as power or rectifier diodes. They convert ac current into dc current for dc power supplies of electronic circuits.
2. as signal diodes in communication circuits for modulation and demodulation of small signals.
3. as Zener diodes in voltage stabilizing circuits.
4. as varactor diodes—for use in voltage controlled tuning circuits as may be found in radio and TV receivers. For this purpose, the diode is deliberately made to have a certain range of junction capacitance. The capacitance of the reversed-biased diode is given by $C=K/\sqrt{V}$.
5. in logic circuits used in computers

Example 29-1. *A silicon diode dissipates 3 W for a forward dc current of 2 A. Calculate the forward voltage drop across the diode and its bulk resistance.*

Solution. Forward drop $V_F=3/2=\mathbf{1{\cdot}5\ V}$

Now, $$V_F=V_j+r_B I_F$$

or $$1{\cdot}5=0{\cdot}7+r_B\times 2 \quad \therefore \quad r_B=\mathbf{0{\cdot}4\ \Omega}$$

Example 29-2. *A silicon diode has a forward voltage drop of 1·2 V for a forward dc current of 100 mA. It has a reverse current of 1 μA for a reverse voltage of 10 V. Calculate*

(a) bulk and reverse resistances of the diode

(b) ac resistance at forward dc current of (i) 2·5 mA and (ii) 25 mA.

Solution (*a*) $$r_B=\frac{V_F-V_j}{I_F}=\frac{1{\cdot}2\ \text{V}-0{\cdot}7\ \text{V}}{100\ \text{mA}}=\mathbf{5\ \Omega}$$

$$R_R=\frac{V_R}{I_R}=\frac{10\text{V}}{1\ \mu\text{A}}=\mathbf{10\ M\Omega}$$

(*b*) (*i*) $$r_j=\frac{25\ \text{mV}}{2{\cdot}5\ \text{mA}}=\mathbf{10\ \Omega}$$

$$r_{ac}=r_B+r_j=5+10=\mathbf{15\ \Omega}$$

(*ii*) $$r_j=\frac{25\ \text{mV}}{25\ \text{mA}}=\mathbf{1\ \Omega}$$

$$r_{ac}=5+1=\mathbf{6\ \Omega}$$

29-21. Zener Diode

(A.M.I.E. Summer 1986)

It is a reverse-biased silicon or germanium *P-N* junction which is operated in the breakdown region where current is limited only by both external resistance and the power dissipation of the diode. Conventional diodes and rectifiers never operate in the breakdown region but the Zener diode makes a virtue of it and operates at this very point like *P* shown in Fig. 29.32. These diodes are analogous

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to gas diodes in that a large current appears when breakdown potential is reached.

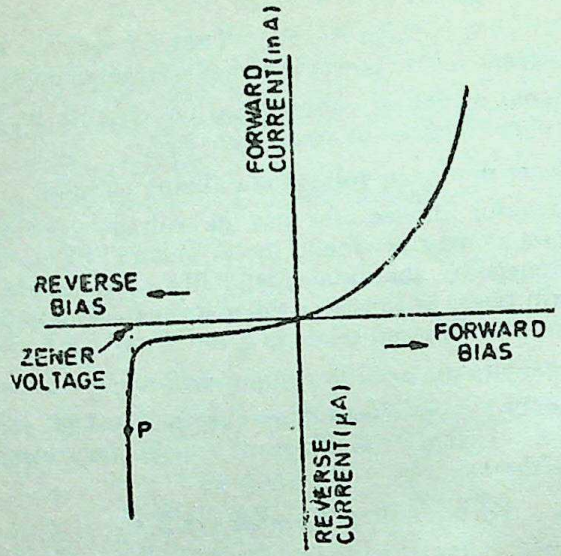

Fig. 29-32

Uses. Zener diodes find numerous applications in transistor circuitry. Some of their uses are as voltage regulators, as peak clippers and for giving protection to a meter movement or circuit against damage from accidental application of excessive voltage.

(*i*) **Voltage Regulation.** It is a measure of a circuit's ability to maintain a constant output voltage even when either input voltage or load current varies. A Zener diode when working in the breakdown region can serve as a voltage regulator. In Fig. 29-33, V_{in} is the input dc voltage whose variations are to be regulated. The Zener diode is reverse-connected across V_{in}. When p.d. across the diode is greater than V_Z, it conducts and draws relatively large current through the series resistance R. The load resistance R_L across which a constant voltage V_{out} is required, is connected in parallel with the diode. The total current I passing through R equals the sum of diode current and load current *i.e.* $I = I_d + I_L$.

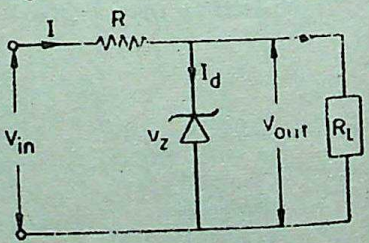

Fig. 29-33

It will be seen that under all conditions, $V_{out} = V_Z$.

Hence, $V_{in} = IR + V_{out} = IR + V_Z$.

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Case 1. Suppose R is kept fixed but supply voltage V_{in} is

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increased slightly. It will increase I. This increase in I will be absorbed by the Zener diode without affecting I_L. The increase in V_{in} will be dropped across R thereby keeping V_{out} constant. Conversely, if supply voltage V_{in} falls, the diode takes a smaller current and voltage drop across R is reduced thus again keeping V_{out} constant. Hence, when V_{in} changes, I and IR drop change in such a way as to keep V_{out} $(=V_Z)$ constant.

Case 2. In this case, V_{in} is fixed but I_L is changed. When I_L increases, diode current I_d decreases thereby keeping I and IR drop constant. In this way, V_{out} remains unaffected.

Should I_L decrease, I_d would increase in order to keep I and hence IR drop constant. Again, V_{out} would remain unchanged because

$$V_{out}=V_{in}-IR=V_{in}-(I_d+I_L)\,R$$

Incidentally, it may be noted that

$$R=\frac{V_{in}-V_{out}}{I_d+I_L}$$

It may also be noted that when diode current reaches its maximum value, I_L becomes zero. In that case

$$R=\frac{V_{in}-V_{out}}{I_{d(max)}}$$

(*ii*) **Meter Protection.** Zener diodes are frequently used in volt-ohm-milliammeters for protecting meter movement against burnout from accidental overloads. If VOM is set to its 2·5-V range and the test leads are accidentally connected to a 25-V circuit, an unprotected meter will be burned out or at least get severely damaged.

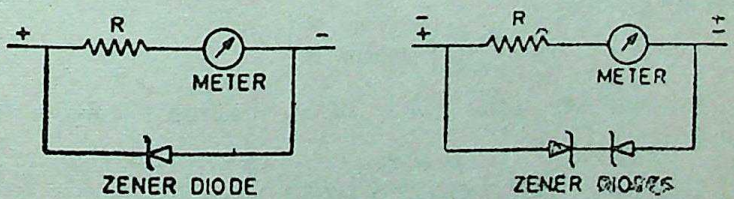

Fig. 29-34

This hazard can be avoided by connecting a Zener diode in parallel with the meter as shown in Fig. 29-34 (*a*). In the event of an accidental overload, most of the current will pass through the diode. Two Zener diodes connected as shown in Fig. 29-34 (*b*) can provide overload protection regardless of the applied polarity.

(*iii*) **Peak Clippers.** Use of Zener diodes in wave-shaping circuits is illustrated in Fig. 29-35. Here, a semi-square wave is produced by clipping a sine wave. Two Zener diodes are shunted across a sine wave source. The shunt resistance remains very high until the positive and negative voltage across the diodes exceeds the Zener value. Thereafter, the diodes become low-resistance shunt

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circuits and increasing voltage values are dropped across the series resistance R. Consequently, the output wave is clipped on both peaks.

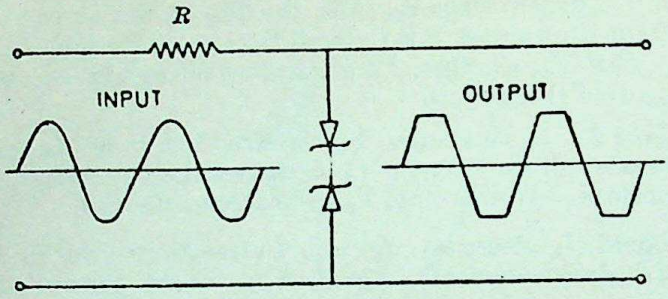

Fig. 29-35

Example 29-3. *A 24-V, 600-mW Zener diode is to be used for providing a 24-V stabilized supply to a variable load. If input voltage is 32 V, calculate*

(*i*) *series resistance R required and*

(*ii*) *diode current when load resistance is 1200* Ω.

Solution.

(*i*) maximum value of I_d *i.e.* $I_{d(max)} = 600/24 = 25$ mA

$$\therefore \quad R = \frac{V_{in} - V_{out}}{I_{d(max)}} = \frac{32 - 24}{25 \times 10^{-3}} = \mathbf{320\ \Omega}.$$

(*ii*) When $R_L = 1200\ \Omega$, $I_L = 24/1200 = 20$ mA

$$\therefore \quad I_d = I - I_L = 25 - 20 = \mathbf{5\ mA}$$

HIGHLIGHTS

1. In Bohr's atomic model, radii of various permitted circular orbits are given by

$$r_n = \frac{\epsilon_0\, n^2 h^2}{\pi m Z e^2} \text{ metre}$$

2. Similarly, total energy of an electron revolving in nth orbit is

$$E_n = -\frac{m\, e^4\, Z^2}{8\epsilon_0^{\,2}\, n^2\, h^2} \text{ joule}$$

3. The energy of the electron in a hydrogen atom when moving in nth orbit is

$$E_n = -\frac{13{\cdot}6}{n^2} \text{ eV}$$

4. In terms of energy bands, insulators are those materials which have

(*i*) full valence band and

(*ii*) an empty conduction band

with large energy gap between the two.

5. Conductors are those in which valence and conduction bands overlap.

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6. Semiconductors are those materials which have an

(*i*) almost filled valence band and

(*ii*) almost empty conduction band

with a very small energy gap between the two.

7. An intrinsic semiconductor is one which is made of semiconductor material in its purest form.

8. In intrinsic semiconductors, number of conduction electrons equals the number of holes. Hence, Fermi level lies at the middle of the energy gap.

9. Extrinsic semiconductors are nothing else but intrinsic semiconductors to which an extremely small amount of impurity or doping agent has been added. They may be further subdivided into

(*i*) *N*—type ...with the help of pentavalent impurity

(*ii*) *P*—type ...with the help of trivalent impurity.

10. A *P-N* junction conducts when forward-biased but not when reverse-biased.

11. Junction breakdown occurs when reverse applied voltage exceeds a certain limit known as breakdown voltage (V_{BR}). It could be

(*i*) Zener breakdown—it occurs in junctions having *narrow* depletion layers and is due to rupturing of covalent bonds by electric field set up in depletion layer.

(*ii*) Avalanche breakdown—it occurs in junctions which have *wide* depletion layers and is due to commulative effect of collisions of minority carriers.

12. Zener diode is a reverse-biased (*P-N*) junction which is operated in the breakdown region.

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TRANSISTORS 30

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30-1 The Junction Transistor*

(A.M.I.E. Summer 1985)

Basically, it consists of two back-to-back P-N junctions manufactured in a single piece of a semiconductor crystal. These two junctions give rise to three regions called *emitter*, *base* and *collector*. As shown in Fig. 30-1, a junction transistor is simply a sandwich of one type of semiconductor material between two layers of the other type. Fig. 30-1 (*a*) shows a layer of N-type material sandwiched between two layers of P-type material. It is described as PNP transistor. Fig. 30-1 (*b*) shows an NPN transistor consisting of a layer of P-type material sandwiched between two layers of N-type material.

*It is a contraction of *transfer resistor*. It is so because a transistor is basically a resistor that amplifies electrical impulses as they are transferred through it from its input to output terminals.

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The emitter, base and collector are provided with terminals which are labelled as E, B and C. The two junctions are: emitter-base (E/B) junction and collector-base (C/B) junction.

The symbols employed for *PNP* and *NPN* transistors are also shown in Fig. 30-1. The arrowhead is *always* at the emitter (not at the collector) and in each case, its direction indicates the *conventional* direction of current flow. For a *PNP* transistor, arrowhead points from emitter to base meaning that emitter is positive with respect tc base (and also with respect to collector)*. For *NPN* transistor, it points from base to emitter meaning that base (and collector as well)* is positive with respect to emitter.

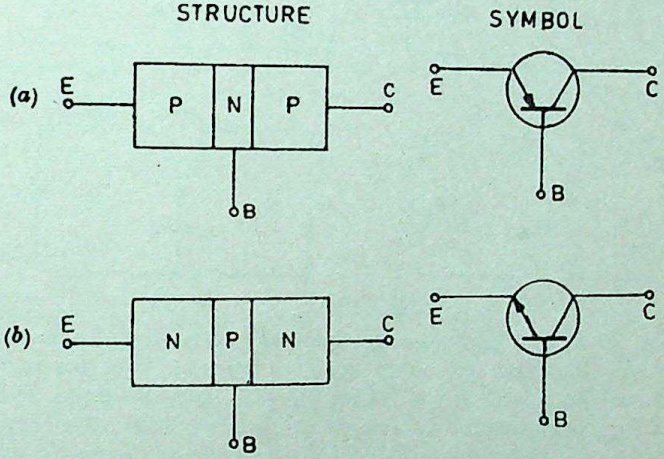

Fig. 30-1

(*i*) Emitter

It forms the left-hand section or region of the transistor as shown in Fig. 30-1. It is more *heavily doped* than any of the other regions because its main function is to supply majority charge carriers (either electrons or holes) to the base.

(*ii*) Base

It forms the middle section of the transistor. It is very thin (10^{-6} m) as compared to either the emitter or collector and is very *lightly doped*.

(*iii*) Collector

It forms the right-hand side section or region of the transistor shown in Fig. 30-1 and its main function is (as indicated by its name) to collect majority charge carriers through the base.

In most transistors, collector region is made physically larger than the emitter region because it has to dissipate much greater power.

It may be noted, in passing, that transistors are made by growing, alloying or diffusing processes.

*In a transistor, for normal operation, collector and base must have reverse polarity with respect to emitter (Art. 30-3).

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30-2 Transistor Biasing

For proper working of a transistor, it is essential to apply voltages of correct polarity across its two junctions. It is worthwhile to remember that for normal operation

(*i*) emitter-base junction is *always* forward-biased and

(*ii*) collector-base junction is *always* reverse-biased.

This type of biasing is known as *FR* biasing.

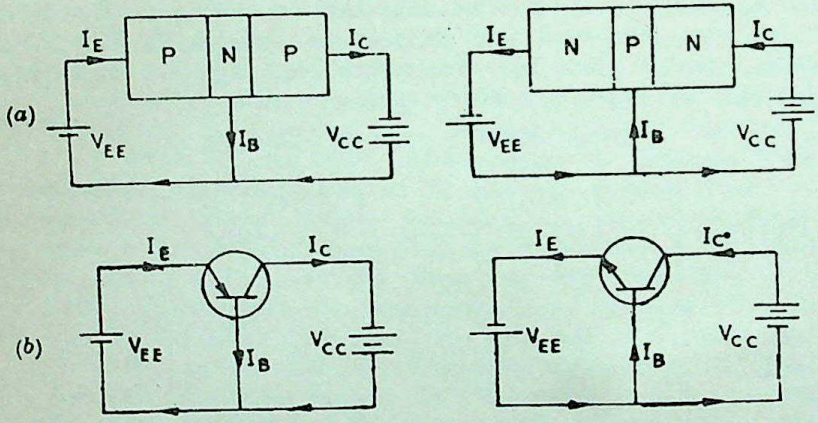

Fig. 30-2

In Fig. 30-2, two batteries respectively provide the dc emitter supply voltage V_{EE} and collector supply voltage V_{CC} for properly biasing the two junctions of the transistor. In Fig. 30-2 (*a*), Positive terminal of V_{EE} is connected to *P*-type emitter in order to *repel* or *push* holes into the base.

The negative terminal of V_{CC} is connected to the collector so that it may attract or pull holes through the base. Similar considerations apply to the *NPN* transistor of Fig. 30-2 (*b*). It must be remembered that a transistor will never conduct any current *if its emitter-base junction is not forward-biased.**

30-3 Important Biasing Rule

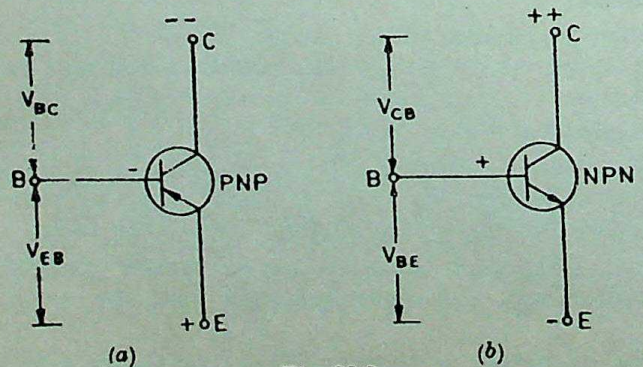

Fig. 30-3

*There would be no current due to majority charge carriers. However, there would be an extremely small current due to minority charge carriers which is called leakage current of the transistor (Art. 29-16).

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For a *PNP* transistor, both collector and base are *negative* with respect to the emitter (the letter *N* of *N*egative being the same as the middle letter of *PNP*). Of course, collector is *more* negative than base [Fig. 30-3 (*a*)]. Similarly, for *NPN* transistor, both collector and base are *positive* with respect to emitter (the letter *P* of *P*ositive being the same as the middle letter of *NPN*). Again, collector is *more* positive than the base as shown in Fig. 30-3 (*b*).

30-4 Transistor Currents

The three primary currents which flow in a properly-biased transistor are I_E, I_B and I_C. In Fig. 30-4 (*a*) are shown the directions of flow as well as relative magnitudes of these currents for a *PNP* transistor connected in the common-base mode. It is seen that

$$I_E = I_B + I_C$$

It means that a small part (about 1—2%) of emitter current goes to supply base current and the remaining major part (99—98%) goes to supply collector current.

Moreover, I_E flows *into* the transistor whereas both I_B and I_C flow out of it.

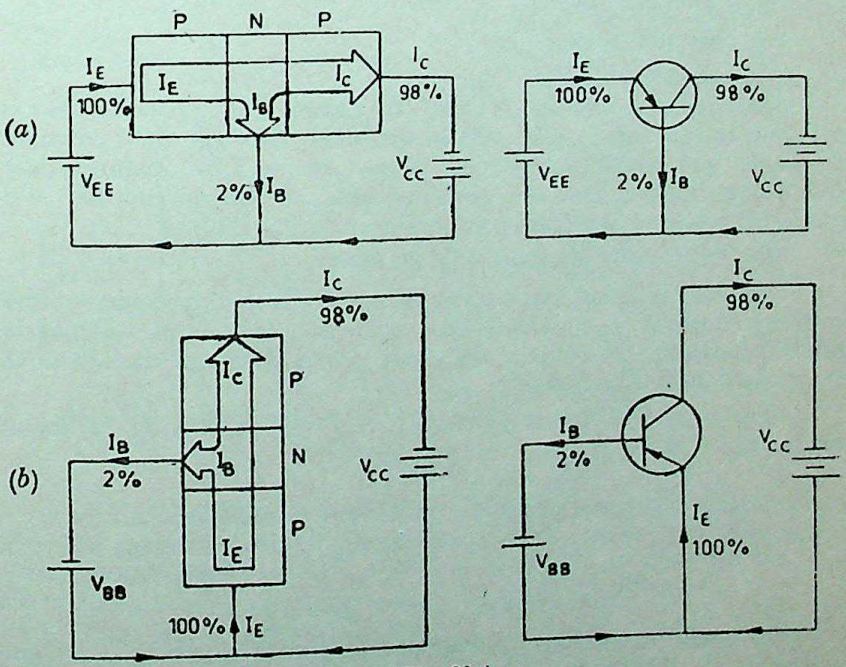

Fig. 30-4

Fig. 30-4 (*b*) shows the flow of currents in the same transistor when connected in the common-emitter mode. It is seen that again

$$I_E = I_B + I_C$$

By normal convention, currents flowing *into* a transistor are taken as positive whereas those flowing *out* of it are taken as *negative*. Hence, I_E is positive whereas both I_B and I_C are negative. Applying Kirchhoff's Current Law, we have

$$I_E + (-I_B) + (-I_C) = 0$$

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or $$I_E - I_B - I_C = 0$$

or $$I_E = I_B + I_C$$

This statement is true *regardless of transistor type or transistor configuration.*

Note. For the time being, we have not taken into account the leakage currents which exist in a transistor.

30-5 Summing Up

The four basic guideposts about all transistor circuits are :

(*a*) conventional current flows along the arrow whereas electrons flow against it

(*b*) E/B junction is always forward-biased

(*c*) C/B junction is always reverse-biased

(*d*) $I_E = I_B + I_C$

30-6 Transistor Circuit Configurations

Basically, there are three types of circuit connections (called configurations) for operating a transistor

(*i*) common-base (*CB*)

(*ii*) common-emitter (*CE*)

(*iii*) common-collector (*CC*)

The term 'common' is used to denote the electrode that is common to the input and output circuits. Because the common electrode is generally grounded, these modes of operation are frequently referred to as grounded-base, grounded-emitter and grounded-collector configurations as shown in Fig. 30-5.

(*a*) **CB Configuration** [Fig. 30-5 (*a*)]

In this configuration, emitter current I_E is the input current and collector current I_C is the output current. The input signal is applied between the emitter and base whereas output is taken out from the collector and base.

The ratio of collector current to emitter current is called dc alpha (α_{dc}) of a transistor.

$$\therefore \quad \alpha_{dc} = \frac{-I_C^*}{I_E} \quad \text{or} \quad I_C = \alpha_{dc} I_E^{**}$$

If we write α_{dc} simply as α, then

$$\alpha = -I_C/I_E \quad \text{or} \quad I_C = \alpha I_E$$

It is also called forward current transfer ratio ($-h_{FB}$). In h_{FB}, subscript F stands for forward and B for common base. The negative sign merely indicates that emitter and collector currents flow in *opposite* directions. The subscript *dc* on α signifies that this ratio is defined from dc values of I_C and I_E.

---

*More accurately, $\alpha_{dc} = \dfrac{I_C - I_{CBO}}{I_E}$ —Art 30-8

**Negative sign has been omitted since we are only concerned with only magnitudes of the currents involved.

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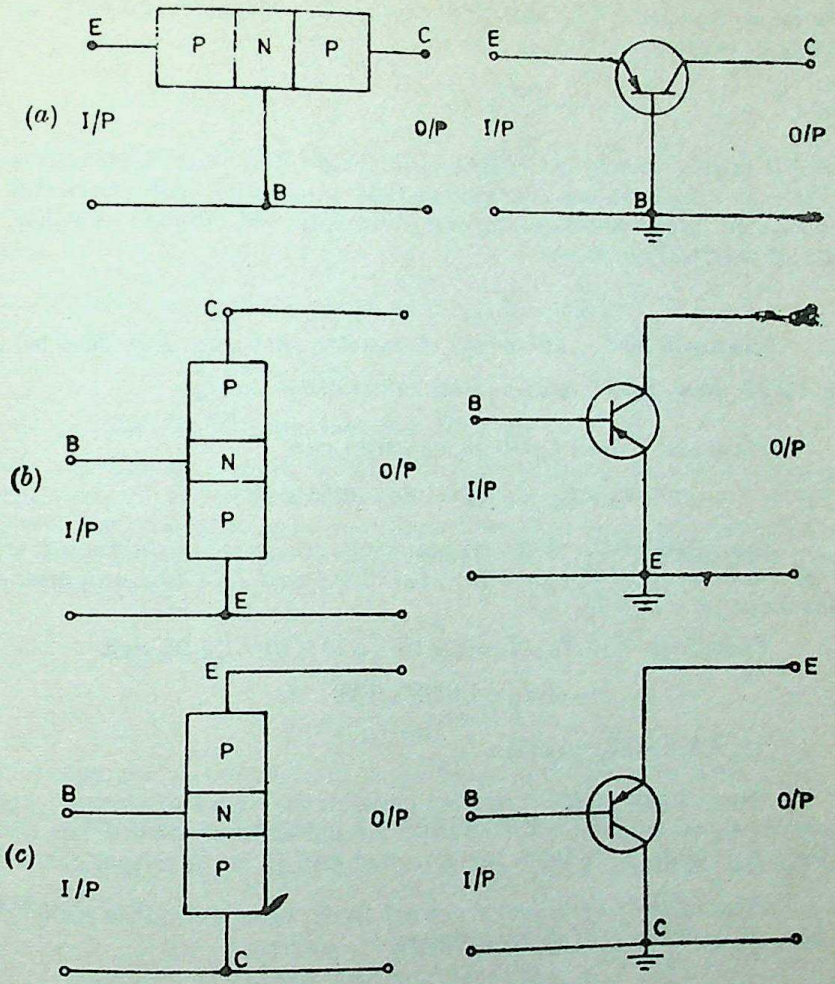

Fig. 30-5

The α of a transistor is a measure of the quality of a transistor : higher the value of α, better the transistor in the sense that collector current more closely equals the emitter current. Its value ranges from 0·95 to 0·999. Obviously, it applies *only to CB configuration of a transistor.* As seen from above,

$$I_C = \alpha I_B$$

Now, $I_B = I_E - I_C$

$$= I_E - \alpha I_E$$

$$\therefore \quad I_B = (1-\alpha) I_E \qquad \text{–Fig. 30-6}$$

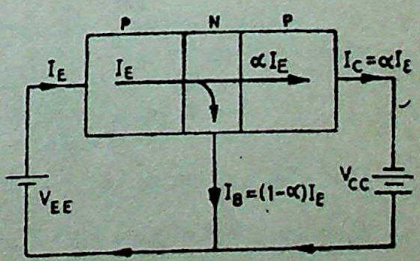

Fig. 30-6

Incidentally, there is also an

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ac α for a transistor. It refers to *change* in collector current to *change* in emitter current.

$$\alpha_{ac} = -\frac{\triangle I_C}{\triangle I_E}$$

It is also known as *short-circuit gain* of a transistor and is written as $-h_{fb}$. It may be noted that upper case subscripts *FB* indicate *dc* value whereas lower case subscripts *'fb'* indicate *ac* value. For all practical purposes,

$$\alpha_{dc} = \alpha_{ac}$$

Example 30.1. *If for the transistor shown in Fig. 30.5 (a),* $\alpha = 0.95$ *and* $I_E = 1$ *mA, find the values of* I_C *and* I_B.

Solution. $I_C = \alpha I_E = 0.95 \times 1 = \mathbf{0.95\ mA}$

$$I_B = I_E - I_C = 1 - 0.95 = \mathbf{0.05\ mA}$$

Example 30.2. *Following current readings are obtained in transistor circuit of Fig. 30.5 (a) :* $I_E = 2$ *mA and* $I_B = 20\ \mu A$. *Compute the values of* α *and* I_C.

Solution $I_C = I_E - I_B = 2 \times 10^{-3} - 20 \times 10^{-6} = \mathbf{1.98\ mA}$

$$\alpha = I_C/I_E = 1.98/2 = \mathbf{0.99}$$

(*b*) CE Configuration

Here, input signal is applied between the base and emitter and output signal is taken out from collector and emitter circuits. As seen from Fig. 30.5 (*b*), I_B is the input current and I_C is the output current.

The ratio of dc collector current to dc base current is called dc beta (β_{dc}) or just β of the transistor.

$$\beta = \frac{I_C}{I_B} \quad \text{or} \quad I_C = \beta I_B$$

It is also called common-emitter dc forward transfer ratio and is written as h_{FE}. It is possible for β to have as high a value as 500.

While analysing ac operation of a transistor, we use ac β which is given by

$$\beta_{ac} = \frac{\triangle I_C}{\triangle I_B}$$

It is also written as h_{fe}.

Relation Between α and β

$$\beta = \frac{I_C}{I_B} \text{ and } \alpha = \frac{I_C}{I_E} \quad \therefore \quad \frac{\beta}{\alpha} = \frac{I_E}{I_B}$$

Now, $I_B = I_E - I_C$

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$$\therefore \qquad \beta = \frac{I_C}{I_E - I_C} = \frac{I_C/I_E}{I_E/I_E - I_C/I_E}$$

$$\therefore \qquad \beta = \frac{\alpha}{1-\alpha} \qquad \ldots(i)$$

Cross-multiplying the above equation and simplifying it, we get

$$\beta(1-\alpha) = \alpha \quad \text{or} \quad \beta = \alpha(1+\beta)$$

$$\therefore \qquad \alpha = \frac{\beta}{1+\beta} \qquad \ldots(ii)$$

It is seen from the above two equations that

$$1-\alpha = \frac{1}{1+\beta}$$

(*c*) **CC Configuration** [Fig. 30-5 (*c*)]

In this case, input signal is applied between base and collector and output signal is taken out from emitter-collector circuit. Obviously, conventionally speaking, here I_B is the input current and I_E is the output current as shown in Fig. 30-5 (*c*). The current gain of the circuit is

$$\frac{I_E}{I_B} = \frac{I_E}{I_C} \cdot \frac{I_C}{I_B} = \frac{1}{\alpha} \cdot \beta$$

$$= \frac{\beta}{\alpha} = \frac{\beta}{\beta/(1+\beta)} = (1+\beta)$$

It means that output current is $(1+\beta)$ times the input current.

30-7 Relations Between Transistor Currents

While deriving various equations, following definitions should be kept in mind.

$$\alpha = I_C/I_E \quad \text{and} \quad \beta = I_C/I_B$$

Also, $\alpha = \beta/(1+\beta)$ and $\beta = \alpha/(1-\alpha)$

(*i*) $I_C = \beta I_B$

$$= \alpha I_E = \frac{\beta}{1+\beta} \cdot I_E$$

(*ii*) $I_B = \dfrac{I_C}{\beta}$

$$= \frac{I_E}{(1+\beta)} = (1-\alpha) I_E$$

(*iii*) $I_E = \dfrac{I_C}{\alpha} = \dfrac{(1+\beta)}{\beta} I_C$

$$= (1+\beta) I_B = \frac{I_B}{(1-\alpha)}$$

Incidentally, it may be noted that for *ac* currents, small letters i_e, i_b and i_c are used.

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30-8. Leakage Currents in a Transistor

(a) CB Circuit

Consider the *CB* transistor circuit shown in Fig. 30-7. The emitter current (due to majority carriers) initiated by the forward-biased emitter-base junction is split into two parts :—

(i) $(1-\alpha)I_E$ which becomes base current I_B in the external circuit and

(ii) αI_E which becomes collector current I_C in the external circuit.

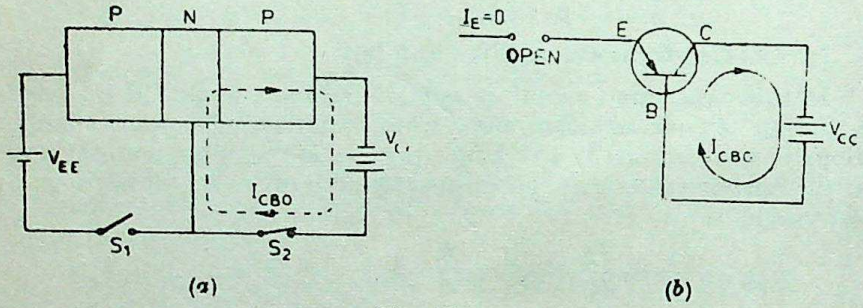

Fig. 30-7

As mentioned earlier (Art. 30-2) though *C/B* junction is reverse-biased for majority charge carriers (*i.e.* holes in this case), it is forward-biased so far as thermally-generated minority charge carriers (*i.e.* electrons) are concerned. This current flows *even when emitter is disconnected from its dc supply* as shown in Fig. 30-7 (*a*) where switch S_1 is open. It flows in the *same* direction* as the collector current of majority carriers. It is called leakage current I_{CBO}. The subscripts *CBO* stand for 'current from *C*ollector to *B*ase with emitter *O*pen'. Very often, it is simply written as I_{CO}.

It should be noted that

(i) I_{CBO} is exactly like the reverse saturation current I_S or I_O of a reverse-biased diode discussed in Art. 29-16.

(ii) I_{CBO} is extremely temperature-dependent because it is made up of thermally-generated minority carriers. As mentioned earlier, I_{CBO} doubles for every 10°C rise in temperature for Ge and 6°C for Si (Art. 29-16).

If we take into account the leakage current, the current distribution in a *CB* transistor circuit becomes as shown in Fig. 30-8 both for *PNP* and *NPN* type transistors.

*Actually, electrons (which form minority charge carriers in collector) flow from negative terminal of collector battery, to collector, then to base through *C/B* junction and finally to positive terminal of V_{CC}. However, conventional current flows in opposite direction as shown by dotted line in Fig. 30-7 (*a*).

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It is seen that total collector current is actually the sum of two components :

(*i*) current produced by normal transistor action *i.e.* component controlled by emitter current. Its value is αI_E and is due to majority carriers

(*ii*) temperature-dependent leakage current I_{CO} due to minority carriers

$$\therefore \quad I_C = \alpha I_E + I_{CO}$$

$$= \underbrace{\alpha I_E}_{\text{majority}} + \underbrace{I_{CO}}_{\text{minority}}$$

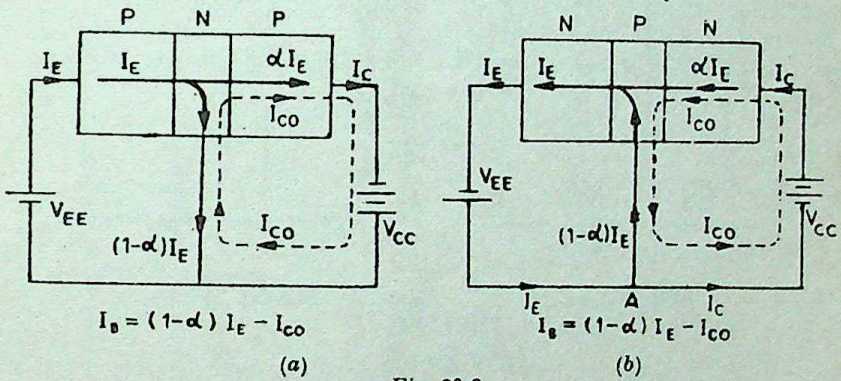

Fig. 30-8

It can also be proved that

$$I_C = \frac{I_{CO}}{1-\alpha} + \frac{\alpha I_B}{1-\alpha}$$

In view of the above, it would be appreciated that

$$\alpha = -\frac{I_C - I_{CO}}{I_E} \cong -\frac{I_C}{I_E}$$

It can also be proved that

$$I_B = (1-\alpha) I_E - I_{CO}$$

(*b*) **CE Circuit**

In Fig. 30-9 (*a*) is shown a grounded-emitter circuit of an *NPN* transistor whose base lead is open. It is found that despite $I_B = 0$,

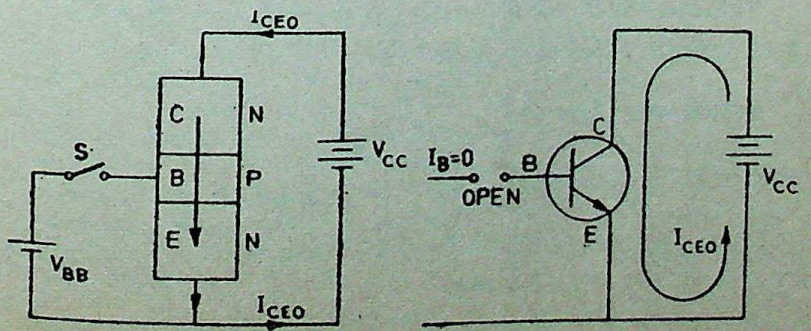

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Fig. 30-9

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there is a leakage current from collector to emitter. It is called I_{CEO}, the subscripts *CEO* standing for '*C*ollector to *E*mitter with base *O*pen.'

Taking this leakage current into account, the current distribution through a *CE* circuit becomes as shown in Fig. 30-10.

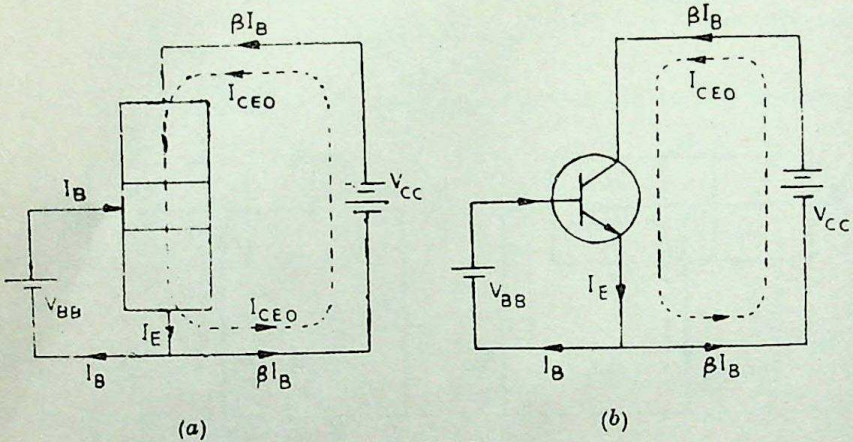

Fig. 30-10

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1+\beta) I_{CO}$$
$$= \beta I_B + I_{CO}/(1-\alpha) = \alpha I_E + (1-\alpha) I_{CEO}$$
$$= \frac{\alpha I_B}{1-\alpha} + \frac{I_{CO}}{1-\alpha}$$

Now, $\beta I_B = \alpha I_E$. Substituting this value above we get,

$$I_C = \alpha I_E + I_{CEO}$$

Also, $I_B = I_E - I_C$

Substitutiug the value of I_C from above, we have

$$I_B = I_E - \alpha I_E - I_{CEO}$$
$$= (1-\alpha) I_E - I_{CEO}$$

Example 30-3. *A transistor has* $\alpha = 0{\cdot}98$, $I_B = 100\ \mu A$ *and* $I_{CO} = 6\ \mu A$. *Calculate* I_C *and* I_E.

Solution. $$I_C = \frac{\alpha I_B}{1-\alpha} + \frac{I_{CO}}{1-\alpha}$$
$$= \frac{0\ 98 \times 100}{1-0{\cdot}98} + \frac{6}{1-0{\cdot}98}$$
$$= 4900 + 300 = 5200\ \mu A$$
$$= \mathbf{5{\cdot}2\ mA}$$
$$I_E = I_C + I_B = 5200 + 100 = 5300\ \mu A$$
$$= \mathbf{5{\cdot}3\ mA}$$

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30.9. Transistor Static Characteristics

These are the curves which represent relationships between different dc currents and voltages of a transistor. These are helpful in studying the operation of a transistor when connected in a circuit. The three important characteristics of a transistor are

(*i*) *input characteristic*

(*ii*) *output characteristic*

(*iii*) *constant-current transfer characteristic*

30.10. Common Base Test Circuit

The static characteristics of an *NPN* transistor connected in common-base configuration can be determined by the use of test circuit shown in Fig. 30-11. Milliammeters are included in series

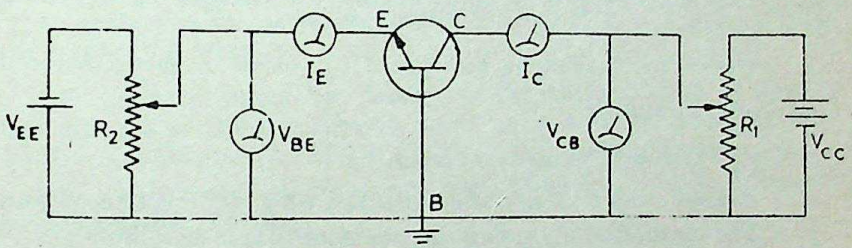

Fig. 30-11

with the emitter and collector circuits to measure I_E and I_C. Similarly, voltmeters are connected across E and B to measure voltage V_{BE}* and across C and B to measure V_{CB}.* The two potentiometer resistors R_1 and R_2 supply variable voltages from the collector and emitter dc supplies.

30.11. Common Base Static Characteristics

(*a*) Input characteristic

It shows how I_E varies with V_{BE} when voltage V_{CB} is held constant. The method of determining the characteristic is as follows :

First, voltage V_{CB} is adjusted to a suitable value with the help of R_1 (Fig. 30-11). Next, voltage V_{BE} is increased in a number of discrete steps and corresponding values of I_E are noted from the milliammeter connected for the purpose. When plotted, we get the input characteristics shown in Fig. 30-12, one for Ge and the

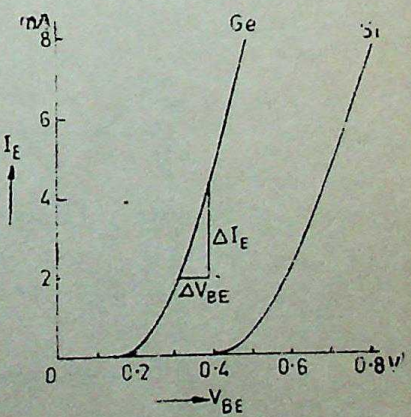

Fig. 30-12

*In these subscripts, first letter indicates that electrode which is at higher potential as compared to the other electrode.

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other for Si. Both curves are exactly similar to the forward characteristic of a *P-N* diode which, in essence, is what the emitter-base junction is.

This characteristic may be used to find the input resistance of the transistor. Its value is given by *the reciprocal of its slope.*

$$R_{in} = \frac{\triangle V_{BE}}{\triangle I_E} \qquad -V_{CB} \text{ constant}$$

Since the characteristic is initially non-linear, R_{in} will vary with the point of measurement. Its value over linear part of the characterstic is about 50 Ω but for low values of V_{BE}, it is considerably greater. This change in R_{in} with change in V_{BE} gives rise to distortion of signals handled by the transistor.

This characteristic is hardly affected by changes either in V_{CB} or temperature.

Note. Strictly speaking, both I_E and V_{BE} should be shown negative for an *NPN* transistor.

(*b*) Output Characteristic

It shows the way I_C varies with V_{CB} when I_E is held constant.

The method of obtaining this characteristic is as follows :

First, movable contact on R_2 (Fig. 30-11) is changed to get a suitable value of V_{BE} and hence I_E. While keeping I_E constant at this value, V_{CB} is increased from zero in a number of steps and the corresponding collector current I_C that flows is noted.

Next, V_{CB} is reduced back to zero, I_E is increased to a value a little higher than before and the whole procedure is repeated. In this way, whole family of curves is obtained, a typical family being shown in Fig. 30-13.

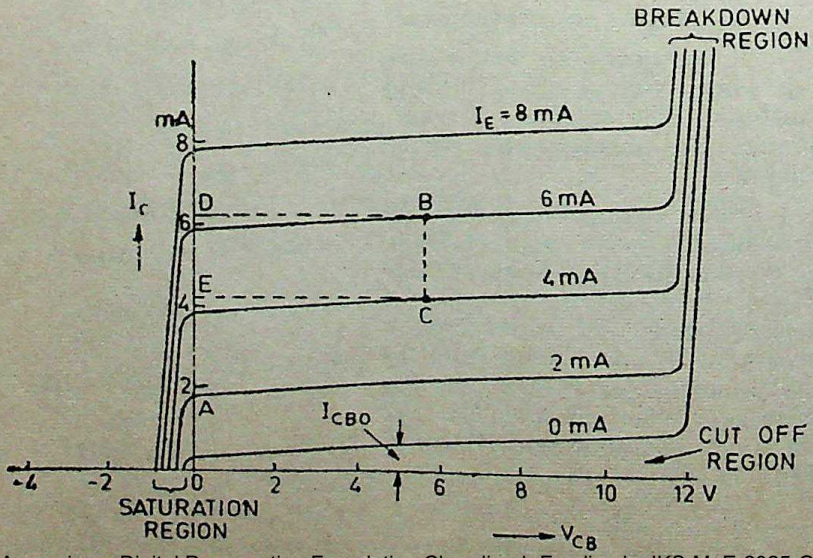

Fig. 30-13

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1. The reciprocal of the slope of the near horizontal part of the characteristic gives the output resistance R_{out} of the transistor which it would offer to an input signal. Since the characteristic is linear over most of its length (meaning that I_C is virtually independent of V_{CB}), R_{out} is very high, a typical value being 500 kΩ.

$$R_{out} = \frac{1}{\Delta I_C / \Delta V_{CB}} = \frac{\Delta V_{CB}}{\Delta I_C}$$

2. It is seen that I_C flows even when $V_{CB}=0$. For example, it has a value $=OA$ corresponding to $I_E = 2$ mA as shown in Fig. 30-13. It is due to the fact that electrons are being injected into the base under the action of forward-biased E/B junction and are being collected by the collector due the action of the internal junction voltage at the C/B junction (Art. 30-2). For reducing I_C to zero, it is essential to neutralize this potential barrier by applying a small forward bias across C/B junction.

3. Another important feature of the characteristic is that a small amount of collector current flows even when emitter current $I_E=0$. As we know (Art. 30-8), it is collector leakage current I_{CBO}.

4. This characteristic may be used to find α_{ac} of the transistor as shown in Fig. 30-13.

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E} = \frac{DE}{BC} = \frac{6{\cdot}2 - 4{\cdot}3}{2} = 0{\cdot}95$$

5. Another point worth noting is that although I_C is practically independent of V_{CB} over the working range of the transistor, yet if V_{CB} is permitted to increase, I_C eventually increases rapidly due to avalanche breakdown as shown.

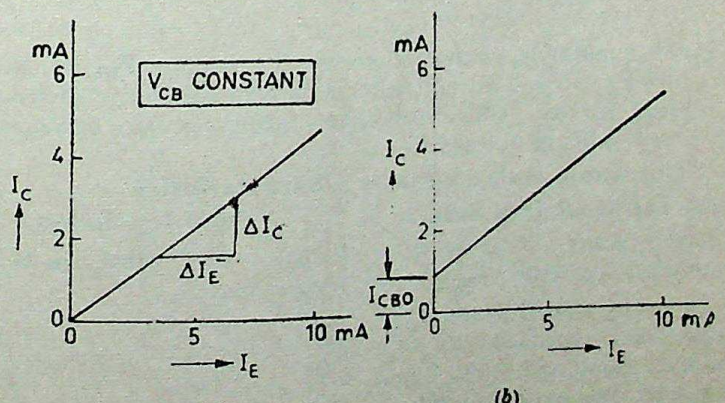

Fig. 30-14

(c) Current Transfer Characteristic

It shows how I_C varies with changes in I_E when V_{CB} is held constant. For drawing this characteristic, first V_{CB} is set to a convenient value and then I_B is increased in steps and corresponding values of I_C noted. A typical transfer characteristic is shown in Fig. 30-14 (*a*). Fig. 30-14 (*b*) shows a more detailed view of the portion near the origin.

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As seen, α_{ac} may be found from the equation

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E}$$

Usually, α_{ac} is found from output characteristic than from this characteristic.

It may be noted in the end that *CB* connection is rarely employed for audio-frequency circuits because firstly, its current gain is less than unity and secondly, its input and output resistances are so different.

Note. Strictly speaking, for an NPN transistor, I_C should be positive but I_E should be negative.

30-12. Common Emitter Test Circuit

The static characteristics of a transistor connected in *CE* configuration may be determined by the use of circuit diagram shown in Fig. 30-15. A milliammeter (or a microammeter in the case of a low-power transistor) is connected in series with the base to measure I_B. Similarly, an ammeter is included in the collector circuit to measure I_C. A voltmeter with a typical range of 0—1 V is connected across base and emitter terminals for measuring V_{BE}.

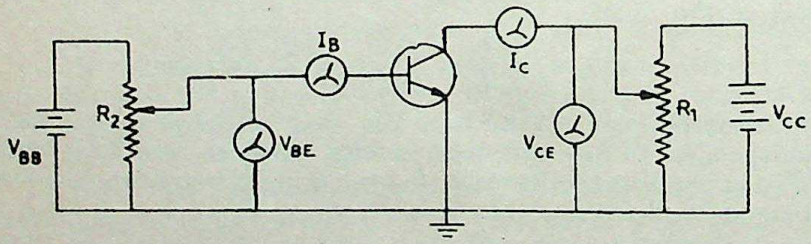

Fig. 30-15

Potentiometer R_2 connected across d.c. supply V_{BB} is used to vary I_B and V_{BE}. A second voltmeter with a typical range of 0—20 V is connected across collector-emitter terminals to measure the output collect-emitter voltage V_{CE}.

30-13. Common Emitter Static Characteristics

(a) Input Characteristic *(A.M.I.E. Summer 1984)*

It shows how I_B varies with change in V_{BE} when V_{CE} is held constant at a particular value.

To begin with, voltage V_{CE} is maintained constant at a convenient value and then V_{BE} is increased in steps. Corresponding values of I_B are noted at each step. The procedure is then repeated for a different but constant value of V_{CE}. A typical input characteristic is shown in Fig. 30-16. Like *CB* connection, the overall shape resembles the forward characteristic of a *P-N* diode.

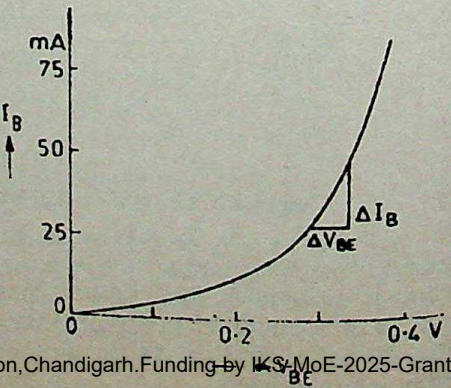

Fig. 30-16

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The reciprocal of the slope gives the input resistance R_{in} of the transistor.

$$R_{in} = \frac{1}{\triangle I_B / \triangle V_{BE}} = \frac{\triangle V_{BE}}{\triangle I_B}$$

Due to initial non-linearity of the curve, R_{in} varies considerably from a value of 4 kΩ near the origin to a value of 600 Ω over the more linear part of the curve.

(*b*) Output or Collector Characteristic

It indicates the way in which I_C varies with changes in V_{CE} when I_B is held constant.

For obtaining this characteristic, first I_B is set to a convenient value and maintained constant and then V_{CE} is increased from zero in steps, I_C being noted at each step. Next, V_{CE} is reduced to zero and I_B increased to another convenient value and the whole procedure repeated. In this way, a family of curves (Fig. 30-17) is obtained.

It is seen that as V_{CE} increases from zero, I_C rapidly increases to a near saturation level for fixed value of I_B. As shown, a small amount of collector current flows even when $I_B = 0$. It is called I_{CEO} (Art. 30-8). Since main collector current is zero, the transistor is said to be *cut-off*.

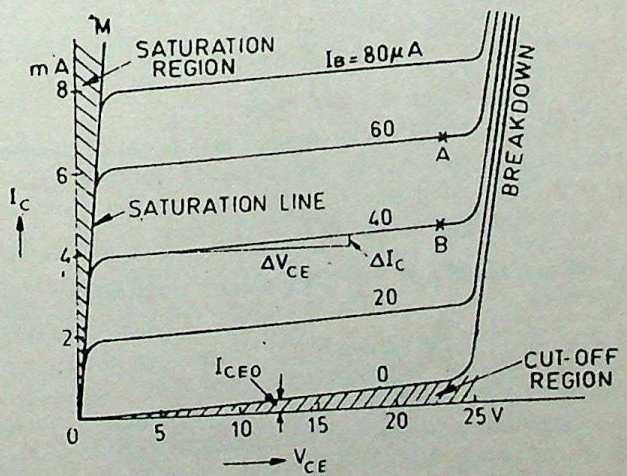

Fig. 30-17

It may be noted that if V_{CE} is allowed to increase too far, *C/B* junction completely breaks down and due to this avalanche breakdown, I_C increases rapidly and may cause damage to the transistor.

When V_{CE} has very low value (ideally zero), the transistor is said to be *saturated* and it operates in the saturation region of the characteristic. Here, change in I_B does not produce a corresponding change in I_C.

This characteristic can be used to find β.

$$\beta = \triangle I_C / \triangle I_B$$

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We may select any two points A and B on the $I_B = 60\ \mu A$ and $40\ \mu A$ lines respectively and measure corresponding values of I_C from the diagram for finding $\triangle I_C$. Since $\triangle I_B = (60 - 40) = 20\ \mu A$, β can be found.

The value of output resistance $R_{out} = \triangle V_{CE} / \triangle I_C$ over the near horizontal part of the characteristic varies from 10 kΩ to 50 kΩ.

(c) Current Transfer Characteristic

It indicates how I_C varies with changes in I_B when V_{CE} is held constant at a given value.

Such a typical characteristic is shown in Fig. 30-18 (*a*). Its slope gives

$$\beta = \triangle I_C / \triangle I_B.$$

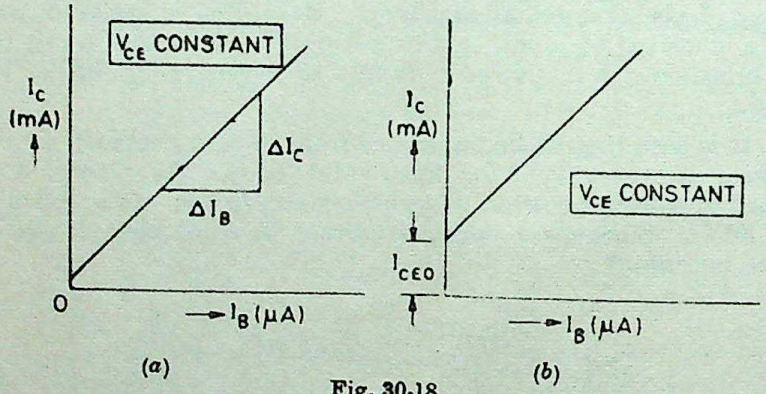

Fig. 30-18

From Fig. 30-18 (*b*), it is seen that a small collector current flows even when $I_B = 0$. It is the common-emitter leakage current $I_{CEO} = (1 + \beta)\ I_{CO}$. Like I_{CO}, it is also due to the flow of minority carriers across the reverse-biased C/B junction.

30-14. Different Ways of Drawing Schematic Transistor Circuits

(A.M.I.E. Summer 1983)

In Fig. 30-19 (*a*) is shown a CB transistor circuit which derives its voltage and current requirements from two independent power sources *i.e.* two different batteries. Correct battery connections can be done by remembering the transistor polarity rule (Art. 30-2) that in an NPN transistor, both collector and base have to be POSITIVE with respect to the emitter. Of course, collector is a litttle more positive than base which means that between themselves, collector is at a slightly higher positive potential with respect to base. Conversely, base is at a little lower potential with respect to collector.

Putting it in a slightly different way, we can say that collector is positive w.r.t. base and conversely, base is negative w.r.t. collector. That is why, potential difference between collector and base is written as V_{CB} (and not V_{BC}) because terminal at higher potential is mentioned first (footnote Art. 30-10). Same reasoning applies to V_{BE}.

Fig. 30-19 (*b*) shows another and more popular way of indicating

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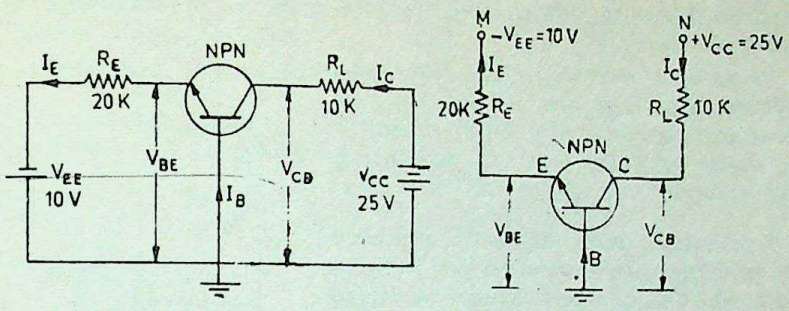


(a) (b)

Fig. 30-19

power supply voltage. Only one terminal of the battery is shown, the other terminal of each power supply is understood to be grounded so as to provide a complete path for the current. For example, negative terminal of V_{CC} and positive terminal of V_{EE} are supposed to be grounded (as is the base) even though not shown in the diagram.

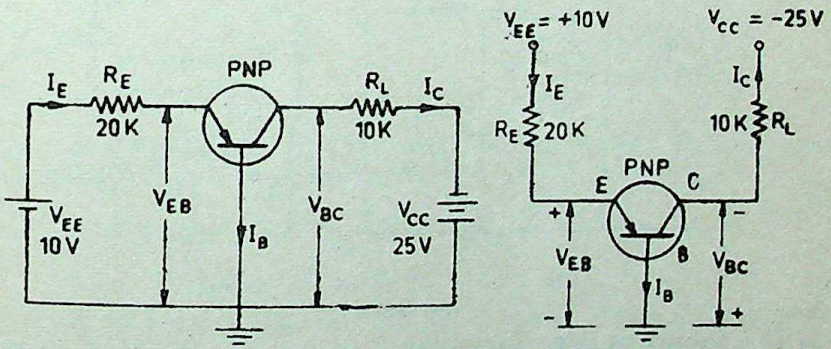
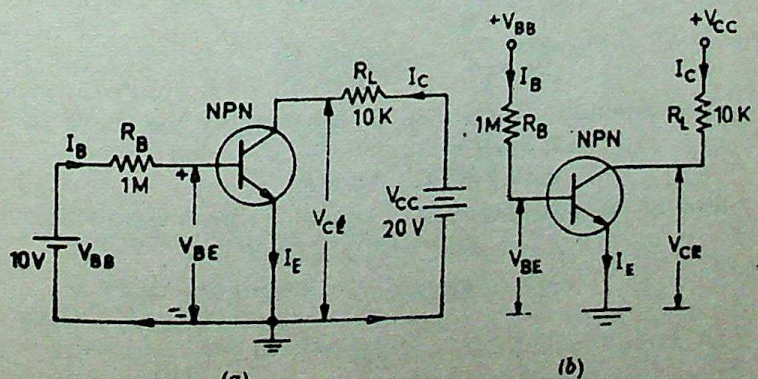


Fig. 30-20

Fig 30-20 shows same circuit drawn for a *PNP* transistor. As expected, in this case, the battery polarities and directions of current flow are opposite to those shown in Fig. 30-19.



(a) (b)

Fig. 30-21

Fig. 30-21 (a) shows an *NPN* transistor connected in *CE*

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configuration with voltage and currents drawn from two independent power sources.

As seen, battery connections and voltage markings are as per the rule given in Art. 30-2. Fig. 30-21 (*b*) shows the more popular way of indicating power-supply voltages.

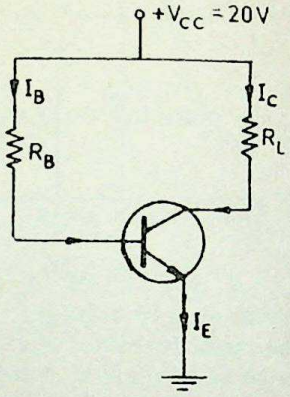

Fig. 30-22

As seen, both collector and base are positive with respect to the common electrode *i.e.* emitter. Hence, a single battery can be used to get proper voltages across the two as shown in Fig. 30-22.

Fig. 30-23 (*a*) shows the *CC* configuration of an *NPN* transistor and Fig. 30-23 (*b*) shows the same circuit drawn differently.

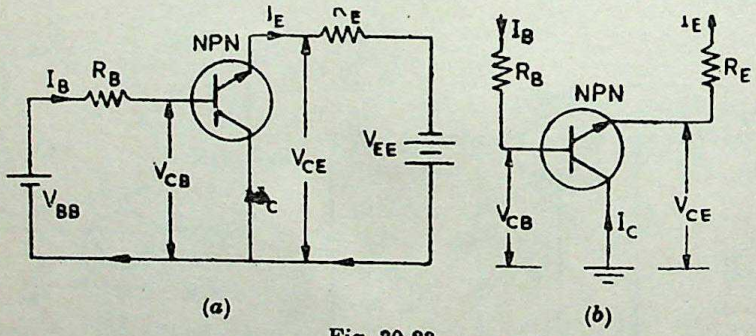

Fig. 30-23

30-15. Common Base Formulas

Let us find the values of different voltages and currents for the circuit shown in Fig. 30-19 (*b*). Consider circuit *MEBM*. Applying Kirchhoff's current law, we get

(*a*) $$I_E = \frac{V_{EE} - V_{BE}}{R_E}$$

where $V_{BE} = 0{\cdot}3$ V —for Ge
$= 0{\cdot}7$ V —for Si

Since generally, $V_{EE} \gg V_{BE}$, we can simplify the above to

$$I_E \cong V_{EE}/R_E$$
$$\cong 10 \text{ V}/20 \text{ K} = 0{\cdot}5 \text{ mA}$$

Taking V_{BE} into account and assuming silicon transistor,

$$I_E = (10 - 0{\cdot}7) \text{ V}/20 \text{ K} = 0{\cdot}465 \text{ mA}$$

(*b*) $I_C = \alpha I_E \cong I_E$ —neglecting leakage current I_{CO} (Art. 30-8)

(*c*) From circuit *NCBN*, we get

$$V_{CB} = V_{CC} - I_C R_L$$
$$\cong V_{CC} - I_E R_L \quad \text{—} \because I_C \cong I_E$$
$$V_{CB} = 25 - 0{\cdot}5 \times 10 = 20 \text{ V}$$

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Example 30-4. *In the circuit of Fig. 30-24 (a), what value of R_L causes $V_{CB}=5$ V ?*

Solution. $I_E \cong V_{EE}/R_E = 10\text{ V}/10\text{ K} = 1\text{ mA}$

$I_C = \alpha I_E \cong I_E = 1\text{ mA}$

Now, $V_{CC} = I_C R_L + V_{CB}$

$$\therefore \quad R_L = \frac{V_{CC}-V_{CB}}{I_C} = \frac{20-5}{1\text{ mA}} = \mathbf{15\text{ K}}$$

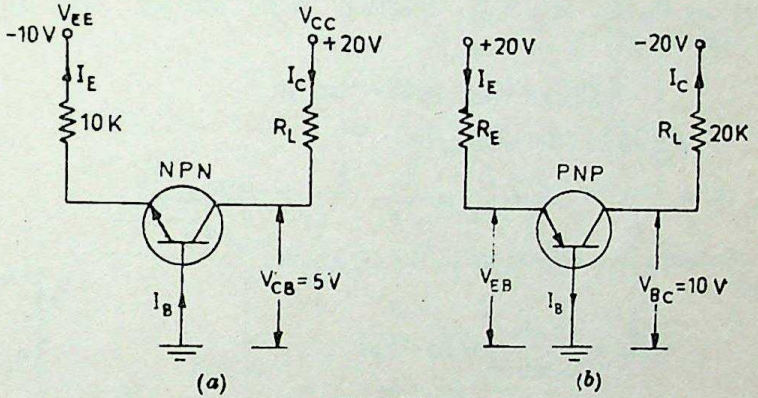

Fig. 30-24

Example 30-5. *For the circuit shown in Fig. 30-24 (b), find the value of R_E which causes $V_{BC}=10$ V.*

Solution. $$I_C = \frac{V_{CC}-V_{BC}}{R_L} = \frac{20-10}{20\text{ K}} = 0{\cdot}5\text{ mA}$$

Now, $I_E = I_C/\alpha \cong I_C = 0{\cdot}5\text{ mA}$

If we neglect V_{EB}, then entire $V_{EE}=20$ V has to be dropped across R_E.

$\therefore \quad 0{\cdot}5\ R_E = 20$ or $R_E = 20/0{\cdot}5\text{ mA} = \mathbf{40\text{ K}}$

30-16. Common Emitter Formulas

Consider the *CE* circuit of Fig. 30-25.

Taking the emitter-base circuit, we have

$$I_B = \frac{V_{BB}-V_{BE}}{R_B} \cong \frac{V_{BB}}{R_B}$$

$I_C = \beta I_B$

—neglecting leakage current I_{CEO} (Art. 30-8)

$V_{CE} = V_{CC} - I_C R_L$

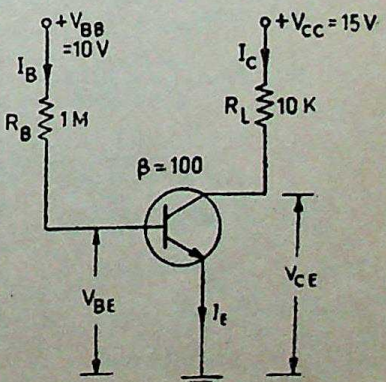

Fig. 30-25

Example. 30-6. *From circuit of Fig. 30-25, find*

(i) I_B (ii) I_C (iii) I_E and V_{CE}. Neglect V_{BE}.

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Solution. (*i*) $I_B = V_{BB}/R_B = 10/1\ \text{M} = \mathbf{10\ \mu A}$

(*ii*) $I_C = \beta\, I_B = 100 \times 10\ \mu\text{A} = \mathbf{1\ mA}$

(*iii*) $I_E = I_B + I_C = 1\ \text{mA} + 10\ \mu\text{A} = \mathbf{1{\cdot}01\ mA}$

(*iv*) $V_{CE} = V_{CC} - I_C R_L = 15 - 1 \times 10 = \mathbf{5\ V}$

Example 30-7. *Find the exact value of emitter current I_E in the two-supply emitter-bias circuit of Fig. 30-26.*

Solution. Let us apply Kirchhoff's voltage law to the loop containing R_B, R_E and V_{EE}. Starting from emitter and going clockwise, we get

$$-I_E R_E + V_{EE} - I_B R_B - V_{BE} = 0$$

or $$I_E R_E + I_B R_B = V_{EE} - V_{BE} \qquad \ldots(i)$$

Now, $$\beta = \frac{I_C}{I_B} \cong \frac{I_E}{I_B} \qquad \therefore \quad I_B \cong \frac{I_E}{\beta}$$

Substituting this value in Eq. (*i*) above, we get

$$I_E R_E + \frac{I_E R_B}{\beta} = V_{EE} - V_{BE}$$

$$\therefore \quad I_E = \frac{V_{EE} - V_{BE}}{R_E + R_B/\beta}$$

Since, in most cases, $(R_B/\beta) \ll R_E$,

$$\therefore \quad I_E \cong \frac{V_{EE} - V_{BE}}{R_E}$$

$$\cong \frac{V_{EE}}{R_E}$$

Also, $$I_B = I_E/(1+\beta)$$

$$\cong I_E/\beta.$$

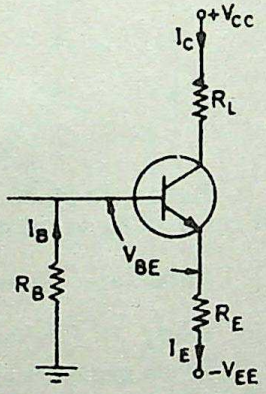

Fig. 30-26

Example 30-8. *In the circuit of Fig. 30-27, find* (*i*) I_E (*ii*) I_B (*iii*) I_C *and* (*iv*) V_{CE}. *Neglect* V_{BE} *and take* $\beta = 100$.

Solution.

(*i*) $$I_E = \frac{V_{EE}}{R_E + R_B/\beta}$$

$$= \frac{30}{30 + 20/100} \cong \mathbf{1\ mA}$$

(*ii*) $$I_B \cong I_E/\beta = 1/100$$

$$= \mathbf{0{\cdot}01\ mA}$$

(*iii*) $$I_C = I_E + I_B$$

$$= \mathbf{1{\cdot}01\ mA}$$

(*iv*) $$V_{CE} = V_{CC} - I_C R_L$$

$$= 30 - 10 \times 1{\cdot}01 \cong \mathbf{20\ V}$$

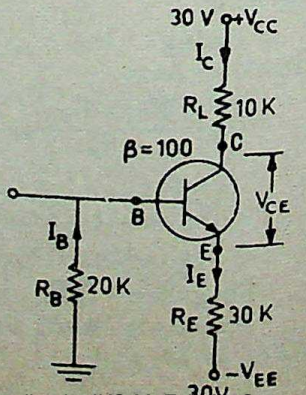

Fig. 30-27

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30-17. Cut-off and Saturation Points

Consider circuit of Fig. 30-28 (*a*). As seen from Art. 30-16

$$V_{CE}=V_{CC}-I_CR_L \qquad ...(i)$$

Since $I_B=0$, $I_C=0$. Hence

$$V_{CE}=V_{CC}$$

Under these conditions, transistor is said to be *cut-off* for the simple reason that transistor does not conduct any current. This value of V_{CE} is written as $V_{CE(cut-off)}$.

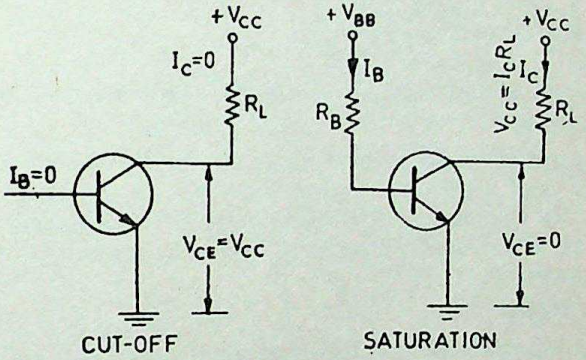

Fig. 30-28

If in Fig. 30-28 (*b*), values of R_B and R_L are such that V_{CE} comes out to be zero, then transistor is said to be saturated. Puting $V_{CE}=0$ in the above equation, we get

$$0=V_{CC}-I_CR_L \quad \text{or} \quad I_C=V_{CC}/R_L.$$

It is obvious that

(*i*) whole of V_{CC} drops across R_L

(*ii*) collector current has maximum possible value called $I_{C(sat)}$.

Normal operation of a transistor lies between the above two extreme conditions of cut-off and saturation.

30-18. D.C. Load Line and Active Region

For drawing the d.c. load line of a transistor, one need to know only its cut-off and saturation points. It is a straight line joining these two points. For the *CE* circuit of Fig. 30-25, the load line is drawn in Fig. 30-29. *A* is the *cut-off* point and *B* is the *saturation* point.

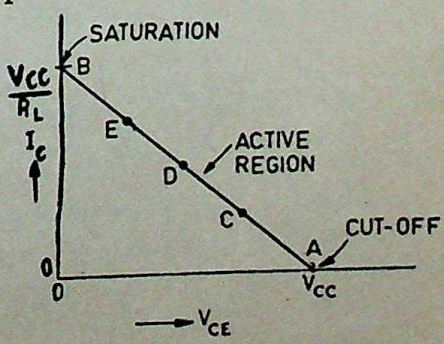

Fig. 30-29

The voltage equation of the collector-emitter circuit is

$$V_{CC}=I_CR_L+V_{CE}$$

$$\therefore\ I_C=\frac{V_{CC}}{R_L}-\frac{V_{CE}}{R_L}$$

Consider the following

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two particular cases :

(*i*) When $I_C=0$; $V_{CE}=V_{CC}$ —cut-off point A

(*ii*) When $V_{CE}=0$, $I_C=V_{CC}/R_L$ —saturation point B

Obviously, load line can be drawn if only V_{CC} and R_L are known.

Incidentally, slope of load line $AB=-1/R_L$.

Note. The above-given equation can be written as

$$I_c=-\frac{V_{CE}}{R_L}+\frac{V_{CC}}{R_L}$$

It is a linear equation similar to

$$y=mx+C$$

The graph of this equation is a straight line whose slope is $m=-1/R_L$.

Active Region

All operating points (like C, D, E etc. in Fig. 30-29 lying between cut-off and saturation points form the active region of the transistor. In this region, E/B junction is forward-biased and C/B junction is reverse-biased—conditions necessary for the proper operation of a transistor.

Quiescent Point

It is a point on the d.c. load line which represents values of I_C and V_{CE} that exist in a transistor circuit when no input signal is applied.

It is also known as the d.c. *operating* point or *working* point. The best position for this point is midway between cut-off and saturation points where $V_{CE}=\frac{1}{2}V_{CC}$ (like point D in Fig. 30-29).

Example 30-9. *For the circuit shown in Fig. 30-30 (a), draw the dc load line and locate its quiescent or d.c. working point.*

Solution. The cut-off point is easily found because it lies along X-axis where $V_{CE}=V_{CC}=20$ V *i.e.* point A in Fig. 30-30 (*b*). At saturation point B, saturation value of collector current is

$$I_{C(sat)}=V_{CC}/R_L=20/5\text{ K}=4\text{ mA}$$ —point B

The line AB represents the load line for the given circuit.

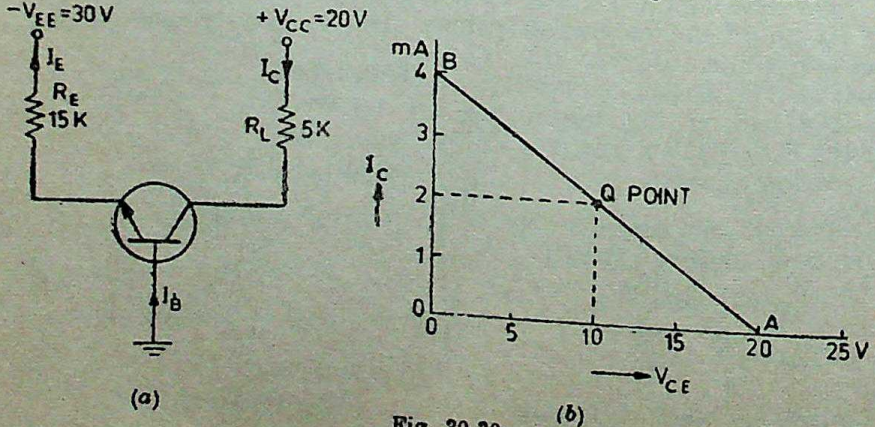

Fig. 30-30

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We will now find the actual operating point.

$$I_E = V_{EE}/R_E = 30/15 \text{ K} = 2\text{mA} \quad \text{—neglecting } V_{BC}$$

$$I_C = \alpha I_E = I_E \cong 2 \text{ mA} \quad \text{—assuming } \alpha = 1$$

$$\therefore \quad V_{CE} = V_{CC} - I_C R_L = 20 - 2 \times 5 = 10 \text{ V}$$

Hence, Q-point is located at (10 V, 2 mA) as shown in Fig. 30-30 (b).

Example 30-10. *For the CE circuit shown in Fig. 30-31 (a), draw the dc load line and mark the d.c. working point on it. Assume* $\beta = 100$ *and neglect* V_{BE}.

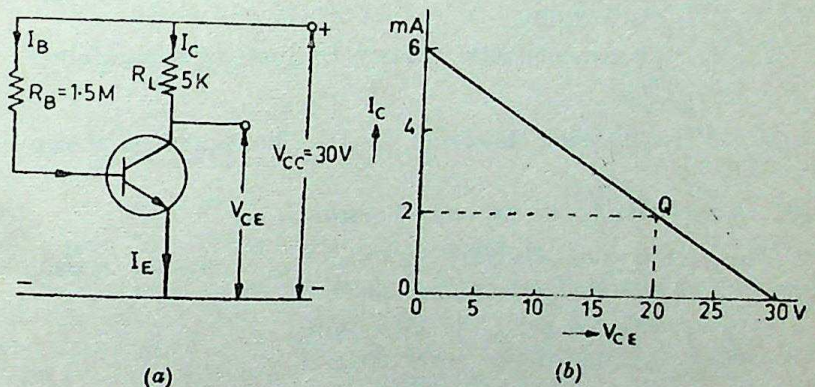
Solution. Cut-off point A is located where

$$I_C = 0 \text{ and } V_{CE} = V_{CC} = 30 \text{ V}$$

Saturation point B is given where

$$V_{CE} = 0 \text{ and } I_{C(sat)} = 30/5 \text{ K} = 6 \text{ mA}$$

Line AB represents the load line in Fig. 30-31 (b).



(a) (b)

Fig. 30-31

Let us find the d.c. working point from the given values of resistances and supply voltage.

$$I_B = 30/1{\cdot}5 \text{ M} = 20 \ \mu\text{A}$$

$$I_C = \beta I_B = 100 \times 20 = 2000 \ \mu\text{A} = 2 \text{ mA}$$

$$V_{CE} = V_{CC} - I_C R_L = 30 - 2 \times 5 = 20 \text{ V}$$

Hence, Q point is (20 V, 2 mA) as shown in Fig. 30-31 (b)

30-19. Need for Biasing a Transistor

For normal operation of a transistor amplifier circuit, it is essential that there should be a

(a) forward bias on emitter-base junction and

(b) reverse bias on the collector-base junction.

In addition, amount of bias required is important for establishing the Q-point which is dictated by mode of operation desired.

If the transistor is not biased correctly it would

(a) work inefficiently and

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(*ii*) produce distortion in the output signal.

Some of the methods used for providing bias for a transistor are

(*a*) *base bias or fixed current bias* (Fig. 30-22)

It is not a very satisfactory method because bias voltages and currents do not remain constant during transistor operation.

(*b*) *base bias with emitter feedback* (Fig. 30-32)

This circuit achieves good stability of d.c. operating point against changes in β with the help of emitter resistor which causes degeneration to take place.

(*c*) *base bias with collector feedback* (Fig. 30-34)

It is also known as collector-to-base bias or collector feedback-bias. It provides better bias stability.

(*d*) *voltage divider bias* (Fig. 30-35)

It is most widely used in linear discrete circuits because it provides good bias stability. It is also called universal bias circuit or base bias with one supply.

Each of the above circuits will now be discussed separately.

30-20. Base Bias

It has already been discussed in Art. 30-14 aud is shown in Fig. 30-22.

30-21. Base Bias With Emitter Feedback

This circuit is obtained by simply adding an emitter resistor to a base bias circuit as shown in Fig. 30-32

(*i*) At saturation, V_{CE} is essentially zero, hence V_{CC} is distributed over R_L and R_E.

$$\therefore \quad I_{C(sat)} = \frac{V_{CC}}{R_E + R_L}$$

(*ii*) I_C can be found as follows :

Consider the supply, base, emitter and ground route. Applying Kirchhoff's voltage law, we have,

$$-I_B R_B - V_{BE} - I_E R_E + V_{CC} = 0$$

or $$V_{CC} = I_B R_B + V_{BE} + I_E R_E \quad \ldots(i)$$

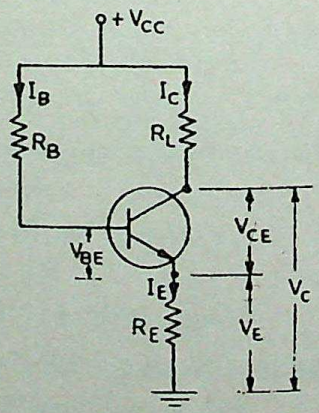

Fig. 30-32

Now, $I_B = I_C/\beta$ and $I_E \cong I_C$

Substituting these values in the above equation, we have

$$V_{CC} \cong \frac{I_C R_B}{\beta} + V_{BE} + I_C R_E$$

$$\therefore \quad I_C \cong \frac{V_{CC} - V_{BE}}{R_E + R_B/\beta}$$

$$\cong \frac{V_{CC}}{R_E + R_B/\beta} \quad \text{—neglecting } V_{BE}$$

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(*iii*) collector-to-ground voltage $V_C = V_{CC} - I_C R_L$

(*iv*) emitter-to-ground voltage, $V_E = I_E R_E \cong I_C R_E$

$\therefore \quad V_{CE} = V_C - V_E$

Example 30-11. *For the circuit shown in Fig. 30-33, find* (*i*) $I_{C(sat)}$ (*ii*) I_C (*iii*) V_C (*iv*) V_E and (*v*) V_{CE}.

Solution. (*i*) $I_{C(sat)} = \dfrac{V_{CC}}{R_E + R_L} = \dfrac{30}{1+2} = \mathbf{10\ mA}$

(*ii*) actual $I_C \cong \dfrac{V_{CC}}{R_E + R_B/\beta}$

$= \dfrac{30}{1+300/100} = \mathbf{7{\cdot}5 mA}$

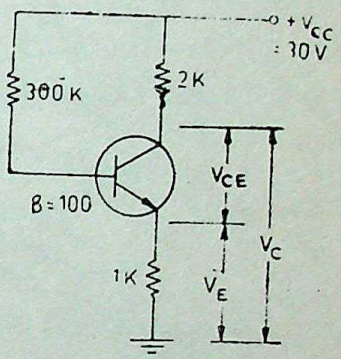

Fig. 30-33

(*iii*) $V_C = V_{CC} - I_C R_L$

$= 30 - 7{\cdot}5 \times 2 = \mathbf{15\ V}$

(*iv*) $V_E = I_E R_E \cong I_C R_E$

$= 7{\cdot}5 \times 1 = \mathbf{7{\cdot}5\ V}$

(*v*) $V_{CE} = V_C - V_E = 15 - 7{\cdot}5$

$= \mathbf{7{\cdot}5\ V}$

30-22. Base Bias With Collector Feedback

This circuit (Fig. 30-34) is like the base bias circuit except that base resistor is returned to collector rather than to the V_{CC} supply. It derives its name from the fact that since voltage for R_B is derived from collector, there exists a negative-feedback effect which tends to stabilise I_C against changes in β. To understand this action, suppose that β increases. It will increase I_C as well as $I_C R_L$ but decrease V_C which is applied across R_B. Consequently, I_E will be decreased which will partially compensate for the original increase in β.

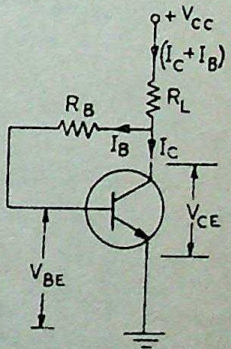

Fig. 30-34

(*i*) $\quad I_{C(sat)} = V_{CC}/R_L \qquad$ —since $V_{CE} = 0$

(*ii*) $\quad V_C = V_{CC} - (I_B + I_C)\, R_L \cong V_{CC} - I_C\, R_L$

Also, $\quad V_C = I_B R_B + V_{BE}$

Equating the two expressions for V_C, we have

$$I_B R_B + V_{BE} \cong V_{CC} - I_C R_L$$

Since $I_B = I_C/\beta$, we get

$$\frac{I_C}{\beta} \cdot R_B + V_{BE} \cong V_C - I_C R_L$$

$$\therefore \quad I_C \cong \frac{V_{CC} - V_{BE}}{R_L + R_B/\beta} \cong \frac{V_{CC}}{R_L + R_B/\beta}$$

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30-23. Voltage Divider Bias

This arrangement is commonly used for transistors incorporated in integrated circuits (ICs). The name 'voltage divider' is derived from the fact that resistors R_1 and R_2 form a potential divider across V_{CC} (Fig. 30-35). The voltage drop V_2 across R_2 forward-biases the emitter whereas V_{CC} supply reverse-biases the collector. As per voltage divider theorem,

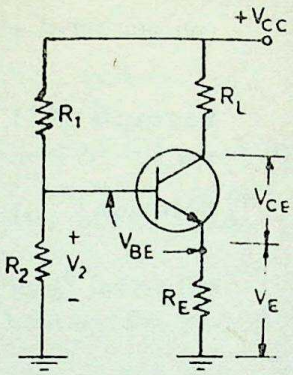

Fig. 30-35

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V_{CC}$$

As seen, $V_E = V_2 - V_{BE}$

$$\therefore \quad I_E = \frac{V_E}{R_E} = \frac{V_2 - V_{BE}}{R_E} \cong \frac{V_2}{R_E}$$

Also, $V_C = V_{CC} - I_C R_L$

$$V_{CE} = V_C - V_E = V_{CC} - I_C R_L - I_E R_E$$
$$= V_{CC} - I_C (R_L + R_E) \qquad \because \ I_C \cong I_E$$

As before, $I_{C(sat)} \cong \dfrac{V_{CC}}{R_L + R_E} \qquad \because \ V_{CE} \cong 0$

Example 30-12. *For the circuit shown in Fig. 30-36 (a), draw the d.c. load line and mark in the Q-point of the circuit. Assume silicon transistor material.*

Solution. Under cut-off condition; $V_{CE} = V_{CC}$.

$\therefore \quad V_{CE(cut\text{-}off)} = V_{CC} = 20$ V —point A in Fig. 30-36 (b)

$$I_{C(sat)} = \frac{V_{CC}}{R_L + R_E} = \frac{20}{2+3}$$
$$= 4 \text{ mA} \qquad \text{—point } B \text{ in Fig. 30-36 } (b)$$

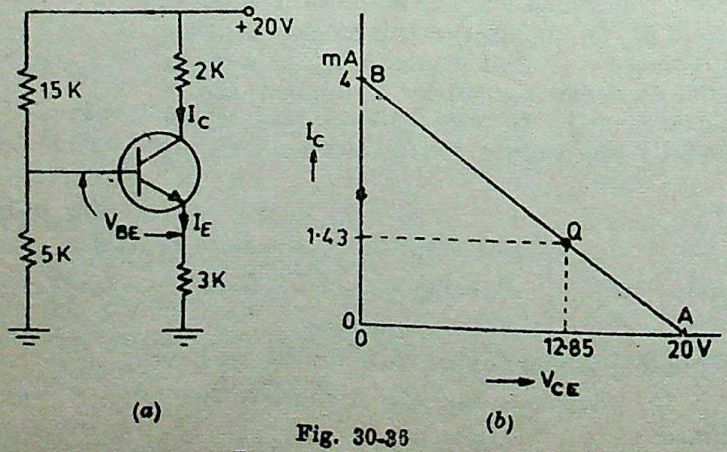

Fig. 30-36

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V_{CC} = \frac{5}{15+5} \times 20 = 5 \text{ V}.$$

$$I_E = \frac{V_2 - V_{BE}}{R_E} = \frac{5 - 0.7}{3} = 1.43 \text{ mA}$$

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Now, $I_C \cong I_B = 1{\cdot}43$ mA

$V_{CE} = V_{CC} - I_C (R_L + R_E)$

$= 20 - 1{\cdot}43\ (2+3) = 12{\cdot}85$ V

The Q-point is shown in Fig. 30-36 (b).

30-24. Load Line and Output Characteristics

In order to study the effect of bias conditions on the performance of a CE circuit, it is necessary to superimpose the d.c. line on the transistor output (V_{CE}/I_C) characteristics. Consider a silicon NPN transistor which is connected in CE configuration (Fig. 30-37) and whose output characteristics are given in Fig. 30-38. Let its $\beta = 100$.

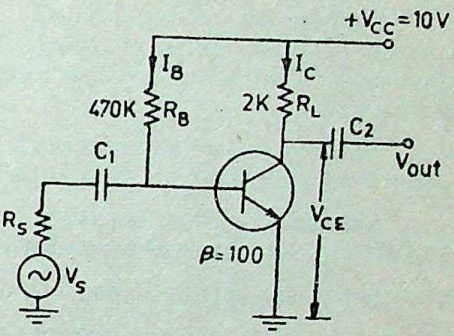

Fig. 30-37

First, let us find the cut-off and saturation points for drawing the d.c. load line and then mark in the Q-point

$I_{C(sat)} = 10/2 = 5$ mA —point B

$V_{CE(cut\text{-}off)} = V_{CC} = 10$ V —point A

The load line is drawn in Fig. 30-38 below.

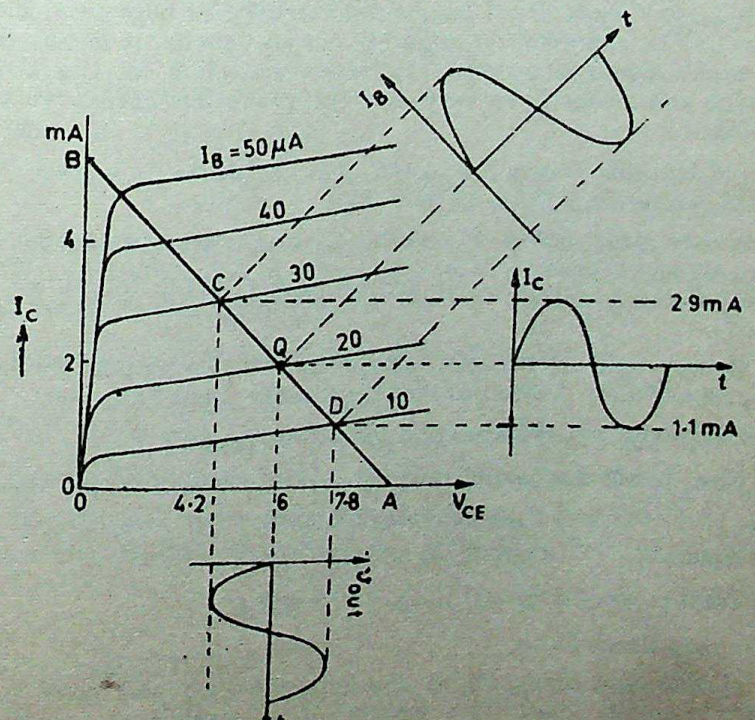

Fig. 30-38

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Actual $$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{10 - 0{\cdot}7}{470} = 20\ \mu A$$

$$I_C = \beta I_B = 100 \times 20 = 2000\ \mu A = 2\ mA$$

$$V_{CE} = V_{CC} - I_C R_L = 10 - 2 \times 2 = 6\ V$$

This locates the Q-point in Fig. 30-38.

Suppose an ac input signal voltage injects a sinusoidal base current of peak value of 10 μA into the circuit of Fig. 30-38. Obviously, it will swing the operating or Q-point up and down along the load line.

When positive half-cycle of I_B is applied, the Q-point shifts to point C which lies on the $(20+10)=30\ \mu A$ line.

Similarly, during negative half-cycle of input base current, Q-point shifts to point D which lies on the $(20-10)=10\ \mu A$ line.

By measurement, at point C, $I_C = 2{\cdot}9$ mA. Hence,

$$V_{CE} = 10 - 2 \times 2{\cdot}9 = 4{\cdot}2\ V$$

Similarly, at point D, I_C measures 1·1 mA.

$$\therefore \quad V_{CE} = 10 - 2 \times 1{\cdot}1 = 7{\cdot}8\ V$$

It is seen that V_{CE} decreases from 6V to 4·2 V *i.e.* by a peak value of $(6-4{\cdot}2)=1{\cdot}8$ V when base current goes positive. On the other hand, V_{CE} increases from 6V to 7·8 V *i.e.* by a peak value of $(7{\cdot}8-6)=1{\cdot}8$ V when input base current signal goes negative. Since changes in V_{CE} represent changes in output voltage, it means that when input signal is applied, I_B varies according to the signal amplitude and causes I_C to vary thereby producing output voltage variations.

Incidentally, it may be noted that variations in voltage drop across R_L are exactly the same as in V_{CE}.

Steady drop across R_L when no signal is applied $= 2 \times 2 = 4$V. When base signal goes positive, drop across $R_L = 2 \times 2{\cdot}9 = 5{\cdot}8$V. When base signal goes negative, $I_C = 1{\cdot}1$ mA and drop across $R_L = 2 \times 1{\cdot}1 = 2{\cdot}2$ V.

Hence, voltage variation $= 5{\cdot}8 - 4 = 1{\cdot}8$V during positive input half-cycle and $= (4 - 2{\cdot}2) = 1{\cdot}8$V during negative input half-cycle.

Obviously, rms voltage variation $= 1{\cdot}8/\sqrt{2} = 1{\cdot}27$ V.

Now, power dissipated in R_L by ac component of output voltage

$$P_{ac} = 1{\cdot}27^2/2 = 0{\cdot}81\ mW$$

Similarly, $$P_{dc} = I_C^2 R_L = 2^2 \times 4 = 8\ mW$$

Total power dissipated in $R_L = 8{\cdot}81$ mW.

30-25. AC Load Line

It is the line along which Q-point shifts up and down when changes in output voltage and current of an amplifier are caused by an ac signal.

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This line is steeper than a dc line but the two intersect at the *Q*-point determined by biasing dc voltages and currents.

AC load line takes into account the ac load resistance whereas dc load line considers only the dc load resistance.

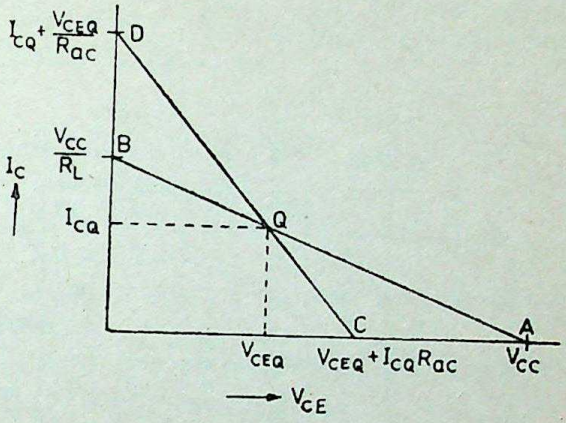

Fig. 30-39

(*i*) DC Load Line

The cut-off point for this line is where

$$V_{CE}=V_{CC}$$

It is also written as $V_{CE(cut-off)}$.

Saturation point is given by

$$I_C=V_{CC}/R_L$$

It is also written as $I_{C(sat)}$.

It is represented by straight line *AQB* in Fig. 30-39.

(*ii*) AC Load Line

The cut-off point is given by

$$V_{CE(cut-off)}=V_{CEQ}+I_{CQ}R_{ac}$$

where R_{ac} is the *ac load resistance.*

Saturation point is given by

$$I_{C(sat)}=I_{CQ}+V_{CEQ}/R_{ac}$$

—as shown in Fig. 30-39.

It is represented by straight line *CQD* in Fig. 30-39.

Example 30-13. *Draw the dc and ac load lines for the CE circuit shown in Fig. 30-40 (a).*

Solution. D.C. Load Line [Fig. 30-40 (*b*)]

$$V_{CE(cut-off)}=V_{CC}=20\text{ V} \quad \text{—point } A$$

$$I_{C(sat)}=V_{CC}/(R_L+R_E)=20/5=4\text{ mA} \quad \text{—point } B$$

Hence, *AB* represents dc load line for the given circuit.

Approximate bias conditions can be quickly found by assuming that I_B is too small to affect the base bias in Fig. 30-40 (*a*).

$$V_2=\frac{4}{4+16}\times 20=4\text{V}$$

If we neglect V_{BE}, $V_2=V_E$

$$\therefore \quad I_E=V_E/R_E=V_2/R_E=4/2=2\text{ mA}$$

Also, $I_C\cong I_E=2$ mA

Hence, $I_{CQ}=2$ mA

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The corresponding value of V_{CEQ} can be found by drawing dotted line in 30-40 (*b*) or may be calculated as under.

$$V_{CEQ}=V_{CC}-I_{CQ}(R_L+R_E)$$
$$=20-2(3+2)=10 \text{ V}$$

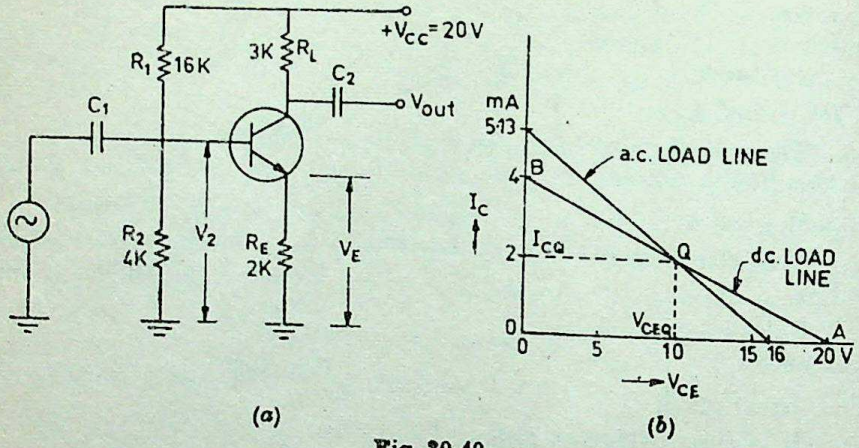

Fig. 30-40

As Load Line

Cut-off point, $V_{CE(cut-off)}=V_{CEQ}+I_{CQ}R_{ac}$

Now, for the given circuit, ac load resistance is

$$R_{ac}=R_L=3\text{K}$$

$\therefore$ Cut-off point $=10+2\times3=16$ V —Fig. 30-40 (*b*)

Saturation point, $I_{C(sat)}=I_{CQ}+V_{CEQ}/R_{ac}$

$$=2+10/3=5{\cdot}13 \text{ mA}$$

Hence, line joining 16-V point and 5·13 mA point gives ac load line as shown in Fig. 30-40 (*b*).

As expected, this line passes through the *Q*-point.

Example 30-14. *Find the dc and ac load lines for the CE circuit shown in Fig. 30-41 (a).*

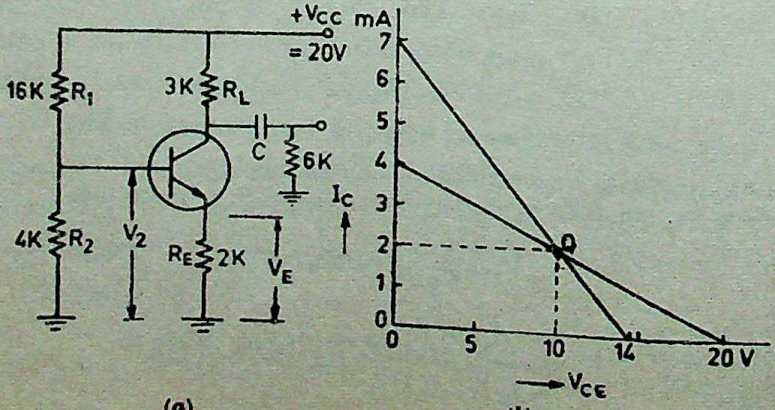

Fig. 30-41

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Solution. The given circuit is identical to that shown in Fig. 30-40 (*a*) except for the addition of 6 K resistor. This makes $R_{ac}=3K \parallel 6K=2K$ because collector *feeds* these two resistors in parallel. The dc load line would remain unaffected. Change would occur only in ac load line.

AC Load Line

$$V_{CE(cut-off)}=V_{CEQ}+I_{CQ}R_{ac}$$
$$=10+2\times 2=14 \text{ V}$$
$$I_{C(sat)}=I_{CQ}+V_{CEQ}/R_{ac}$$
$$=2+10/2=7 \text{ mA}$$

Example 30-15. *Draw the dc and ac load lines for the CB circuit shown in Fig. 30-42 (a).*

Solution. The dc load line passes through cut-off point of 30V and saturation point of $V_{CC}/R_L=1$ mA.

Now, $I_E \cong 20/40=0{\cdot}5$ mA $\therefore$ $I_C \cong I_E=0{\cdot}5$ mA

$\therefore$ $I_{CQ}=0{\cdot}5$ mA

$$V_{CE}=V_{CC}-I_CR_L=30-0{\cdot}5\times 30=15 \text{ V}$$

$\therefore$ $V_{CEQ}=15$V

Hence, dc operating point or *Q*-point is (15V, 0·5 mA) as shown in Fig. 30-42 (*b*).

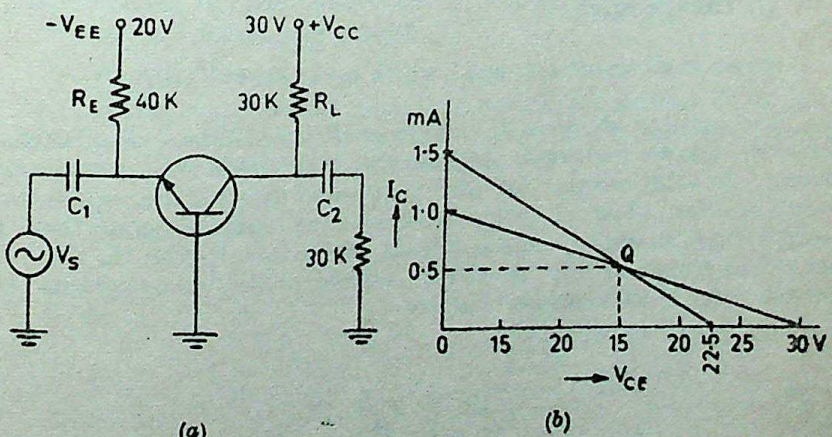

Fig. 30-42

AC Load Line

The cut-off point for ac load line is

$$=V_{CEQ}+I_{CQ}.R_{ac}$$

Since collector *sees* an ac load of two 30K resistors in parallel

$\therefore$ $R_{ac}=30K \parallel 30K=15K$

$\therefore$ $V_{CEQ}+I_{CQ}R_{ac}=15+0{\cdot}5\times 15=22{\cdot}5$ V

Saturation current for ac line is

$$=I_{CQ}+V_{CEQ}/R_{ac}=0{\cdot}5+15/15=1{\cdot}5 \text{ mA}$$

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The line joining these two points (and also passing through Q) gives the ac load line as shown in Fig. 30-42 (b).

Note. Knowing ac cut-off point and Q-point, we can draw the ac load line. Hence, we need not find the value of saturation current for this purpose.

TRANSISTOR EQUIVALENT CIRCUITS

30-26. Rule Regarding Effect of β

Before proceeding with the analysis of equivalent circuits, it would be worthwhile to study the effect of β on circuit values. Following two important rules should be remembered :—

(*i*) when 'looking' into the base terminal of a transistor, any resistance 'seen' in the emitter leg becomes $(1+\beta)$ times larger at the base.

(*ii*) when 'looking' into the emitter terminal of a tansistor, any resistance 'seen' in the base becomes $(1+\beta)$ times smaller at the emitter.

30-27. Ideal Transistor Equivalent Circuits

(*a*) *DC Equivalent Circuit*

(*i*) CB Circuit

In an ideal transistor, $\alpha=1$ which means that $I_C=I_E$.

The emitter diode acts like any forward-biased ideal diode. However, due to transistor action, collector diode acts as a current source. In other words, for the purpose of drawing d.c. equivalent circuit, we can view an ideal transistor as nothing more than a rectifier diode in emitter and a current source in collector. In the d.c. equivalent circuit of Fig. 30-43 (b), current arrow always points in the direction of conventional current.

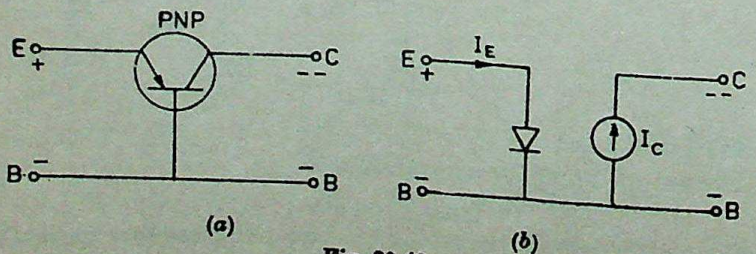

Fig. 30.43

As per the polarities of transistor terminals (Art. 30-2) shown in Fig. 30-43 (a), emitter current flows from E to B and collector current from B to C.

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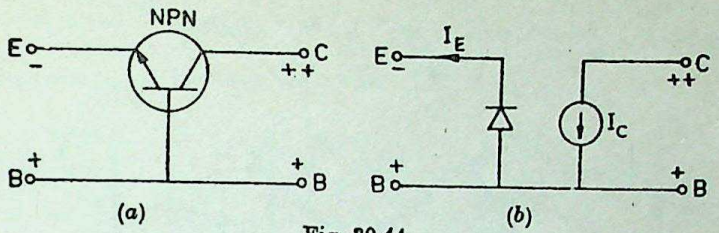

Fig. 30-44

The d.c. equivalent circuit shown in Fig. 30-44 for an *NPN* transistor is exactly similar except that direction of current flow is opposite.

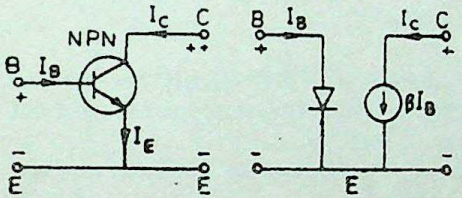

Fig. 30-45

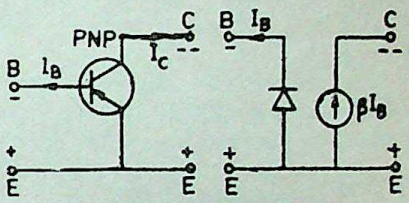

Fig. 30-46

(ii) CE Circuit

Fig. 30-45 shows the d.c. equivalent circuit of an *NPN* transistor when connected in the *CE* configuration and Fig. 30-46 that of a *PNP* transistor. Direction of current flow can be easily found by remembering the transistor polarity rule given in Art. 30-2.

(*b*) *AC Equivalent Circuit*

(i) CB Circuit

In the case of *small* input a.c. signals, the emitter *diode does not rectify*, instead it offers resistance called *a.c. resistance.* As usual, collector diode acts as a current source.

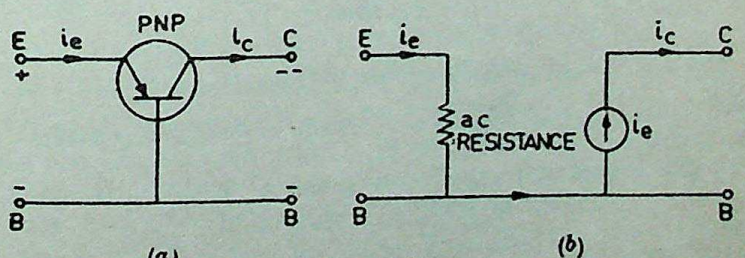

Fig. 30-47

Fig. 30-47 (*b*) shows the a.c. equivalent circuit of a transisto connected in the *CB* configuration*. Here. a.c. resistance offered by the emitter diode is

$$r_{ac} = \text{junction resistance } (r_j) + \text{bulk resistance } (r_B)$$

$$= r_j \qquad \because \; r_B \text{ is negligible.}$$

$$= \frac{25 \text{ mV}}{I_E} \qquad \text{—where } I_E \text{ is dc emitter current}$$

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*This circuit is valid both for *PNP* as well as *NPN* transistors because difference in the direction of ac currents does not matter.

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It is written as r_e' signifying junction resistance of the *emitter* diode *i.e.* a.c. resistanee *looking into the emitter.*

$$\therefore \qquad r_e' = \frac{25 \text{ mV}}{I_E}$$

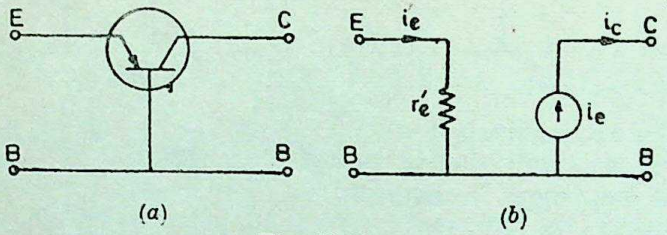

Fig. 30-48

Hence, the a.c. equivalent circuit of a *CB* circuit becomes as shown in Fig. 30-48. Since changes in collector current are almost equal to changes in emitter current, $\triangle i_c \cong \triangle i_e$.

(*ii*) CE Circuit

Fig. 30-49 (*b*) shows the equivalent circuit when transistor has been connected in the *CB* configuration.

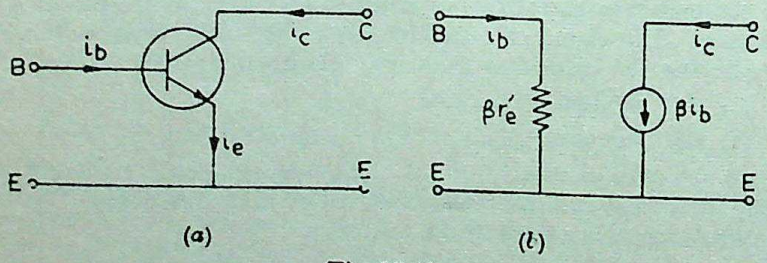

Fig. 30-49

The a.c. resistance *looking into the base* is

$$r_{ac} = \frac{25 \text{ mV}}{I_B} \qquad \text{—dc current is } I_B \text{ not } I_E.$$

$$= \frac{25 \text{ mV}}{I_C/\beta} = \beta . \frac{25 \text{ mV}}{I_C}$$

$$\cong \beta . \frac{25 \text{ mV}}{I_E} = \beta r_e'$$

Strictly speaking, $r_{ac} = (1+\beta)\, r_e' \cong \beta r_e'$ —Art. 30-26.

The a.c. collector current is β times the base current *i.e.* $i_c = \beta i_b$.

30-28. Equivalent Circuit of a CB Amplifier

In Fig. 30-50 (*a*) is shown the circuit of a common base amplifier. As seen, emitter is forward-biased by V_{EE} and collector is

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reverse-biased by V_{CC}. The a.c. signal voltage v_{in} drives the emitter. It produces small fluctuations in transistor voltages and currents in the output circuit. It will be seen that a.c. output voltage is amplified because it is more than v_{in}.

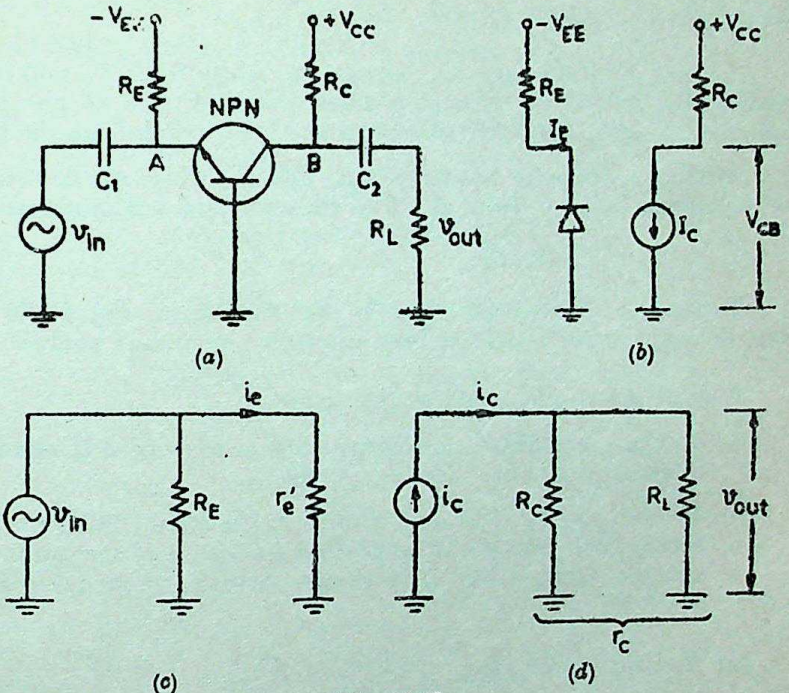

Fig. 30-50

(*a*) DC Equivalent Circuit

For drawing d.c. equivalent circuit, following procedure should be adopted :

(*i*) short all a.c. sources *i.e.* reduce them to zero.

(*ii*) open all capacitors because they block d.c.

If we do this, then as seen from Fig. 30-50 (*a*), neither emitter current can pass through C_1 nor collector current can pass through C_2. These are confined to their respective resistances R_E and R_C (earlier we had been designating it as resistance R_L)

Here, $I_C \cong I_E$ and $V_{CB} = V_{CC} - I_C R_C$

Hence, the d.c. equivalent circuit becomes as shown in Fig. 30-50 (*b*).

(*b*) AC Equivalent Circuit

For drawing a.c. equivalent circuit, following procedure is adopted :

(*i*) all d.c. sources are shorted *i.e.* they are treated as ac ground.

(*ii*) all coupling capacitors like C_1 and C_2 in Fig. 30-50 (*a*) are shorted and

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(*iii*) emitter diode is replaced by its a.c. resistance* $r_{ac}=r_e'$

$$r_e'=\frac{25\text{ mV}}{I_E}$$

where I_E is d.c. emitter current.

As seen by the input a.c. signal, it has to feed R_E and r_e' in parallel [Fig. 30-50 (*c*)]. As seen from point A, R_E is grounded through V_{EE} which has been shorted and r_e' is grounded via the base.

Similarly, collector has to feed R_C and R_L which are connected parallel across it *i.e.* at point B. The collector load resistance is

$$r_c=R_C\|R_L.$$

Hence, a.c. equivalent circuit is as shown in Fig. 30-50 (*d*). Here, collector diode itself has been shown by a current source.

Following two points are worth noting

(*i*) changes in collector current are very nearly equal to changes in emitter current. Hence, $\triangle i_c\cong\triangle i_e$

(*ii*) directions of a.c. currents shown in the circuit diagram are those which correspond to *positive* half-cycle of the ac input voltage. That is why i_c is shown flowing upwards in Fig. 30-50 (*d*).

(*c*) Voltage Gain (A_v)

Since a.c. input voltage drives the emitter diode,

$$\therefore \qquad i_e=v_{in}/r_e' \quad \text{or} \quad v_{in}=i_e\,r_e'$$

The collector current drives a load** $r_c=R_C\|R_L$.

$$\therefore \qquad v_{out}=i_c r_c=i_c\,(R_C\|R_L).$$

Voltage gain, $$A=\frac{v_{out}}{v_{in}}=\frac{i_c\,r_c}{i_e\,r_e'}$$

$$\cong\frac{r_c}{r_e'} \qquad \because \quad i_e\cong i_c$$

---

*Emitter diode ac resistance is the sum of junction resistance (r_j) and bulk resistance (r_B). Since r_B is negligible,

$$r_{ac}=r_j=25\text{ mV}/I_E$$

**On the emitter side, we are only concerned with emitter input a.c. current i_e and not with that going via R_E. Total a.c. current supplied by a.c. voltage would be definitely more than i_e. On the collector side, i_c has to pass through both R_L and R_C. Hence, these two parallel resistances constitute collector load resistance r_c.

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Example 30-16. *For the CB amplifier circuit shown in Fig. 30-51, find*

(i) emitter ac resistance r_e'

(ii) ac output voltage v_{out} and

(iii) voltage gain A_v Take $V_{BE}=0.7$ V

Solution. Here, $I_E=(20-0.7)/30=0.64$ mA

(i) $r_e'=25 \text{ mV}/I_E=25 \text{ mV}/0.64=39\Omega$

(ii) $i_e=v_{in}/r_e'=1.6/39=0.04$ mA

$i_c \cong i_e=0.04$ mA

$r_c=R_C \parallel R_L=15\text{K} \parallel 30\text{K}=10\text{K}$

$\therefore \quad v_{out}=i_c r_c=0.04\times 10=0.4$ V

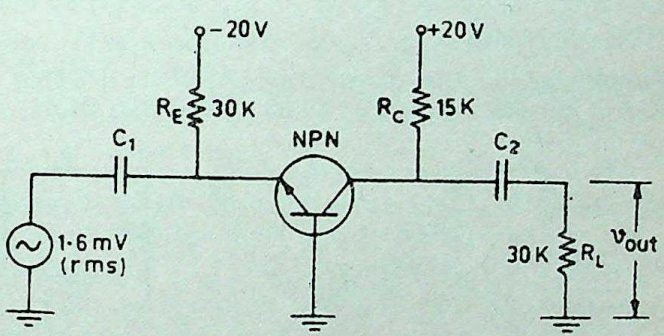

Fig. 30-51

(iii) $$A_v=\frac{v_{out}}{v_{in}}=\frac{0.4}{1.6\times 10^{-3}}=250$$

or $$A_v=\frac{r_c}{r_e'}=\frac{10\text{K}}{39\Omega}=250$$

Example 30-17. *For the circuit shown in Fig. 30-52, find*

(i) voltage gain A_v and

(ii) output voltage v_{out}.

Solution. This circuit is exactly similar to the one shown in Fig. 30-51 except for the addition of a.c. voltage source resistance R_S.

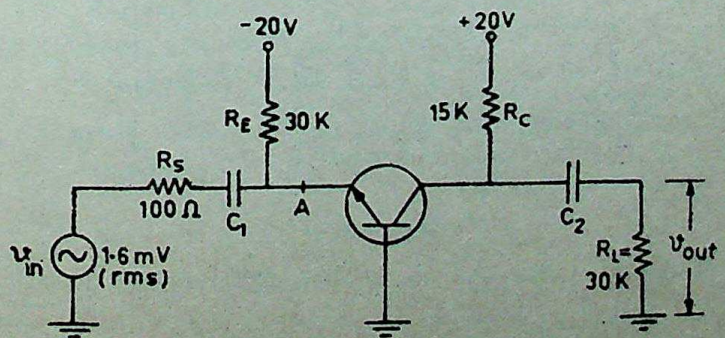

Fig. 30-52

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Since R_S and r_e' are in series so far as input ac signal is concerned,

input resistance $= R_S + r_e'$

(*i*) $\therefore \quad A_v = -\dfrac{r_c}{R_S + r_e'} = \dfrac{10\text{K}}{(100+39)\Omega} = 72$

It is seen that overall voltage gain is drastically reduced *i.e.* from 250 to 72.

(*ii*) $v_{out} = v_{in} \times A_v = 1{\cdot}6 \times 72 = 115 \text{ mV} = 0{\cdot}115 \text{ V}$

30-29. Equivalent Circuit of a CE Amplifier

Consider the simple *CE* circuit of Fig. 30-53 (*a*) in which base bias has been employed.

Remembering the rules given in Art. 30-27 for drawing d.c. and a.c. equivalent circuits, we get Fig. 30-53 (*b*) and Fig. 30-54.

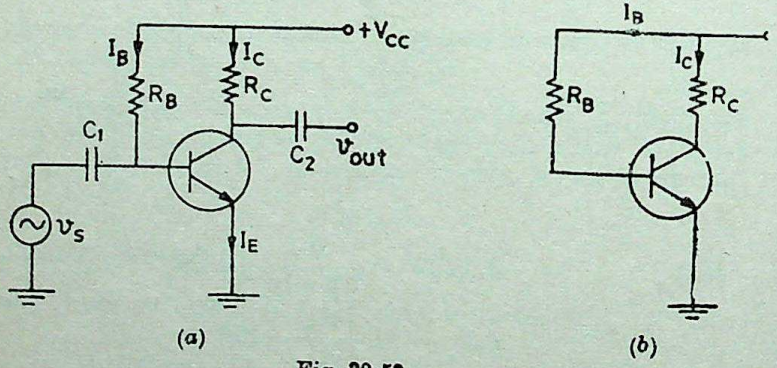

Fig. 30-53

In Fig. 30-53 (*b*), $I_B \cong V_{CC}/R_B$ and $I_C = \beta I_B$.

Hence, $I_E \cong I_C = \beta I_B$

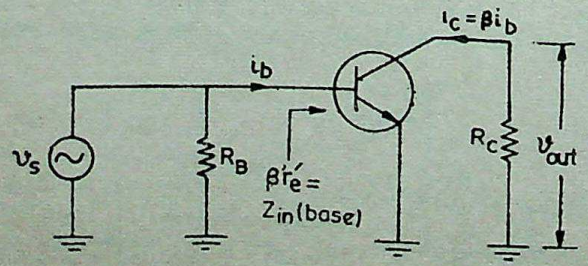

Fig. 30-54

Let us now analyse the a.c. equivalent circuit of Fig. 30-54.

Looking into the base, the a.c. input signal sees an ac resistance which is called $Z_{in(base)}$

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$$Z_{in(base)} = \beta r_e' = \beta \times 25 \text{ mV}/I_E$$

The a.c. input signal voltage of rms value v_s acts across R_B as well as $Z_{in(base)}=\beta r_e'$ because the two are connected in parallel across it.

$\therefore$ a.c. input base current is

$$i_b=v_s/Z_{in(base)}=v_s/\beta r_e'$$

(or $v_s=\beta r_e'.i_b$))

a.c. collector current $i_c=\beta i_b$.

a.c. output voltage, $v_{out}=i_cR_L=\beta i_bR_C$

$\therefore$ $$A_v=\frac{v_{out}^*}{v_s}=\frac{v_{out}}{v_{in}}=\frac{\beta i_bR_C}{\beta r_e' i_b}$$

$$=\frac{R_C}{r_e'}$$

Note. In case, some load resistance R_L is connected across collector (Fig. 30-55) then

$v_{out}=i_cr_c$ —where $r_c=R_C \parallel R_L$

$\therefore$ $A_v=\frac{r_c}{r_e'}$ —as usual

Example 30-18. *If in CE circuit of Fig. 30-53 (a), $V_{CC}=20$ V, $R_C=10$K, $R_B=1$ M, $v_s=2$ mV and $\beta=50$, find*

(*i*) $Z_{in(base)}$

(*ii*) i_b

(*iii*) i_c

(*iv*) v_{out} *and*

(*v*) *voltage gain* A_v *Neglect* V_{BE}.

Solution. $I_B \cong V_{CC}/R_B=20/1\text{ M}=20\ \mu\text{A}$

$I_C=\beta I_B=50\times 20\ \mu\text{A}=1\text{ mA}$

$r_e'=25\text{ mV}/1\text{ mA}=25\ \Omega$

(*i*) $Z_{in(base)}=\beta r_e'=50\times 25=\mathbf{1250\ \Omega}$

(*ii*) a.c. base current is

$$i_b=v_s/Z_{in(base)}=2/1250=\mathbf{1{\cdot}6\ \mu A}$$

(*iii*) a.c. collector current $=\beta i_b=50\times 1{\cdot}6=\mathbf{80\ \mu A}$

(*iv*) $v_{out}=i_cR_C=80\ \mu\text{A}\times 10\text{ K}=\mathbf{0{\cdot}8\ V}$

(*v*) $A_v=v_{out}/v_s=0{\cdot}8\text{V}/0{\cdot}2\text{ mV}=\mathbf{400}$

or $A_v=R_C/r_e'=10\text{K}/25\ \Omega=400$ (Here, $r_c=R_C$)

Example 30-19. *For the circuit shown in Fig. 30-55, compute*

(*i*) $Z_{in(base)}$

(*ii*) v_{out}

(*iii*) A_v

(*iv*) Z_{in}

Neglect V_{BE} *and take* $\beta=200$.

---

*Here $v_s=v_{in}$ because there is no source resistance R_S. If R_S is there, then $v_{in}<v_s$ and is given by $v_{in}=v_s$—ac drop across R_S.

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Solution. It may be noted that voltage divider bias has been used in the circuit.

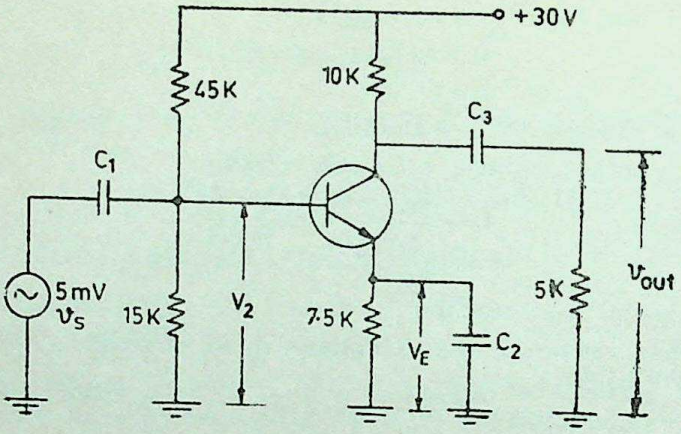

Fig. 30-55

$$V_2 = \frac{15}{15+45} \times 30 = 7.5 \text{ V}$$

$$V_E = V_2 - V_{BE} \cong V_2 = 7.5 \text{ V}$$

$$I_E = 7.5/7.5\text{K} = 1 \text{ mA}$$

$$r_e' = 25 \text{ mV}/1 \text{ mA} = 25 \ \Omega$$

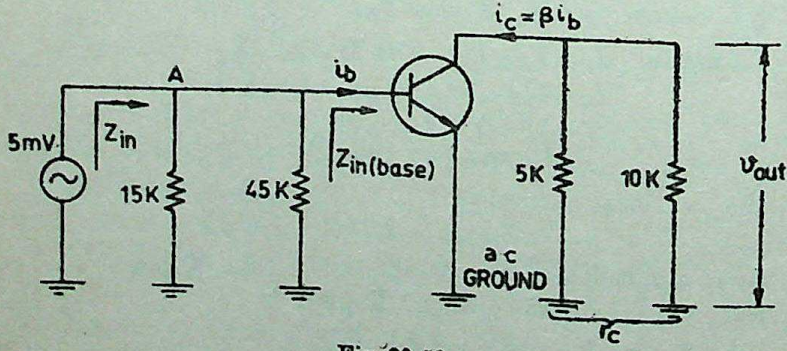

Fig. 30-56

The a.c. equivalent circuit is shown in Fig. 30-56. Since d.c. source is shorted, 45K resistor is ac grounded. On the input side, three resistances become parallel across v_s (*i*) 15 K (*ii*) 45 K and (*iii*) $\beta r_e'$ or $Z_{in(base)}$.

Capacitor C_2 grounds the emitter, so does C_3 to 5K and V_{CC} to 10 K.

(*i*) $Z_{in(base)} = \beta r_e' = 200 \times 25 = 5\text{K}$

(*ii*) $i_b = 5 \text{ mV}/5\text{K} = 1 \ \mu\text{A}$

(Obviously, i_s is not the current which leaves the source but that part of source current i_s which enters the base)

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$$i_c = \beta i_b = 200 \times 1 = 200\ \mu A = 0{\cdot}2\ mA$$

Now, collector load resistance is

$$r_c = 10K \parallel 5K = 10/3\ K = 3{\cdot}33\ K$$

$$v_{out} = i_c r_c = 0\ 2 \times 3{\cdot}33 = \mathbf{0{\cdot}667\ V}$$

(*iii*) $A_v = v_{out}/v_{in} = 0{\cdot}667/5\ mV = \mathbf{133}$

or $A_v = r_c/r_e' = 3{\cdot}33\ K/25\ \Omega = 133$

(*iv*) Z_{in} means the input ac resistance as seen from the source *i.e.* from point *A* in Fig. 30-56. It is different from $Z_{in(base)}$. Obviously, Z_{in} is equal to the equivalent resistance of three resistances connected in parallel

$$Z_{in} = 15K \parallel 45K \parallel 5K = \mathbf{3{\cdot}54\ K}$$

30-30. Transistor Models

Although many transistor equivalent circuits have been suggested and widely used, the equivalent *T*-model is the easiest to understand because, in this representation, component parts retain their identity in all configurations leading to rapid appreciation of a given network.

30-31. T-model

(*a*) CB Circuit

In Fig. 30-57 is shown the low-frequency *T*-equivalent circuit of a transistor connected in *CB* configuration. Here, r_e represents the a.c. resistance of the forward-biased emitter-base junction. Its value is

$$r_e = \frac{25\ mV}{I_E} \quad \text{—for Ge}$$

$$= \frac{50\ mV}{I_E} \quad \text{—for Si}$$

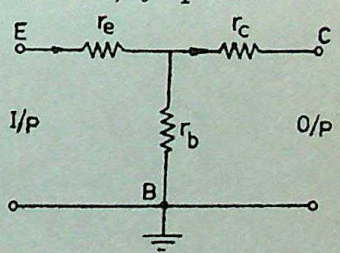

Fig. 30-57

This resistance is fairly small and depends on I_E. Also, r_c represents the a.c. resistance of the reversed-biased *C/B* junction. It is of the order of a few MΩ. Finally, r_b represents the resistance of the base region which is common to both junctions. Its value depends on the degree of doping. Usually, r_b is larger than r_e but much smaller than r_c.

However, circuit shown in Fig. 30-57 is not complete because it does not illustrate the forward current transfer ratio. Since current in the output of a transistor depends on the current at the input, a current-generator in parallel with r_c must be included as shown in Fig. 30-58.

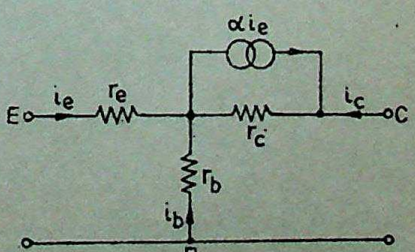

Fig. 30-58

As per convention, all currents are shown flowing towards the transistor.

The current generator may be replaced by a voltage generator

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with the help of Thevenin's theorem as shown in Fig. 30-59 (*a*). In that case, the *T*-equivalent circuit becomes as shown in Fig. 30-59 (*b*). The generator has a voltage of $\alpha\, i_e\, r_c = r_m\, i_e$ where $r_m = \alpha\, i_c$.

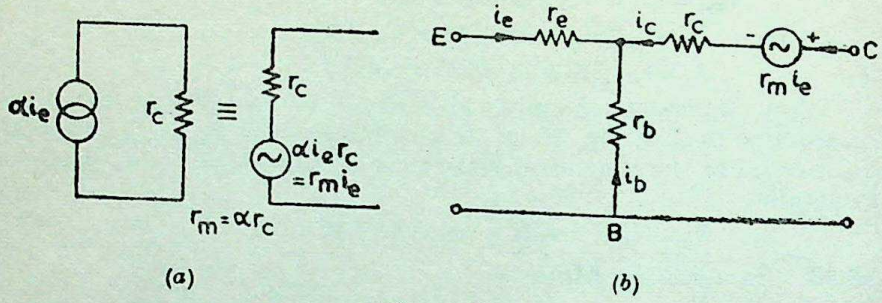

Fig. 30-59

(*b*) CE Circuit

The *T*-equivalent circuit for such a configuration is shown in Fig, 30-60. Whereas circuit shown in Fig. 30-60 contains a parallel

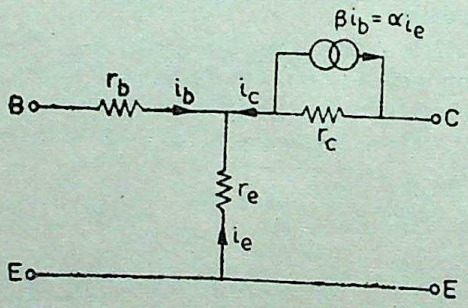

Fig. 30-60

current-generator that shown in Fig. 30-61 contains a series-voltage generator.

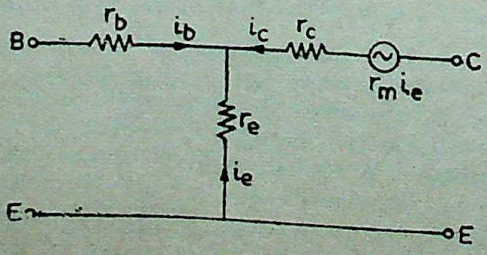

Fig. 30-61

It is important to note the direction of current supplied by the generator in Fig. 30-60. If direction of current flow is reversed, we get the circuit of Fig. 30-62.

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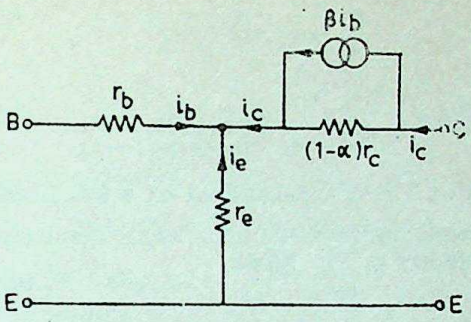

Fig. 30-62

As seen, resistance of collector has been reduced to $r_c\,(1-\alpha)$ in this case.

30.32. Formulas for T-equivalent of a CB Circuit

In Fig. 30-63 is shown a small-signal low-frequency *T*-equivalent circuit for *CB* configuration. The a.c. input signal source has a resistance of R_S and voltage of v_s.

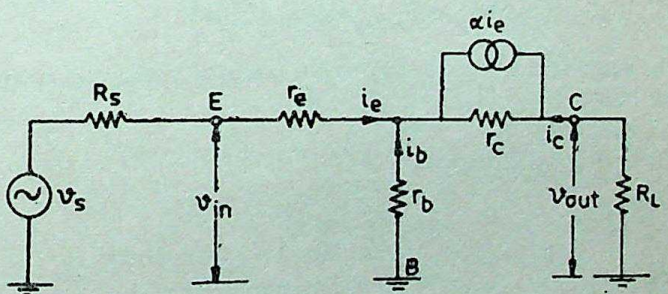

Fig. 30-63

Of course, d.c. biasing circuit has been omitted and only a.c. equivalent circuit shown. The approximate expressions for input and output resistances and voltages and current gains are given below without derivation :

(*i*) $$r_{in} = r_e + r_b\,(1-\alpha)$$
$$= r_e + \frac{r_b}{(1+\beta)}$$

(*ii*) $$r_{out} = r_c - \frac{\alpha\, r_b r_c}{r_e + r_b + R_S}$$

(*iii*) $$A_i = \alpha$$

(*iv*) $$A_v = \frac{v_{out}}{v_{in}} = \frac{\alpha\, R_L}{r_e + r_b\,(1-\alpha)}$$
$$= \frac{v_{out}}{v_s} = \frac{\alpha\, R_L}{(r_e + R_S) + r_b(1-\alpha)}$$

(*v*) $$G = A_i\, A_v$$

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$$= \frac{\alpha^2 R_L}{r_e + r_b (1-\alpha)}$$

or

$$= \frac{\alpha^2 R_L}{(r_e + R_S) + r_b (1-\alpha)}$$

30-33. Formulas For T-equivalent of a CE Circuit

The low-frequency small-signal T-equivalent circuit for such a configuration is shown in Fig. 30-64.

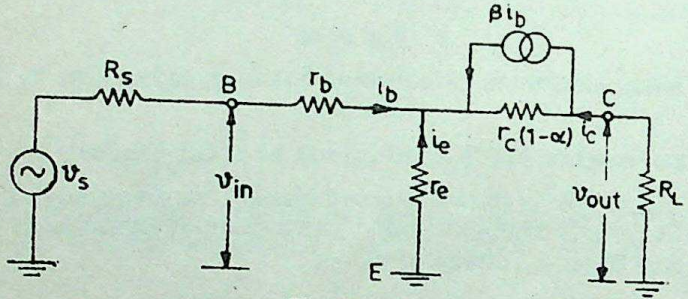

Fig. 30-64

The approximate expressions for various resistances and gains are given below :

(*i*) $$r_{in} = r_b + \frac{r_e}{(1-\alpha)}$$

$$= r_b + (1+\beta)\, r_e$$

(*ii*) $$r_{out} = r_c (1-\alpha) + \frac{\alpha r_c r_e}{r_b + r_e + R_S}$$

(*iii*) $$A_i = \beta = \alpha/(1-\alpha)$$

(*iv*) $$A_v = \frac{v_{out}}{v_{in}} = \frac{\alpha R_L}{r_e + r_b (1-\alpha)}$$

$$= \frac{v_{out}}{v_s} = \frac{\alpha R_L}{r_e + (r_b + R_S)(1-\alpha)}$$

(*v*) $$G = A_v A_i$$

$$= \frac{\alpha^2 R_L}{(1-\alpha)[r_e + r_b (1-\alpha)]}$$

or

$$= \frac{\alpha^2 R_L}{(1-\alpha)[r_e + (r_b + R_S)(1-\alpha)]}$$

Example 30-20. *A junction transistor has $r_e = 50\ \Omega$, $r_b = 1\ K$, $r_c = 1\ M$ and $\alpha = 0{\cdot}98$. It is used in common-base circuit with a load resistance of 10 K. Calculate the current, voltage and power gains and the input resistance.* *(A.M.I.E., Summer 1980)*

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Solution. (*i*) $A_i = \alpha = 0.98$

(*ii*) $$A_v = \frac{\alpha R_L}{r_e + r_b(1-\alpha)}$$
$$= \frac{0.98 \times 10{,}000}{50 + 1000(1-0.98)} = 140$$

(*iii*) $G = A_v A_i = 140 \times 0.98 = 137$

(*iv*) $r_{in} = r_e + r_b(1-\alpha) = 50 + 1000(1-0.98) = 70\ \Omega$

Example 30-21. *Calculate the input and output resistances, overall current, voltage and power gains for a CE connected transistor having following r-parameters :*

$$r_b = 30\ \Omega,\ r_e = 400\ \Omega,\ r_c = 0.75\ \text{M},\ \alpha = 0.95$$
$$R_L = 10\ K \quad \text{and} \quad R_S = 400\ \Omega$$

Also calculate the power gain in decibels.

Solution. (*i*) $$r_{in} = r_b + \frac{r_e}{1-\alpha}$$
$$= 30 + 400/(1-0.95) = 8030\ \Omega$$

(*ii*) $$r_{out} = r_c\ (1-\alpha) + \frac{\alpha\, r_c r_e}{r_b + r_e + R_S}$$
$$= 750{,}000\ (1-0.95) + \frac{0.95 \times 750{,}000 \times 400}{(30+400+400)}$$
$$= 380{,}870 = 0.38\ \text{M}$$

(*iii*) $A_i = \beta = \alpha/(1-\alpha) = 0.95/(1-0.95) = 19$

(*iv*) $$A_v = \frac{\alpha R_L}{r_e + (r_b + R_S)(1-\alpha)}$$
$$= \frac{0.95 \times 10 \times 10^3}{400 + (30+400) \times 0.05} = 22.5$$

(*v*) $G = A_v A_i = 22.5 \times 19 = 427.5$

Power gain in decibels

$$= 10 \log_{10}^{427.5} = 26.3\ \text{dB}$$

30-34. The h-parameters of a Transistor

Every linear circuit has a set of parameters associated with it which completely describe its behaviour. When *small* a.c. signals are involved, a transistor behaves like a linear device because its output a.c. signal varies directly as the input signal. Hence, for small a.c. signals, each transistor has its own characteristic set of *h*-parameters or constants. The letter '*h*' has come from the word 'hybrid' which means mixture of distinctly different items. These constants are hybrid because they have different units.

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The four h-parameters of a transistor are as under :—

h_i = Input impedance with **output shorted**

h_f = Forward current gain with **output shorted**

h_r = Reverse voltage ratio with **input open**

h_o = Output admittance with **input open.**

Consider a transistor connected in *CE* configuration as shown in Fig. 30-65 (*a*). In this case, output circuit has been shorted. The input impedance under this condition is given by

$$h_i = \frac{v_b}{i_b}$$

The forward current gain is given by

$$h_f = \frac{i_c}{i_b}$$

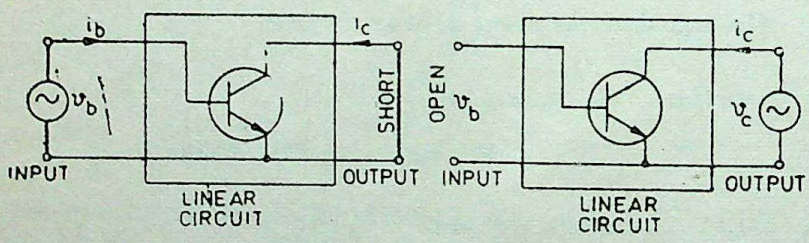

Fig. 30-65

In Fig. 30-65 (*b*), the output collector circuit is driven with an a.c. voltage of v_c which produces an a.c. current of i_c. Meanwhile, input circuit has been left open. Though no current can flow in the open input circuit, a voltage v_b does appear at its terminals. The reverse voltage ratio is given by

$$h_r = \frac{v_b}{v_c}$$

Also, under this condition, admittance of the output circuit is given by

$$h_o = \frac{i_c}{v_c}$$

It may be noted that a second subscript is added to the above parameters to indicate the particular circuit configuration used.

For example, for *CE* connection, the four parameters are written as h_{ie}, h_{fe}, h_{re} and h_{oe}

Similarly, for *CB* connection, these are written as h_{ib}, h_{fb}, h_{rb} and h_{ob} and for *CC* connection as h_{ic}, h_{fc}, h_{ec} and h_{oc}.

30-35. Hybrid Equivalent Circuits

(*a*) CB Circuit

In Fig. 30-66 (*a*) is shown an *NPN* transistor connected in *CB* configuration. Its equivalent circuit employing h-parameters is shown

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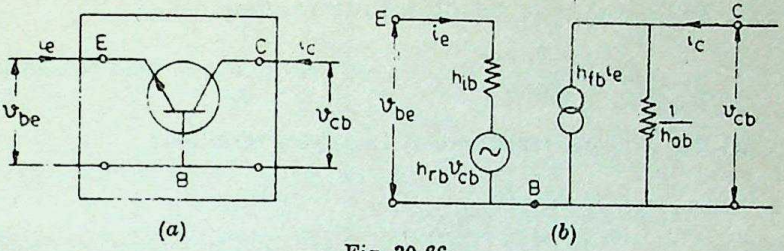
in Fig. 30-66 (*b*). It should be noted that no external biasing resistor or any signal source (with its internal resistance R_S) has been shown connected to the transistor.



(*a*) (*b*)

Fig. 30-66

However, if we connect such an a.c. signal source across input terminals and also add a load resistance R_L across output terminals, then amplifier circuit becomes as shown in Fig. 30-67. Here, R_{in} represents the input resistance of the *transistor only*. Similarly, R_{out} represents output resistance of *transistor itself* whereas R_{out}' takes into account load resistance R_L. Hence,

$$R_{out}' = R_{out} \| R_L.$$

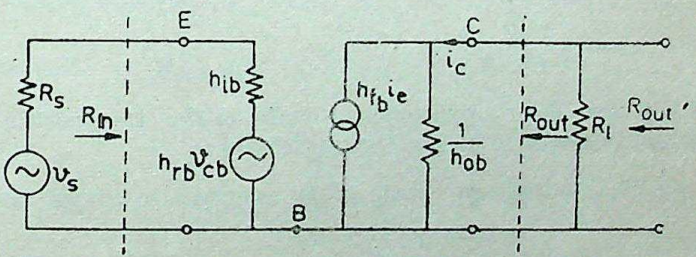
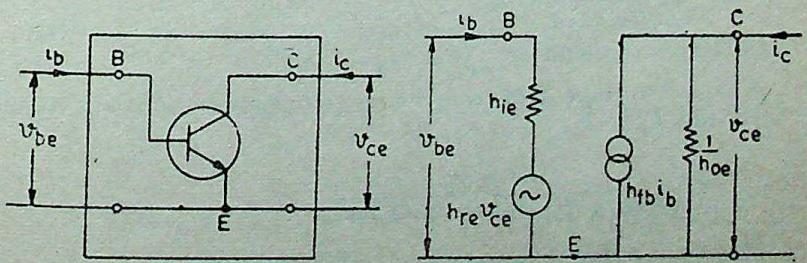


Fig. 30-67

(*b*) CE Circuit

The hybrid equivalent circuit of the transistor *alone* when connected in *CE* configuration is shown in Fig 30-68 (*b*). As before,



(*a*) (*b*)

Fig. 30-68

we may connect input signal source across the input terminals and load resistance across the output terminals.

30-36. Hybrid Formulas

Sufficiently accurate formulas for the various gains and

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resistances of a transistor amplifier circuit in terms of h-parameters are given below. These are applicable to all transistor configurations.

(*i*) R_{in} = input resistance at transistor terminals

$$= h_i - \frac{h_r\, h_f}{h_o + 1/r_L}$$ —where r_L is a.c. load resistance

(*ii*) R_{out} = output resistance at transistor terminals

$$= \frac{h_i + R_S}{(h_i + R_S)\, h_o - h_f\, h_r}$$

$$\cong \frac{h_i + R_S}{h_o R_S + \triangle h}$$

It is worth noting that *input resistance depends on ac load resistance* r_L and *output resistance depends on input signal source resistance* R_S.

(*iii*) A_i = transistor current gain from input terminals *to load*

$$= \frac{-h_f}{1 + h_o\, r_L}$$

where h_f is the short-circuit current gain of the transistor *alone i.e. from input teminals to output terminals only.*

(*iv*) A_v = voltage gain from input terminals to output terminals

$$= \frac{-h_f\, r_L}{h_i\,(1 + h_o r_L) - h_f h_r r_L}$$

$$= \frac{-h_f\, r_L}{h_i + \triangle h\,.\, r_L}$$

$$= \frac{-h_f}{R_{in}\,(h_o + 1/r_L)}$$

In the above formulas, $\triangle h = h_o h_i - h_f\, h_r$.

If overall voltage gain from *source* to output terminals is required, then add R_S to h_i.

$$\therefore \quad A_v' = \frac{-h_f\, r_L}{(h_i + R_S)\,(1 + h_o r_L) - h_f h_r r_L}$$

It may be noted that a.c. load resistance r_L will depend on the transistor connections. For *CB* and *CE* connections, $r_L = r_c$ where r_c is the a.c. load seen by the collector. For a *CC* connection, $r_L = r_E$

30.37. Approximate Hybrid Formulas

The *approximate* hybrid formulas for the three connections are listed below.

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Item	*CE*	*CB*	*CC*
R_{in}	h_{ie}	$\frac{h_{ie}}{h_{fe}}$	$h_{fe} \cdot r_E$
R_{out}	$1/h_{oe}$	$\frac{h_{fe}}{h_{oe}}$	$\frac{h_{ie}+R_s}{h_{fe}}$
A_i	h_{fe}	1	h_{fe}
A_v	$\frac{h_{fe} r_c}{h_{ie}}$	$\frac{h_{fe} r_c}{h_{ie}}$	1

Example 30-22. *A transistor used in CB circuit has the following set of parameters.*

$$h_{ib}=36\ \Omega,\ h_{fb}=-0{\cdot}98,\ h_{rb}=5\times10^{-4},\ h_{ob}=10^{-6}\ mho$$

With $R_S=2K$ *and* $r_L=10\ K$, *calculate*

(*i*) R_{in} (*ii*) R_{out} (*iii*) A_i and (*iv*) A_v

Solution. (*i*) $$R_{in}=h_{ib}-\frac{h_{rb}\,h_{fb}}{h_{ob}+1/r_L}$$

$$=36-\frac{-0{\cdot}98\times5\times10^{-4}}{10^{-6}+1/10\times10^{3}}$$

$$=36+4{\cdot}9=\mathbf{40{\cdot}9\ \Omega}$$

(*ii*) $$R_{out}=\frac{h_{ib}+R_S}{h_o(h_{ib}+R_S)-h_{fb}h_{rb}}$$

$$=\frac{36+2000}{10^{-6}(36+2000)-(-0\ 98)\times5\times10^{-4}}=\mathbf{0{\cdot}8\ M\,\Omega}$$

(*iii*) $$A_i=\frac{h_{fb}}{1+h_{ob}\,r_L}=\frac{-0{\cdot}98}{1+10^{-6}\times10^{4}}=-\mathbf{0{\cdot}97}$$

(*iv*) $$A_v'=\frac{-h_{fb}r_L}{(h_{ib}+R_S)\,(1+h_{ob}\,r_L)-h_{fb}.h_{rb}.r_L}$$

$$=\frac{-(-0\ 98)\times10^{4}}{(36+2000)(1+10^{-6}\times10^{4})-(-0{\cdot}98)\times5\times10^{-4}\times10^{4}}$$

$$=\mathbf{4{\cdot}76}$$

Note. If we consider A_v for transistor *alone*, then putting $R_S=0$, we get

$$A_v=\frac{-(-0{\cdot}98)\times10^{4}}{36(1+10^{-6}\times10^{4})-(-0\ 98)\times5\times10^{-4}\times10^{4}}$$

$$=249$$

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Example 30-23. *A transistor used in CE connection (Fig. 30-69) has the following set of h-parameters*

$$h_{ie}=1K,\ h_{fe}=100,\ h_{re}=5\times10^{-4}\ and\ h_{oe}=2\times10^{-5}\ mho$$

With $R_S=2K$, $r_L=5K$, *determine*

(*i*) R_{in} (*ii*) R_{out} (*iii*) A_i *and* (*iv*) A_v.

Solution. (*i*) $$R_{in}=h_{ie}-\frac{h_{re}\,h_{fe}}{h_o+1/r_L}$$

$$=1000-\frac{5\times10^{-4}\times100}{2\times10^{-5}+1/5\times10^{3}}=723\ \Omega$$

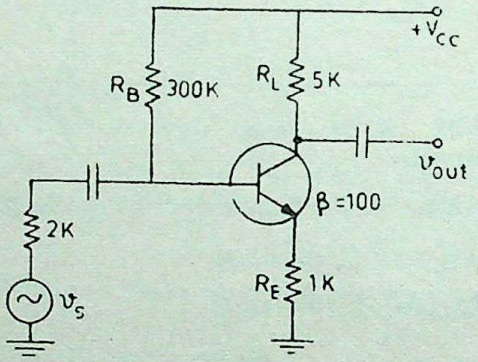

Fig 30-69

(*ii*) $$R_{out}=\frac{h_{ie}+R_S}{(h_{ie}+R_S)h_{oe}-h_{fe}h_{re}}$$

$$=\frac{1000+2000}{(1000+2000)\times2\times10^{-5}-100\times5\times10^{-4}}$$

$$=30{,}000\ \Omega=30\ \text{k}\Omega$$

(*iii*) $$A_i=\frac{h_{fe}}{1+h_{oe}\,r_L}=\frac{100}{1+2\times10^{-5}\times5\times10^{3}}=\mathbf{91}$$

(*iv*) $$A_v=\frac{-h_{fe}\,r_L}{(h_{ie}+R_S)(1+h_{oe}\,r_L)-h_{fe}\,h_{re}r_L}$$

$$=\frac{-100\times5\times10^{3}}{(1000+2000)(1+2\times10^{-5}\times5\times10^{3})-100\times5\times10^{-4}\times5\times10^{3}}$$

$$=-164$$

The negative sign indicates that there is 180° phase shift between the input and output a.c. signals.

Example 30-24. *In the CE circuit shown in Fig. 30-70, the transistor parameters are :*

$$h_{ie}=2K,\ h_{fe}=100,\ h_{re}=5\times10^{-4},\ h_{oe}=2\times10^{-5}\ mho.$$

Calculate (*i*) $Z_{in(base)}$ (*ii*) $Z_{in(stage)}$ (*iii*) Z_{out} (*iv*) Z_{out}' (*v*) A_i and (*vi*) A_v.

Solution. The hybrid equivalent circuit is shown in Fig. 30-71. We will use approximate formulas given in Art. 30-35.

(*i*) $Z_{in(base)}=Z_{in}\cong h_{ie}=\mathbf{2\ K}$

(*ii*) $Z_{in(stage)}=Z_{in}'=2\text{K}\parallel250\ \text{K}=\mathbf{1{\cdot}98\ K}$

(*iii*) $Z_{out}\cong1/h_{oe}=1/2\times10^{-5}=\mathbf{50\ K}$

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It is the output impedance of the transistor *only*.

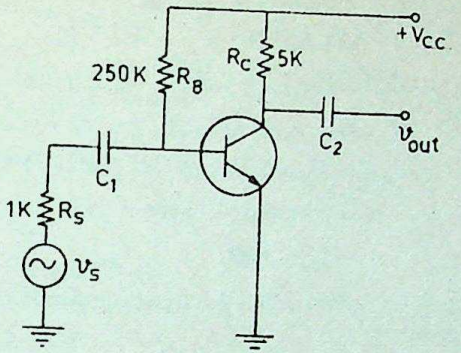

Fig. 30-70

(*iv*) $Z_{out}' = 50\ \text{K} \parallel 5\text{K} = 4{\cdot}54\ \text{K}$

This impedance takes into account collector load.

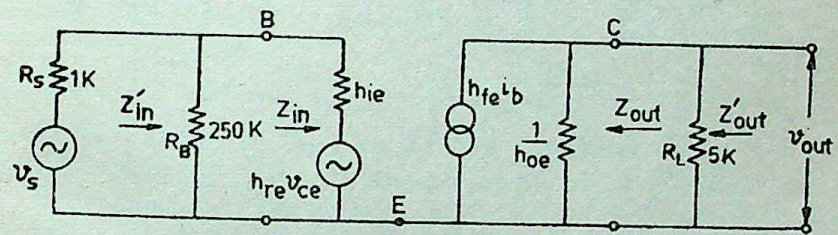

Fig. 30-71

(*v*) $A_i \cong h_{fe} = 100$

(*vi*) $A_v = \dfrac{-h_{fe} r_L}{h_{ie}} = \dfrac{-100 \times 5}{2} = -250$

Example 30-25. *In the CE amplifier circuit shown in Fig. 30-72, the transistor parameters are :*

$h_{ie} = 1{\cdot}5\ K$, $h_{fe} = 100$,

$h_{re} = 5 \times 10^{-4}$ and

$h_{oe} = 2 \times 10^{-5}$ *mho*

Calculate the approximate values of

(*i*) $Z_{in(base)}$

(*ii*) $Z_{in(stage)}$

(*iii*) Z_{out}

(*iv*) Z_{out}'

(*v*) A_i

(*vi*) A_v

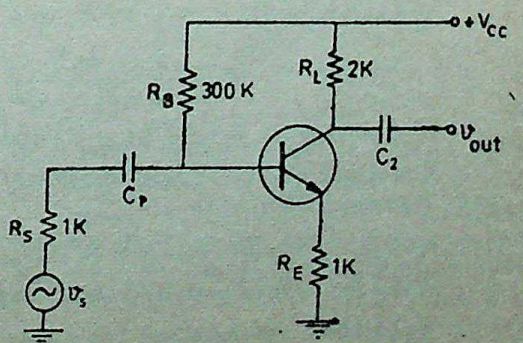

Fig. 30-72

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Solution. (*i*) $Z_{in(base)} = Z_{in} = h_{ie} + (1+\beta)\, R_E$*

$= h_{ie} + (1+h_{fe})\, R_E$

$= 1{\cdot}5 + 101 \times 1 = \mathbf{102{\cdot}5\ K}$

(*ii*) $Z_{in(stage)} = Z_{in}' = Z_{in(base)} \parallel R_B = 102{\cdot}5 \parallel 300 = \mathbf{76\ K}$

(*iii*) $Z_{out} \cong 1/h_{oe} = 1/2 \times 10^{-5} = \mathbf{50\ K}$

(*iv*) $Z_{out}' = Z_{out} \parallel R_L = 50\text{K} \parallel 2\text{K} = \mathbf{1{\cdot}92\ K}$

(*v*) A_i = transistor current gain

$= h_{fe} = \mathbf{100}$

Even when R_E is taken into account, gain of transistor *alone* would remain the same.

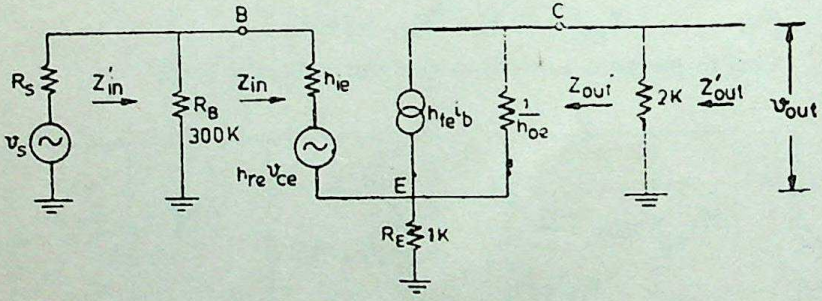

Fig. 30-73

The *circuit* current gain is

$$A_i' = \frac{h_{fe} R_B}{R_B + Z_{in(base)}} = \frac{100 \times 300}{300 + 102{\cdot}5} = \mathbf{74{\cdot}5}$$

(*vi*) Voltage gain of transistor *alone* is

$$A_v = -h_{fe} R_L / h_{ie} = -100 \times 2/1{\cdot}5 = \mathbf{-133{\cdot}3}$$

Voltage gain when R_E is taken into account is

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (1+h_{fe}) R_E} \cong \frac{-R_L}{R_E} = \frac{-2}{1} = \mathbf{-2}$$

It is so because $h_{ie} \ll (1+h_{fe})\, R_E$ and $h_{fe} \gg 1$.

Example 30-26. *The transistor of Fig. 30-74 has the following set of h-parameters :*

$h_{ie} = 2K$, $h_{fe} = 100$, $h_{re} = 5 \times 10^{-4}$, $h_{oe} = 2{\cdot}5 \times 10^{-5}$ *mho*

Find the voltage gain and the ac input impedance of the stage.

Solution. Using exact formulas given in Art. 30-34, we have,

$$Z_{in(base)} = Z_{in} = h_{ie} - \frac{h_{fe}\, h_{re}}{h_{oe} + 1/r_L}$$

*Please refer to the p-rule given in Art. 30-25.

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Now, collector load $r_L = 10K \parallel 30K = 7{\cdot}5K$

$$\therefore \quad Z_{in(base)} = 2000 - \frac{100 \times 5 \times 10^{-4}}{2{\cdot}5 \times 10^{-5} + 1/7{\cdot}5 \times 10^{3}}$$

$$= 2000 - 316 = \mathbf{1684\ \Omega}$$

The a.c. input impedance of the stage *i.e.* impedance when looking into point *B* is

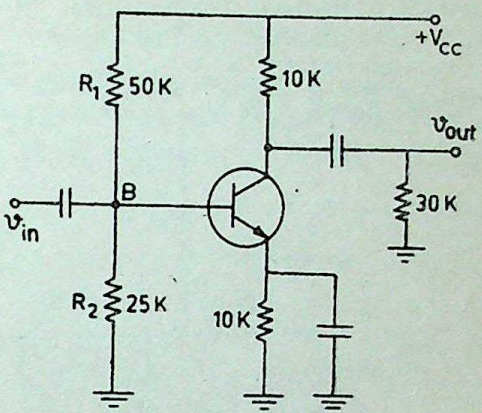

Fig. 30-74

$$Z_{in(stage)} = Z_{in}'$$

$$= Z_{in(base)} \| R_1 \| R_2$$

$$= 1{\cdot}684 \parallel 50 \parallel 25$$

$$= \mathbf{1{\cdot}53K}$$

$$A_v = \frac{-h_{fe}}{Z_{in(base}(h_{oe} + 1/r_L)}$$

Now, $r_L = 10K \parallel 30\ K$

$$= 7{\cdot}5\ K$$

$$\therefore \quad A_v = \frac{-100}{1684(2{\cdot}5 \times 10^{-5} + 1/7500)} = \mathbf{375}$$

30-38. Common Base Transistor Amplifier

Both Fig. 30-75 and 30-76 show the circuit of a *CB* single-stage *RC* coupled amplifier using a *PNP* transistor. As seen, input a.c. signal is injected into the emitter-base circuit and output is taken from the collector-base circuit. The *E/B* junction is forward-biased

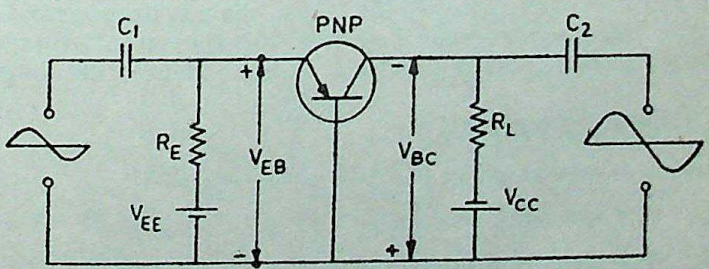

Fig. 30-75

by V_{EE} and *C/B* junction is reverse-biased by V_{CC}. The *Q*-point or d.c. operating conditions are determined by power sources together with R_E and R_L. In other words, before any signal is applied, do values of I_B, I_C, V_{EB} and V_{BC} are determined by V_{EE}, V_{CC}, R_E and R_L. Let us now see what happens when we apply an a.c. signal to the *E/B* junction.

Working

When positive half-cycle of the input signal is applied, then

1. forward bias is *increased* because V_{EB} is *positive* with respect to ground

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2. I_E and hence I_C are increased
3. the drop $I_C R_L$ is increased
4. V_{CB} is decreased.*

Since it is *negative* with respect to ground, it means it becomes less negative. Hence, in effect, it produces a *positive* output half-cycle.

Since a positive-going input signal produces a positive-going output signal, there is *no phase reversal between the two.*

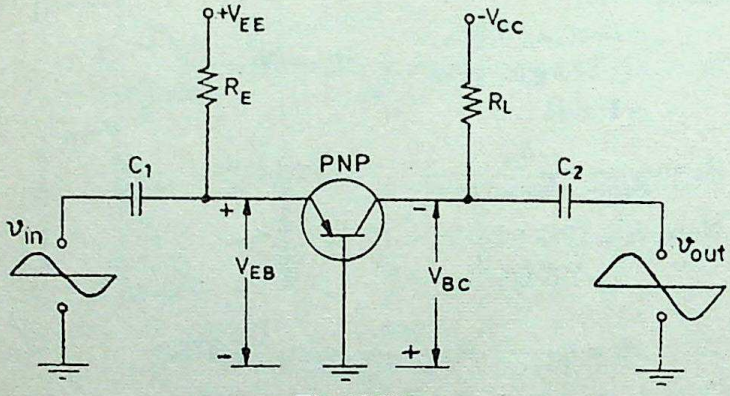

Fig. 30-76

Voltage amplification in this circuit is possible by reason of relative input and output circuitry rather than current gain (α) which is always less than unity. The input circuit has low resistance whereas output circuit has very large resistance. Although changes in input and output currents are the same, the a.c. drop across R_L is very large. Hence, changes in V_{CB} (which is the output voltage) are much larger than changes in input a.c. signal. Hence, the amplification.

CHARACTERISTICS

Common-base amplifier has

1. very low input resistance
2. very high output resistance
3. a current gain $\alpha < 1$
4. large voltage gain of about 1500
5. power gain of upto 30 dB
6. no phase reversal between input and output voltages

30-39. Common Emitter Transistor Amplifier

(*A.M.I.E., Winter 1986*)

Amplifiers employing transistors most frequently use the *CE* configuration because (*i*) it provides very large gains in current,

*$V_{CC} = V_{CB} + I_C R_L$ or $V_{CB} = V_{CC} - I_C R_L$

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voltage and power and (*ii*) its input and output impedance characteristics are suitable for many applications.

As shown in Fig. 30-77 and 30-78, base is the driven element. The input signal is injected into the base-emitter circuit whereas output signal is taken from collector-emitter circuit. Same power source V_{CC} provides forward bias to E/B junction and reverse bias to the C/B junction in Fig. 30-78 though two separate power sources have been used in Fig. 30-77. In the absence of input signal, the quiescent or working values of I_B and I_C are determined by V_{CC} together with resistors R_B and R_L.

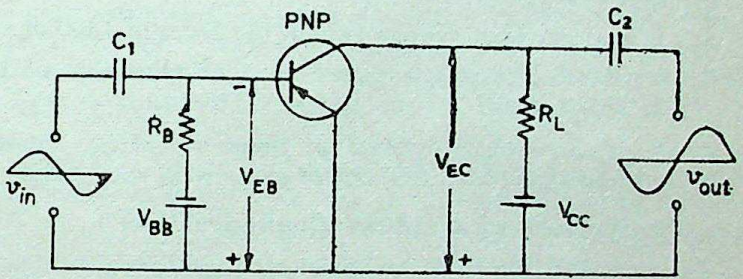

Fig. 30-77

WORKING

Let us see what happens when an a.c. input signal is injected into the base-emitter circuit. When positive half-cycle of the signal is applied, V_{EB} is reduced because it is negative w.r.t. ground as shown in Fig. 30-78. This reduction leads to

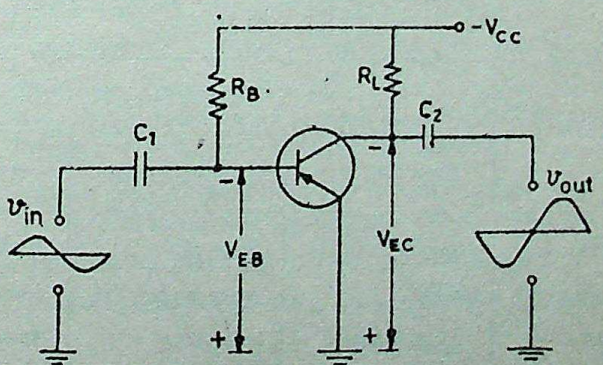

Fig. 30-78

1. a decrease in forward bias of base-emitter junction
2. a small decrease in I_B
3. a large decrease in I_E and I_C*
4. a decrease in $I_C R_L$ drop and consequently
5. an *increase* in V_{EC} which represents output voltage.**

---

*$I_C = \beta I_B$ and $I_E = (1+\beta) I_B$

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**$V_{CC} = V_{EC} + I_C R_L$ or $V_{EC} = V_{CC} - I_C R_L$

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Since V_{EC} is negative w.r.t ground, it means it becomes more negative. Hence, in effect, it produces a negative output half-cycle. Consequently, a positive-going input signal becomes a negative-going output signal.

CHARACTERISTICS

A CE transistor amplifier has the following characteristics :

1. it has moderately low input resistance (1K to 2K)
2. its output resistance is moderately large (50K or so)
3. its current gain (β) is high
4. it has very high voltage gain of the order of 1500 or so
5. it produces very high power gain of the order of 10,000 times or 40 dB
6. it produces phase reversal of input signal *i.e.* input and output signals are 180° out of phase with each other.

30-10. Comparison of Amplifier Configurations

The basic building blocks of transistor amplifiers are single-stage common-base, common-emitter and common-collector circuits. The choice of configuration and type of transistor for a given application will depend largely upon the desired input and output impedances, voltage, current and power gains and frequency response. For convenience, these values are tabulated in Table No. 30-1. Here, the input and output parameters are shown as resistances and apply only to low-frequency conditions.

Table 30-1

Characteristic	*Type of Circuit*		
	Common base	*Common emitter*	*Common collector*
Current gain	nearly unity (α)	high (β)	highest ($\beta+1$)
Voltage gain	high	very high	nearly unity
Power gain	moderate	highest	lowest
Input impedance	lowest	moderate	highest
Output impedance	highest	moderate	lowest
Phase reversal	No	Yes	No

In each case, the amplifying action depends on low-power input circuits controlling the high-power output circuits.

In the common-base circuit, although the input and output current gains are nearly equal, the low-impedance emitter circuit absorbs far less power as compared to that which is available at the high-impedance collector.

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The low base current flowing into the common-emitter circuit (where the impedance is a few kilohms) gives rise to much higher collector current flowing out of the high-impedance output circuit.

The common-collector circuit with approximately equal input and output resistances requires a low input current to control much larger output current.

30-41. Function of an Oscillator

An oscillator is a circuit which converts electric energy at d.c. (zero frequency) to electric energy at frequency varying from a few Hz to gigahertz (GHz). Stated simply, it is a source of alternating voltage or current. An electronic oscillator is, essentially, a feedback amplifier that supplies its own input *i.e.* it requires no external signal to initiate or maintain the energy conversion process.

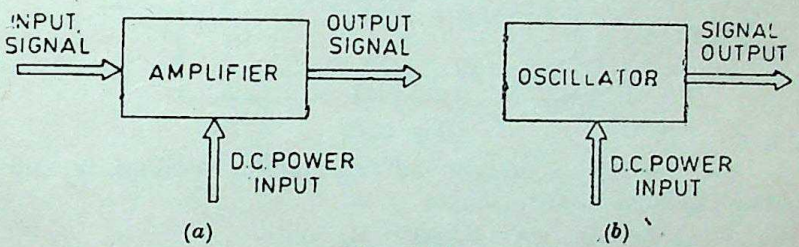

Fig. 30-79

The basic difference between an amplifier and an oscillator is shown in Fig. 30-79.

30-42. Classification of Oscillator Circuits

Oscillator circuits may be broadly divided into two groups :

(*a*) sinusoidal (harmonic) oscillators—such circuits are used as electronic timing and control circuits in TV, radar, oscilloscope and industrial control equipment.

(*b*) non-sinusoidal (relaxation) oscillators—they produce output which has square or rectangular or sawtooth waveform.

Sine-wave oscillators may by further subdivided into :

(*i*) *LC* feedback oscillators such as Hartley, Colpitts and Clapp etc.

(*ii*) *RC* phase-shift oscillators such as Wienbridge oscillator

(*iii*) negative-resistance oscillators such as tunnel diode oscillator

However, only *LC* feedback oscillators would be considered in this book.

30-43. Essentials of a Feedback Oscillator

The essential components of a feedback oscillator shown in Fig. 30-80 (*a*) are :

(*i*) a resonator which consists of an *LC* circuit. It is also known as frequency determining network (FDN) or tank circuit.

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(*ii*) an amplifier whose function is to amplify the oscillations produced by the resonator.

(*iii*) a positive feedback network (PFN) whose function is to transfer part of the output energy to the resonant *LC* circuit in proper phase. The amount of energy fed back is sufficient to meet I^2R losses in the *LC* circuit.

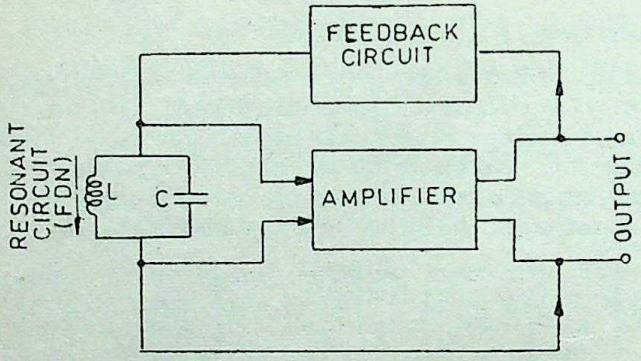

Fig. 30-80

The essential condition for maintaining oscillations and for finding the value of frequency is

$$\beta A = 1 + j0$$

It means that

(*i*) the open loop gain $| \beta A | = 1$

(*ii*) the net phase shift around the loop is $= 0°$ or $360°$. In other words feedback should be *positive*.

30-44. Tuned Base Oscillator

Such an oscillator using a transistor in *CE* configuration is shown in Fig. 30-81, Resistors R_1, R_2 and R_3 determine the d.c. bias of the circuit. The mutually-coupled coils L_1 and L forming primary and secondary of an *RF* transformer provide the required feedback between the collector and base circuits. The amount of feedback depends on the coefficient of coupling between the two coils. The *CE* connected transistor itself provides a phase shift of 180° between its input and output circuits. The transformer provides another 180° phase shift thus producing a total phase shift of 360° which is an essential condition for producing oscillations.

The parallel-tuned *LC* circuit connected between base and emitter is the frequency-determining network (FDN) *i.e.* it generates the oscillations at its resonant frequency.

Circuit Action

The moment switch *S* is closed, collector current is set up which tends to rise to its quiescent value. This increase in I_C is accompanied by :

1. an expanding magnetic field through L_1 which links with *L* and
2. an induced e.m.f. called feedback voltage in *L*.

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Two immediate reactions of this feedback voltage are :

(*i*) increase in emitter-base voltage (and current) and

(*ii*) a further increase in collector current I_C.

It is followed by a succession of cycles of

1. an increase in feedback voltage
2. an increase in emitter-base voltage and
3. an increase in I_C until saturation is reached

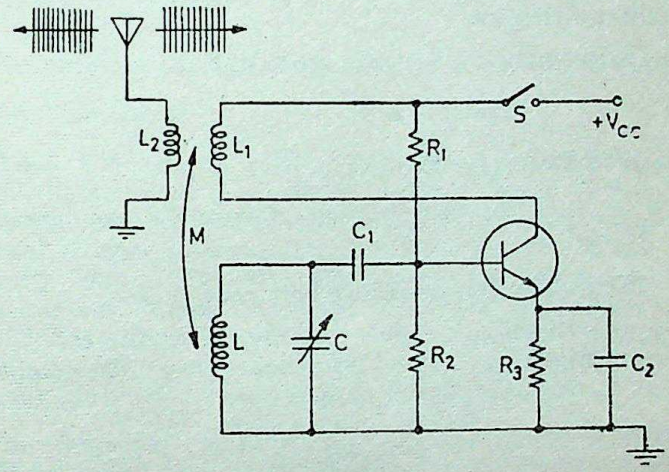

Fig. 30-81

Meanwhile, C gets charged. As soon as I_C ceases to increase, magnetic field of L_1 ceases to expand and thus no longer induces feedback voltage in L. Having been charged to maximum value, C starts to discharge through L. However, decrease in voltage across C causes the following sequence of reactions :

1. a decrease in emitter-base bias
2. a decrease in I_C
3. a collapsing magnetic field in L_1
4. an induced feedback voltage in L though in *opposite* direction
5. further decrease in emitter-base bias.

and so on till I_C reaches its cut-off value.

During this time, the capacitor having lost its original charge, again becomes fully charged though with *oppsite* polarity. Transistor being in cut-off, the capacitor will again begin to discharge through L. Since polarity of capacitor charge is opposite to that when transistor was in saturation, the sequence of reactions now will be

1. an increase in emitter-base bias
2. an increase in I_C

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3. an expanding magnetic field in L_1
4. an induced feedback voltage in L
5. a further increase in emitter-base and
6. so on till I_C increases to its saturation value.

This cycle of operation keeps repeating so long as enough energy is supplied to meet losses in $L\,C$ circuit.

The output can be taken out by means of a third winding L_2 magnetically coupled to L_1. It has approximately the same waveform as collector current.

The frequency of oscillation is given by

$$f_0 = 1/2\pi \sqrt{LC}$$

30.45. Tuned Collector Oscillator

Such an oscillator using a transistor in *CE* configuration is shown in Fig. 30-82.

(*i*) Frequency Determining Network (FDN).

It is made up of a variable capacitor C_1 and a coil L_1 which forms primary winding of a step-down transformer. The combination of L_1 and C_1 forms an oscillatory tank circuit.

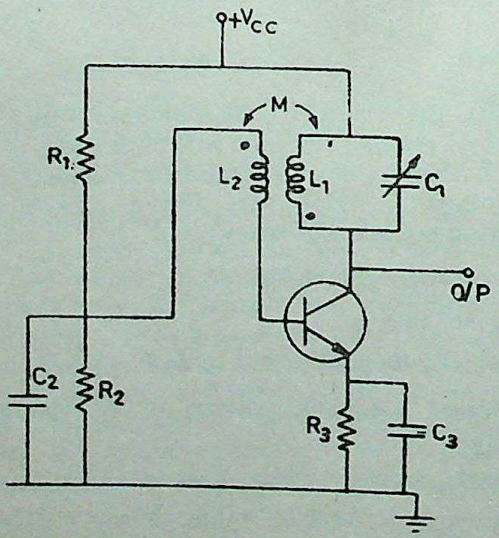

Fig. 30-82

(*ii*) Positive Feedback

Feedback between the collector-emitter circuit and base-emitter circuit is provided by the transformer secondary winding L_2 which is mutually-coupled to L_1. As far as a.c. signals are concerned, L_2 is connected to emitter via low-reactance capacitors C_2 and C_3.

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Since transistor is connected in *CE* configuration, it provides a phase shift of 180° between its input and output circuits. Another phase shift of 180° is provided by the transformer thus producing a total phase shift of 360° between the output and input voltages resulting in positive feedback between the two.

(*iii*) Amplifying Action

The transistor amplifier provides sufficient gain for oscillator action to take place.

The resistors R_1, R_2 and R_3 are used to d.c. bias the transistor.

(*iv*) Working

When the supply is first switched on, a transient current is developed in the tuned L_1 C_1 circuit as the collector current rises to its quiescent value. This transient current initiates natural oscillations in the tank circuit. These natural oscillations induce a small e.m.f. into L_2 by mutual induction which causes corresponding variations in base current. These variations in I_B are amplified β times and appear in the collector circuit. Part of this amplified energy is used to meet losses taking place in the oscillatory circuit and the balance is radiated out in the form of electromagnetic waves.

The frequency of oscillatory currents is almost equal to the resonant frequency of the tuned circuit.

$$\therefore \qquad f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

Example 30-27. *A tuned-collector has a fixed inductance of 100 μF and has to be tunable over the frequency band of 500 kHz to 1500 kHz. Find the range of variable capacitor to be used.*

Solution. Resonant frequency is given by

$$f_0 = 1/2\pi\sqrt{LC} \quad \text{or} \quad C = 1/4\pi^2 f_0^2 L$$

where L and C refer to the tank circuit.

When f_0 = 500 kHz

$$C = 1/4\pi^2 \times (500 \times 10^3)^2 \times 100 \times 10^{-6} = \mathbf{1015\ pF}$$

When f_0 = 1500 kHz

$$C = 1015/(1500/500)^2 = \mathbf{113\ pF}.$$

Hence, capacitor range required is **113—1015 pF**.

Example 30-28. *The resonant circuit of a tuned-collector transistor oscillator has a resonant frequency of 5 MHz. If value of capacitance is increased by 50%, calculate the new resonant frequency.*

Solution. Using the equation for resonant frequency, we have

$$5 \times 10^6 = 1/2\pi\sqrt{LC}$$

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$$f_0 = 1/2\pi\sqrt{L \times 1{\cdot}5\, C}$$

$$\therefore \quad \frac{f_0}{5\times 10^6} = \frac{1}{\sqrt{1{\cdot}5}} \quad \text{or} \quad f_0 = 4{\cdot}08 \text{ MHz}.$$

30-46. Hartley Oscillator *(A.M.I.E. Summer 1981)*

In Fig. 30-83 (*a*) is shown a transistor Hartley oscillator using *CE* configuration. Its general principle of operation is similar to the tuned-collector oscillator discussed in Art. 30-45.

It uses a single tapped-coil having two parts marked L_1 and L_2 instead of two separate coils. So far as a.c. signals are concerned, one side of L_2 is connected to base via C_1 and the other to emitter via C_3 and C_2. Similarly, one end of L_1 is connected to collector and the other to common emitter terminal via C_3 and C_2. These two parts are inductively-coupled and form an auto-transformer or a split-tank inductor. Feedback between the output and input circuits is accomplished through autotransformer action which also introduces a phase reversal of 180°. Since transistor itself introduces a phase shift of 180°, the total phase shift becomes 360° thereby making the feedback positive or regenerative which is essential for oscillations (Art. 30-43). As seen, positive feedback is obtained from the

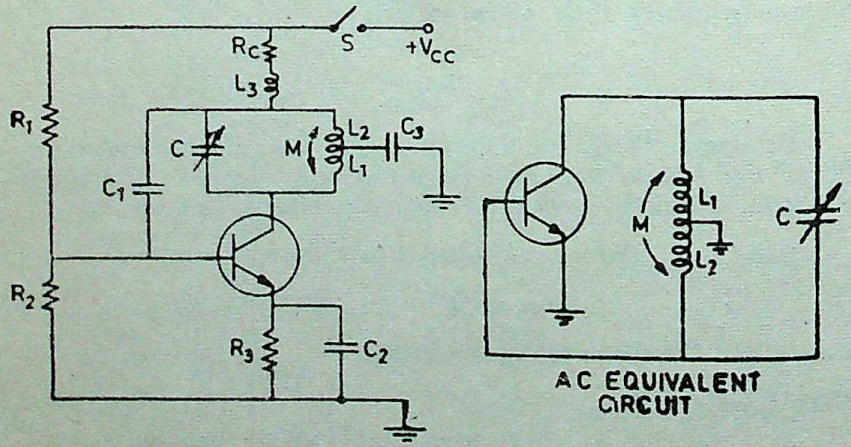

Fig. 30-83

tank circuit and is coupled to the base via C_1. Fig. 30-83 (*b*) shows the a.c. equivalent circuit of Hartley oscillator.

Resistors R_1 and R_2 form a voltage divider for providing the base bias and R_3 is an emitter swamping resistor to add stability to the circuit. Capacitor C_2 provides a.c. ground thereby preventing any degeneration while still providing temperature stabilisation. Collector load resistor R_C is necessary for amplifier action and inductor L_3 is necessary to keep a.c. currents out of d.c. supply V_{CC}.

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When V_{cc} is first switched on through S, an initial bias is established by R_1-R_2 and oscillations are produced because of positive feedback from the LC tank circuit (L_1 and L_2 constitute L). The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where $$L = L_1 + L_2 + 2M.$$

The output from the tank may be taken out by means of another coil coupled to either to L_1 or L_2.

30-47. Colpitts Oscillator

This oscillator is essentially the same as Hartley oscillator except for one difference. Colpitts oscillator uses tapped capacitance whereas Hartley oscillator uses tapped inductance.* Fig. 30-84 (*a*) shows the complete circuit with its power source and d.c. biasing circuit whereas Fig. 30-84 (*b*) shows its equivalent a.c. circuit. The two series capacitors C_1 and C_2 form the voltage divider used for providing the feedback voltage (the voltage drop across C_1 constitutes the feedback voltage).

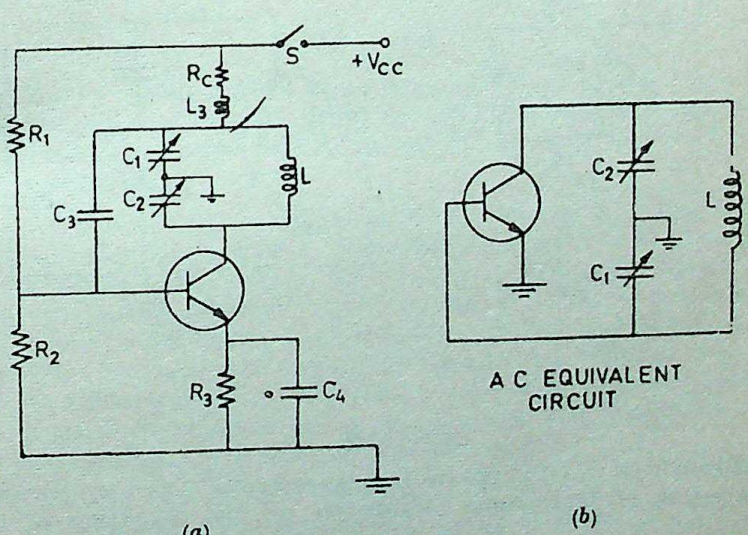

Fig. 30-84

The tank circuit consists of two ganged capacitors C_1 and C_2 and a single fixed coil. The frequency of oscillation (which does not depend on mutual inductance) is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where $$C = \frac{C_1 C_2}{C_1 + C_2}$$

*As an aid to memory, remember that Hartley begins with *H* for Henry (coil) and Colpitts begins with *C* for Capacitor.

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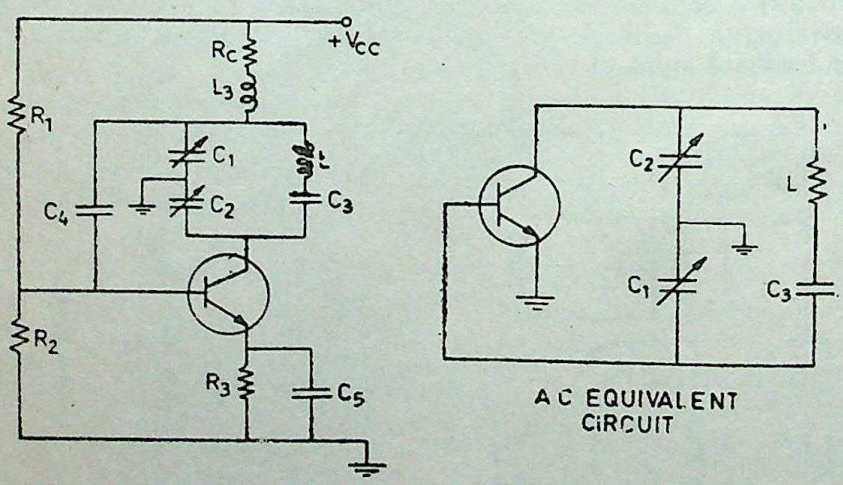
Transistor itself produces a phase shift of 180°. Another phase shift of 180° is provided by the capacitive feedback thus giving a total phase shift of 360° between the emitter-base and collector-base circuits.

Resistors R_1 and R_2 form a voltage divider across V_{CC} for providing base bias, R_3 is for emitter stabilisation and R_C is collector load necessary for amplifier action. Inductor L_3 keeps a.c. currents out of d.c. supply. Capacitor C_4 is a by-pass capacitor whereas C_3 conveys feedback from the collector-to-base circuit.

Colpitts oscillator is widely used in commercial signal generators upto 1 MHz. Frequency of oscillation is varied by gang-tuning the two capacitors C_1 and C_2.

30-48. Clapp Oscillator

It is a variation of Colpitts oscillator and is shown in Fig. 30-85 (*a*). It differs from Colpitts oscillator in respect of capacitor C_3 only which is joined in series with the tank inductor.



(*a*) (*b*)

Fig. 30-85

Addition of C_3 (*i*) improves frequency stability and (*ii*) eliminates the effect of transistor's parameters on the operation of the circuit.

The frequency of oscillation is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where $$1/C = 1/C_1 + 1/C_2 + 1/C_3$$

30-49. Crystal Controlled Oscillator

Fig. 30-86 shows the use of a crystal to stabilise the frequency of a tuned-collector oscillator which has a crystal (usually quartz) in the feedback circuit.

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The Colpitts circuit has a frequency of oscillation $f_0 = 1/2\pi\sqrt{LC}$. The circuit is adjusted to have a frequency nearabout the desired

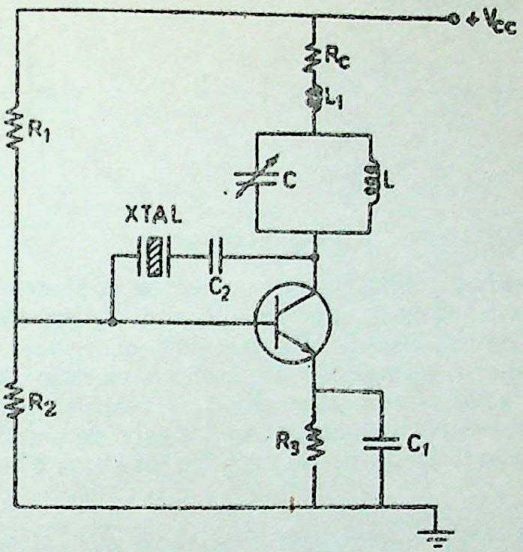

Fig. 30-86

operating frequency but the exact frequency is set by the crystal and stabilized by the crystal. For example, if natural frequency of vibration of the crystal is 27 MHz, the *LC* circuit is made to resonate at this frequency.

As usual, resistors R_1, R_2 and R_3 provide a voltage-divider stabilised d.c. bias circuit. Capacitor C_1 by-passes R_3 in order to maintain large voltage gain. *RFC* coil L_1 prevents a.c. signals from entering d.c. line whereas R_c is the required d.c. load of the collector. The coupling capacitor C_2 has negligible impedance at the operating frequency but prevents any d.c. link between collector and base. Due to extreme stability of crystal oscillations, such oscillators are widely used in communication transmitters and receivers where frequency stability is of prime importance.

30-50 OPAMP Oscillator Circuits

IC chips are finding increasingly greater use in oscillator circuits. One has only to buy an *OPAMP* to obtain an amplifier circuit of stablised gain and incorporate some means of signal feedback to produce an oscillator.

30-51. Thyristor*

(A.M.I.E., Winter 1986)

(*i*) **Construction.** It is a four-layer, three-junction transistor and is the solid-state equivalent of gas-filled switching valve known as thyratron. As shown in Fig. 30.87 (*a*), the four layers are made of alternately *P*-type and *N*-type silicon.

*The name has been derived from both THYRatron and transISTOR i.e. Thyristor.

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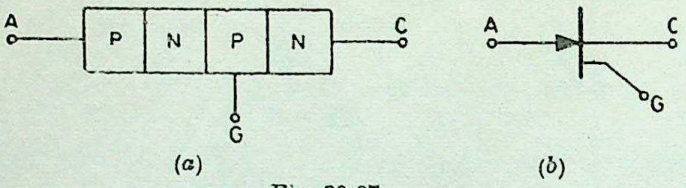
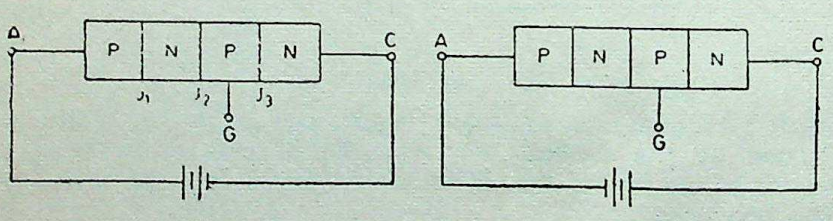
The outer *P*-type layer is called the anode and the *N*-type layer the cathode. There is an extra control electrode called 'gate' which is connected to the inner *P*-type layer. Fig. 30-87 (*b*) shows the symbol of a thyristor.



(*a*) (*b*)

Fig. 30-87

(*ii*) **Working.** With battery connection as shown in Fig. 30-88 (*a*) and with gate left open, junctions J_1 and J_3 are forward-biased whereas J_2 is reverse-biased. Hence, flow of current is blocked. However, as anode voltage is increased, a certain critical value called forward breakover voltage (V_{BO}) is reached [Fig. 30-89 (*a*)] when thyristor switches suddenly to a highly conducting state and the voltage across it drops to a low value (of about 1 volt). In this



(*a*) (*b*)

Fig. 30-88

conducting state, current through the device is limited only by the supply voltage and the load resistance. Current continues flowing indefinitely until the circuit is interrupted briefly just as in the case of a thyratron.

When thyristor is operated with reverse voltage (anode negative and cathode positive) as shown in Fig. 30-88 (*b*), it again blocks the flow of current until reverse voltage reaches Zener value when avalanche or breakdown occurs [Fig. 30-89 (*a*)].

(*iii*) **Firing.** Usually, thyristor is operated with an anode voltage slightly less than the forward breakover voltage V_{BO} and it is then turned on or triggered into a high conduction state by means of a low-power pulse (typically 1·5 V and 30 mA) applied between the gate and cathode. Once switched on to the conducting state, the gate has no further influence over the anode-cathode current until the anode circuit is completely interrupted for about 20 μs. Gate signals can be (*a*) d.c. firing signals (*b*) pulse signals and (*c*) a.c. phase signals.

(*iv*) **Applications.** Thyristor behaves as a bistable switch, either non-conducting or conducting. In practice, it is used as a high-speed switch and in power applications is likely to take the

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place of thyratrons and mercury-arc rectifiers, because it eliminates moving contacts and arcing and reduces power losses.

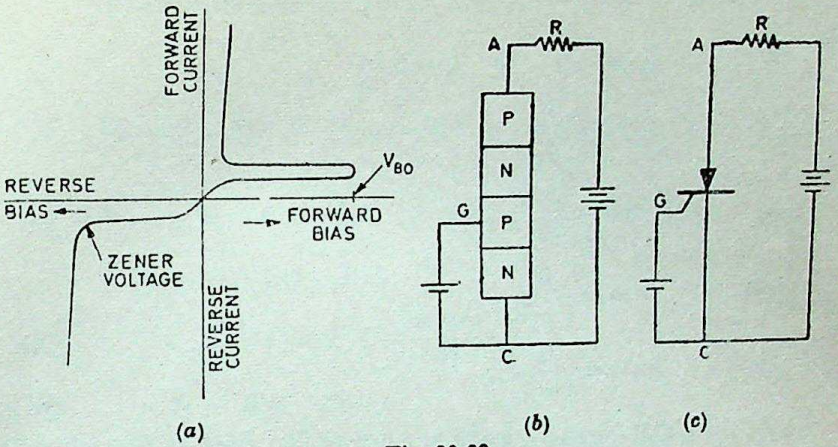

Fig. 30-89

At present, it is available in ratings of as much as 1300 V and 500 A. Since a thyristor can be caused to conduct by a pulse with as little power as 0·5 W, it will be seen that its power gain is more than one million.

Because it exerts control over very large currents by means of a small gate pulse, it is very useful in relay, switching and control applications. Typical applications are in regulated power supplies, d.c. to a.c. invertors, radar modulators, servo systems and latching relays etc. The most important advantage of a thyristor is, as mentioned earlier, that switching is extremely rapid and requires no moving parts.

HIGHLIGHTS

1. A junction transistor is made from a single piece of semiconductor material but has three differently-doped regions called emitter, base and collector.

2. For normal operation, *E/B* junction is forward-biased whereas *C/B* junction is reverse-biased.

3. Both in *PNP* as well as in *NPN* transistors, collector and base have *same* polarity with respect to the emitter.

4. $I_E = I_B + I_C$ – both for *PNP* and *NPN* transistors (neglecting leakage currents).

5. For *CB* configuration if we neglect leakage current,

(*i*) $\alpha_{dc} = \dfrac{-I_C}{I_E}$

(*ii*) $I_C = \alpha I_E$ (neglecting minus sign)

(*iii*) $I_B = (1-\alpha) I_E$

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(iv) $\alpha_{ac} = \dfrac{-\triangle I_C}{\triangle I_B}$

6. For *CE* configuration if we neglect leakage current,

(i) $\beta_{dc} = \dfrac{I_C}{I_B}$ or $I_C = \beta I_B$

(ii) $\beta_{ac} = \dfrac{\triangle I_C}{\triangle I_B}$

(iii) $\beta = \dfrac{\alpha}{1-\alpha}$

(iv) $\alpha = \dfrac{\beta}{1+\beta}$

(v) $1-\alpha = \dfrac{1}{1+\beta}$

7. In a *CB* circuit, leakage current is I_{CBO} or I_{CO} which is temperature-dependent. Hence, in such a circuit

$$I_C = \underset{\text{majority current}}{\alpha I_E} + \underset{\text{minority current}}{I_{CO}}$$

8. In a *CE* circuit, leakage current is I_{CEO}. Hence, in such a circuit,

$$I_C = \underset{\text{majority current}}{\beta I_B} + \underset{\text{minority current}}{I_{CEO}}$$

9. A transistor is said to be cut-off when

(i) $I_B = 0$

(ii) $I_C = 0$

(iii) hence, $V_{CE} = V_{CC}$

10. A transistor is said to be in saturation when

(i) collector current has maximum possible value

$I_{C(sat)} = V_{CC}/R_L$

(ii) $V_{CE} = 0$

(iii) $V_{CC} = I_C R_L$

11. AC load line of a transistor is steeper than d.c. load line.

12. Formulas for *T*—equivalent of a *CB* amplifier circuit are :—

(i) $r_{in} = r_e + r_b\,(1-\alpha)$

$= r_e + \dfrac{r_b}{(1+\beta)}$

(ii) $r_{out} = r_c - \dfrac{\alpha\, r_b\, r_c}{r_e + r_b + R_S}$

(iii) $A_i = \alpha$

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(*iv*) $$A_v = \frac{\alpha R_L}{r_e + r_b(1-\alpha)}$$

or $$= \frac{\alpha R_L}{(r_e + R_s) + r_b(1-\alpha)}$$

(*v*) $$G = A_i A_v$$

13. Formulas for T−equivalent of a *CE* amplifier circuit are :—

(*i*) $$r_{in} = r_b + \frac{r_e}{(1-\alpha)}$$

$$= r_b + (1+\beta) r_e$$

(*ii*) $$r_{out} = r_c(1-\alpha) + \frac{\alpha r_e r_c}{r_e + r_b + R_S}$$

(*iii*) $$A_i = \beta = \alpha/(1-\alpha)$$

(*iv*) $$A_v = \frac{\alpha R_L}{r_e + r_b(1-\alpha)}$$

or $$= \frac{\alpha R_L}{r_e + (r_b + R_s)(1-\alpha)}$$

(*v*) $$G = A_v A_i$$

14. Hybrid formulas for transistor amplifier circuit are :—

(*i*) $$R_{in} = h_i - \frac{h_r h_f}{h_o + 1/r_L}$$

(*ii*) $$R_{out} = \frac{h_i + R_s}{(h_i + R_s) h_o - h_f h_r}$$

$$\cong \frac{h_i + R_s}{h_o R_s + \triangle h}$$

(*iii*) $$A_i = \frac{-h_f}{1 + h_o r_L}$$

(*iv*) $$A_v = \frac{-h_f r_L}{h_i + \triangle h . r_L}$$

$$= \frac{-h_f}{R_{in}(h_o + 1/r_L)}$$

where r_L = a.c. load resistance

$\triangle h = h_o h_i - h_f h_r$

15. The essential condition for maintaining oscillations in an oscillator is

$$\beta A = 1 + j0 = 1 \angle 0°$$

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It means that

(*i*) open-loop gain $|\beta A| = 1$

(*ii*) net phase shift around the loop is 0° or 360°

16. In a tuned-base oscillator circuit, the oscillatory tank circuit is connected across the base and emitter terminals.

17. In a tuned-collector oscillator, the oscillatory tank circuit is connected across the collector and emittter terminals.

18. Hartley oscillator is similar to tuned-collector oscillator except that it uses a tapped coil oscillatory tank circuit.

19. Colpitts oscillator is essentially the same as Hartley oscillator except that it uses tapped capacitance instead of tapped inductance.

20. Clapp oscillator is a variation of Colpitts oscillator.

21. Crystal-controlled oscillator is a tuned-collector oscillator which uses a crystal not only for frequency stabilization but for feedback purposes as well.

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ILLUMINATION 31

1. Production of Light 2. Definitions 3. Laws of Illuminance for Point Sources 4. Practical Lighting Schemes 5. Design of Lighting Schemes 6. Calculations Based on Lumen Method 7. Highlights.

31-1. Production of Light

The different methods of producing light by electricity may, in a broad sense, be divided into three groups :

1. By temperature incandescence. In this method, an electric current is passed through a filament of thin wire placed in vacuum or an inert gas. The current generates enough heat to raise the temperature of the filament to luminosity.

Incandescent tungsten filament lamps are examples of this type and since their output depends on the temperature of their filaments, they are known as temperature radiators.

2. By establishing an arc between two carbon electrodes. The source of light, in their case, is the incandescent electrode.

3. Discharge Lamps—in these lamps, gas or vapour is made luminous by an electric discharge through them. The colour and intensity of light *i.e.* candle-power emitted depends on the nature of the gas or vapour only. It should be particularly noted that these discharge lamps are luminiscent-light lamps and do not depend on temperature for higher efficiencies. In this respect, they differ radically from incandescent lamps whose efficiency is dependent on temperature. Mercury-vapour lamp, sodium-vapour lamp, neon gas lamp and fluorescent lamps are examples of light sources based on discharge through gases and vapours.

Discharge lamps can be further subdivided into two types :

(*a*) those in which the colour of the light is the same as produced by the discharge through the gas or vapour.

Examples are : neon gas lamps, mercury vapour lamps and sodium vapour lamps.

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(*b*) those which use the phenomenon of flourescence. In their case, the discharge through the vapour produces invisible ultra-violet rays (especially of $\lambda = 2537$ Å) which cause flourescence in certain internally-coated materials known as **phosphors**. These phosphors absorb these incident ultra-violet rays and then re-radiate them at longer wavelengths of the visible spectrum. The colour of the flourescent light depends on the type of phosphor used.

The wavelength of red light is $\lambda = 0{\cdot}78\ \mu m$ (or 780 nm or 7800×10^{-10} m) and that of violet light is $0{\cdot}39\ \mu m$ (or 390 nm or 3900×10^{-10} m).

31-2. Definitions

1. **Steradian** (sr). It is the unit of solid angle which is defined as the angle substended by a certain surface at a given point. As shown in Fig. 31-1, if A is the area of the spherical surface, then solid angle

$$\omega = \frac{A}{r^2} \text{ steradian}$$

If $A = 1$ m^2, $r = 1$ m, then $\omega = 1$ steradian (sr).

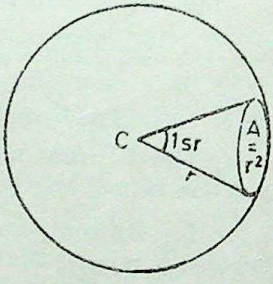

Fig. 31-1

Hence, steradian may be defined as the angle subtended at the centre of a sphere of radius 1 m by a part of its surface having an area of 1 m^2. Obviously, solid angle subtended at the centre by whole of the spherical surface of a sphere of radius r is $= 4\pi\, r^2/r^2 = 4\,\pi r$ sr.

2. **Candela** (cd). It is the unit of luminous intensity of a *source*.

A light source has a luminous intensity of one candela if it omits 1 lumen (lm) per steradian. As shown in Fig. 31-2, the source has a luminous intensity of one candela. Since it emits a flux of one lumen in a solid angle of one steradian, total luminous flux emitted by it alround is

$$= 4\pi \times 1 = 4\pi \text{ lumen}$$

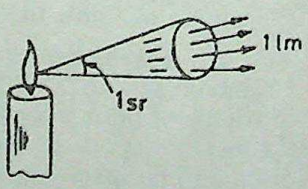

Fig. 31-2

It is so because solid angle subtended by the surrounding space at a point is 4π steradian.

3. **Luminous Flux** (F or Φ). It is the total light energy radiated out per second from a luminous source.

Its unit is *lumen* (lm). Since, it is a rate of flow of light energy, it is a sort of *power* unit.

One lumen is equal to the flux given out in a solid angle of one steradian by a source of one candela.

4. **Luminous Intensity** (I). It is defined as the luminous flux radiated out per unit solid angle by a light source.

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If $d\Phi$ is the luminous flux emitted by a source within a solid angle of $d\omega$ in any particular direction, then

$$I = \frac{d\Phi}{d\omega} \text{ lm/sr or cd}$$

It may be noted that if a source has a luminous intensity of I candela, then total flux radiated out by it in the space around it is

$$= 4\pi \times I \text{ lumen}$$

5. **Illuminance or Illumination** (E). When luminous flux falls on a surface, it is said to be illuminated.

The illumination of a surface is measured by the normal flux per unit area received by it. If $d\Phi$ is the luminous flux incident normally on an area dA, then

$$E = \frac{d\Phi}{dA} \text{ lm/m}^2 \text{ or lux}$$

In general, $$E = \frac{\Phi}{A} \text{ lux}$$

31-3. Laws of Illuminance for Point Sources

The illuminance (E) of a *surface* depends on the following factors. The light source is assumed to be a point source or is otherwise sufficiently away from the surface to be regarded as such.

(*i*) E is *directly* proportional to the luminous intensity (I) of the *source i.e.* $E \propto I$.

(*ii*) E is *inversely* proportional to the square of the distance between the source and the surface. *i.e.* $E \propto 1/r^2$

(*iii*) E is *directly* proportional to the cosine of the angle made by the normal to the illuminated surface with the direction of the incident flux.

Combining the above three factors, we have

$$E = \frac{I \cos\theta}{r^2}$$

Let a lamp L of uniform luminous intensity I be suspended at a height of h above the working plane as shown in Fig. 31-3. Let us apply the above law to calculate illuminance E at point A immediately below the lamp and at point B.

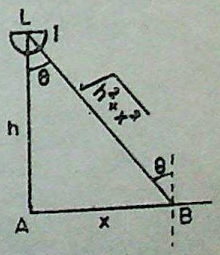

Fig. 31-3

Point A

Here, $\theta = 0$, $\cos\theta = 1$

$$E_A = \frac{I \cos\theta}{r^2} = \frac{I}{h^2}$$

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Point B

Here, $$\cos\theta = \frac{h}{LB} = \frac{h}{\sqrt{h^2+x^2}}$$

$$\therefore \quad E_B = \frac{I}{LB^2} \times \frac{h}{LB} = I \cdot \frac{h}{LB^3}$$

We may also express E_B in terms of E_A as under :

$$E_B = I\,\frac{h}{LB^3} = \frac{I}{h^2} \cdot \frac{h^3}{LB^3} = E_A \cos^3\theta$$

Example 31-1. *An incandescent lamp having a luminous intensity of 60 cd in all directions gives an illumination of 15 lux at the surface of a table vertically below it.*

(a) What distance is the lamp from the table ?

(b) What illumination would be given if 60 cd lamp is replaced by a similar 100 cd lamp ?

Solution. (a) $E = I/h^2$ or $15 = 60/h^2$; $h = \mathbf{2\ m}$

(b) Since $E \propto I$, $E_1 \propto I_1$ and $E_2 \propto I_2$

$$\therefore \quad \frac{E_2}{E_1} = \frac{I_2}{I_1} \quad \text{or} \quad \frac{E_2}{15} = \frac{100}{60} \quad \text{or} \quad E_2 = \mathbf{25\ lux}$$

Example 31-2. *A lamp giving out 1260 lumen in all directions is suspended 8 m above the working plane. Calculate the illumination at a point on the working plane 6 m away from the foot of the lamp.*

Solution.

$$\Phi = 4\pi I \text{ lumen}$$

$$1260 = 4\,\pi I \text{ or } I = 100 \text{ cd}$$

As seen from Fig. 31-4,

$$LB = \sqrt{8^2+6^2} = 10 \text{ m}$$

$$\cos\theta = LA/LB = 8/10 = 0{\cdot}8$$

Now. $$E = I\cos\theta/r^2$$

$$= 100 \times 0{\cdot}8/10^2 = \mathbf{0{\cdot}8\ lux}$$

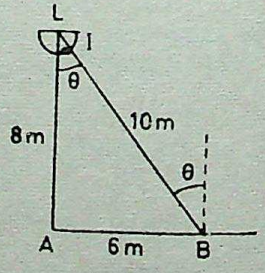

Fig. 31-4

Example 31-3. *A certain lamp, giving equal illuminance below the horizontal hangs from the ceiling of a room. The illuminance received on a small horizontal screen lying on a bench vertically below the lamp is 60 lux. When the screen is moved a distance of 1·2 m along the bench, the illumination is 30·72 lux.*

Calculate the luminous intensity of the lamp and its vertical distance from the bench.

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Solution. As seen from Fig. 31-5,

$E_1 = I/h^2$ or $I = E_1 h^2$ ...(*i*)

$E_2 = I \cos\theta/d^2 = Ih/d^3$

or $I = E_2 d^3/h$ ...(*ii*)

From (*i*) and (*ii*), we get

$E_1 h^3 = E_2 d^3/h$ or $E_2/E_1 = h^3/d^3$

Now, $h/d = \cos\theta$

$\therefore \quad \cos^3\theta = E_2/E_1$

or $\cos\theta = (30{\cdot}72/60)^{1/3} = 0{\cdot}8$

$\therefore \quad \theta = 36{\cdot}8°$

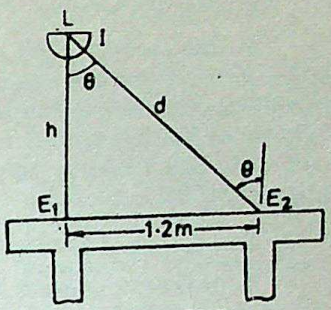

Fig. 31-5

Now, $\tan\theta = 1{\cdot}2/h \quad \therefore \quad h = 1{\cdot}2/\tan 36{\cdot}8° = \mathbf{1{\cdot}6\ m}$

Substituting this value in Eq. (*i*), we have

$$I = 60 \times 1{\cdot}6^2 = \mathbf{154\ candela}$$

31-4. Practical Lighting Schemes

The simple inverse-square-law calculations are applicable only to point sources of light or where there are no reflecting or absorbing surfaces.

For indoor illumination, we have to consider not only the light received directly from the source itself but also that reflected from walls and ceilings etc.

Different lighting schemes are classified as under :

1. Direct Lighting

In this case, light from the source falls directly on the surface or object to be illuminated. Most of the light is directed in the lower hemisphere with the help of shades, globes and reflectors of various types. It is essential to keep the lamps and fittings clean otherwise decrease in effective illumination due to dirty bulbs or reflectors may amount to about 20 to 25% in offices and homes. This type of lighting is liable to cause glare and hard shadows.

2. Indirect Lighting

In this case, light does not reach the working plane *directly* from the source but indirectly by diffuse reflection. Lamps and tubes are placed either behind the cornice or in suspended opaque bowls. In this way, maximum light is thrown upwards on the ceiling from where it is distributed all over the room by diffuse reflection. Indirect lighting provides shadowless illumination which is vary suitable for drawing offices, composing rooms and in workshops.

3. Semi-direct Lighting

This system utilizes luminaries which send most of the light downwards directly on the working plane though a considerable amount reaches the ceiling and walls also. Such a system is best suited to rooms with high ceilings where a high level of uniformly distributed illumination is desirable.

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4. Semi-indirect Lighting

In this system, light is partly received by *diffuse* reflection and partly *direct* from the source. Instead of using opaque bowls with reflectors, *translucent* bowls without reflectors are used. Most of the light is, as before, directed upwards to the ceiling for diffuse reflection and the rest reaches the working plane directly except for some absorption by the bowl.

5. General Diffusing System

In this system, such luminaries are employed which have almost equal light distribution downwards and upwards.

31·5. Design of Lighting Schemes

Practical lighting schemes are based on the *lumen method* which takes into account the following factors :

1. Utilization Factor

It is well-known that part of the lumen output of a source is lost in the fittings. Some output is directed towards the walls and ceiling where it is partly absorbed and partly reflected. Consequently, only a portion of the lumen output reaches the working plane.

Utilization factor is the ratio of lumens actually received by a particular surface to the total lumens emitted by a luminous source.

$$\therefore \quad \eta = \frac{\text{lumens actually received on working plane}}{\text{lumens emitted by the light source}}$$

2. Maintenance Factor (m)

This factor allows for the fact that luminous intensity of all lamps or luminaries deteriorates owing to blackening and/or collection of dust or dirt on the globes and reflectors etc. Similarly, walls and ceilings etc. also do not reflect as much light as when they are clean.

$$m = \frac{\text{illumination when everything is perfectly clean}}{\text{illumination under actual conditions}}$$

Its value varies from 1·3 to 1·5.

Occasionally, the term depreciation factor (p) is used which is reciprocal of maintenance factor.

$$\text{depreciation factor} = \frac{1}{\text{maintenance factor}}$$

or

$$p = 1/m$$

3. Space-height Ratio

It is given by

$$\frac{\text{horizontal distance between two lamps}}{\text{mounting height of lamps}}$$

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This ratio depends on the nature of the polar curve when *used*

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along with its reflector. For reflectors normally used in indoor lighting, the value of this ratio lies between 1 and 2.

31-6. Calculations Based on Lumen Method

Taking into account the above-given factors, total number of lumens required for a certain building are

$$\text{total flux required } \Phi = \frac{E \times A}{\eta \times p} = \frac{E \times A}{\eta \times 1/m} = \frac{mEA}{\eta}$$

If a group of light units is given, then illuminance produced on a given surface is

$$E = \frac{\Phi \times \eta \times p}{A} = \frac{\Phi \times \eta}{m \times A}$$

Example 31-4. *A workshop measures 10m × 12m and is lighted by 20 lamps of 100 W each. Taking depreciation factor of 0·75, coefficient of utilisation 0·6 and efficiency of lamps as 15 lm/W, find the illumination on the working plane.*

Solution. $A = 10 \times 12 = 120$ m², $p = 0{\cdot}75$

$\eta = 0{\cdot}6$, $\Phi = (20 \times 100) \times 15 = 30{,}000$ lm

$E = ?$

Now $\Phi = \dfrac{EA}{\eta p}$ or $30{,}000 = \dfrac{E \times 120}{0{\cdot}75 \times 0{\cdot}6}$

$\therefore$ **$E = 112{\cdot}5$ lux**

Example 31-5. *A light assembly shop, 15 m long, 9 m wide and 3 m upto trusses is to be illuminated to a level of 200 lux. The utilisation and depreciation factors are respectively 0·9 and 0·8. Using tungsten lamps and dispersive metallic reflectors, draw their spacing lay-out. Assume a lamp efficiency of 13 lm/W and space/height ratio of unity.*

Solution. $E = 200$ lux, $A = 15 \times 9 = 135$ m²

$\eta = 0{\cdot}9$, $p = 0{\cdot}8$, $\Phi = ?$

$$\Phi = \frac{200 \times 135}{0{\cdot}9 \times 0{\cdot}8} = 37{,}500 \text{ lm}$$

$$S/H = 1 \quad \text{or} \quad S/3 = 1 \quad \text{or} \quad S = 3 \text{ m}$$

Obviousiy, 5 lamps can be mounted along the length and 3 along the width of the assembly shop as shown in the layout diagram of Fig. 31-6. Total number of lamps required is $5 \times 3 = 15$.

If W is the wattage of each lamp, then total wattage is $15 \times W$. Taking lamp efficiency into account, total lumens output

$$= 15 \times W \times 13 \text{ lumens}$$

$\therefore$ $15 \times 13 \times W = 37{,}500$

$\therefore$ **$W = 192{\cdot}3$ watt $\simeq$ 200 watt**

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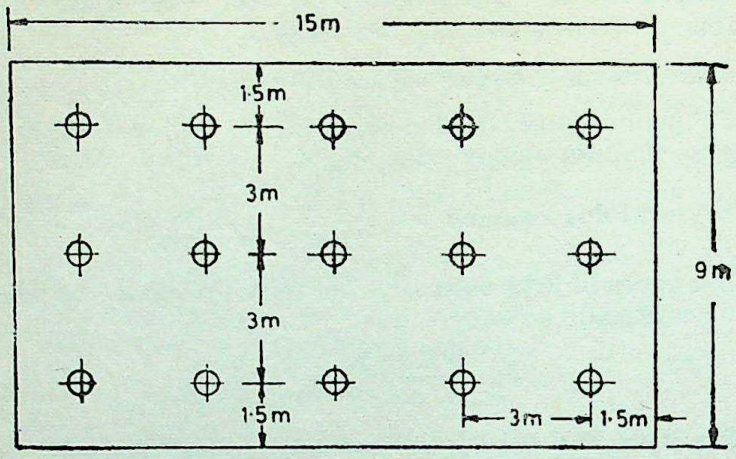

Fig. 31-6

HIGHLIGHTS

1. Candela is the unit of luminous intensity of a source.
2. Luminous intensity of a light source is given by luminous flux radiated out by it per unit solid angle.
3. Illuminance of a surface is measured by the normal flux per unit area received by it.
4. Laws of illuminance are given by the expression

$$E = \frac{I \cos \theta}{r^2}$$

5. For indoor lighting, illuminance formula is

$$E = \frac{\Phi \times \eta}{m \times A} \quad \text{or} \quad \frac{\Phi \times \eta \times p}{A}$$

where η = utilization factor
m = maintenance factor
p = depreciation factor
$= 1/m$

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THERMOELECTRICITY 32

Production of Thermo-electromotive Force—Seebeck Effect—Thermo-electric Effect and Contact Potential Difference—Peltier Effect—Explanation of Peltier Effect—Peltier Effect and Joulé Heating Effect—Demonstration of Peltier Effect—Definition of Peltier Coefficient—Measurement of Peltier Co-efficient—Thomson Effect—Demonstration of Thomson Effect—Definition of Thomson Coefficient—The Value of E.M.F. in a Thermocouple—Thermo-electromotive Force in Terms of Peltier and Thomson Coefficients—Laws of Thermo-electricity—Equation of Thermoelectric Curve—Thermo-electric Power and Power Lines—Thermo-electric Diagram—Thermodynamics Applied to Thermo-electric Circuit—Applications of Thermo-electric Effect—Thermocouples as Current Sources—Highlights—Objective Tests.

32-1. Production of Thermo-electromotive Force—Seebeck Effect

In Fig. 32-1 is shown a copper-iron couple consisting of bars of copper and iron joined together at their ends A and B with a microvoltmeter M included in the circuit. The junctions may be soldered or welded togethre. Suppose that *initially* the two junctions A and B are at 0°C. Now, keeping junction A at 0°C as before, let junction B be gradually heated. It is found that microvoltmeter starts showing deflection which increases with temperature ndicating that some e.m.f. and hence current is produced in the circuit. The direction of flow of the current is as shown in Fig. 32-1. iIt flows from copper to iron at the hot junction* and from iron to

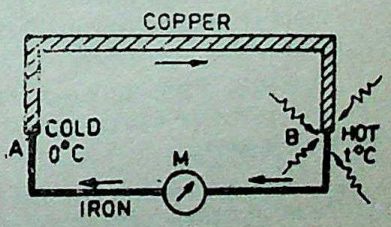

Fig. 3·21

*It is useful to remember the word 'Chi' meaning that current flows from Copper to Iron at Hot junction.

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copper at the cold junction. Incidentally, for an antimony-bismuth thermocouple, current flows from Antimony to Bismuth at the Cold junction (remember the three letters *ABC* as an aid to memory) and bismuth to antimony at the hot junction.

This current is called *thermo-electric current* and is, obviously, due to the production of an e.m.f. called thermo-e.m.f. (of the order of microvolt). The pair of **dissimilar** metals is known as a *thermo-couple*. The generation of thermo-e.m.f by a thermo-couple when one of its junctions is heated is called **Seebeck effect***.

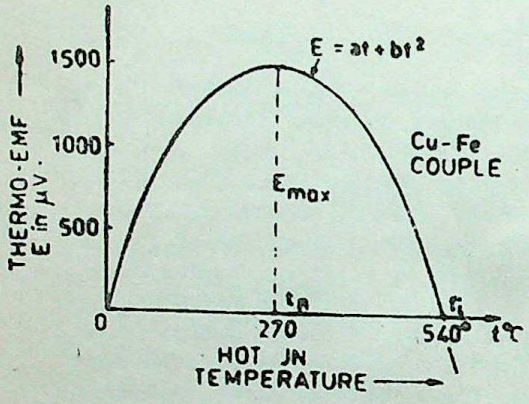

Fig. 32-2

It is found that for a copper-iron couple, the value of this thermo-e.m.f. goes on increasing *parabolically* as junction *B* is further heated till a temperature of 270°C is reached. At this temperature the value of thermo-e.m.f. is **maximum** as shown in the thermo-electric curve of Fig. 32-2. This temperature is known as the *neutral* temperature (t_n) and may be defined as *that temperature of the hot junction of a thermocouple for which the thermo-e.m.f. produced is maximum.*

If heating of junction *B* (Fig. 32-1) is continued beyond the neutral temperature, the thermo-e.m.f. starts *decreasing* more or less parabolically, till it becomes zero at 540° which is called the *temperature of inversion* (t_i).

If junction *B* is heated beyond 540°, thermo-*e.m.f.* again starts increasing but in the *reversed* direction (Fig. 32-2). Now, the current flows from iron to copper at the hot junction *B* and from copper to iron at the cold junction *A*.

It is seen from the above discussion that a thermocouple is just like a generator or energy converter *i.e. one which converts heat energy into electric energy.*

A few points about the neutral temperature and the temperature of inversion etc. are worth noting.

1. The neutral temperature is *constant for a given thermo-couple.* It depends only on the pair of metals chosen but is *independent of the temperature of the cold junction.* For copper-iron thermo-couple, the value of neutral temperature is about 270° whatever the temperature of the cold junction may be.
2. However, the temperature of inversion does **not** have a fixed value for a given couple. It is *variable* and **depends on the**

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*After Seebeck who discovered this phenomenon in 1821.

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temperature of the *cold junction*. But it is always *as much above the neutral temperature as the neutral temperature is above the temperature of the cold junction.*

For example, as shown in Fig. 32-3 if cold junction is kept at 10°C, then thermo-e.m.f. will again be maximum at 270° *i.e.* t_n would remain the same. But the temperature of inversion would become 530°C because it has to be as much above t_n as t_n is above the temperature of the cold junction.

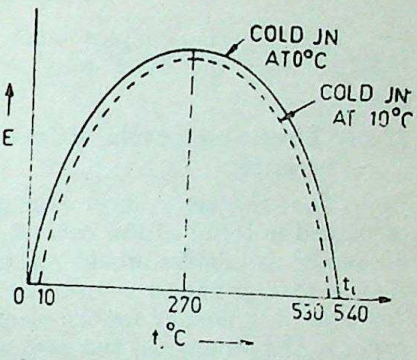

Fig. 32.3

3. The slope of the thermo-e.m.f. curve at any point is called the thermoelectric **power** of that couple at that temperature*. As seen, its value is highest for the initial portion of the curve *i.e.* at 0°C and falls off at higher temperatures *becoming zero at neutral temperature* (Fig. 32-14).
4. If we replace one of the metals by a semi-conductor, the thermo-e.m.f. is *increased substantially* because of much higher electron concentration in semiconductors.
5. The equation of the parabolic curve of Fig. 32-2 is given by

$$E = at + bt^2 \quad \text{microvolt } (\mu V)$$

where t = temperature of the hot junction in °C

a, b = constants for the particular pair of metals

However, it is more usual to write the above equation as

$$E = at + \tfrac{1}{2}bt^2 \quad \mu V$$

In the above equation, cold junction has been assumed to be at 0°C so that 't' actually represents the temperature of the hot junction (or, in general, temperature *difference* of the two junctions).

6. In case, cold junction is at t_1°C and hot junction at t_2°C, then the above equation becomes

$$E = a(t_2 - t_1) + \tfrac{1}{2}b(t_2 - t_1)^2 \ \mu V$$

7. If temperatures are T_1°K and T_2°K, then the above equation is written as

$$E = a(T_2 - T_1) + \tfrac{1}{2}b(T_2 - T_1)^2 \ \mu V$$

8. Depending on the type of metals used, both a and b can have either positive or negative values. Moreover, lot of variations are possible in the value of b.
9. It may be noted that for most of the thermo-couples in practical use, the thermo-e.m.f. is very nearly proportional to

*The name 'power' is misleading because the unit of this thermoelectric power is V/°C (usually μV/°C) and not watt.

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the temperature difference between the two junctions. In that case, the above equation can be simplified to

$$E = a\,t \qquad \text{—in °C}$$
$$= a(t_2 - t_1) \qquad \text{—in °C}$$
$$= a(T_2 - T_1) \qquad \text{—in °K}$$

Such thermo-couples work below the neutral temperature especially over the early part of the curve which is assumed to be linear.

32-2. Thermo-electric Effect and Contact Potential Difference.

The thermo-electric phenomenon discussed above can be explained in terms of the *contact potential difference* developed when *dissimilar* metals are brought in touch with each other. It is found that as soon as the two dissimilar metals touch each other, there is developed a tendency for the electrons to go (or diffuse) from one metal to the other. In the case of copper and iron, more electrons go from iron to copper than from copper to iron. The result is that iron, having lost more electrons, is left at somewhat higher *positive potential* than copper. Hence, this unequal mutual exchange of electrons gives rise to the establishment of a potential difference between iron and copper which is known as the contact potential difference *with iron at slightly higher potential.*

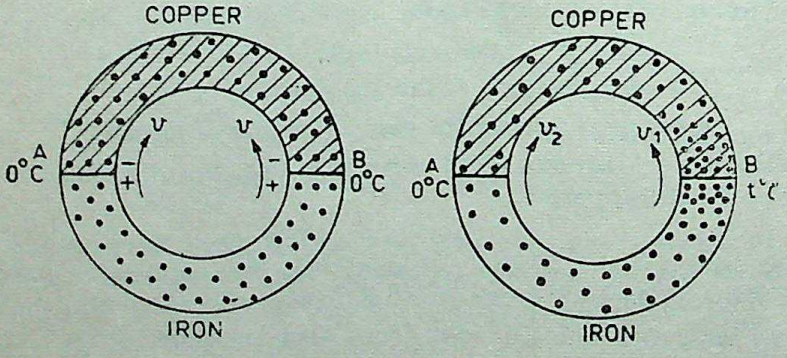

Fig. 32-4

Since in Fig. 32-4 (*a*), junctions *A* and *B* are at the same temperature, then clearly, with respect to the whole circuit, the contact potentials at junctions *A* and *B* are equal *i.e.* *v* each but opposite. Hence, the resultant e.m.f. in the circuit is zero. But when junction *B* is at a higher temperature as shown in Fig. 32-4(*b*) then, due to increased thermal agitation, the transfer (or diffusion) of electrons at junction *B* is more easily accomplished than at the cold junction *A* with the result that a net e.m.f. acts round the circuit. The value of this net e.m.f. is equal to the difference between the junction potentials at *A* and *B* *i.e.* $(v_1 - v_2)$. However, it should be noted that *Peltier e.m.f. acts in the same direction at both junctions* *i.e.* from iron to copper in Fig. 32-4.

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32-3. Measurement of Thermo-electric EMF

(A.M.I.E. Winter 1984)

The most accurate method of measuring thermoelectric e.m.f. is the one which uses a sensitive potentiometer. In its simplest form, it consists of a German silver (or manganin) wire usually one metre long and stretched between two terminals A and C as shown in Fig. 32-5.

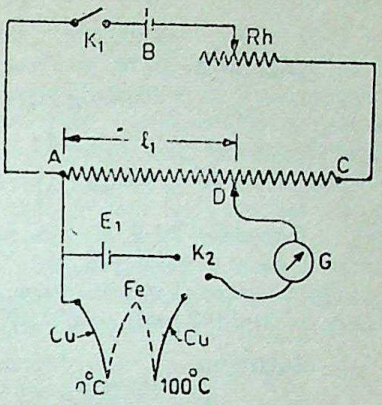

Fig. 32-5

This wire is connected in series with a suitable rheostat Rh and battery B which sends a certain steady current through the resistance wire AC. As the wire is of uniform cross-section throughout, the fall in potential across it is uniform so that drop between any two points on the wire is proportional to the distance between them. As seen, part of the battery voltage drops on the rheostat and the rest over the resistance wire AC. Hence, starting from point A, as we go along AC, there is a progressive *fall* of potential. If ρ is the resistance per cm of this wire, l its length in cm and I the current in amperes passing through it, then total potential drop over $AC = \rho lI$ volt. A standard cell of known e.m.f. E_1 and the thermocouple are joined as shown in Fig. 32-5 *always remembering that positive terminal of E_1, that of the thermocouple and of battery B must all be joined together at the common point A*. The cell and thermocouple can be joined to the galvanometer G in turn through a two-way key K_2. The other end of G is connected to a movable contact on wire AC. The experiment is started by first closing key K_1 so that battery sends a current of I through the circuit. Then, with the help of the movable end, a point like D is found when there is no current in and hence no deflection of G with its one end connected to E_1 through K_2. Then, it means that E_1 just balances the potential drop across AD due to the battery current I passing through it.

If $AD = l_1$, then drop across $AD = \rho l_1 I$. Hence, $E_1 = \rho l_1 I$

Now, thermocouple is connected to G through K_2 and again a balance point is obtained on AC corresponding to thermo-e.m.f. E_2. If, in this case, the balance (or null) point is situated at a length of l_2 from A, then thermoelectric e.m.f. $E_2 = \rho l_2 I$.

$$\therefore \quad \frac{E_2}{E_1} = \frac{\rho l_2 I}{\rho l_1 I} = \frac{l_2}{l_1}$$

$$\therefore \quad E_2 = E_1 \frac{l_2}{l_1}$$

If E_1 is known and l_1 and l_2 are measured experimentally, then thermoelectric e.m.f. E_2 can be easily found from the above equation.

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Example 32-1 *One junction of copper-iron thermocouple is at at 0°C. Find the value of thermo-e.m.f. produced when the other junction is at 200°C. The values of the two constants for the couple are*

$$a = -14 \text{ and } b = +0{\cdot}042$$

Solution

$$E = at + \tfrac{1}{2}bt^2 \quad \mu V$$
$$= -14 \times 200 + \tfrac{1}{2} \times 0{\cdot}042 \times 200^2$$
$$= -1960\ \mu V = \mathbf{-1{\cdot}96\ mV}$$

Example 32-2 *A thermo-couple is made of iron and constantan. Find the e.m.f. developed per °C difference of temperature between the junctions given that thermo-e.m.f. of iron and constantan against platinum are +1600 and −3400 μV per 100°C difference of temperature.*

Solution. e.m.f. of iron against platinum

$$= 1600\ \mu V \text{ per } 100°C$$

$$\therefore \quad E_{Fe}^{Pt} = 16\ \mu V/°C$$

Similary $E_{Con}^{Pt} = -34\ \mu V/°C$

$$\therefore \quad E_{Fe}^{Con} = E_{Fe}^{Pt} - E_{Con}^{Pt} \qquad \text{[Art. 32-15 (iii)]}$$

$$= 16 - (-34) = \mathbf{50\ \mu V/°C}$$

32-4. Peltier Effect

It was discovered by Peltier in 1834 and is a *complementary phenomenon to Seebeck effect.* In Fig. 32-6, again a Cu-iron couple is shown with its two junctions *A* and *B initially at the same temperature.* Suppose that now a battery is included in the circuit such that current flows from copper to iron at junction *B*. It is found that heat energy is *absorbed* at junction *B* and *evolved* at *A*. Hence, junction *B* is cooled and *A* is heated up. It is seen that this effect is *reverse of the Seebeck effect.*

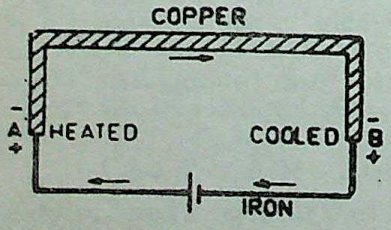

Fig. 32-6

This generation or absorption of heat energy at a junction of a thermocouple when a current is passed through it is known as Peltier effect.

It is interesting to compare Fig 32-6 with Fig. 32-1. It is seen that in Fig. 32-6 that particular junction is cooled which must be heated (in Fig. 32-1) in order to give the thermoelectric current *in the same direction* as the battery current. If, in Fig. 32-6, the direction of current is reversed by reversing the battery connections, a complete interchange of the effects at the two junctions takes

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place. It is obvious that Peltier effect is basically the *transformation of electrical energy into heat energy.*

32-5. Explanation of Peltier Effect

Peltier effect can be regarded as a necessary consequence of junction potential difference. It has been pointed out in Art. 32-2 that when iron touches copper, there is developed a contact potential difference between the two, with iron being at a little higher potential than copper. Hence, when battery current passes through junction B, it goes from a place of lower potential to one at a higher potential *i.e. it flows against an upward gradient of potential.* For this purpose, work (or energy) is needed, which is taken from the junction B. This explains why junction B is cooled.

At junction A, current flows 'down hill' *i.e.* along the downward gradient of potential, hence energy is evolved which, consequently, heats up the junction A.

32-6. Peltier Effect and Joule Heating Effect

It is extremely important to bear in mind that this heating and cooling of junctions is in no way due to the Joule's heating effect ($=I^2Rt/J$ kcal). It is only due to Peltier effect which is basically and totally different from Joule's heating effect. It is well-known that Joule's heating effect is

(*i*) proportional to the square of the current and

(*ii*) is irreversible *i.e.* it produces heat whichever way the current flows.

On the other hand, Peltier effect varies *directly* as the current and is *reversible i.e.* when the current direction is reversed, the thermal effects are also reversed.

Another essential difference between the two is that whereas heating due to Joule's effect occurs *throughout* the circuit, the heating or cooling due to Peltier effect is *localised at the junctions only.*

32-7. Demonstration of Peltier Effect

The apparatus shown in Fig. 32-7 may be utilized for demonstrating the Peltier effect. A differential air thermoscope is placed with its two bulbs on junctions A and B of an antimony-bismuth* couple such that the index lies symmetrically at the centre of the connecting tube. When battery current is passed in the direction shown in Fig. 32-7, then due to Peltier effect, junction A is heated whereas B is cooled. The result is that the index moves *away* from the bulb in which heat is generated. Since heat produced

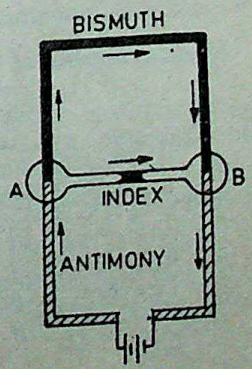

Fig. 32.7

*Antimony is at a higher contact potential than bismuth (Art. 32-1)

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due to Joule effect is the same in both bulbs, it does not effect the position of the index in the connecting tube. Reversal of current through the couple will, obviously, cause the index to move in the opposite direction.

32-8. Definition of Peltier Coefficient (π)

Peltier coefficient is defined as

the quantity of energy evolved or absorbed reversibly at the junction of a thermocouple when one coulomb of charge flows across that junction.

It is denoted by π and its unit is joules/coulomb (which is the same thing as volt). Its value is different for different pairs of metals and even for a given pair, its value is *not constant but depends on temperature.*

It is easy to prove that Peltier coefficient is equal to the contact potential difference at the junction considered.

Proof

Suppose the contact p.d. at a junction is V volts. When a current of I amperes passes through this junction for t seconds, the amount of energy evolved or absorbed (depending on whether this current flows along or against this p.d.) is

$$= VIt \text{ joules} = Vq \text{ joules}$$

Since the Peltier coefficient represents the amount of energy evolved or absorbed per unit charge, the amount of energy evolved or absorbed during the passage of q coulombs of charge

$$= \pi q \text{ joules}$$

Equating the two amounts of energy, we get

$$\pi q = Vq$$

$$\therefore \quad \pi = V \text{ volts}$$

In this case, obviously, the unit of Peltier coefficient is volt.

32-9. Measurement of Peltier Coefficient

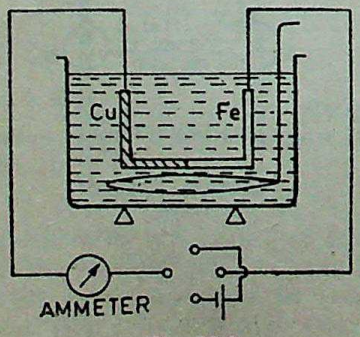

Fig. 32-8

The measurement of Peltier coefficient is difficult in practice because this effect becomes masked by Joule's heating effect. However, the apparatus shown in Fig. 32-8 may be used for measuring the value of Peltier coefficient. The junction of a copper-iron couple is immersed in water contained in a copper calorimeter of known capacity. First, a current of I amperes is passed for t seconds through the junction from iron to copper. Let

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H_1 be the total heat produced. It is made up of two parts :

(*i*) I^2Rt/J —due to Joule's heating effect and

(*ii*) $\pi It/J$ —due to Peltier effect

$$\therefore \quad H_1 = \frac{I^2Rt}{J} + \frac{\pi It}{J}$$

or
$$JH_1 = I^2Rt + \pi It \qquad \ldots(i)$$

Next, the current is reversed and this reversal effects the Peltier effect which is reversible but not Joule's heating effect which, as said earlier, is irreversible. Hence, this time, cooling is produced due to Peltier effect. If H_2 is the total heat produced, then, as before

$$JH_2 = I^2Rt - \pi It \qquad \ldots(ii)$$

Subtracting Eq. (*ii*) from Eq. (*i*), we get

$$\pi = \frac{J(H_1 - H_2)}{2It}$$

The amounts H_1 and H_2 can be calculated if the mass of water, the water equivalent of the calorimeter and the rise in temperature are known. Hence, π can be calculated.

32-10. Thomson Effect

Consider a single metal conductor AB which is unequally heated *i.e.* whose various parts are not at the same but *different* temperatures (Fig. 32-9) It is found that its various parts are at different electric potentials. In other words, the *temperature* gradient along the length of the rod is found responsible for setting up a potential gradient along its length. If, now, current is sent through such an unequally heated conductor along AB, heat energy would be either evolved or absorbed depending on whether the current is flowing along the *downward* or *upward* gradient of potential.

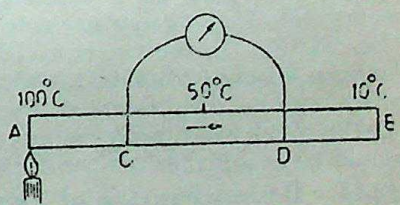

Fig. 32-9

This absorption or evolution of heat energy when current flows through an unequally heated conductor is known as Thomson effect (or Kelvin effect).

It has been found that Thomsom effect is *reversible.*

In the case of substances like copper, zinc, silver, antimony and cadmium etc., the hotter parts are at a higher electric potential than

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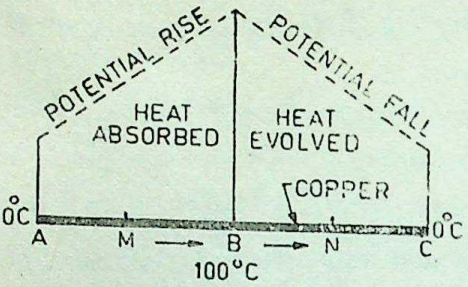

Fig. 32-10

colder parts. Hence, in the case of such substances, if current flows from hotter to colder parts, heat energy is evolved but if it flows from colder to hotter parts, energy is absorbed. Consider a copper conductor being heated at its middle (Fig. 32-10). Obviously, heat energy is absorbed over the part AB and evolved over the portion BC. Thomson effect for such metals is said to be *positive*.

However, in the case of substances like iron, bismuth, cobalt, platinum and nickel etc. colder parts are at higher electrical potential than hotter parts. Hence, in their case, heat energy is evolved when current flows from colder to hotter parts. Consider an iron conductor being heated at its middle (Fig. 32-11). As shown, heat energy is evolved over the portion AB and absorbed over the portion BC. Thomson effect for such substances is said to be *negative*. It should be noted that Thomson effect *for lead (Pb) is nil*. Hence, it is used as a reference metal while drawing thermoelectric power lines for various other metals (Art. 32-15). In the two cases considered above, usually the Joule's heating effect predominates the Thomson effect. But Thomson effect can be demonstrated by considering points M and N (Fig. 32-10) which are equally distant from point B. Obviously, point N would be at a higher temperature than M.

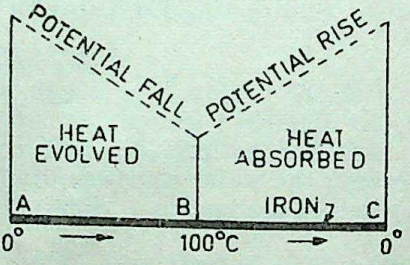

Fig. 32-11

32-11. Demonstration of Thomson Effect

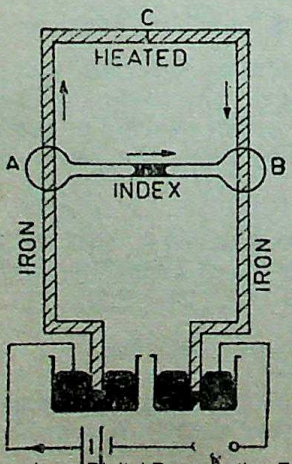

Fig. 32-12

This effect may be demonstrated by the apparatus shown in Fig. 32-12. An iron rod is so bent that it can be heated in the middle while its two ends are kept at a low temperature by immersing them in mercury cups. A differential air thermoscope is placed with its bulbs at parts A and B so that the index lies symmetrically at the centre of the connecting tube. When a heavy current is passed through the iron rod in the direction as shown in Fig. 32-12, then due to Thomson effect, the index moves towards A to B in the connecting tube. It

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shows that heat is *evolved* in the left hand side portion of the rod where current is flowing along the *downward* gradient of potential (remember iron has negative Thomson effect). But heat energy is *absorbed* in the right-hand side portion of the rod where current flows *against* the upward gradient of potential.

Obviously, when current is reversed through the rod, the index moves in the opposite direction.

Similarly, if we take a copper rod (which has positive Thomson effect), then the effect produced would be just the opposite.

32-12. Definition of Thomson Coefficient (σ)

Thomson or Kelvin coefficient is defined as :

the amount of heat energy absorbed or evolved reversibly when one coulomb of charge flows between two points which differ by 1°C in any unequally heated metallic conductor.

It is denoted by σ and its unit is joules per coulomb per °C. Since joules/coulomb means volt, hence its alternative unit is volt per °C (V/°C).

It is quite simple to deduce that σ is equal to the difference of potential per °C in an unequally heated conductor.

Proof

Let there be two points A and B on an unequally heated conductor differing in potential by V volts and in temperature by 1°C. When q coulombs of charge pass between them, heat energy evolved or absorbed due to Thomson effect is

$$= \sigma q \text{ joules}$$

Since the p.d. is V volts, heat energy absorbed or evolved is

$$= Vq \text{ joules}$$

Equating the two amounts of energy, we get

$$\sigma q = Vq$$

$$\therefore \quad \sigma = V \text{ volt/°C}$$

Hence, Thomson coefficient is equal to *the difference of potential per °C.*

It should be particularly noted that, like Peltier coefficient, Thomson coefficient is *not constant but is a function of temperature.*

Let us see how we can calculate the energy absorbed or evolved due to Thomson effect when one coulomb of charge passes along a conductor from a point where temperature is T_1 to a point where temperature is T_2. If the value of σ did not depend on temperature, then this energy would have been $= \sigma(T_2 - T_1)$ joules. But value of σ varies with temperature. Hence, in such cases, the usual procedure is to divide the temperature interval $(T_2 - T_1)$ into infinitely small steps each of value dT. The energy absorbed or evolved in each such interval is σdT. The total energy can be found by integrating the above expression between proper limits.

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∴ Total energy absorbed or evolved

$$= \int_{T_1}^{T_2} \sigma dT \text{ joule}$$

∴ Thomson e.m.f.

$$= \int_{T_1}^{T_2} \sigma dT \text{ volt}$$

The value of this e.m.f. for a thermocouple of metals A and B would be

$$= \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\, dT \text{ volt}$$

32-13. The Value of E.M.F. in a Thermocouple

Consider the Cu-iron couple of Fig. 32-1. When the current starts flowing round the circuit, the following four amounts of energy are evolved :

1. When current passes from cold junction A to hot junction B through copper, then heat energy is *absorbed* due to Thomson effect (Art. 32-10).

2. When current traverses through junction B from Cu to Fe, then again heat energy is *absorbed* due to Peltier effect (Art. 32-4).

3. In iron, current goes from hot to cold junction, hence again energy is *absorbed* due to Thomson effect (Art. 32-10).

4. When current traverses cold junction A from iron to copper, heat is *evolved* due to Peltier effect because current is flowing along the downward gradient of potential (Art. 32-4).

It is seen that, on the whole, more heat is absorbed than evolved. The difference between the two provides the source of energy to which the current is due.

32-14. Thermo-e.m.f. in Terms of Peltier and Thomson Coefficients

Let us now express the thermo-e.m.f. in terms of the heat energies evolved due to both Peltier and Thomson effects. Consider a thermocouple made up of metals A and B (both of which are assumed to have positive Thomson effect) and whose junctions are maintained at temperatures of T_1 and T_2 °C (such that $T_2 > T_1$). Let π_1 and π_2 be the Peltier coefficients at T_1 and T_2 respectively. Assume further that current flows from A to B at the hot junction. Let us now write down the various amounts of energies either absorbed or evolved when *one coulomb of charge passes round the*

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circuit. We will take the heat energy absorbed as positive and evolved as negative.

(*i*) π_2 joules—absorbed at hot junction due to Peltier effect.

(*ii*) $-\pi_1$ joules—evolved at the cold junction due to Peltier effect.

(*iii*) $+\int_{T_1}^{T_2} \sigma_A \, dT$ joules—absorbed in metal A due to Thomson effect.

(*iv*) $-\int_{T_1}^{T_2} \sigma_B \, dT$ joules—evolved in metal B due to Thomson effect.

Hence, total energy *gained* by the charge after once passing round the circuit is

$$=(\pi_2-\pi_1)+\int_{T_1}^{T_2} (\sigma_A-\sigma_B)\, dT$$

By definition, this energy represents the thermo-e.m.f. produced in the thermocouple

Now, $\pi_2-\pi_1$ = Peltier e.m.f.

$\int_{T_1}^{T_2} (\sigma_A-\sigma_B)\, dT$ = Thomson e.m.f.

$\therefore$ Seebeck e.m.f. = Peltier e.m.f. + Thomson e.m.f.

or
$$E=(\pi_2-\pi_1)+\int_{T_1}^{T_2} (\sigma_A-\sigma_B)\, dT \qquad ...(i)$$

The differential form of the above equation is

$$dE=d\pi+(\sigma_A-\sigma_B)\, dT \qquad ...(ii)$$

It is seen that Seebeck e.m.f. in a thermocouple is partly due to Peltier e.m.f. at the *junctions* and partly due to Thomson e.m.f. generated *along* the wires themselves.

32-15. Laws of Thermo-electricity

There are three experimental laws which govern the phenomenon of thermoelectricity.

(*i*) Law of Intermediate Metals

This law states that if, at a given temperature, a number of metals are in successive contact so as to form a chain of elements in

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series, then the thermo-e.m.f. between the extreme metals, if placed in direct contact, is the *sum* of the thermo-e.m.f.'s between the successive adjacent elements.

As an illustration. suppose a number of elements A, B, C,...N are in successive contact, then

$$E_A^N = E_A^B + E^B + E_C^D + \ldots\ldots E_M^N$$

provided all the intermediate junctions in the series are *at the same temperature*. In the above equation, E_A^B, E_B^C, E_C^D etc. represent the thermo-e.m.f.'s for the couples with elements A and B, B and C, and C and D etc.

Or

Put in a simple way, this law states :

if a third element is introduced in a thermocouple, then provided the two junctions resulting from its two ends are kept at *the same temperature as the point of insertion*, there will be no change in the total thermo-e.m.f. of the thermocouple.

Suppose a third element B is inserted between the two elements A and C of a thermo-couple. Then, according to this law

$$E_A^C = E_A^B + E_B^C$$

This law is of extreme practical importance because it shows that the junctions of the two metals may be soldered or a microvoltmeter may be introduced in the thermolectric circuit (Fig. 32-1) without, in any way, effecting the total thermo-e.m.f. produced in the circuit.

(*ii*) Law of Intermediate Temperatures

According to this law :

for a given couple, thermo-e.m.f. for any specific range of temperature is the **sum** *of the thermo-e.m.f's. for any number of successive steps (whether equal or not) into which this given range of temperature may be divided.*

$$\therefore \quad E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + E_{t_3}^{t_4} \ldots\ldots E_{t_{n-1}}^{t_n}$$

where $(t_n - t_1)$ is the specific range of temperature and t_2, t_3.....t_{n-1} are the succcessive temperatures lying intermediate between the extreme temperatures t_1 and t_n.

(*iii*) Law of Combination of E.M.F's.

This law can be stated as follows :

if the thermo-e.m.f's. of two metals with respect to a third metal are given, then the thermo-e.m.f. due to the two metals when brought in

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direct contact is equal to the algebraic **difference** *of their thermo-e.m.f's. relative to the third metal.*

For example, if E_A^B and E_C^B represent the thermo-e.m.f's. of metals A and C with respect to third metal B, then the thermo-e.m.f. between A and C *i.e.* E_A^C is given by

$$E_A^C = E_A^B - E_C^B .$$

Suppose, we are given the e.m.f's. of copper and iron with respect to lead, then e.m.f. of copper with respect to iron can be found thus

$$E_{Cu}^{Fc} = E_{Cu}^{Pb} - E_{Fe}^{Pb}$$

32-16. Equation of Thermoelectric Curve

The curve showing relation between the total thermo-e.m.f. of a thermocuple and the temperature of its hot junction (when the cold junction is kept at constant temperature) is called the *thermoelectric curve* of that couple. Its shape is as shown in Fig. 32-2. It is reproduced as Fig. 32-13. The curve is found to approximate very closely a parabola with its axis parallel to the axis of e.m.f. If the cold junction is kept at 0°C and the hot junction at any temperature t°C, then its equation can be written as

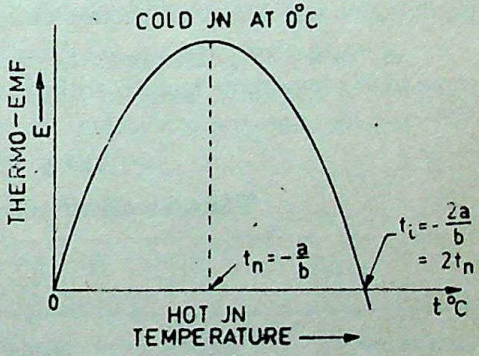

Fig. 32-13

$$E = at + \tfrac{1}{2}bt^2 \qquad \ldots(i)$$

However, if the cold junction instead of being at°C is at t_1°C and the hot junction at t_2°C, the above equation becomes

$$E = a(t_2 - t_1) + \tfrac{1}{2}b(t_2 - t_1)^2$$

where a and b are constants characteristics of the combination of metals A and B under consideration and are known as *thermo-electric coefficients.*

Now, let us see how, by utilizing the above equation, we can calculate the values of neutral temperature and the temperature of inversion.

As discussed in Art. 32-1, neutral temperature is defined as that temperature of the hot junction for which the thermo-e.m.f. produced in the couple is *maximum.* Since, the value of e.m.f. is maximum at neutral temperature, obviously its rate of change with temperature *i.e.* dE/dt must be zero at that temperature.

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Differentiating Eq. (*i*) given above, we get

$$\frac{dE}{dt}=a+bt$$

At neutral temperature t_n, dE/dt must be zero. Hence,

$$0=a+b\ t_n$$

$$\therefore \qquad t_n=-\frac{a}{b}$$

The temperature of inversion t_i is the temperature at which the e.m.f. is zero and starts reversing. Hence, putting $E=0$ in Eq. (*i*) above, we get

$$0=at_i+\tfrac{1}{2}bt_i^2$$

or

$$t_i=-2\ \frac{a}{b}$$

It is seen that t_i is twice t_n—a fact to which attention has been already drawn in Art. 32-1.

In Eq. (*i*), we may regard a as representating the Peltier effect contribution and b that of Thomson effect.

In Table 32-1 are given the values of constants a and b for some of the important metals with respect to lead as the reference metal.

TABLE 32-1

Thermo-electric Coefficients

Metal	a	b
Antimony	+35·5	+0·145
Bismuth	−74·5	+0·032
Constantan	−34·8	−0·09
Copper	+ 2·78	+0·012
Iron	+16·2	−0·03
Platinum	− 3·1	−0·03

The e.m.f.'s calculated by using these values would be given in micro-volts. It may be noted that values of a and b especially of b vary considerably from the above values because effect of factors like the presence of impurity, the heat treatment of the metals and the process used in making the hot junction etc. is much more for b than for a.

The values of a and b for the two metals can be found from their given values with respect to lead as shown below. As seen from Table 32-1, the values for copper-iron thermocouple would be

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$$a_{Cu-Fe} = a_{Cu-Pb} - a_{Fe-Pb}$$
$$= 2{\cdot}78 - 16{\cdot}2 = -13{\cdot}42$$
$$b_{Cu-Fe} = b_{Cu-Pb} - b_{Fe-Pb}$$
$$= 0{\cdot}012 - (-0{\cdot}03) = 0{\cdot}042$$

It may be noted that values for *iron-copper* couple would be

$$a_{Fe-Cu} = +13{\cdot}42 \text{ and } b_{Fe-Cu} = -0{\cdot}042$$

As seen, the signs of the two constants have reversed.

Example 32-3. *One junction of an iron-copper thermocouple is maintained at 210°C and the other at 10°C. Calculate the thermo-electric e.m.f. generated. The thermo-electric coefficients a and b for iron are +16·2 and −0·03 and for copper +2·78 and +0·009 respectively.*

Calculate also the neutral temperature for the above.

Solution. With respect to lead (Art. 32-10)

for iron : $a = 16{\cdot}2$ and $b = -0{\cdot}03$

for copper : $a = 2{\cdot}78$ and $b = +0{\cdot}009$

Let us now find out the values of these constants for an iron-copper couple. Using the Law of Intermediate metals, we get

$$a_{Fe-Cu} = 16{\cdot}2 - 2{\cdot}78 = 13{\cdot}42$$
$$b_{Fe-Cu} = -0{\cdot}03 - 0{\cdot}009 = -0{\cdot}039$$

Now $E = a\,(t_2 - t_1) + \frac{1}{2}b\,(t_2 - t_1)^2$

Here $t_1 = 10°C,\ t_2 = 210°C$

$a = 13{\cdot}42$ and $b = -0{\cdot}039$

$\therefore$ $E = 13{\cdot}42\,(210 - 10) + \frac{1}{2}(-0{\cdot}039)(210 - 10)^2$

$= 2684 - 780 = \mathbf{1904}\ \mu\mathbf{V}$

Now, neutral temperature is given by

$$t_n = -a/b$$
$$= -13{\cdot}42/(-0{\cdot}039) = \mathbf{343°C}$$

Example 32-4. *The hot and cold junctions of a certain copper-iron thermocouple are at 200°C and 0°C respectively. If the constants for the couple are $a_{Cu-Fe} = -14{\cdot}0$ and $b_{Cu-Fe} = 0{\cdot}05$, find the values of (i) neutral temperature (ii) inversion temperature and (iii) thermo-e.m.f.*

Solution.

(i) $$t_n = -\frac{a}{b} = -\frac{-14{\cdot}0}{0{\cdot}05} = \mathbf{280°C}$$

(ii) $$t_i = -\frac{2a}{b} = 2 \times 280 = \mathbf{560°C}$$

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(*iii*) $$E = at + \tfrac{1}{2}bt^2$$
$$= -14 \times 200 + \tfrac{1}{2}(0{\cdot}05) \times 200^2$$
$$= -1800\ \mu V = \mathbf{-1{\cdot}8\ mV}$$

Example 32-5. *The thermoelectric power of an iron-copper couple at 100°C is 6·68 μV/°C and the neutral temperature of the two metals is 282°C. Find the thermoelectric power of the thermocouple at 0°C.* (*A.M.I.E. Winter 1983*)

Solution.

We know that

$$\frac{dE}{dt} = a + bt \qquad \text{—Art. 32-16}$$

$$t_n = -\frac{a}{b} \qquad \text{—Art. 32-16}$$

$$\therefore \quad 282 = -\frac{a}{b} \quad \text{or} \quad b = -\frac{a}{282}$$

Replacing b in the above equation, we get

$$\frac{dE}{dt} = a - \frac{at}{282} = a\left(1 - \frac{t}{282}\right)$$

Substituting the given values, we get

$$6{\cdot}68 = a\left(1 - \frac{t}{282}\right) = a\left(1 - \frac{100}{282}\right)$$

$$\therefore \quad a = \mathbf{10{\cdot}35}$$

Knowing the value of a, dE/dt at $t = 0$°C can be found from above.

$$\frac{dE}{dt} = a + b \times 0$$
$$= a = \mathbf{10{\cdot}35\ \mu V/°C}$$

Example 32-6. *Thermo-electric power for iron with respect to lead falls from 16·5 μV/°C at 0°C to 8·5 at 150°C. Its value for copper rises from 2·8 μV/°C at 0°C to 4·4 at 150°C. Calculate the value of thermo-e.m.f. developed by an iron-copper couple when one junction is at 0°C and the other at 150°C.*

Solution.

Average thermoelectric power for iron

$$= \frac{16{\cdot}5 + 8{\cdot}5}{2} = 12{\cdot}5\ \mu V/°C$$

Hence, total e.m.f. generated by iron-lead couple over temperature range of 0—150°C is

$$= 12{\cdot}5\ \frac{\mu V}{°C} \times 150°C = 1875\ \mu V$$

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Average thermoelectric power for copper-lead couple

$$=\frac{2\cdot8+4\cdot4}{2}=3\cdot6\ \mu V/°C$$

Total e.m.f. produced by this couple over the 0—150°C range is

$$=3\cdot6\times150=540\ \mu V$$

∴ Thermo-e.m.f. of iron-copper couple is

$$=1875-540=\mathbf{1335\ \mu V}$$

Example 32-7. *In an experiment with a thermocouple with one junction at 0°C, the following results were obtained with the hot junction at θ°C*

θ	*25*	*50*	*150*	*200°C*
E	*225*	*400*	*600*	*400 μV*

Assuming a parabolic relation find

(*i*) *neutral temperature*

(*ii*) *thermo-electric power at 50°C*

Solution.

Let the parabolic relation be

$$E=a\theta+\tfrac{1}{2}b\theta^2 \qquad ...(i)$$

Substituting the corresponding values for E and θ for 25°C and 50°C, we get

$$225=25a+\frac{625}{2}b \qquad ...(ii)$$

$$400=50a+1250\ b \qquad ...(iii)$$

Eliminating 'a', we get, $b=-0\cdot08\ \mu V/°C^2$

Substituting this value in Eq. (*iii*) above, we have,

$$a=10\ \mu V/°C$$

When we substitute the values of a and b in Eq. (*i*), we get the parabolic equation as

$$E=10\theta-0\cdot04\ \theta^2$$

(*i*) The neutral temperature occurs where $dE/d\theta=0$

$$\therefore \quad \frac{dE}{d\theta}=10-0\cdot08\ \theta=0$$

$$\theta=10/0\cdot08=\mathbf{125°C}$$

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(*ii*) Thermo-electric power is given as

$$\frac{dE}{d\theta} = 10 - 0{\cdot}08\,\theta$$

For $\theta = 50°C$

$$dE/d\theta = 10 - 0{\cdot}08 \times 50 = 6\ \mu V/°C$$

32-17. Thermo-electric Power and Power Lines

Thermoelectric power of a thermocouple is defined as *the rate of change of its e.m.f. with temperature i.e.* dE/dt.

Its unit is V/°C though it is generally expressed in μV/°C.

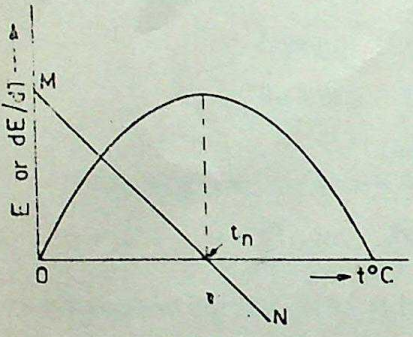

Fig. 32-14

Suppose the two junctions of a thermocouple are initially at t°C. Let the temperature of one junction be increased by dt so that the e.m.f. generated is dE. Then, the ratio dE/dt is known as the thermo-electric power of the metals of the thermo-couple at temperature t°C —an unfortunate use of the term *power*.

Thermoelectric power *changes with temperature*. As the temperature of the hot junction is increased, thermo-electric power is *decreased* till it becomes zero at the neutral temperature of the couple and negative at temperatures beyond the neutral temperature (Fig. 32-14).

It is obvious that thermo-electric power (dE/dt) is just the *slope of the thermoelectric curve*. The slope of this curve is maximum in the beginning and then goes on decreasing till at neutral temperature it is reduced to zero. Beyond that temperature, the slope becomes negative.

It will now be proved that variations of dE/dt with temperature are linear. The equation of the thermo-e.m.f. is (Art. 32-16)

$$E = at + \tfrac{1}{2}bt^2$$

$$\therefore \quad \frac{dE}{dt} = a + bt$$

or $$y = bt + a \qquad \ldots(i)$$

where $y = dE/dt$—the thermoelectric power

In terms of degrees Kelvin (°K), the above relation may be written as

$$\frac{dE}{dT} = a + bT$$

where T is the temperature *difference* of the two junctions in °K.

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It is seen from Eq. (*i*) above that thermo-electric power is a linear function of temperature T. The graph of dE/dT against the temperature T for any thermocouple is a straight line—this line being known as the *thermoelectric power line*. It is shown in Fig. 32-14 as line MN. It is seen that a thermo-electric power line must cut the temperature axis at the neutral temperature for the given couple.

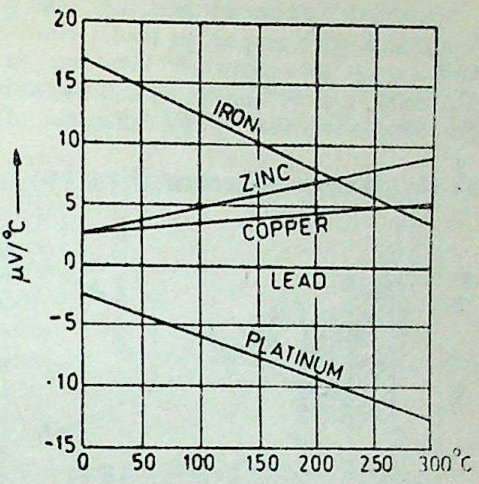

Fig. 32-15

In Fig. 32-15 are shown thermoelectric power lines for various metals with lead taken as the base line. It is worth noting that in the case of metals like copper and zinc etc. which have positive Thomson effect, the lines slope upwards whereas for metals like iron and nickel which have negative Thomson effect, the lines slope downwards.

It may be noted that an alternate expression for thermo-electric power can be derived from Eq. (*ii*) of Art. 32-14 which gives the differential form of the equation for the thermo-e.m.f. as

$$dE = d\pi + (\sigma_A - \sigma_B)\, dT$$

$$\therefore \quad \frac{dE}{dT} = \frac{d\pi}{dT} + (\sigma_A - \sigma_B).$$

Thermo-electric power can be used for finding the total thermo-e.m.f. developed in a thermocouple whose two junctions are maintained at temperatures of T_1 and T_2. This total e.m f. can be found by the expression given below

$$E = \int_{T_1}^{T_2} \frac{dE}{dT}\, dT \qquad \ldots(ii)$$

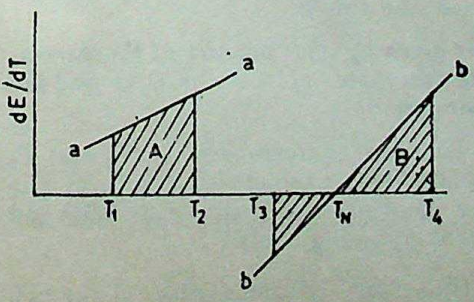

Fig. 32-16

As in the case of $P-V$ diagrams, the area under the thermoelectric power line and the temperature axis and between ordinates at T_1 and T_2 gives *the total thermo-e.m.f.* In Fig. 32-16, line *aa* represents the thermoelectric power line for a couple of metal A and lead. The total e.m.f. edveloped in such a couple is given by the vertical shaded area.

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Similarly, line *bb* represents the thermoelectric power line for a metal *B* with respect to lead. Since the neutral temperature for this couple lies somewhere between its junction temperature T_3 and T_4, hence it cuts the temperature axis at T_N. The total e.m.f. in the couple is given by the difference of the two shaded areas.

32-18. Thermo-electric Diagram

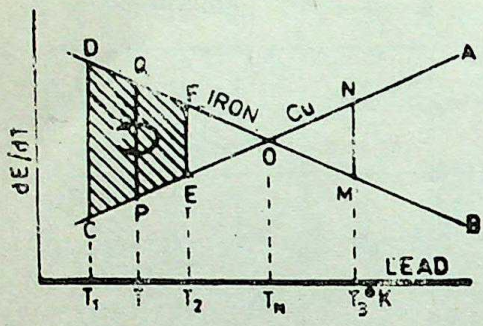

Fig. 32-17

A thermoelectric diagram is a graph which shows relation between *the thermo-electric power and the corresponding temperature*. It is invariably a straight line and is known as *thermoelectric power line*. While constructing such diagrams for different metals, lead is always taken as the second metal because Thomson effect for lead is nil.

In Fig. 32-17, straight lines *AC*, *BD* and the horizontal axis represent the thermo-electric power lines for Cu, iron and lead respectively. Obviously

$\therefore$ T_1C = thermo-electric power for Cu and Pb at T_1

T_1D = thermo-electric power for Fe and Pb at T_2

$\therefore$ CD = thermo-electric power for Cu and Fe at T_1

Similarly, EF = thermolectric power for Cu and Fe at T_2

Consider now a Cu-iron couple with junctions at T_1 and T_2 °K. Thermo-current will flow from Cu to iron at the hot junction. The total thermo-e.m.f. for such a couple is given by the area of the trapezium *CEFD*. This area is equal to $(T_2 - T_1) \times PQ$. Now, PQ is the mean ordinate and represents the thermoelectric power of the couple at the mean temperature T and $(T_1 - T_2)$ is the difference of the junction temperatures. Hence, the rule is

the total e.m.f. of a couple is given by the product of the thermo-electric power at the mean temperature of the junctions and the difference in the temperature of the junctions.

If the Cu-iron couple had junction temperatures of T_1 and T_3 (so that T_N lies in between them), then the total e.m.f. in the circuit would have been given by the *difference* of the areas *COD* and *MON* because these areas represent opposing e.m.f.'s.

It may also be noted that the point of intersection of the two thermoelectric power lines drawn with reference to lead gives the neutral temperature for the couple.

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32-19. Thermodynamics Applied to Thermoelectric Circuit

(A.M.I.E. Summer 1983)

Since the Peltier and Thomson effects occurring in a thermocouple are reversible, the laws of reversible heat engine can be justifiably applied to a thermo-electric circuit provided the flow of charge round the circuit is assumed to be *extremely slow* so that the only irreversible effect *i.e.* Joule's heating effect is negligible. Application of these laws leads to some very important relations.

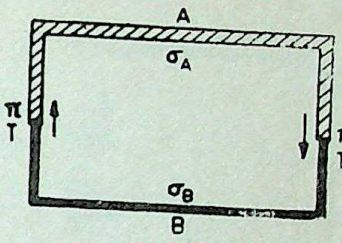

Fig. 32-18

In Fig. 32-18 is shown a thermocouple consisting of metals A and B and having its cold junction at T°K and hot junction at $(T+dT)$°K. Let π and $(\pi+d\pi)$ be the Peltier coefficients for the pair of metals at the two respective temperatures and σ_A and σ_B the Thomson coefficients for the two metals which are both assumed to be positive. Let one coulomb of charge *flow very slowly* through the hot junction from metal A and B.

(*i*) Heat energy *absorbed* at $(T+dT)$°K at the hot junction due to Peltier effect $=(\pi+d\pi)$ joules

(*ii*) Heat energy *evolved* at T°K at the cold junction due to Peltier effect $=\pi$ joules

(*iii*) Heat energy *absorbed* in metal A at the mean temperature of T°K due to Thomson effect

$$=\sigma_A\, dT \text{ joules}$$

(*iv*) Heat energy *evolved* in metal B at the mean temperature of T°K due to Thomson effect

$$=\sigma_B\, dT \text{ joules}$$

We will consider the heat energy *absorbed as positive and that evolved as negative.* Since in a reversible cycle of operation, the net change in entropy* is zero, hence

$$\frac{\pi+d\pi}{T+dT}+\frac{\sigma_A dT}{T}-\frac{\pi}{T}-\frac{\sigma_B dT}{T}=0$$

or
$$\frac{\pi+d\pi}{T+dT}-\frac{\pi}{T}+\frac{(\sigma_A-\sigma_B)dT}{T}=0$$

The differential equation of the thermo-e.m.f. as given in Art. 32-14 is

$$dE=d\pi+(\sigma_A-\sigma_B)dT$$

$$\therefore \quad (\sigma_A-\sigma_B)dT=dE-d\pi$$

*Change of entropy is given by the ratio of heat given or taken and the temperature at which it is given or taken.

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Substituting this value in Eq. (*i*) above and neglecting terms containing dT^2, we get

$$\pi = T \cdot \frac{dE}{dT} \qquad \ldots(ii)$$

= mean tempt. × thermo-electric power

Now, differentiating Eq. (*ii*) with respect to temperature, we get

$$\frac{d\pi}{dT} = T\frac{d_2E}{dT^2} + \frac{dE}{dT}$$

$$\frac{dE}{dT} - \frac{d\pi}{dT} = -T\frac{d_2E}{dT^2} \qquad \ldots(iii)$$

Now $$dE = d\pi + (\sigma_A - \sigma_B)\, dT$$

or $$\frac{dE}{dT} = \frac{d\pi}{\pi T} + (\sigma_A - \sigma_B)\frac{dT}{T}$$

or $$\frac{dE}{dT} - \frac{d\pi}{dT} = (\sigma_A - \sigma_B)\frac{dT}{T} \qquad \ldots(iv)$$

From Eq. (*iii*) and (*iv*), we get

$$\sigma_A - \sigma_B = -T\frac{d_2E}{dT^2} \qquad \ldots(v)$$

Example 32-8. *The e.m.f. in a thermocouple whose one junction is kept at 0°C is given by* $E = at + \frac{1}{2}bt^2$. *Find the neutral temperature and the Peltier and Thomson coefficients.*

Solution.

Expressing the e.m.f equation in terms of degree Kelvin (°K), we get

$$E = a(T-273) + \tfrac{1}{2}b(T-273)^2$$

$$\therefore \quad \frac{dE}{dT} = a + b(T-273) \quad \therefore \quad \frac{d_2E}{dT^2} = b$$

(*i*) Neutral Temperature

At neutral temperature, $dE/dT = 0$

$$\therefore \quad (T_n - 273)°\text{K} \text{ or } t_n\,°\text{C} = -\frac{a}{b}$$

(*ii*) Peltier Coefficient

$$\pi = T \cdot \frac{dE}{dT} = T\,[a + b(T-273)]$$

(*iii*) Thomson Coefficient

$$\sigma = T \cdot \frac{d_2E}{dT^2} = bT$$

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Example 32-9. *Compute the Peltier and Thomson e.m.f.'s and hence Seebeck e.m.f. for the copper-iron thermocouple given in Ex. 32-4.*

Solution.

Peltier E.M.F.

As seen from Art. 32-19

$$\pi = T.\frac{dE}{dT} \quad \text{...in °K}$$

$$= (t+273)\,.\,\frac{dE}{dt} \quad \text{...in °C}$$

$$= (t+273)(a+bt) \quad \text{...Art. 32-17}$$

$\therefore$ π at 200°C $= (200+273)(-14+0{\cdot}05\times 200)$

$= -1892\ \mu V$

π at 0°C $= 273a$

$= 273\times(-14) = -3822\ \mu V$

Net Peltier e.m.f. $= -1892-(-3822) = \mathbf{1930\ \mu V}$

Thomson E.M.F.

$$E = at + \tfrac{1}{2}bt^2$$

$$\frac{dE}{dt} = a+bt \text{ and } \frac{d_2E}{dt^2} = b$$

Thomson e.m.f. $= \int_{T_1}^{T_2} (\sigma_A - \sigma_B)\,dT$ ...Art. 32-12

$$= \int_{T_1}^{T_2} -T\,.\,\frac{d_2E}{dT^2}\,dT \quad \text{...Art. 32-19}$$

$$= \int_{T_1}^{T_2} -bT\,dT = \frac{b}{2}\,(T_1^2 - T_2^2)$$

$$= \frac{0{\cdot}05}{2}\,(273^2 - 473^2) = \mathbf{-3730\ \mu V}$$

Seebeck E.M.F.

The net or Seebeck E.M.F. is

E = Peltier E.M.F. + Thomson E.M.F.

$= 1930 + (-3730) = \mathbf{-1800\ \mu V}$

It is exactly the same result as found in Ex. 32-4.

Example 32-10. *The thermo-electric power of iron is (1734—4·877t) μV/°C and that of copper is (1430+0·95 t) μV/°C. Show that*

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the e.m.f. of the iron-copper thermocouple, the junctions of which are at 0°C and 100°C, is 1265 μV.

Solution.

Thermo-electric power is given by

$$P=\frac{dE}{dT} \quad \text{or} \quad dE=P.dT=P.dt$$

Hence, for iron

$$E_0^{100}=\int_0^{100} dE=\int_0^{100} P\,dt=\int_0^{100}(1734-4\cdot877\ t)dt$$

$$=1734t-4\cdot877\times\frac{t^2}{2}$$

$$=1734\times100-\frac{4\cdot877}{2}\times100^2$$

$$=149{,}015\ \mu V$$

For copper

$$E_0^{100}=1430\times100+\tfrac{1}{2}\times0\cdot95\times100^2$$

$$=147{,}750\ \mu V$$

$\therefore$ Total e.m.f. of iron-copper couple is

$$=149{,}015-147{,}750=1265\ \mu V.$$

32-20. Applications of Thermo-electric Effect

Some of the applications of thermoelectric effect are given below :

(*i*) **Thermo-electric thermometer or pyrometer.** It has been shown that the thermo-e.m.f. in a thermo-couple depends on the difference of temperatures of the two junctions of the couple. Hence, the e.m.f. may be used as a measure of the temperature of the hot junction when the other is kept at a constant temperature. This arrangement gives the thermo-electric thermometer *i.e.* pyrometer (Fig. 32-19).

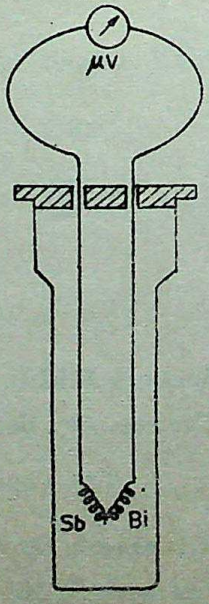

Fig. 32-19

A simple sketch for such a thermometer is shown in Fig. 32-19 where a bismuth-antimony couple has been used. The hot junction is used to measure the unknown temperature. The junction is enclosed in a protective tube which is either of glass or porcelain depending on the temperature to be measured. The microvoltmeter connected in the circuit can be calibrated directly in temperature by bringing the hot junction at known temperatures.

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(*ii*) **Thermopile.** It consists of a number of bismuth-antimony couples arranged in series in order to multiply the thermo-e.m.f's thereby enhancing the sensitivity of the apparatus regarding the detection of minute quantities of heat. As shown in Fig. 32-20, one set of junctions is protected whereas the other set is exposed to the heat radiations. The temperature of the hot body sending the heat radiations can be read directly on the millivoltmeter provided it has been directly calibrated in temperature by sighting the thermopile to different sources of known temperature. Very sensitive thermopiles capable of detecting temperature differences of the order of 0·001°C are commonly used.

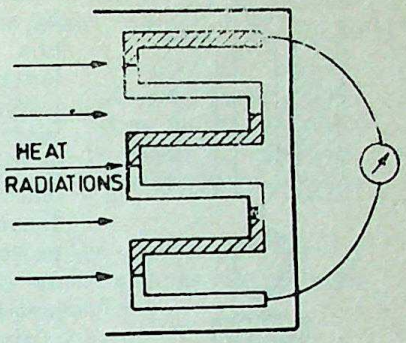
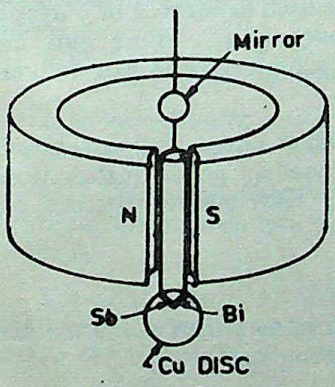


Fig. 32-20

(*iii*) **Boy's Radio-micrometer.** It is a very sensitive instrument used for the detection of heat radiations and, essentially, consists of an antimony-bismuth thermocouple alongwith a sensitive galvanometer. Two short rods of antimony and bismuth are soldered to a blackened copper disc and are then connected to a loop of Cu wire. This copper loop is then suspended by a fine quartz fibre between the poles of a strong permanent magnet.



Fgi· 32-21

When the heat radiations from the hot body fall on the blackened disc, they raise the temperature of the junction of the antimony-bismuth couple. Hence, an e.m.f. and so an electric current flows through the loop of copper wire. This current-carrying loop of copper wire, since it lies in a strong field, experiences a torque which deflects it, the amount of deflection depending on the strength of the thermo-current. The angle of deflection can be measured by the reflection of a beam of light from the mirror attached to the loop of copper wire.

Such instruments have been made with an amazingly high sensitivity in the matter of heat detection. They can detect heat radiations coming from a candle placed at a distance of 3 km or so !

(*iv*) **Duddell's Thermo-galvanometer.** It is, in fact, a modification of Boy's radio-micrometer and is used for measuring

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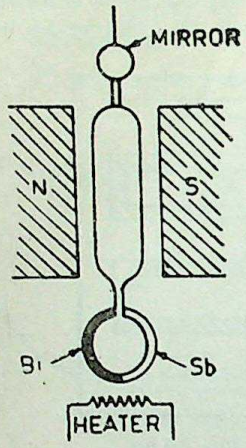

Fig. 32-22

currents, especially alternating ones. As shown in Fig. 32-22 it consists of a single loop of copper or silver wire suspended by a fine quartz fibre between the poles of a powerful permanent magnet. The lower ends of the loop are connected to bismuth-antimony couple. Below the junction of the bismuth-antimony couple is kept a 'heater' which consists of a fine wire of platinised quartz fibre. When the current to be measured is passed through the heater, it heats up and sends heat radiations to the junction of the thermo-couple, thereby raising its temperature. The thermo-current generated in the couple produces deflection of the loop of silver wire which can be measured as discussed before. In this way, the strength of the current passing through the 'heater' can be measured.

32-21. Thermocouples as Current Sources

Though thermocouples have low efficiency (about one percent) and low e.m.f. (microvolts) still they can be used as sources of current particularly when combined to form a thermopile. These can be operated from any heat source, say, a kerosene lamp and can deliver sufficient power to run a radio receiver.

Silicon-germanium thermopiles are being successfully operated from the intense heat generated in a nuclear power installation called *Romashkha* designed in 1964 by Kurchatov Institute of Atomic Energy, USSR.

HIGHLIGHTS

1. The production of a thermo-e.m.f. when one of the junctions of a thermocouple is heated is called the Seebeck effect.

2. The thermo-electric curve *i.e.* the curve showing relation between the thermo-e.m.f. produced and the temperature of the hot junction is, more or less, a parabola. Its equation is given by

$$E = at + \tfrac{1}{2}\, bt^2 \ \mu V \qquad -°C$$

$$= a\,(T-273) + \tfrac{1}{2} b\,(T-273)^2 \qquad -°K$$

3. The neutral temperature (t_n) is defined as that temperature of the hot junction of a thermocouple for which the thermo-e.m.f. produced is maximum.

4. Temperature of inversion is that temperature at which the thermo-e.m.f. becomes zero and starts reversing.

5. Whereas the neutral temperature is fixed for a thermo-couple, the temperature of inversion is not. It depends on the

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temperature of the cold junction and lies as much above t_n as t_n is above the temperature of the cold junction.

6. The generation or absorption of heat energy at the junction of a thermocouple when current is passed through it is known as Peltier effect.

7. Peltier effect is fundamentally different from Joule's heating effect because Joule's heating effect varies as the square of current, occurs throughout the circuit and is irreversible. However, Peltier effect varies directly as current, is localised at junctions and is reversible.

8. Peltier coefficient (π) is defined as the quantity of energy evolved or absorbed reversibly at a junction of a thermocouple when one coulomb of charge flows across the junction.

It is expressed in joules/coulomb or volts. It is numerically equal to the contact potential difference at that junction.

9. Thomson effect occurs in an unequally heated conductor. The absorption or evolution of heat energy when current flows through an unequally heated conductor is known as Thomson (or Kelvin) effect.

10. Thomson coefficient (σ) is defined as the amount of heat energy absorbed or evolved reversibly when one coulomb of charge flows between two points which differ by 1°C in an unequally heated conductor.

It is expressed in joules per coulomb per °C or volts/°C.

It is numerically equal to the *p.d.* between two points which differ in temperature by 1°C.

11. Thomson effect for metals like copper, zinc and silver etc. is positive, for metals like iron and nickel etc. is negative and for lead is nil.

12. The value of the thermo-e.m.f. of a couple can be expressed either in terms of Peltier and Thomson effects or solely in terms of the temperatures of the junctions as given below

$$(i)\quad E=\pi_2-\pi_1+\int_{T_1}^{T_2}(\sigma_A-\sigma_B)\,dT$$

or $$dE=d\pi+(\sigma_A-\sigma_B)\,dT$$ —differential form

$$(ii)\quad E=a+\tfrac{1}{2}\,bt^2$$ —cold junction at 0°C

$$=a\,(T_2-T_1)+\tfrac{1}{2}\,b\,(T_2-T_1)^2$$ —cold junction at T_1 and hot at T_2

$$=a\,(t_2-t_1)+\tfrac{1}{2}\,b\,(t_2-t_1)^2$$ —cold junction at t_1°C and hot junction at t_2°C

where a and b are known as thermoelectric constants which are characteristics of the two metals A and B.

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13. The three experimental laws of thermoelectricity can be summed up mathematically as follows :

(*i*) *Law of Intermediate Metals*

$$E_A^N = E_A^B + E_B^C + E_C^D + \ldots\ldots\ldots\ldots E_N^M$$

(*ii*) **Law of Intermediate Temperatures**

$$E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + E_{t_3}^{t_4} + \ldots\ldots\ldots\ldots E_{t_{n-1}}^{t_n}$$

(*iii*) **Law of Combination of E.M.F.'s**

$$E_A^C = E_A^B - E_C^B$$

14. The values of the neutral temperature and temperature of inversion, in terms of thermoelectric constants, are as under :

$$t_n = -a/b \quad \text{and} \quad t_i = -2a/b$$

15. Thermo-electric power is defined as the rate of change of thermo-e.m.f. with respect to temperature *i.e.* dE/dT. It is not constant but varies with temperature, having zero value at the neutral temperature.

16. Thermo-electric power line is a graph between thermo-electric power and the temperature of hot junction. It is a straight line whose equation is

$$\frac{dE}{dT} = y = a + bT$$

Also $\frac{dE}{dT} = \frac{d\pi}{dT} + (\sigma_A - \sigma_B)$ —in terms of Peltier and Thomson coefficients.

17. By applying laws of thermodynamics and concept of entropy to a thermocouple, the following relations can be deduced :

(*i*) $$\pi = T\frac{dE}{dT} = TP$$

(*ii*) $$\sigma_A - \sigma_B = -T\frac{d_2E}{dT_2}$$

18. The phenomenon of thermo-electricity has been applied for the following :

(*i*) Thermo-electric thermometers —for measuring high temperatures

(*ii*) Thermopile — for detecting small amounts of heat radiations

(*iii*) Boy's radio-micrometer — for measuring extremely small quantities of heat radiations as in the heat spectra of hot bodies

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(*iv*) Duddell's thermogalvanometer—for measuring small currents especially the alternating ones

OBJECTIVE TESTS

A. Fill in the blanks with most appropriate word/s.

1. At neutral temperature in a thermo-couple, the thermo-electric e.m.f. is ——.
2. Neutral temperature for a given couple is —— but temperature of inversion is variable.
3. Temperature of inversion of a thermocouple depends on the temperature of its —— junction.
4. Temperature of inversion is as much —— the neutral temperature as the neutral temperature is above the temperature of the cold junction.
5. The slope of the thermo-e.m.f. curve represents thermo-electric —— of the couple at that temperature.
6. Peltier coefficient is equal to the —— potential difference at the junction considered.
7. The absorption or evolution of heat energy when current flows through an —— heated conductor is called Thomson effect.
8. Like Peltier coefficient, the Thomson coefficient is a function of ——
9. Thermo-electric power of a couple is —— at neutral temperature.
10. In regard to thermo-electricity, $dE/d\,T$ is called —— —— (*A.M.I.E. Summer 1984*)
11. Peltier coefficient of any junction is the product of —— and thermo-electric power. (*A.M.I.E. Summer 1984*)
12. Thermo-electric power line of a couple is a —— line.
13. Seebeck e.m.f. is the —— of Peltier e.m.f. and Thomson e.m.f.
14. Thomson e.m.f. exists between different parts of the —— metal if it is heated unequally.
15. Temperature gradient along a metallic rod produces —— gradient.
16. Thermo-electric power is just the —— of the thermo-electric curve.

Multiple Choice Tests

B. Tick off the most appropriate choice.

1. In a thermocouple, thermo-e.m.f. is produced when its
   (*a*) one junction is at 0°C and the other at 100°C
   (*b*) both junctions are at the same temperature
   (*c*) two junctions are at different temperatures
   (*d*) two wires are made of the same metal

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2. The unit of Peltier coefficient is
   (*a*) volt (*b*) coulomb
   (*c*) kilocalorie (*d*) volt/°C

3. Peltier effect in a thermo-couple is
   (*a*) irreversible (*b*) localised at junctions
   (*c*) similar to Joule's heating effect
   (*d*) of constant value for a given couple

4. Thomson effect is produced when current is passed through
   (*a*) a thermo-couple
   (*b*) a heated rod
   (*c*) a metallic conductor at room temperature
   (*d*) an unequally-heated metallic conductor

5. Like Peltier coefficient, Thomson coefficient is also
   (*a*) expressed in μV/°C (*b*) irreversible
   (*c*) function of temperature (*d*) constant in value

6. Neutral temperature of a thermo-couple is defined as the temperature where thermo-e.m.f. is
   (*a*) maximum (*b*) zero
   (*c*) negative (*d*) minimum

7. Temperature of inversion of a thermo-couple
   (*a*) is twice the neutral temperature
   (*b*) has negative value
   (*c*) is constant for a given thermo-couple
   (*d*) is as much above t_n as t_n is above the temperature of its cold junction

8. Seebeck e.m.f. is the sum of Peltier e.m.f. and
   (*a*) battery e.m.f. (*b*) Thomson e.m.f.
   (*c*) contact potential difference
   (*d*) drop across the wires of the couple

ANSWERS TO OBJECTIVE TESTS

A. Fill in the blanks

1. maximum
2. constant
3. cold
4. above
5. power
6. contact
7. unequally
8. temperature
9. zero
10. thermo electric power
11. temperature
12. straight
13. sum
14. same
15. potential
16. slope

B. Multiple Choice

1. *c* 2. *a* 3. *b* 4. *c* 5. *c* 6. *a* 7. *a* 8. *b*

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